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Passive flutter suppression with rotary nonlinear energy sink

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Abstract

The control of aeroelastic phenomena, such as flutter, is of great interest due to its high amplitude self-excited characteristic. Nonlinear Energy Sinks (NES) are vibration absorbers whose nonlinear coupling to the structure contributes to broad excitation ranges for passive suppression. This paper investigates the attachment of a rotary-type NES (RNES) to an aeroelastic typical section to suppress nonlinear flutter oscillations passively. An unsteady aerodynamic loads model is used based on the Theodorsen and Wagner approaches. Pitching structural nonlinearity is added, inducing limit cycle oscillations in the airfoil. The system is modeled and numerically simulated. A dynamic characterization is done, obtaining the mechanisms of action of RNES through a regime identification, and its typical bifurcation behavior is accessed. A parametric analysis based on the system bifurcation over different RNES configurations is used to understand how each design parameter influences vibration mitigation performance and how absorption regimes correlate with RNES effectiveness. An energy analysis is carried through to conceive the activation of the Targeted Energy Transfer suppression mechanism and an energy-based parametric analysis of RNES performance. The results indicate that NES efficiency for flutter postponement is related mainly to low-radius devices located near the leading edge. RNES mass and angular damping parameters also present an impact but are limited due to subcritical behavior.

Keywords: Rotary Nonlinear Energy Sink, Flutter, Passive control, Nonlinear typical section, Aeroelasticity

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1. Introduction

The interest in nonlinear absorbers for vibration mitigation has recently increased due to their ability to absorb and dissipate structural energy quickly and irreversibly over a broader excitation range compared to traditional linear absorbers (Vakakis et al., 2008). In this sense, Nonlinear Energy Sinks (NES) arose as oscillation absorbers based on a nonlinear attachment to the main structure to promote the so-called Targeted Energy Transfer (TET). The TET consists of the transference of energy through the nonlinear coupling to a secondary mass and its passive dissipation. Various NES configurations have been proposed, such as cubic stiffness (Vakakis, 2001), bistable cubic stiffness (AL-Shudeifat, 2014), rotary inertial mass (Gendelman et al., 2011) and a variant rotary-oscillatory NES (Saeed et al., 2019), among others. More NES configurations can be found in Lu et al. (2018) and Ding and Chen (2020).

The problems involving aeroelastic phenomena are interesting among potential NES applications due to their characteristic self-excitation and self-sustained oscillations. For instance, flutter instabilities occur in airfoils from a critical airflow speed, leading to high amplitude structural pitch and plunge oscillations and, consequently, fatigue or catastrophic failures of components (Dowell, 2014). An elementary flutter model consists of a linear or nonlinear elastically supported airfoil subjected to pitch and plunge motions. In the linear case, exponential divergent oscillation occurs past the critical airspeed. If nonlinearities are considered, limit cycle oscillations (LCO) are induced (Nayfeh and Mook, 1995).

Several flutter control approaches have been proposed over the years, from passive to active alternatives (Livne, 2018; Chai et al., 2021). Nonlinear Energy Sinks (NES) for flutter control are promising due to their capacity for vibration suppression over a high excitation range for high amplitudes. Saeed et al. (2023) presents an overview of NES applications for diverse vibration phenomena, amongst them aeroelastic applications, such as flutter suppression.

The cubic NES for flutter control was first introduced in Lee et al. (2007a) to suppress a typical section numerically and analytically through a complexification-averaging technique. The authors considered pitch and plunge nonlinearities in the airfoil and a quasi-static aerodynamic model in their analysis. The paper identified different absorption mechanisms and analyzed the bifurcation behavior of the system. The results by Lee et al. (2007a) were
recently confirmed with wind tunnel experiments (Lee et al., 2007b). Later, Lee et al. (2008) introduced a multi dof NES for enhancing flutter suppression robustness. The authors conducted parametric analysis for different NES configurations and presented a reduced weight robust nonlinear design fit for flutter limit cycle oscillations mitigation.

More recently, Bichiou et al. (2016) studied the effectiveness of a cubic NES in flutter suppression numerically and through normal form quantification, identifying the NES mass and relative position in the airfoil influences on NES performance. The authors indicate that although effective, there is a limited influence on NES performance for flutter alleviation. Ebrahimzade et al. (2016) numerically compared linear absorber and NES performances for different proposed configurations. Yan et al. (2018) studied the application of a cubic NES in a transonic flutter by coupling the elastic model to a CFD solver, indicating its capacity to fully or partially suppress oscillations. The cubic NES was also analyzed in Silva et al. (2023), in which a dynamic characterization was carried through asymptotic analysis of the airfoil-NES bifurcations and TET parametric analysis. Lastly, cubic-piezoelectric hybrid NES for energy harvesting and passive vibration suppression were studied in Fasihi et al. (2022) and Zhang et al. (2023).

Recently, a so-called flap-NES device was proposed in Amar (2017) by adding a cubic nonlinearity in the flap control surface, with promising results in flutter speed postponement and oscillations alleviation without adding mass to the wing structure. This innovative device was studied experimentally by Escudero (2021) and numerically by García Pérez et al. (2022).

