Bearing capacity of circular footing on sand: A critical state approach

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Research Article

Keywords: bearing capacity, circular footing, sand, critical friction angle, hypoplastic model

Posted Date: October 10th, 2023

DOI: https://doi.org/10.21203/rs.3.rs-3420884/v1

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Bearing capacity of circular footing on sand: A critical state approach

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Abstract

There has been extensive research into the bearing capacity of circular footing under vertical centric load. Although the scatter values for the bearing capacity factor $N_\gamma$ that have been suggested by different methods, the classical solution is still being widely used in the design codes. Sand particle morphology, footing diameter $D$, mean effective stress level $p'$, and sand relative density $D_r$, are some of the variables influencing the surface footing bearing capacity in sandy soil. Different sands may have various mobilization friction angles and, thus, various stress-strain responses, although they may have the same relative densities $D_r$ and mean effective stresses $p'$. Therefore, it is possible that estimating bearing capacity factors based on peak friction angle may not accurately reflect the actual bearing capacity. In the current study, a three-dimensional finite element model (3D-FEM) was utilized. Hypoplastic sand model has been used to simulate sand behavior. It can effectively simulate the compression and shear behavior of sands over a broad confining pressure and density range. The model has been validated using experimental centrifuge and one element tests available in literature. Critical friction angle $\phi_c$ has been considered as a shear strength parameter that is independent on $p'$ and $D_r$. Further research is done using parametric analysis. The main objective is to assess the expected bearing capacity factor $N_\gamma$ of various sand characteristics and to offer a solution that can be valid for a wide range of sand properties.

Keywords

bearing capacity, circular footing, sand, critical friction angle, hypoplastic model
1 Introduction

The topic of bearing capacity of shallow circular footings on sand surfaces is a well-established subject in the field of foundation engineering, which has been the subject of extensive research over the past few decades. The preceding inquiries predominantly utilized laboratory tests conducted under one-gravity conditions [1–4]. Failure to adhere to similitude rules may lead to an imprecise emulation of soil behavior [5,6]. Numerous empirical equations were formulated based on the deceptive testing. The use of such tests to validate analytical techniques such as the characteristics and limit state approaches led to a misinterpretation of the behavior of sand beneath shallow footings. According to [7], the sand bearing capacity solutions that were initially proposed through large-scale field experiments were not effective in predicting the actual behavior of the field. Utilizing bearing capacity factors from laboratory studies conducted under one-gravity conditions to predict prototype or field scale footings would be deemed unsuitable. Subsequently, the geotechnical centrifuge modeling technique was employed to conduct physical modeling in geotechnical engineering through experimental model tests, as evidenced by various studies [8–17]. This finding enhances the comprehension of the behavior of sand in the presence of shallow footings. Numerous scholars have achieved success in formulating empirical equations or design methodologies to estimate the bearing capacity of soil. Furthermore, centrifuge experiments were employed to authenticate analytical methodologies. However, when attempting to comprehend the behavior of sand, it is crucial to consider additional variables when analyzing test results.

2 Factors Affecting Sand Bearing Capacity

According to the classical bearing capacity solution, the ultimate bearing capacity; \( q_u \) for a footing on sand surface can be calculated as:

\[
q_u = \frac{1}{2} \gamma' D N_y
\]
where, $\gamma'$ is the effective unit weight of the sand, $D$ is the footing diameter, and $N_G$ is the bearing capacity factor, which is dependent on sand unit weight. The classical solution suggests that $N_G$ can be calculated as a function of peak friction angle, $\phi_p$. However, peak friction angle of sand $\phi_p$ is a function of mean effective stress $p'$. As can be seen in Figure (1), $\phi_p$ decreases as the mean effective stress $p'$ increases. Also, shown in Fig. (1) that $\phi_p$ decreases as $D_r$ decreases. Therefore, $q_u$ should not be dependent on variable parameter as $\phi_p$. There are several proposed estimating techniques since there is no clear agreement on the most effective way to define, $N_G$. Most of the methods available are analytical (e.g., limit equilibrium, upper limit, lower limit, or characteristic methods). They are all derived based on Terzaghi solution [2] or similar solutions [1,4]. Because the value of, $N_G$ can vary significantly depending on the estimation technique, this has emerged as one of the primary causes of discrepancy among methods used to estimate $q_u$. The study by [18] shows variances in maximum and minimum $N_G$ values derived using 60 different estimating techniques up to 267% for equal $\phi_p$ values.

The comprehension of the correlations between strength and dilation in sandy soils [7], prompted a reassessment of the existing solutions for bearing capacity. The maximum angle of friction exhibited by sand is determined by two distinct contributing factors. The initial factor pertains to the critical state or the angle of constant volume friction. The determination of this angle is primarily contingent upon the morphology of sand particles. As suggested by various scholars [7,19–23], it is not dependent upon stress level or sand density. The second component pertains to the dilation of sand particles. The extent of sand dilation is contingent upon not only the stress level and density of the sand, but also its morphology. Sand is a material that is dependent on both stress and density. The maximum effective friction angle exhibited by sand, $\phi_p$, is observed increases with a decrease in the mean effective stress $p'$, and an increase in the relative density $D_r$ of the sand. By extrapolating this phenomenon to the bearing capacity of shallow footings situated on sand surfaces, one can gain insight into the non-linear nature of sand behavior when subjected to footing loading. The application of vertical loading induces continuous variations in both $p'$ and $D_r$, resulting in alterations to the mobilized sand friction.
angle, and consequently, the shear strength of the sand in the proximity of the footing. As the loading increases, certain segments along the anticipated slip surface will attain the peak of the friction angle and undergo softening, while other segments will remain at a low strength level. Upon completion of the loading process, the shear strain levels attain a magnitude that is sufficient to cause certain sections of sand located along the slip surface to attain the critical friction angle, prior to the limit load on the footing being achieved. The occurrence being referred to is commonly known as progressive failure, as documented by various authors [1,16,17,24–28]. The phenomenon of progressive failure has been documented in various experiments, including those conducted under one-gravity conditions [1,3,24,28,29] and in centrifuge tests [8–11,14–17,30–32].

