# Supplementary Information

## Maximizing Consumers' Satisfaction with Risk Decision in Distributed Energy Systems: A Proof-of-Prospect Blockchain Consensus Mechanism

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# 1 Supplementary Figure

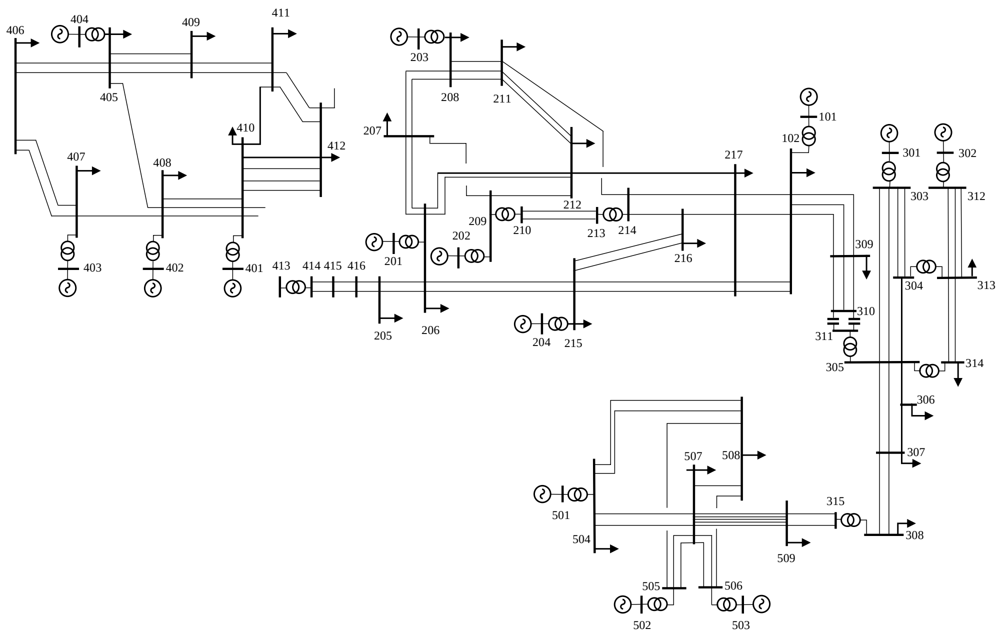


**Fig. 1**: The Simplified 14-Generator Australian Power Grid.

# 2 Supplementary Tables

**Table 1**: Symbol definition

| Symbol | Definition |
|---|---|
| $\mathbb{P}$ | The set of producers |
| $\mathbb{C}$ | The set of consumers |
| $p_i$ | The index of producer |
| $c_j$ | The index of consumer |
| $s_i$ | Energy supplied to other consumers from $p_i$ |
| $d_j$ | Energy allocated to $c_j$ from other producers |
| $s_i^{\max}$ | Maximum electricity supply of $p_i$ |
| $d_j^{\min}$ | Minimum electricity demand of $c_j$ |
| $pr_i$ | Expected electricity supply price (ESP) of $p_i$ |
| $cr_j$ | Expected electricity demand price (EDP) of $c_j$ |
| $r_{ij}$ | Energy allocation price between $p_i$ and $c_j$ |
| $s_{ij}$ | Energy volume that $p_i$ allocate to $c_j$ |
| $d_{ij}$ | Energy volume that $c_j$ received from $p_i$ |
| $pv_i$ | Prospect value of $p_i$ |

**Table 2**: Parameters for initializing the PoP consensus mechanism

| $\alpha$ | $\beta$ | $\lambda$ | $\tau$ | $N_a$ | $l_1$ | $T$ | $I$ | $J$ | $a$ | $r_l$ | $r_u$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.88 | 0.88 | 2.25 | 100 | 10 | 0.80 | 5 | 14 | 29 | $0.05 \times 10^{-2}$ | 7.00 c/kW·h | 15.00 c/kW·h |