The rotary Nonlinear Energy Sink (RNES) consists of a horizontal pendulum where a mass can rotate over a fixed-radius orbit. The device was first proposed by Gendelman et al. (2012), in which its ability to resonate with any primary structure excitation was discussed, thus acting as an NES. Over the past years, RNES configurations for Fluid-Induced Vibrations have been proposed and studied, comprising Vortex-Induced Vibrations (VIV) and galloping phenomena (Saeed et al., 2023).

The coupling of an RNES for the VIV phenomenon was first proposed by Blanchard et al. (2017) on a laminar VIV phenomenon. The system was numerically studied through CFD and the authors performed a dynamic characterization and an analysis of the RNES capacity to perform targeted energy transfer (TET). The influence of locking and releasing the RNES...
on vortex-shedding and VIV was investigated by Blanchard and Pearlstein (2020), and an expanded analysis to three-dimensional turbulent-flow VIV was studied by Blanchard et al. (2020). The suppression of one-dof and two-dof VIV phenomenon was studied by Ueno and Franzini (2019). Phenomenological models were employed, and a numerical characterization and parametric analysis were done. The work results indicated the ability of the RNES to alleviate Vortex-Induced Vibrations. Energy harvesting from a cylinder subjected to VIV was studied by Araujo et al. (2022) for a variant rotary NES with an electric generator attached to its axis of rotation, indicating the absorber’s potential for simultaneous passive suppression and electricity generation.

A RNES attached to a square prism subjected to galloping was proposed by Selwanis et al. (2021) on a wind tunnel experimental study, evidencing its capacity to postpone galloping oscillations. More recently, the authors presented a subsequent numerical study over the RNES suppression mechanism and efficiency in galloping alleviation (Selwanis et al., 2023). A multi-ball rotary NES configuration was proposed and studied for galloping on Selwanis et al. (2022). The device consists of several spheres rotating on the same circular track, free to rotate and collide. The authors concluded that the impacts impede highly efficient rotary regimes, leading to less efficient performance of the variant device.

NES has largely been explored for single and multiple dof cubic configurations and the recent flap-NES, particularly for flutter suppression. Still, to the best of the author’s knowledge, there is a lack of understanding of how other absorbers behave for this phenomenon, such as the rotary NES. RNES dynamics holds promising flutter suppression performance once the coupling between its rotational motion and the airfoil torsion can lead to efficient flutter passive suppression.

This work proposes the application of a rotary-type NES for passive suppression of an aeroelastic typical section undergoing flutter oscillations. For this, the aeroelastic section with the attachment of the RNES is mathematically modeled and subjected to flutter oscillations with the coupling of an unsteady aerodynamic model. A pitch hardening is also considered in the model. The aeroelastic equations of motion are numerically integrated, and characterization is performed for uncontrolled and controlled models with bifurcation and regime identification. Then, a parametric study is done through bifurcations for RNES parameter variation to understand the absorber performance over different designs. TET is
studied on time histories of the system’s energies, and a TET-based parametric analysis is carried through.

2. Mathematical model

The model proposed is presented in Figure 1 and consists of an airfoil with semi-chord length $b$, mass per unit length $m$, mass moment of inertia $I_\alpha$, and static moment $S_\alpha$. The origin $O$ of the coordinate system is defined in the airfoil mid-chord at a dimensionless distance $a$ to the elastic axis. The airfoil is supported by an elastic suspension of stiffness $k_h$ and $k_\alpha$ at the elastic axis regarding the plunge $h(t)$ (positive downward) and pitch $\alpha(t)$ motions, respectively. A hardening nonlinearity is considered in the pitch stiffness $k_\alpha$, which employs a cubic coefficient $H_\alpha$. Such a nonlinear effect gives the model a more realistic dynamical behavior. When the airfoil is subjected to the airflow of density $\rho$ and speed $U$, the aerodynamic excitation comprises an unsteady lift of $L(t)$ in the plunging motion and a pitching moment of $M_\alpha(t)$.

![Figure 1: Schematic of a typical aeroelastic section coupled to a RNES.](image)

The RNES device is introduced to the structure as a pendulum with lumped mass $m_n$, with a rotational motion defined by the fixed-radius orbit of $r_0$ in the $\theta(t)$ variable (positive clockwise). The RNES center of rotation is positioned at a distance $d$ from the elastic axis along the airfoil chord length. No structural dissipation is considered for the airfoil, but
torsional viscous damping of coefficient $c_\theta$ is assumed for the RNES, enabling its suppression mechanism. Considering the model proposed in Figure 1, the position of the pendulum lumped mass $m_n$ is written as:

$$
\begin{align*}
  x_n(t) &= d \cos(\alpha) - r_0 \sin(\theta) \\
  y_n(t) &= h - d \sin(\alpha) - r_0 \cos(\theta)
\end{align*}
$$