Considerable investigation, including both empirical and computational/analytical methods, has been carried out regarding the phenomenon of scale effect. Two distinct constituents have influence on the scale. The peak friction angle of sand, $\phi_p$, is dependent upon the value of $p'$. A greater diameter of the footing is likely to result in a more extensive zone of influence. This indicates that an increase in the average mean stress $p'$ beneath the footing will result in a decrease in the average $\phi_p$. A smaller footing diameter will result in a smaller zone of influence. This implies a decrease in average mean stress $p'$ and a higher average $\phi_p$. The diameter of the footing has the ability to regulate the bearing capacity with respect to the mean stress and shear strength of the soil. The second element primarily pertains to the small-scale testing typically conducted by researchers to examine the bearing capacity. The increase in $N_t$ is observed to be at an increasing rate as the ratio $D/d_{50}$ decreases, where $d_{50}$ represents the average sand particle size. This phenomenon has been illustrated in the works of [24,33]. This phenomenon is commonly denoted as the particle size effect. However, the influence of particle size is not a consequence of pressure. According to [33,34], it is recommended that the ratio between the diameter of the footing and the size of the sand particles should not be less than 150. The load-carrying capacity of a footing can be enhanced by increasing its diameter, although this increase is not directly proportional to the standard bearing capacity solutions [1,2,4,17,35].
The previous discussion suggests that several variables, including particle morphology, footing diameter, mean effective stress level, relative density, and progressive failure, have an influence on the bearing capacity of a surface footing on sandy soil. The structural composition of sand may be the primary determinant of its behavior under loading conditions. It is important to comprehend that the composition of various types of sand exhibits significant variations in their structural configuration. Numerous sand shear testing devices have confirmed that the sand shear strength behavior can be influenced by the initial fabric state [27]. The shear strength of sand is influenced by the mean grain size $d_{50}$ and uniformity coefficient $C_u$, as evidenced by studies conducted by [20,22]. Conversely, the dilation behavior of sand is expected to be impacted by sand angularity. Hence, it can be inferred that sands exhibiting identical peak friction angles exhibit dissimilar behavior. Variations in mobilized friction angles and stress-strain responses can be observed among sands that share identical relative densities ($D_r$) and mean effective stresses ($p'$). Consequently, the determination of actual bearing capacity may be imprecise if bearing capacity factors are computed solely on the basis of $\phi_p$. In this manner, bearing capacity is dependent on a variable parameter $\phi_p$. In order to develop a distinctive methodology for calculating sand bearing capacity, it is imperative to incorporate the primary factors of progressive failure, namely $p'$ and $D_r$. Furthermore, it is necessary to incorporate a shear strength parameter that is not or less dependent on either $p'$ or $D_r$ in order to accurately depict the impact of sand structure. A footing with identical size, placed on Toyoura sand will exhibit a greater limit load compared to a footing situated on clean, medium density Ottawa sand at the same soil unit weight, and relative density [27]. Both samples exhibit uniform grain size distributions and are composed of silica sands. However, notable differences exist in their respective mean particle sizes and angularities. It is possible that the shear strengths of the Ottawa sand with round to sub-round grain sizes are lower than those of the Toyoura sand with sub-angular to angular grain sizes. The accurate portrayal of this occurrence is demonstrated through the distinct critical-state friction angle $\phi_{cr}$ measurements of the two sands.
In the current study, a three-dimensional finite element model (3D-FEM) is used to simulate a circular footing on a dry sand surface. Sand is modeled using hypoplastic sand model in order to take the effects of stress-density behavior into account. Different critical state friction angles $\phi_{cr}$ are thought to exist for different types of sands. A wide range of sand critical friction angles can help to expand and improve the method suggested by [27]. Additionally, a parametric study is performed. The fundamental goal of the current research is to analyze the predicted bearing capacity factors of various sand characteristics and to provide a solution that can be applied for a wide range of sand properties.

3 Problem Definition

To conduct this research, a Finite Element Model (FEM) was created in PLAXIS 3D. The program supports numerous soil constitutive models and allows users to add their own material subroutines. Details of the model geometry, meshing, and constitutive models are presented below.

3.1 Geometry Boundary Conditions and Meshing

Due to the centric vertical loading's symmetry, just one-quarter of the model is simulated. As per the study results of [36], the selection of boundary conditions was made while ensuring an adequate distance from the edge of the footing. According to Figure (2), the dimensions of the model base and vertical sides are approximately 7 and 10 times the diameter of the footing far from footing edge, respectively. The vertical sides are subject to constraints that limit their horizontal displacement but allow for unrestricted vertical displacement. The model base is constrained from all displacements in any direction. The mesh of the model has been designed in such a way that a finer mesh is positioned in proximity to the region of elevated stresses and deformation across the entire foundation. The mesh refinement in the specified zone is three times the diameter of the footing, surpassing the level of refinement in the rest of the model. The dimensions of the zone were determined through an analysis of various models featuring distinct footing diameters, with the aim of ensuring that the failure surface remains contained.
within this designated area. The models have undergone control measures to ensure that their meshes possess a minimum size of 0.05 of the diameters of the footings, as prescribed by [36]. Furthermore, a sensitivity analysis was conducted to assess the impact of mesh size and model size on the results. The variation in maximum bearing capacity as a function of minimum mesh size and model size is shown in Figure (3).