**Table 3**: Power parameters of prosumers (kW·h)

| | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ | $p_7$ | $p_8$ | $p_9$ | $p_{10}$ | $p_{11}$ | $p_{12}$ | $p_{13}$ | $p_{14}$ | $c_1$ | $c_2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Units | 101 | 201 | 202 | 203 | 204 | 301 | 302 | 401 | 402 | 403 | 404 | 501 | 502 | 503 | 102 | 205 |
| $P^{max}$ | 60 | 50 | 55 | 55 | 50 | 45 | 50 | 30 | 20 | 25 | 15 | 100 | 100 | 100 | 35 | 30 |
| $s^{max}/d^{min}$ | 248.3 | 600 | 500 | 490 | 536 | 600 | 393 | 350 | 290 | 380 | 272 | 300 | 200 | 150 | 450 | 390 |
| $pr/cr$ | 8.31 | 13.43 | 9.52 | 11.28 | 8.34 | 11.87 | 9.13 | 12.29 | 12.58 | 13.05 | 10.64 | 7.68 | 8.85 | 14.39 | 7.5 | 11.63 |
| | $c_3$ | $c_4$ | $c_5$ | $c_6$ | $c_7$ | $c_8$ | $c_9$ | $c_{10}$ | $c_{11}$ | $c_{12}$ | $c_{13}$ | $c_{14}$ | $c_{15}$ | $c_{16}$ | $c_{17}$ | $c_{18}$ |
| Units | 206 | 207 | 208 | 211 | 212 | 215 | 216 | 217 | 306 | 307 | 308 | 309 | 312 | 313 | 314 | 405 |
| $P^{max}$ | 20 | 35 | 30 | 40 | 40 | 15 | 45 | 30 | 35 | 30 | 40 | 40 | 25 | 30 | 35 | 35 |
| $s^{max}/d^{min}$ | 130 | 1880 | 210 | 1700 | 1660 | 480 | 1840 | 1260 | 1230 | 650 | 655 | 195 | 115 | 2405 | 250 | 990 |
| $pr/cr$ | 7.42 | 12.84 | 8.2 | 7.07 | 7.47 | 10.17 | 9.11 | 8.22 | 11.75 | 9.94 | 9.68 | 12.39 | 7.73 | 11.21 | 11.18 | 8.43 |
| | $c_{19}$ | $c_{20}$ | $c_{21}$ | $c_{22}$ | $c_{23}$ | $c_{24}$ | $c_{25}$ | $c_{26}$ | $c_{27}$ | $c_{28}$ | $c_{29}$ | | | | | |
| Units | 406 | 407 | 408 | 409 | 410 | 411 | 412 | 504 | 507 | 508 | 509 | | | | | |
| $P^{max}$ | 30 | 40 | 40 | 25 | 35 | 35 | 40 | 35 | 30 | 20 | 35 | | | | | |
| $s^{max}/d^{min}$ | 740 | 0 | 150 | 260 | 530 | 575 | 1255 | 300 | 1000 | 800 | 200 | | | | | |
| $pr/cr$ | 9.81 | 6.18 | 6.02 | 9.54 | 11.37 | 12.51 | 6.58 | 9.82 | 9.08 | 5.71 | 8.11 | | | | | |

# 3 Supplementary Notes

**Supplementary Note 1: Motivations of selfish delegates.**

Selfish delegates are motivated by self-interest, and we have analyze the utility of delegates here. The delegates compete to come up with the optimal solution to $P1$. The solution given by delegate $n (n \in \mathbb{N})$ is $(s_{ij,n})_{I\times J}$, in which $s_{ij,n}$ is the allocation volume in delegate $n$'s solution. In PoW, delegates spend their computational power to find the solution to the hash puzzle and get the block rewards. Nevertheless, in this system, delegates act as electricity producers in electricity allocation, and the solution to $P1$ affects the delegate's utility. The delegates may not purely want to find the optimal solution of $P1$ but want to propose a solution to maximize the sum of the utility in the electricity allocation and block rewards.

We suppose the other identity of delegate $n$ is producer $p_i$. The utility of producer $p_i$ in the electricity allocation is

$$U_i(s_{ij}) = \sum_{j=1}^{J} s_{ij} r_{ij} + \left( s_i^{\max} - \sum_{j=1}^{J} s_{ij} \right) r_l.$$

The two terms are the earnings from allocating electricity to consumers and the utility grid. The utility of delegate $n$ is

$$U_n = R_n + U_i\left((s_{ij,n^*})_{I\times J}\right),$$

where $R_n$ is the block rewards of delegate $n$ and $(s_{ij,n^*})_{I\times J}$ is the optimal solution of $P1$. $R_n$ depends on whether delegate $n$ is the validator or not:

$$R_n = \begin{cases} R, & n \text{ is the validator} \\ 0, & n \text{ is not the validator}, \end{cases}$$

where $R$ is the block rewards. The block reward is related to the power generation situation in the electricity system. If there is an excess of new electricity generation in the electricity system, a larger $R$ factor is designed for the use of renewable electricity. If there is more thermal power generation, the block reward $R$ is smaller.