(1)

and the velocity components as,

$$
\begin{align*}
  \dot{x}_n(t) &= -d \dot{\alpha} \sin(\alpha) - r_0 \dot{\theta} \cos(\theta) \\
  \dot{y}_n(t) &= \dot{h} - d \dot{\alpha} \cos(\alpha) + r_0 \dot{\theta} \sin(\theta)
\end{align*}
$$

(2)

The total kinetic energy of the system can be expressed as:

$$
T = T_s + T_n,
$$

(3)

with $T_s$ and $T_n$ representing, respectively, the structural and NES contributions, which are written as:

$$
\begin{align*}
  T_s &= \frac{1}{2} m \dot{h}^2 + S_\alpha \dot{h} \dot{\alpha} + \frac{1}{2} I_\alpha \dot{\alpha}^2 \\
  T_n &= \frac{1}{2} m_n (\dot{x}_n^2 + \dot{y}_n^2) \\
  &= \frac{1}{2} m_n (d^2 \dot{\alpha}^2 + r_0^2 \dot{\theta}^2 + \dot{h}^2 + 2 d r_0 \sin(\alpha - \theta) \dot{\alpha} \dot{\theta} - 2 d \cos(\alpha) \dot{h} + 2 r_0 \sin(\theta) \dot{\theta} \dot{h})
\end{align*}
$$

(4)

Since neither the torsional stiffness nor gravitational contributions are considered in the RNES, the potential energy is as follows:

$$
V = \frac{1}{2} k_h h^2 + \frac{1}{2} k_\alpha \dot{\alpha}^2 + \frac{1}{4} k_\alpha H \dot{\alpha}^4 .
$$

(5)

Furthermore, by neglecting structural dissipation from the typical section, the dissipation effect is only from the torsional damping in the RNES axis, that is:

$$
\mathcal{D} = \frac{1}{2} c_\theta \dot{\theta}^2 .
$$

(6)
The equation of motion can be derived from the Lagrange equations:

\[
\begin{align*}
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\mathbf{q}}} \right) - \frac{\partial L}{\partial \mathbf{q}} + \frac{\partial D}{\partial \dot{\mathbf{q}}} &= -L(t) \\
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\mathbf{\alpha}}} \right) - \frac{\partial L}{\partial \mathbf{\alpha}} + \frac{\partial D}{\partial \dot{\mathbf{\alpha}}} &= M_{\alpha}(t) \\
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\mathbf{\theta}}} \right) - \frac{\partial L}{\partial \mathbf{\theta}} + \frac{\partial D}{\partial \dot{\mathbf{\theta}}} &= 0
\end{align*}
\]

with \( \mathcal{L} \) denoting the Lagrangian of the system, which is defined as \( \mathcal{L} = T - V \).

The RNES-airfoil system equations of motion result in the following set of nonlinear differential equations:

\[
\begin{align*}
(m + m_n)\ddot{h} + S_\alpha\dot{\alpha} + k_h\dot{h} + m_n d (\sin(\alpha)\dot{\alpha}^2 - \cos(\alpha)\ddot{\alpha}) + \\
+ m_n r_0 (\sin(\theta)\ddot{\theta} + \cos(\theta)\dot{\theta}^2) &= -L(t) \\
S_\alpha\ddot{h} + (I_n + m_n d^2)\dot{\alpha} + k_a\dot{\alpha} + H_\alpha\dot{\alpha}^3 - m_n d\dot{\theta}\cos(\alpha) + \\
+ m_n d r_0 (\sin(\alpha - \theta)\dot{\theta} - \cos(\alpha - \theta)\ddot{\theta}) &= M_{\alpha}(t) \\
m_n r_0^2\ddot{\theta} + c_0 \dot{\theta} + m_n r_0 h\sin(\theta) + m_n d r_0 (\sin(\alpha - \theta)\dot{\alpha} + \cos(\alpha - \theta)\ddot{\alpha}) &= 0
\end{align*}
\]

The unsteady aerodynamic solution is determined by generalizing the Theodorsen approach and expressing \( L(t) \) and \( M_{\alpha}(t) \) in terms of an aerodynamic system state. For this, the Theodorsen function is expanded by applying the indicial response using Wagner function (Fung, 2008) and then the Padé interpolation approach. Consequently, the augmented aerodynamic states are assessed (Vasconcellos et al., 2012), resulting in the unsteady aerodynamic loading as follow:

\[
L(t) = \pi \rho b^3 \left[ \xi + \left( \frac{U}{b} \right) \dot{\alpha} - a \ddot{\alpha} \right] + 2\pi \rho U b^2 \{ (c_0 - c_1 - c_3) \left[ \left( \frac{U}{b} \right) \alpha + \dot{\xi} + (1/2 - a) \ddot{\alpha} \right] + \\
+ \left( \frac{U}{b} \right)^2 \{ c_2 c_4 (c_1 + c_3) \ddot{x} + \left( \frac{U}{b} \right) (c_1 c_2 + c_3 c_4) \dot{x} \} \}
\]

\[
M_{\alpha}(t) = \pi \rho b^4 \left[ a \xi - \left( \frac{U}{b} \right) (1/2 - a) \dot{\alpha} - (1/8 + a^2) \ddot{\alpha} \right] + \\
+ 2\pi \rho U b^3 \{ (c_0 - c_1 - c_3) \left[ \left( \frac{U}{b} \right) \alpha + \dot{\xi} + (1/2 - a) \ddot{\alpha} \right] + \\
+ \left( \frac{U}{b} \right)^2 \{ c_2 c_4 (c_1 + c_3) \ddot{x} + \left( \frac{U}{b} \right) (c_1 c_2 + c_3 c_4) \dot{x} \} \}
\]

where \( c_1 = 0.165, c_2 = 0.0455, c_3 = 0.335, \) and \( c_4 = 0.3 \) are coefficients of Wagner function according to Sear’s approach, and \( \ddot{x} \) is the augmented aerodynamic variable.