The simulation of footing involves the consideration of a rigid body that experiences a vertical displacement that is predetermined. The rotation of all axes and the horizontal displacement are subject to constraints. As per [18] empirical findings, the bearing capacity of footings under static vertical loads is minimally affected by the base roughness. Consequently, the present study employs an interface element to simulate rough conditions between the soil and footing elements. The Mohr friction criterion is employed for the purpose of modeling the interface. A value of 28° for the interface friction angle $\phi_{int}$, which is equivalent to a friction coefficient of 0.5 was suggested by [27]. There was no tension permitted for the interface elements.

### 3.2 Hypoplastic Sand Constitutive Model

In contrast to the conventional elastoplasticity approach, hypoplasticity is an advanced constitutive modeling technique. It provides a framework for property-based asymptotic constitutive modeling of soils. The constitutive relation may be established from the fundamental characteristics of granular materials within this theoretical framework. The constitutive relation employs a singular tensorial equation to articulate the stress evolution in relation to deformation. The model integrates conventional concepts from soil mechanics, such as critical states, barotropy, pyknotropy, and a stress dilatancy relationship. The hypoplastic constitutive model is significantly dependent on the concept of a critical state. Despite their relatively straightforward mathematical structure, many complex phenomena related to barotropy (the dependence of stiffness and strength on the stress level) and pyknotropy (the dependence of stiffness and strength on density) have been well described. This makes the hypoplastic model an efficient tool for simulating the behavior of soil in a wide range of
engineering applications. Furthermore, the stress dilatancy relationship facilitates precise
anticipation of soil deformation and collapse in different loading scenarios.

Typically, in elastoplastic constitutive modeling of granular soil, materials with varying initial
densities are considered different, despite the fact that the constituent grains remain unchanged.
It is preferable to perform calibration of the constitutive model for a singular initial void ratio,
as the resulting constants will hold true across the entire range of densities. It is required to be
recalibrated for different density and pressure states. To accomplish this objective, it is
necessary for the constitutive model to be dependent upon the void ratio. The initial version of
the hypoplastic model designed for sands only integrated stress state as a state variable, which
resulted in its inadequacy in precisely capturing soil behavior. Consequently, the subsequent
laws pertaining to hypoplasticity were formulated with the aim of incorporating the effects of
barotropy and pycnotropy in soil. In order to attain these objectives, a novel state variable was
incorporated into the rate tensorial equation, more specifically a void ratio.

3.2.1 Model formulation

The concept of critical state is established in the hypoplastic sand model through the
incorporation of a unique correlation between the mean stress $p$ and the limiting void ratios at
its minimum density $e_i$, at the critical state $e_c$, and at the maximum density $e_d$ as

$$
e_i/\epsilon_{i0} = e_c/\epsilon_{c0} = e_d/\epsilon_{d0} = \exp\left[-\left(\frac{3p}{h_s}\right)^n\right]$$

where $h_s$ and $n$ are constitutive parameters as described in Table (1).

The tensorial equation was modified to incorporate barotropy and pycnotropy, which are
represented by two scalar functions $f_b$ and $f_d$, respectively, according to

$$\dot{T} = f_b(L\cdot D + f_d N \parallel D\parallel)$$

where $L$ is a fourth-order constitutive tensor, $N$ is a second-order constitutive tensor and $D$ is a
stretching tensor. The scalar factors $f_b$ and $f_d$ are utilized to incorporate the impact of average
pressure and density, respectively. These factors are also referred to as barotropy and
pyknotropy factors, respectively. Soil stiffness is regulated by the factor $f_s$. The calculation can be derived from the two variables $f_b$ and $f_e$.

$$f_s = f_b \cdot f_e$$  \hspace{1cm} (3.a)

$$f_b = \frac{h_s}{n} \left( \frac{1+e_i}{e_i} \right) \left( \frac{e_{f0}}{e_{co}} \right)^\beta \left( \frac{\tau r}{h_s} \right)^{1-n} \left[ 3 + a^2 - a\sqrt{3} \left( \frac{e_{f0}-e_d}{e_{co}-e_d} \right)^\alpha \right]^{-1}$$  \hspace{1cm} (3.b)

where:

$$a = \frac{\sqrt{3} (3 - \sin \phi_c)}{2\sqrt{2} \sin \phi_c}$$  \hspace{1cm} (3.c)

$$f_e = \left( \frac{e_{co}}{e} \right)^\beta$$  \hspace{1cm} (3.d)

$f_e$ increases the stiffness as void ratio decreases. This increase is controlled by parameter $\beta$.

$$f_d = \left( \frac{e-e_d}{e_{co}-e_d} \right)^\alpha = r_e^\alpha$$  \hspace{1cm} (3.e)

where $a$ is a parameter that controls peak friction angle and $r_e$ is the relative void ratio.

### 3.2.2 Model parameters validation

The literature comprises a significant quantity of effectively calibrated models for hypoplastic sand. Sands with a wide spectrum of critical friction angle ($\phi_c$) and mean grain size ($d_{50}$) were selected for the present study. Table (2) displays the parameters of the hypoplastic sand model for the sands that were chosen. Table (3) displays the physical properties of the referenced sands. The hypoplastic sand model has been implemented in Plaxis 3D software as a user defined material [37].

The sands, that were chosen, have been subjected to proper calibration utilizing a standardized methodology [38]. The calibration technique has been expounded in detail by various scholars [38–40]. The calibration procedures employed in all of the aforementioned sources were consistent. Moreover, specific physical models were employed to validate some of these models.
The calibration of various types of sand, including Karlsruhe ($d_{50} = 0.4$ mm), Hostun, Zbraslav, and Ottawa 50-70 sands had been reported by [39,41]. Calibration procedure for Ottawa F65 sand had been conducted by employing elemental testing and physical model tests [42]. The assessment of liquefaction potential in Berlin sand [43] and Nevada sand [35] subjected to seismic loading was conducted by employing triaxial tests and centrifuge tests. UWA silica sand calibration was achieved by employing centrifuge modeling, spudcan penetration, and pile driving simulations [44,45].

Automatic calibration tools have been implemented to calibrate hypoplastic sand model using soil element tests. The process of calibrating Komorany sand is executed through the utilization of Excalibre, an online calibration instrument created by [37]. This tool is available for access at (https://soilmodels.com/excalibre/). The methodology employed entails the utilization of a sensitivity analysis approach that shares similarities with the stochastic calibration technique. In the process of calibration, the primary focus is on the objective error function, with comparatively lower emphasis on the physical significance of the model parameters. The parameter bounds imposed are crucial in ensuring the physical significance, as noted by [46].

Calibration of Hochstetten sand have been presented using an optimization framework that utilizes a genetic algorithm [47].

The study conducted by [48] utilized triaxial and physical model tests to perform a thorough examination of Leighton Buzzard sand (fraction $E; d_{50} = 0.14$ mm), with the aim of calibration. To validate the results of a prior empirical investigation on a group of piles subjected to seismic loads, the adjusted characteristics of multiple hypoplastic sand models were compared. Furthermore, in the present study, a comparative study utilized the results of centrifuge tests conducted by [49] on circular footing on a Leighton Buzzard sand surface comprising coarser fractions (fraction $C; d_{50} = 0.40$ mm, fraction $D; d_{50} = 0.19$ mm, Mix (1); $d_{50} = 0.37$ mm, consists of fractions $C, D$ and $B$ (fraction $B$ has $d_{50} = 0.37$ mm) of different ratios, and Mix (2); $d_{50} = 0.38$ mm, consists of the same fractions as Mix (1) but with different fractions ratios). The comparison shows a good agreement between the FEM using parameters of the fine fraction
(E) by [48] with the results of [49] for the coarser fractions as shown in Fig. (4). The results follow the same trend suggested by [49] in terms of increasing bearing capacity of footing as $d_{50}$ decreases (sand becomes finer).

Fine Karlsruhe sand was examined through a series of oedometer and triaxial compression tests in a study conducted by [50]. A hypoplastic sand model was calibrated based on a comprehensive database that covers a wide range of relative densities.

Considerable investigation has been carried out on Toyoura sand, which has been employed for calibrating hypoplastic sand models. Numerous studies have reported on the calibration of Toyoura sand, [39–41,51–53]. The current investigation utilizes finite element method (FEM) simulations of circular footing on the surface of dense Toyoura sand centrifuge test, as previously conducted by [14], in order to determine suitable model parameters. A thorough analysis was carried out on the range of values associated with the parameters of the hypoplastic model. The validation of the model parameters suggested by [51] was carried out through a centrifuge test on footing diameters of 2m and 3m, resulting in similar findings. To achieve consistency with the experimental data, it was imperative to revise the critical friction angle $\phi_{cr}$ to 31.5°. This little alteration has been suggested by prior research endeavors [21,27,54] as well.

Additionally, it is important to mention that the design methodology put forth in this study will undergo a comparative evaluation with physical model experiments and established numerical models in the existing literature. The present study will undertake an examination of a centrally loaded vertical footing on a sand surface.

### 3.3 Ultimate Bearing Capacity $q_u$ Selection

The ultimate soil bearing capacity refers to the greatest average pressure of contact that a foundation can exert on the soil without inducing shear failure. Upon reaching a state of constant stress, the point of bearing capacity has been attained. The foundation load tests typically do not attain the maximum stress level owing to the repositioning of soil particles beneath the foundation during the progressive failure or the inherent limitations of field or experimental equipment. As a result, several definitions of bearing capacity criteria have been
proposed in an attempt to attain consistent values. The bearing capacity can be denoted by the ratio of 10% vertical footing displacement to footing diameter \( (w/D\%) \), as noted by [54,55]. An additional interpretation of this notion refers to the point at which the two linear sections of the load-displacement graph intersect, as observed on a standard or logarithmic axis [54,56]. The utilization of the latter method, as implemented in this study, is supported by findings from various centrifuge and field experiments conducted by [10,11,14,16,17,31,32]. All results are presented in terms of bearing capacity factor \( N_{\gamma} \) without considering shape factor effect for circular footing where:

\[
N_{\gamma} = 2q_{u}/\gamma' D
\]

where; \( \gamma' \) is the effective unit weight of the sand.

4 Parametric Study Results and Discussion

The finite element method (FEM) was utilized to conduct a comprehensive set of parametric investigations, comprising nearly 260 simulations. In addition to the critical friction angle \( \phi_{cr} \), which serves as the primary parameter in this investigation, additional parameters must be considered when examining the load-bearing capacity of a circular foundation on a sand surface subjected to a vertical centric load. These parameters have a significant impact on the behavior of sand. The study considers the diameters of footings \( D \) measuring 3, 4.36, 7, and 10 meters. The chosen diameters of the footings have been determined through centrifuge experiments conducted by [14,16,31]. Sand relative density \( D_r \) has been identified as a significant parameter in controlling the vertical bearing capacity of circular footings on sand. The study also incorporates sand relative densities of 50%, 60%, 70%, 80%, and 90%. The variability of sand relative densities can offer a distinctive bearing capacity resolution for sand of medium to high density.