The aerodynamic augmented variable follows a second-order differential equation and is
a function of the airfoil pitch and plunge motion variables (Vasconcellos et al., 2012), as
follows:

\[
\ddot{x} - \dot{\xi} - \left(\frac{1}{2} - a\right) \dot{\alpha} + (c_2 + c_4) \frac{U}{b} \ddot{x} - \frac{U}{b} \alpha + c_2 c_4 \frac{U^2}{b^2} \ddot{x} = 0 .
\]  

(11)

Following a convenient dimensionless parameters, that is: \( \xi = \frac{b(\dot{t})}{b}, \chi_\alpha = \frac{S_\alpha}{mb}, r_\alpha^2 = \frac{I_\alpha}{mb^2}, \omega_h^2 = \frac{k_h}{m}, \omega_\alpha^2 = \frac{k_\alpha}{I_\alpha}, \hat{m} = \frac{m_n}{m}, \hat{r} = \frac{r_n}{b}, \hat{d} = \frac{d}{b} \) and \( \lambda_n = \frac{c_\omega}{m_n r_\alpha^2} \), the final set of equations of motion become:

\[
\begin{align*}
(1 + \hat{m}) \ddot{\xi} + \chi_\alpha \dot{\alpha} + \omega_h^2 \xi + \hat{m} \ddot{d} (\sin(\alpha) \dot{\alpha}^2 - \cos(\alpha) \ddot{\alpha}) + \\
\chi_\alpha \dot{\xi} + (r_\alpha^2 + \hat{m} \ddot{d}^2) \dot{\alpha} + r_\alpha^2 \omega_\alpha^2 (\alpha + H_\alpha \alpha^3) - \hat{m} \ddot{d} \cos(\alpha) + \\
\hat{m} \ddot{r} (\sin(\alpha - \theta) - \cos(\alpha - \theta) \ddot{\theta}) = \frac{M_\alpha(t)}{mb^2} .
\end{align*}
\]  

(12)

Since the RNES suppression mechanism consists of energy pumping from the structure to a secondary mass for passive dissipation, it is important to understand how the system’s energies behave. For that, reducing the energy expressions to a dimensionless form is convenient. The significant energy components to completely comprehend the TET phenomenon are the mechanical energy contained on the airfoil \( (E_s) \), the energy contained on the RNES \( (E_n) \), the fluid injected energy by the aerodynamic loads \( (E_a) \) and the energy dissipated by the absorber \( (E_d) \).

The structural energy contained by the airfoil \( (E_s) \) can be expressed in terms of kinetic and potential energy components, as well as RNES-induced kinetic terms due to the coupling. These components are expresses as \( \mathcal{T}_s, \mathcal{V}_s \) and \( \mathcal{T}_{sn} \), as presented in:

\[
\begin{align*}
\mathcal{T}_s &= \frac{1}{2} \dot{\xi}^2 + \chi_\alpha \dot{\alpha}^2 + \frac{1}{2} \hat{r}_\alpha^2 \dot{\alpha}^2 , \\
\mathcal{V}_s &= \frac{1}{2} \omega_h^2 \dot{\xi}^2 + \frac{1}{2} r_\alpha^2 \omega_\alpha^2 \dot{\alpha}^2 + \frac{1}{2} \hat{r}_\alpha^2 \omega_\alpha^2 H_\alpha \alpha^4 , \\
\mathcal{T}_{sn} &= \frac{1}{2} \hat{m} \ddot{d} \dot{\alpha}^2 - \hat{m} \ddot{d} \cos(\alpha) \dot{\alpha} , \\
E_s &= \mathcal{T}_s + \mathcal{V}_s + \mathcal{T}_{sn} .
\end{align*}
\]  

(13)

The instantaneous RNES energy is expressed on its kinetic energy, excluding the com-
ponents regarding RNES-induced structural energy, as follows:

$$E_n = \frac{1}{2} \dot{m} \dot{r}^2 + \dot{m} \ddot{r} \sin(\alpha - \theta) \ddot{\theta} + \dot{m} \dot{r} \sin(\theta) \ddot{\xi}.$$ (14)

The input energy injected by the airflow is computed through the work done by the aerodynamic loads on the airfoil:

$$E_a = \int_0^t \left( -\frac{L(t)}{mb} \ddot{\xi} + \frac{M_a}{mb^2} \dot{\alpha}(t) \right) dt.$$ (15)

Similarly, the cumulative dissipated energy by the RNES is considered through the work done by RNES angular damping:

$$E_d = \int_0^t \dot{m} \dot{r}^2 \lambda_n \dot{\theta}^2 \, dt.$$ (16)

### 3. Results and discussion

The study was carried out through numerical simulations of the system, which were done by directly integrating the equations of motion. The equations of motion were solved through a fourth-order Runge-Kutta method with a time-step of 0.01s and a total simulation time of 100 seconds. An arbitrary initial pitch perturbation of \(\alpha(0) = 1^\circ\) was chosen to ensure oscillations occur in the airfoil. The initial RNES angle was defined as \(\theta(0) = \pi/2\), perpendicular to the plunge motion, to enable the pendulum’s energy transference from the structure to the RNES. All other initial conditions were considered null (i.e., \(\xi(0) = \dot{\xi}(0) = \dot{\alpha}(0) = \dot{\theta}(0) = \ddot{\xi}(0) = \dot{\ddot{\xi}}(0) = 0\)).

The reference case study of the aeroelastic model presented in Figure 1 was assumed with parameters: \(b = 0.5m\), \(a = -0.15\), \(\omega_h = 2\pi\text{rad/s}, \omega_\alpha = 6\pi\text{rad/s}, \chi_\alpha = 0.25, r_\alpha = 0.75, m = 20\text{kg/m}\) and \(\rho = 1.0\text{kg/m}^3\). Regarding the RNES parameters, they are taken in the ranges of \(\dot{m} \in ]0, 0.15[, \dot{r} \in ]0, 0.3[, \dot{d} \in ]-0.5, 0.8[\), and \(\lambda_n \in ]0, 3.0[\). A base RNES parameters case was chosen with \(\dot{m} = 0.1, \dot{r} = 0.02, \dot{d} = 0.5\), and \(\lambda_n = 1.5\). This reference case is studied concerning characteristic bifurcation behavior over the fluid-flow speed, with a step of \(dU = 0.001\) in the range of \(U \in [0.95, 1.3]\).

An uncontrolled typical aeroelastic section initially characterizes the linear flutter phenomenon, and the critical flutter speed is computed through an eigenvalue analysis for vary-
ing fluid-flow speeds. After the linear analysis of flutter speed, the nonlinear aeroelastic model was studied considering a hardening nonlinearity in pitch with constant $H_\alpha = 7.5$. All further simulations are done considering the critical flutter speed obtained in the linear analysis for both uncontrolled and RNES configurations.

A parametric analysis inspects the bifurcation behavior for varying RNES parameters. With this, the influence of each RNES parameter was observed in terms of amplitude and corresponding regime boundaries. The TET behavior is also analyzed by observing the energy of each system component for different oscillation regimes with and without RNES. Furthermore, TET parametric analysis is done from variations of absorber design parameters with a fixed airspeed for low and high perturbations obtained in characteristic bifurcation.

3.1. Characterization of the uncontrolled flutter phenomenon

Initial characterization is done over the uncontrolled typical aeroelastic section linear model regarding the reference case study. For this, a linear system was considered with no pitch-hardening contribution. Figure 2 presents the eigenvalue ($\lambda$) evolution over rising airspeed, which indicates that the critical flutter speed happens at $U = 26.89\text{m/s}$ (mode crossing the speed axis; real($\lambda$) = 0).

From the linear analysis, the nonlinear flutter phenomenon was studied with the addition of pitch hardening nonlinearity ($H_\alpha = 7.5$), which induces supercritical LCOs on the airfoil dynamic response over airspeed bifurcation. The nonlinear uncontrolled results served as a base for performance analysis of the RNES-attached system. Throughout all simulations, flutter critical speed is considered to be $U = 26.89\text{m/s}$, as obtained in the linear characterization.

3.2. Characterization of the nonlinear flutter phenomenon with the application of RNES

Five typical regimes are observed throughout the simulations and denoted as $R_i$ for $i = 0, ..., 4$. Figure 3 gathers the five cases regarding the airfoil dynamical responses and the RNES angular time history. All these examples were taken from the reference case RNES parameters for different flow speeds.

The first regime presented is the pre-flutter response, denoted as $R_0$ (cf. Figure 3(a)). This response is usually associated with lower excitation and amplitudes, mainly in $U/U_c < 1$
Figure 2: Eigenvalue evolution over airspeed for linear airfoil subjected to flutter oscillations.

Figure 3: Response examples for (a) $R_0$ (b) $R_1$ (c) $R_2$ (d) $R_3$ (e) $R_4$ (— RNES response, —— uncontrolled LCO amplitude).

regions and represents. However, in some RNES configurations, this response happens in a higher range, which implies a postponement of the critical flutter speed.
The second response, namely \( R_1 \), presented is the modulated regime (cf. Figure 3(b)). In this case, the RNES alternates between constant-speed rotation and brief moments in which it ceases its motion, with cycles of increasing and decreasing energy absorption performance and amplitude modulations.