4.1 Critical Friction Angle of Sand \( \phi_{cr} \)

Sands exhibit variations in their mineral composition, distribution of grain sizes, average grain size \( d_{50} \), and characteristics related to particle shape, such as roundness and angularity. The concept of the sand critical friction angle, denoted as \( \phi_{cr} \), has been proposed as a convenient
parameter to encompass various physical characteristics, as discussed by [20,22]. The sand
samples utilized in Table (1) and Table (2) were sourced from diverse geographical locations
across the globe. The relationship between the ultimate bearing capacity ($q_u$) or the bearing
capacity factor ($N_f$) and the critical friction angle ($\phi_{cr}$) can be observed by examining the load
displacement curves of sands with varying $\phi_{cr}$ values ranging from 30° to 35° (Figure 5-a). The
ultimate bearing capacity of sand, $q_u$, decreases as the critical friction angle, $\phi_{cr}$, decreases,
while keeping the relative density, $D_r$, and footing diameter, $D$, constant. (Fig. 5-a). At $\phi_{cr} = 33'$
the load displacement curves for Leighton Buzzard sand and fine Karlsruhe sand do not
have the same starting and ending slopes. Using the chosen approach to select $q_u$ or $N_f$ reveals
that despite this demonstrated variance in load displacement curve, both sands have nearly close
values of $N_f$ of difference range about 5-10% for all cases (Fig. 5-b). Similar results are found
for a $\phi_{cr}$ value of 30°- 31.5°, but with a smaller difference range (Fig. 5-a). Sands of the same
$\phi_{cr}$ may differ in particle morphology and consequently the dilation behavior. However, the
difference in $N_f$ will be within a limited range that $\phi_{cr}$ could still have the main control on the
bearing capacity (Fig. 6).

The effect of sand critical friction angle can be shown by presenting void ratio distribution for
different sands, Fig. (7). At different loading stages; $w/D = 10\%, 15\%$, and $33\%$; the difference
between sands of $\phi_{cr} = 30°$, 33.1°, and 35° can be observed. For all cases, a wedge zone is
generated underneath the footing where sand is highly dense. However, as load progresses, the
wedge zone is limited, and shear bands are generated starting from the footing corner according
to the failure criterion. At small value of $\phi_{cr} = 30°$, local shear failure is generated. Void ratio
distribution shows an increase in void ratio at the wedge boundary where shear bands are
generated. By increasing the displacement, local shear failure still exists with continuous
increase in void ratio at the wedge boundary. Increasing $\phi_{cr}$ to 33.1°, fine Karlsruhe sand shows
the same local shear failure with stiffer response. However, at high $\phi_{cr} = 35°$, it is clear that
general shear failure has been generated at $w/D = 33\%$ where void ratio reached a limit or
critical state corresponding to sand critical friction angle along shear failure slip surface.
4.2 Sand Relative density $D_r$

Numerous studies have examined the connection between relative density $D_r$ and ultimate bearing capacity of sand $q_u$, and the findings highlight the significance of $D_r$. Other studies have demonstrated, and this one confirms, that ultimate bearing capacity of sand, $q_u$, increases with its relative density $D_r$ (Fig. 8). Increasing $D_r$ results in a steeper initial slope of the load displacement curve. A higher $D_r$, however, results in a shallower final slope. So, as $D_r$ increases, there is more certainty about the ultimate bearing capacity. This relates to the phenomenon of progressive failure. The significance and clarity of such phenomena increase as $D_r$ increases.

Figure (9) shows the progressive failure phenomenon for Komorany sand ($\phi_{cr} = 35^\circ$) at different relative densities. The progressive failure phenomenon can clearly be observed at high relative density $D_r = 90\%$ (Fig. 9-a). Void ratio, mobilized friction angle, and incremental shear strain distributions are all showing the same behavior where shear slip surface extends up to a horizontal distance of footing diameter. At slip shear surface, critical friction angle is reached, while mobilized friction angle increases out of the shear zone reaching more than $60^\circ$ where passive pressure is generated. By decreasing sand relative density, $D_r$ to 70% and 50%, local shear failure exists (Fig. 9-b and c). The slip surface is almost generated at $w/D\% = 33\%$ for $D_r = 70\%$. However, no slip surface is clear at the same displacement ratio for $D_r = 50\%$, which may need further displacement to be developed.

4.3 Footing Diameter $D$

The chance of progressive failure is increased under low confinement conditions or when using smaller footing diameters. The bearing capacity factor $N_f$ tends to be higher for smaller footing sizes, as shown in Figure (10). The bearing capacity factor of smaller diameter footings is expected to be higher than that of larger diameter footings due to the observed increase in shear strength at lower confining stresses or shallower depths. The phenomenon under consideration can be clarified by examining the incremental shear strain distributions and the void ratio at $w/D$ ratio of 33% for $D_r$ of 90% (refer to Figure 11). The occurrence of general shear failure can be observed at a footing diameter of $D = 3$ m, where the void ratio reaches a critical value.
along the shear slip surface. Comparable behavior can be observed at $D = 4.37\text{m}$. However, when the foundation diameter is increased to 7 m or 10 m, the general shear failure is unclear. While the mobilization of bearing capacity in both scenarios experiences a gradual failure, the presence of a slip surface is less obvious in cases involving larger footing diameters. In order to fully mobilize the entire capacity and make the phenomena more evident, it may be necessary to increase the magnitude of the footing displacement. The occurrence of mesh distortion, a prevalent phenomenon in finite element analysis, imposes limitations in these particular instances.

4.4 Settlement – Diameter ratio $w/D\%$

The results of this study demonstrate that the settlement required to achieve the ultimate bearing capacity falls within the range of $w/D$ percent = 8 - 12\%. It is observed that lower values are associated with higher relative density and a higher critical friction angle $\phi_{cr}$. On the other hand, as the relative density $D_r$ and critical friction angle $\phi_{cr}$ decrease, the ultimate bearing capacity is reached at higher settlement. The range of observations presented in this study matches previous findings in the centrifuge literature. Specifically, A range of 7-9\% was observed by [9] at a relative density ($D_r$) of 94\%, while a range of approximately 10\% at $D_r$ of 80\% was reported by [49]. The load-displacement curves observed in this study exhibit similarities to the results obtained from centrifuge tests conducted by [31] on silica sand with a relative density ($D_r$) of approximately 90\% (Figure 5-b). Additionally, the load-displacement curves for Toyoura sand with $D_r$ of approximately 92\% [14] also demonstrate comparable trends (Figure 8). The initial slopes of the curves and the ultimate bearing capacity exhibit a favorable correspondence with the empirical findings within the range of critical friction angle $\phi_{cr}$ for sand, which spans from 31.5˚ to 33˚.