In the \( R_2 \) response regime (cf. Figure 3(c)), the RNES moves almost constantly. This response was observed to be the best performing for amplitude mitigation, promoting a limit cycle oscillation of small amplitudes in the plunge and pitch motion. It is observed that \( R_2 \) regime is basically a more efficient and organized modulated response, in which RNES present motion and pause cycles at briefer time intervals and on only one direction rotation, leading to a nearly constant-speed rotary response.

The \( R_3 \) happens when the RNES rotates towards an equilibrium angle and starts oscillating around it, leading to a high amplitude limit cycle oscillation in both the pendulum and airfoil (cf. Figure 3(d)). This response is usually inefficient in vibration mitigation and related to higher excitation airspeeds.

Lastly, the \( R_4 \) response is caused by irregular and unpredictable oscillations in the RNES (cf. Figure 3(e)), and, such as the \( R_3 \) response, is related to high excitation. Even so, it presents rotation in the pendulum. It is usually a highly inefficient response for vibration mitigation.

Therefore, for RNES design purposes, it is desirable to increase mainly \( R_0 \), \( R_1 \), and \( R_3 \) regime boundaries, with emphasis on the first and latter, which are the most efficient. In addition, an effective RNES design comprehends the passive suppression over lower airspeed, aiming to postpone the growth of amplitudes to higher flow speeds to minimize the effects of flutter.

It is worth noticing that an abnormal and highly inefficient response happens when the RNES rotates towards an equilibrium angle and, instead of oscillating around it as in \( R_3 \), it becomes stable and ceases its motion. It is a rather rare response and happens only in the case of \( \dot{d} = 0 \), which removes the pitching contributions from the RNES angle \( dof \), in a way that it is independent of the airfoil torsion. Even if it is a rare and specific case, if this response occurs, the RNES becomes unable to reactivate its rotation and, thus, useless in vibration suppression. Hence, the attachment of the absorber to the elastic axis must be avoided in order not to make this regime possible.
In Figure 4, the pitch and plunge bifurcation over the airspeed for the base case with $\dot{m} = 0.1$, $\dot{r} = 0.02$, $\dot{d} = 0.5$, and $\lambda_n = 1.5$ are presented, as well as the regime identification corresponding to the bifurcation. It is interesting to notice that the two regions have an amplitude separation with divergent regimes.

Initially, $R_0$ regimes delay the beginning of flutter oscillations up to about 5% of flutter critical speed. The first subcritical region appears in this region, separating two regimes: low amplitudes $R_0$ and higher amplitude $R_1$. With the rise of the airspeed, the highly efficient $R_2$ takes place, with supercritical behavior from 5% up to 7% of $U_c$. After this, between 7% and 13%, there is the second subcritical range with a divergence of high efficiency $R_2$ and the ineffective $R_4$ response. As airspeed grows beyond 10% of critical flutter speed, the RNES can no longer absorb and dissipate structural energy, and the amplitudes continuously grow. It is noteworthy that even for high excitation regimes, there is a subtle amplitude alleviation for the pitch dof, which is caused by the great coupling of $\alpha(t)$ and $\theta(t)$ due to the positioning of the RNES relative to the elastic axis, which can be observed in Equation (12).

![Figure 4: Reference case bifurcation analysis with $\dot{m} = 0.1$, $\dot{r} = 0.02$, $\dot{d} = 0.5$ and $\lambda_n = 1.5$ for (a) Plunge motion (b) Pitch motion (c) Response regime.](image_url)
3.3. Bifurcation parametric analysis

3.3.1. Influence of RNES mass ratio

Figures 5 and 6 present, respectively, the behavior of plunge and pitch amplitudes in airspeed bifurcation and its corresponding response type under the influence of a variation in the RNES mass ratio parameter. For this, the reference case parameters $\hat{r} = 0.02$, $\hat{d} = 0.5$ and $\lambda_n = 1.5$ were fixed as $\hat{m}$ was varied.

It can be inferred from the curves that the increase in mass ratio leads to more significant amplitude suppression in forward bifurcation, postponing the flutter speed to 20% higher than the uncontrolled critical flutter speed when $\hat{m} = 0.15$. This behavior happens due to the widening of the $R_0$ and $R_2$ regimes, in which the best suppression performance is observed. Past the suppression region, $R_3$ and $R_4$ responses take over the absorber dynamics, and an amplitude growth is observed. In the backward bifurcation, the suppression is considerably lower, as a higher range is controlled by $R_3$ and $R_4$ responses, reducing the $R_2$ boundary. Besides that, the $R_0$ region is also affected, with the appearance of the slightly higher amplitude region of $R_1$.

The broadening of the efficient suppression region results from the increase in RNES inertia. However, even if the best performance is observed for higher mass ratios, it is worth highlighting that the addition of structural mass is often undesirable, and this must be pondered altogether with the benefits of the RNES in vibration mitigation. In addition to that, subcritical behavior contributes to the limited influence of $\hat{m}$ over NES performance, with greater impact over low initial conditions but the low overall change in efficiency flutter phenomenon postponement from a certain value of $\hat{m}$.