5 Proposed Design Method

By re-evaluating the results of the current study, it is now possible to incorporate a stress level parameter ($\gamma D/p_a$) [3,8,12,14,16,57,58] into the analysis. Here, $p_a$ represents the atmospheric pressure, approximately equal to 100 kPa, and is utilized to consider the influence of both sand
density and footing diameter. The presentation of results in relation to this metric enables a more comprehensive analysis. The variation of the bearing capacity factor, $N_γ$, is presented in Figure (12), as a function of $γD/p_a$. The observable impact of the critical angle of sand, $ϕ_{cr}$, is readily evident. Reducing $ϕ_{cr}$ from 35˚ to 30˚ leads to a significant decrease in the value of $N_γ$ within the $γD/p_a$ range of 0.5 to 2.0. As the critical angle $ϕ_{cr}$ decreases, there is a corresponding decrease in the rate of reduction. Moreover, it can be demonstrated that the relative density of sand has a substantial impact. A noticeable reduction in the value of $N_γ$, specifically from 212 at $γD/p_a = 0.6$ to approximately 100 at $γD/p_a$ values greater than 1.6, can be observed when $D_r$ is 90%. In the case of lower sand relative density ($D_r = 50\%$), the observed differences within the given range exhibit a comparatively smaller magnitude, falling within the range of 40 to 20.

The bearing capacity factor $N_γ$ for sand subjected to a centric vertical load is primarily influenced by the critical friction angle $ϕ_{cr}$, relative density $D_r$, and $γD/p_a$. All the results of the current study are plotted in Figure (12) as a three-dimensional plot of the sand critical friction angle $ϕ_{cr}$, stress level parameter $γD/p_a$, and bearing capacity factor $N_γ$ for varying sand relative densities $D_r$. Results reveal that $N_γ$ is the highest value at a critical friction angle $ϕ_{cr}$ of 35˚ and $D_r = 90\%$. The minimum value of $N_γ$ occurs at a friction angle of 30˚ and $D_r = 50\%$. The bearing capacity factor $N_γ$ reduction rate is reduced steadily up to around 50%, after which the rate of reduction slows down to a negligible level.

The primary aim of this study is to contribute towards the development of unique, reliable, and standardized solution for determining the ultimate bearing capacity of vertically loaded footings on sand surfaces. The results of the present study have been fitted using a nonlinear surface fitting function. A surface is fitted for each specific relative density value. A single function can be proposed to accommodate various values of sand relative density ($D_r$), with respect to different values of critical state friction angle ($ϕ_{cr}$) and effective stress ratio ($γD/p_a$).

$$N_γ = e^{[A+(B \tan ϕ_{cr})+(-0.437 \ln(γD/p_a))] \}}$$

(4)
Where both $A$, and $B$ are variable parameters as a function of sand relative density $D_r$. They can be calculated according to the following equations:

\[ A = 12.33 \left( \frac{D_r}{100} \right) + 2 \]  \hspace{1cm} (4.a)

\[ B = -6.29 \left( \frac{D_r}{100} \right) + 0.155 \]  \hspace{1cm} (4.b)

The plotted function has been displayed in Figure 13, alongside the Finite Element Method (FEM) results. The coefficient of determination ($R^2$) for the regression ranges from approximately 93% to 98%, with a higher value observed at $D_r = 90\%$ and a lower value observed at $D_r = 50\%$. Based on the examined ranges of the parameters, it can be determined that the proposed method possesses validity, though with certain limitations, within the specified ranges of $D_r$, $\gamma D/p_a$, and $\phi_{cr}$, as will be explained. Figure 14 illustrates the Finite Element Method (FEM) results in comparison to the predicted results obtained through the presently suggested approach. It is evident that the proposed method aligns well with the Finite Element Method (FEM) results.

### 6 Comparison with previous work

There are several models in the literature, both experimental and numerical. However, caution is needed when comparing the existing approach to the new recommended one. The initial step is to have acceptable values of the friction angle at the critical condition $\phi_{cr}$. It is recommended that a stress-path triaxial test be considered in order to obtain a suitable sand critical friction angle $\phi_{cr}$.

#### 6.1 Experimental work comparison

The bearing capacity of shallow foundations has been extensively researched in literature. As centrifuge tests are physical models that follow similitude laws, the comparison will only take into account results from well-documented centrifuge tests of surface footings on sand under central vertical loading. Using Monterey 0/30 sand ($d_{50} = 0.4$ mm) and relative density ranges between 93% and 95%, centrifuge experiments were conducted by [9]. The diameters of the prototype footings were...
The range for the stress level parameter was 0.15 to 0.32. The critical friction angle \( \phi_{cr} \) of Monterey 0/30 should be used to compare their findings to the currently recommended approach. According to [21], Monterey 0 has a critical friction angle \( \phi_{cr} \) of roughly 32°. However, a value of 32.7° ± 0.7° had been reported by [59]. The current method is compared with the results of centrifuge tests using a value of 33° and an average relative density value of 94%. Both experimental and predicted results are in good agreement (Fig. 15-a).

Inagi sand with a relative density of roughly 80%, had been used in centrifuge tests by [11] to study the behavior of surface circular footing under vertical centric load. The stress level parameter was in the 0.14 to 0.43 range. The diameter of the footings varied from 0.87 to 2.67 m. Loose Inagi sand has a peak friction angle of 33° and no dilatation, according to [60]. Additionally, they display the critical state which corresponds to \( \phi_{cr} = 32.75° \). Another study by [61] shows a value of about 30°. Given the wide range of available \( \phi_{cr} \) values and centrifuge results discrepancies, the currently proposed approach, which uses average value \( \phi_{cr} = 32° \), exhibits a very good agreement with centrifuge tests (Fig. 15-a).