3.3.2. Influence of RNES radius ratio

Figures 7 and 8 present the influence of the RNES radius ratio for the plunging and pitching bifurcation and the corresponding forward and backward bifurcation. RNES parameters $\hat{m} = 0.10$, $\hat{d} = 0.5$ and $\lambda_n = 1.5$ were maintained fixed, as $\hat{r}$ was varied.

It can be observed that the variation of $\hat{r}$ has a significant impact on the airspeed range in which the amplitude mitigation takes place, with the absorption happening better on high flow speeds for greater radii. This behavior is physically explained by the radius’s quadratic relation with the RNES inertia, making greater available energy needed for great efficiency
regimes, such as $R_1$ and $R_2$. Also, the low-speed $R_0$ regime is unaffected by $\hat{r}$, which is dependent only on other RNES parameters.

An interesting phenomenon is a critical radius around $\hat{r} = 0.1$ in which the $R_2$ regime efficiency zone changes from forward to backward bifurcation for increasing $\hat{r}$. However, when aiming to postpone flutter oscillations to higher airspeed, it is beneficial to maintain lower radii to dissipate energy at lower excitation more effectively. In this sense, Figure 9 presents the bifurcation behavior for lower radius ratios.

It is clear from Figure 9 that below $\hat{r} < 0.05$ subcritical behavior is predominant with great forward bifurcation efficiency and higher amplitude backward bifurcation. With the increase of the radius ratio, the subcritical range continuously decreases until it becomes
Figure 7: Influence of the RNES radius in the bifurcation behavior, considering $\hat{m} = 0.10$, $\hat{d} = 0.5$ and $\lambda_n = 1.5$ (--- without RNES, ■ stable LCO, ●→ forward, ■→ backward).

Figure 8: Influence of the RNES radius in type of response in forward and backward bifurcation, considering $\hat{r} = 0.02$, $\hat{d} = 0.5$ and $\lambda_n = 1.5$ ($\rightarrow R_0$, $\rightarrow R_1$, $\rightarrow R_2$, $\rightarrow R_3$$\rightarrow R_4$).

almost supercritical. This behavior would be interesting for RNES design due to its independence from the initial conditions if not for the considerable amplitude increase.

Therefore, regarding RNES radius design, the most interesting vibration mitigation behavior occurs for low values of $\hat{r}$. Nonetheless, it is noteworthy that shallow values of $\hat{r}$ combined with higher $m_r$ may not be feasible in real applications depending on the airfoil dimensions, despite having a good vibration suppression efficiency.

3.3.3. Influence of RNES distance to the elastic axis ratio

The distance to the elastic axis in which the RNES is positioned influences its absorption abilities, as is presented in Figures 10 and 11 (following the same fashion as in previous
Figure 9: Detail of influence of lower RNES radii in the bifurcation, considering $\hat{m} = 0.10$, $\hat{d} = 0.5$ and $\lambda_n = 1.5$ (without RNES, stable LCO, forward, backward).

Sections). All other parameters were fixed as $\hat{m} = 0.10$, $\hat{d} = 0.5$ and $\lambda_n = 1.5$.

It is noticeable that negative relative distances reduce the $R_0$ boundary and are less efficient for vibration suppression. Accordingly, only $\hat{d} > 0$ should be considered in RNES design. In the region between $\hat{d} = 0$ and $\hat{d} = 0.75$, the suppression is enhanced continuously, analogously as observed in the mass ratio influence. In the equations of motion (i.e., Equation (12)), there is a relation between the airfoil inertia and $\hat{d}$, as well as in the coupling terms of $\theta(t)$ and $\alpha(t)$ coordinates, which justifies the best absorption performance for increasing distance ratios.

Figure 10: Influence of the RNES distance to the elastic axis in the bifurcation behavior, considering $\hat{m} = 0.10$, $\hat{r} = 0.02$ and $\lambda_n = 1.5$ (without RNES, stable LCO, forward, backward).
3.3.4. Influence of RNES damping

Figures 12 and 13 present the bifurcation diagrams corresponding the RNES angular viscous damping $\lambda_n$. For that, the base case parameters $\hat{m} = 0.10$, $\hat{r} = 0.02$ and $\hat{d} = 0.5$ were kept fixed while $\lambda_n$ was varied.

Regarding Figure 12, low damping values, although presenting supercritical behavior with some vibration mitigation, have somewhat low efficiency. Then, there is a threshold in which the system acquires a subcritical behavior with optimal suppression. As the damping parameter continuously grows, the subcritical range of the $R_2$ regime continuously decreases, with amplitude growth at lower airspeed.

It is understood that sufficiently low absorber damping provides the best vibration mitigation, consistently transferring and dissipating structural energy on the appended mass. Thus, for design purposes, damping must be kept low enough to ensure RNES performance but at a safe margin to avoid becoming inefficient.

3.4. Energy transfer analysis

The RNES suppression mechanism consists of pumping energy from the fluid-induced structural energy on the airfoil to the secondary mass and its successive dissipation. This energy transfer phenomenon is analyzed in this section by computing the energy components present in the system in time histories and parametric variation.