A series of centrifuge tests on circular footings on dense sand with a 90% relative density were conducted by [16,17,31]. The diameters of the footings were 4.37 m, 5 m, 7 m, and 10 m. They reported a varied critical friction angle \( \phi_{cr} \) as a function of mean stress. However, the load displacement relationship of their centrifuge test of \( D = 7 \) m (Fig. 5-b) shows close behavior to other sands of \( \phi_{cr} = 33° \). Using a critical friction angle \( \phi_{cr} \) of 33°, there is an excellent agreement between the currently proposed approach and centrifuge testing, as shown in Fig. (15-a).

In centrifuge testing, Toyoura sand had been used by [14,15]. Ueno [15] conducted the tests at a relative density of 80%–85% while taking into account a stress level parameter of 0.1–0.47, which corresponds to footing diameters of 1.5 m and 3 m. While Okamura [14] tested compacted Toyoura sand in a centrifuge test at a relative density of 86% - 98% range (≈ 92 %). As previously discussed, a critical friction angle \( \phi_{cr} \) of 31.5° is taken into account by modeling centrifuge experiments by [14]. The current proposed method and the results of centrifuge tests show very strong agreement in Fig. (15-b).
Centrifuge tests were performed using Leighton Buzzard sand of different fractions C, D, and B [49]. These fractions have $d_{50}$ of 0.4, 0.19, and 0.37-0.38 for fractions C, D, and a mixture of all, as discussed before. Using direct shear tests, they suggested $\phi_{cr}$ of 31.4°, 30.7°, and 32.6° for Leighton Buzzard sand fractions C, D, and the mixture. Fraction E of the same sand ($d_{50} = 0.14$) has a critical friction angle $\phi_{cr}$ of 33.4° using triaxial tests [62]. Using direct shear tests, both of the coarse fraction A ($d_{50} = 2.24$) and fraction D have about 32° [63]. The centrifuge results at relative density of about 80% show that fraction C has the lowest ultimate bearing capacity factor and fraction D has the highest value, while the mixture has a median value.

These results reveal that the finer fraction has higher critical friction angle than the coarser fraction. This was also the case for Karlsruhe sand, where the finer fraction ($d_{50} = 0.14$) has $\phi_{cr} \approx 33$° and the coarse fraction ($d_{50} = 0.4$) has $\phi_{cr} = 30°$. After adjusting the critical friction angle based on the previous discussion, the current proposed method shows very well agreement, with an error range of -1.2% to -13.6%.

Additionally, to examine a wider range of sand relative densities, centrifuge strip footing tests on Toyoura sand have been considered for the comparison. Relative densities of 58 %, 74 %, and 88 % were considered in centrifuge tests by [8]. The bearing capacity factor $N_γ$ of a strip footing is converted to a circular footing one using the shape factor proposed by [27]. As observed in (Fig. 15-c), the current approach at a variety of sand relative densities and stress level parameters exhibits good agreement.

The chosen centrifuge experiments for comparison included thorough documentation that allowed for the use of the critical friction angle $\phi_{cr}$. Other tests could not be compared with them because either the ultimate bearing capacity [55] did not reached or stress-density dependency was not observed [64].

### 6.2 Numerical work comparison

In addition to the experimental work, a bearing capacity prediction method [27] were proposed using FEM parametric study. The method is based on progressive failure phenomenon. It can predict $N_γ$ based on both sand critical friction angle $\phi_{cr}$ and sand relative density $D_r$. However,
the method was based on Toyoura sand ($\phi_{cr} = 31.5^\circ$) and Ottawa sand ($\phi_{cr} = 30^\circ$) only. The present method is compared with. Their method has been plotted as dashed line in Figs. (15-a, b, c). It can be seen that their method underestimates the capacity for some experimental centrifuge tests. However, both methods show stress-density dependency. Both are dependent on the main three factors; $\phi_{cr}$, $D_r$, and $\gamma D/p_a$. Other analytical or numerical methods are dependent on sand peak friction angle, either as a constant value or variable as a function of only mean effective stress. Such methods could not be considered in the comparison as the proposed method is not dependent on the peak friction angle of sand.

7 Conclusion

In the present study, FEM is investigated to study the bearing capacity on vertically centric footing on sand surface. The objective is to present a unique valid solution that can fit different sand conditions; mineralogy, grain size distribution, mean grain size $d_{50}$, and particle shape (i.e. roundness and angularity). Hypoplastic sand model has been used in the simulations to consider stress-density dependency effect. Sand critical friction angle ($\phi_{cr} = 30^\circ – 35^\circ$), sand relative densities ($D_r = 50\% - 90\%$) and footing diameters ($D = 3, 4.36, 7, \text{and } 10 \text{m}$) are the main variable parameters in the study. The results have been presented in terms of bearing capacity factor $N_\gamma$. A three-dimensional plot as a function of stress level parameter $\gamma D/p_a$, sand friction angle $\phi_{cr}$, and sand relative density $D_r$ is considered in presenting the results. It is found that $N_\gamma$ increases significantly by increasing all variable parameters. The increasing rate is high at higher $D_r$ and decreases gradually up to $D_r = 50\%$. A nonlinear surface function is extracted from the results proposing a prediction method. The prediction method has been compared with available centrifuge tests in the literature. It has a good agreement with the centrifuge tests. However, care should be taken when using the proposed method in the selection of sand critical friction angle. In addition, an available method based on numerical study has been compared with as well. The current method could be valid with a limitation to the range of the parameters used in the study. Based on the comparison with the available centrifuge tests, the proposed method provides good results for $\gamma D/p_a = 0.1 - 2$, $\phi_{cr} = 30^\circ – 35^\circ$, and $D_r = 50\% - 90\%$.  