Figure 14 present time histories of the relevant energy components, comprising the input
energy from the airflow ($E_a$), the mechanical energy contained in the airfoil structure ($E_s$), the instantaneous absorbed energy on the RNES ($E_n$) and its dissipated energy ($E_d$). This energy analysis was done over the five characterized regimes in Figure 3. Regarding the RNES performance, for all regimes, the dissipated energy grows according to the input airflow energy, which indicates the ability of the device to dissipate the injected energy otherwise converted into structural vibration passively. Once again, best suppression behavior was observed for $R_0$, $R_1$, and $R_2$, with emphasis on $R_2$ for maintaining low structural energy over higher airspeeds. When purely analyzing the energy contained in the airfoil, it is more perceptible the organized modulated behavior of the $R_2$ regime.

Figure 15 presents a comparison between RNES-attached airfoil and the corresponding
Figure 14: Energy transfer and dissipation process for each absorption regime with $\hat{m} = 0.1$, $\hat{r} = 0.02$, $\hat{d} = 0.5$, and $\lambda_n = 1.5$. 

...uncontrolled nonlinear flutter response mechanical energies contained in the structure for the different characterized regimes. The results show that regimes $R_0$, $R_1$, and $R_2$ successfully maintain mechanical structural energy at a considerably lower rate compared to when the...
device is inactive. A reduction is perceptible for the $R_4$ and $R_5$ regimes, although with lower efficiency, which is explained due to its usual occurrence at high excitation.

Figure 15: Energy contained in the airfoil for uncontrolled and with RNES cases for absorption regimes with $\hat{m} = 0.1$, $\hat{r} = 0.02$, $\hat{d} = 0.5$, and $\lambda_n = 1.5$.

As for the TET-based parametric analysis, Figure 16 presents a parametric variation for
an airspeed of $U/U_c = 1.09$, corresponding to a section of the airflow bifurcation. Reference case parameters where adopted ($\hat{m} = 0.1$, $\hat{r} = 0.02$, $\hat{d} = 0.5$, and $\lambda_n = 1.5$), except for each varied parameter and values of mean structural energy were accessed on each configuration.

For the low perturbation case (forward bifurcation), the relation between the $R_2$ regime and energy pumping by the NES, with a greater reduction of mechanical energy on the airfoil, is clear. The RNES performance becomes more complex for higher perturbations and lower energy reduction is achieved, although some high amplitude $R_1$ and $R_2$ regimes are observed.

Figure 16: TET parametric analysis with $\hat{m} = 0.1$, $\hat{r} = 0.02$, $\hat{d} = 0.5$, and $\lambda_n = 1.5$ at $U/U_c = 1.09$ for forward and backward bifurcation.
The anomalous behavior of the radius variation is understood by the critical value of $\hat{r}$ in which RNES effectiveness shifts to higher excitation, as discussed in Section 3.3.2.

### 4. Conclusions

This paper investigated the coupling of an aeroelastic typical section with a rotary Nonlinear Energy Sink. The model consists of a two *dof* pitch and plunge airfoil, in which the RNES is attached as a rigid arm pendulum inside its structure. An unsteady aerodynamic loading is considered for the flutter phenomenon modeling, and a hardening nonlinearity is assumed in the pitch *dof*. The study was done through numerical simulations of the equations of motion, and regime identification, parametric bifurcation, and TET analysis were carried out.

It is observed in the regime identification that there is a response $R_0$ in which full suppression occurs, leading to the postponement of the flutter critical airspeed. This would be the ideal response once it completely prevents flutter oscillations. Besides that, $R_1$ and $R_2$ present partial vibration suppression, activating RNES TET efficiently either with modulated or near-constant pendulum rotation regimes. The TET effect was later verified on time histories of the system’s energies for each regime. The typical system bifurcation indicates the subcritical behavior of the nonlinear system, which restricts RNES robustness to a lower airspeed range, even though oscillation alleviation might be possible.

In the bifurcation and TET parametric analysis, it was possible to identify the influence of each RNES design parameter in the oscillation behavior over airspeed variation and its correspondent regimes. The RNES’ mass and relative distance to the elastic axis are observed to mainly influence the amount of energy absorbed, as the radius defines the airspeed region in which the absorption occurs. RNES angular damping presents a critical value in which suppression is maximized, growing more inefficient as damping rises. Essentially, the ideal design would be a low radius RNES positioned close to the airfoil’s leading edge with sufficiently low angular damping and enough mass to ensure efficiency. However, subcritical behavior typically appears in higher excitation regions, which may lead to non-robust performance from certain airspeeds and compromise the RNES effectiveness.

Overall, RNES was able to absorb and suppress oscillations in an aeroelastic typical section subjected to flutter oscillations. Nevertheless, this region of effectiveness is restricted
to a certain range in airspeed defined by the design parameters chosen, which is commonly
dependent on configurations whose viability depends on the real application.

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Data Availability

The authors confirm that the data supporting the findings of this study are available
within the article. Additional data are available from the corresponding author on reasonable
request.

Conflict of interests

The authors declare that they have no conflict of interest.

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