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Acknowledgement

This project was funded by the Deanship of Scientific Research (DSR), King Abdulaziz University, Jeddah, under grant No. G:609-829-1439. The authors, therefore, acknowledge with thanks DSR for technical and financial support.

Conflict of Interest

On behalf of all authors, the corresponding author states that there is no conflict of interest.
Table 1 Hypoplastic Sand Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Calibration Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_c(\degree) )</td>
<td>Critical friction angle</td>
<td>angle of repose or drained triaxial test</td>
</tr>
<tr>
<td>( h_s(\text{MPa}) )</td>
<td>Granular hardness</td>
<td></td>
</tr>
<tr>
<td>( n )</td>
<td>controlling the curvature of the compression line</td>
<td>Oedometer test at ( e_{max} )</td>
</tr>
<tr>
<td>( e_{do} )</td>
<td>Void ratio at the lowest density at ( p^\prime=0 ) kPa</td>
<td>Cyclic shearing or standard ( e_{min} ) test</td>
</tr>
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<td>( e_{cs} )</td>
<td>Void ratio at the critical state line at ( p^\prime=0 ) kPa</td>
<td>standard ( e_{max} ) test</td>
</tr>
<tr>
<td>( e_{io} )</td>
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<td>( 1.15 - 1.2 \ e_{max} )</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>controlling the peak friction angle</td>
<td></td>
</tr>
<tr>
<td>( \beta )</td>
<td>controlling the bulk and shear stiffness</td>
<td>Drained triaxial test</td>
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</table>
Table 2 Hypoplastic Sand Parameters Values of Different Sands

<table>
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<th>φc (°)</th>
<th>$h_s$ (MPa)</th>
<th>n</th>
<th>$e_{do}$</th>
<th>$e_{co}$</th>
<th>$e_{io}$</th>
<th>α</th>
<th>β</th>
<th>Reference</th>
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</tr>
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<td>1.212</td>
<td>0.14</td>
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<td>0.60</td>
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<td>0.791</td>
<td>0.13</td>
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<td>[43]</td>
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<tr>
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<td>1</td>
<td>[41]</td>
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*Adjusted based on the current study validation
Table 3 Sands Physical Properties

<table>
<thead>
<tr>
<th>Sand Name</th>
<th>$d_{50}$ (mm)</th>
<th>$C_u$</th>
<th>$G_s$</th>
<th>$e_{min}$</th>
<th>$e_{max}$</th>
<th>Angularity</th>
<th>Reference</th>
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<td>UWA silica</td>
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<td>0.49</td>
<td>0.79</td>
<td>rounded</td>
<td>[44,65]</td>
</tr>
<tr>
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<td>1.58</td>
<td>2.65</td>
<td>0.613</td>
<td>1.014</td>
<td>sub-rounded</td>
<td>[48]</td>
</tr>
<tr>
<td>(fr. E)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Ottawa 50-70</td>
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<td>sub-rounded</td>
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</table>
Fig. 1 Variation of $\phi_p$ with $p'$, $\phi_{cr}$ and $D_r$. 

$D_r = 90\%$

$D_r = 50\%$
Fig. 2 Finite element geometry and meshing
Fig. 3 Geometry and Meshing Optimization (a) Variation of element size \((le)\) with ultimate bearing capacity \(q_u\) (b) Variation of model geometry size \((x)\ or \(z)\) with ultimate bearing capacity \(q_u\)
Fig. (4) Normalized ultimate bearing capacity versus sand mean diameter ($d_{50}$) for Leighton Buzzard Sand
Fig. 5 Normalized ultimate bearing capacity – settlement curves for different sands (a) $Dr = 70\%$, $D = 4.37$ m, $\phi_{cr} = 30^\circ$ - $35^\circ$, (b) $Dr = 90\%$, $D = 7$ m, $\phi_{cr} = 31.5^\circ$, $32^\circ$, $33^\circ$. 

(a)

(b)
Fig. 6 Normalized ultimate bearing capacity versus sand critical friction angle $\phi_{cr}$ for different footing diameters ($D_r = 70\%$)
<table>
<thead>
<tr>
<th>Ottawa Sand 50-70 ($\phi_{cr} = 30^\circ$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Karlsruhe fine Sand ($\phi_{cr} = 33.1^\circ$)</td>
</tr>
<tr>
<td>Komorany Sand ($\phi_{cr} = 35^\circ$)</td>
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</table>

Fig. 7 Distribution of void ratio at different loading stages at w/D % (a) 10 %, (b) 15%, and (c) 33% ($D = 3$m)
Fig. 8 Normalized ultimate bearing capacity – settlement curves for Toyoura sand ($D = 3$ m)
Fig. 9 Distribution of void ratio, mobilized friction angle, and incremental shear strain at different $D_r$, (a) 90%, (b) 70%, and (c) 50% for Komorany sand
Fig. 10 Normalized ultimate bearing capacity – settlement curves for Toyoura sand ($D_r = 70\%$)
Fig. 11 Distribution of (a) void ratio, and (b) incremental shear strain at different footing diameters $D$ for Komorany Sand
Fig. 12 Normalized ultimate bearing capacity versus stress level parameter for $D_r = 90\%$ (solid lines) and 50\% (dashed lines)
Fig. 13 Three-dimensional plot of $\gamma D/p_a$, $\tan \phi_{cr}$ and $N_\gamma$ at different $D_r$. 
Fig. 14 $N_y$ (FEM) versus $N_y$ (predicted)
Fig. 15 Comparison of the proposed method with other centrifuge tests (solid line is the current proposed method, dashed line is the proposed method by [27])
References