**Supplementary Note 2: Proof of Theorem 1.**

*Proof.* For any function $g(x, y)$, if for any $x_1 > x_2$ and $y_1 > y_2$, the following inequality hold

$$g(x_1, y_1) + g(x_2, y_2) \geq g(x_1, y_2) + g(x_2, y_1), \tag{1}$$

then $g(x, y)$ is said to be a supermodular function. If $g(x, y)$ is differentiable, (1) is equivalent to

$$\frac{g(x, y)}{\partial x \partial y} \geq 0.$$

Similarly, if

$$\frac{g(x, y)}{\partial x \partial y} \leq 0,$$

$g(x, y)$ is said to be a submodular function. To prove that $P1$ is both a supermodular and a submodular function, it is only necessary to prove that $v_j(.)$ is a non-convex and non-concave function to $U_j^t(s_{ij}^t)$. We need to prove that

$$v(x, x_0, \alpha, \beta, \lambda) = \begin{cases} (x - x_0)^\alpha, & \text{if } x \geq x_0 \\ -\lambda (x_0 - x)^\beta, & \text{if } x \leq x_0 \end{cases}$$

is a non-convex and non-concave function to $x$. We take the second derivative of $v(.)$ with respect to $x$:

$$\frac{\partial^2 v(.)}{\partial x^2} = \begin{cases} \alpha(\alpha - 1)(x - x_0)^{\alpha - 2}, & \text{if } x \geq x_0 \\ -\lambda\beta(\beta - 1)(x_0 - x)^{\beta - 2}, & \text{if } x \leq x_0. \end{cases}$$

Because $0 < \alpha, \beta < 1$ and $\lambda > 0$,

$$\begin{cases} \dfrac{\partial^2 v(.)}{\partial x^2} < 0, & \text{if } x \geq x_0 \\ \dfrac{\partial^2 v(.)}{\partial x^2} > 0, & \text{if } x \leq x_0. \end{cases}$$

Thus, $(x_0, v(x_0, x_0, \alpha, \beta, \lambda))$ is the inflection point of $v(.)$, where $v(.)$ is convex upward for $x$ with $x \geq x_0$ and concave downward for $x$ with $x \leq x_0$. Therefore, $v(.)$ is a non-convex and non-concave function to $x$ and $P1$ is a non-convex and non-concave problem. □

**Supplementary Note 3: Necessary and sufficient conditions for the local optimal solution of $P1$.**

Firstly, for readability, we use some simple symbols to represent the equations in $P1$. We denote $\mathbf{s} = (s_{ij})_{I \times J}$, $\mathbf{s}^* = (s_{ij,n^*})_{I \times J}$, and

$$H_i^1(\mathbf{s}) = s_{ij} - P_i^{min}, \forall p_i \in \mathbb{P},$$

$$H_i^2(\mathbf{s}) = P_i^{max} - s_{ij}, \forall p_i \in \mathbb{P},$$

$$H_i^3(\mathbf{s}) = \sum_{j=1}^{J} s_{ij} - s_i^{\max}, \forall p_i \in \mathbb{P},$$

$$H_j^4(\mathbf{s}) = \rho_j s_{ij} - P_i^{min}, \forall c_j \in \mathbb{C},$$

$$H_j^5(\mathbf{s}) = P_i^{max} - \rho_j s_{ij}, \forall c_j \in \mathbb{C},$$

Denote the producers with $U_j^t\left(s_{ij}^t\right) \geq U_j^t(pre)$ as set $\mathbb{P}_1^t$, and the other producers as set $\mathbb{P}_2^t$. Then

$$SU^t\left(s_{ij}^t\right) = \sum_{c_j^t \in \mathbb{P}_1^t} \left(U_j^t\left(s_{ij}^t\right) - U_j^t(pre)\right)^\alpha - \sum_{c_j^t \in \mathbb{P}_2^t} \lambda\left(U_j^t(pre) - U_j^t\left(s_{ij}^t\right)\right)^\beta.$$

According to the Karush-Kuhn-Tucker (KKT) condition, for the non-convex and non-concave optimization problem $P1$, solution $\mathbf{s}^*$ is a local optimum if and only if (2) holds.

$$\left.\frac{\partial L}{\partial \mathbf{s}}\right|_{\mathbf{s}=\mathbf{s}^*} = 0 \tag{2a}$$

$$\mathbf{d}^T \left.\frac{\partial^2 L}{\partial \mathbf{s}^2}\right|_{\mathbf{s}=\mathbf{s}^*} \mathbf{d} > 0, \forall \mathbf{d} \neq 0 \tag{2b}$$

$$H_i^k \left(\mathbf{s}^*, \Theta^{H_i}\right) \leq 0, \forall p_i \in \mathbb{P}, k = 1, 2, 3 \tag{2c}$$

$$H_j^k \left(\mathbf{s}^*, \Theta^{H_j}\right) \leq 0, \forall c_j \in \mathbb{C}, k = 4, 5 \tag{2d}$$

$$\mu_i^{k,*} H_i \left(\mathbf{s}^*, \Theta^{H_i}\right) = 0, \forall p_i \in \mathbb{P}, k = 1, 2, 3 \tag{2e}$$

$$\gamma_j^{k,*} H_i \left(\mathbf{s}^*, \Theta^{H_j}\right) = 0, \forall c_j \in \mathbb{C}, k = 4, 5 \tag{2f}$$

$$\mu_i^{k,*} \geq 0, \forall p_i \in \mathbb{P}, k = 1, 2, 3 \tag{2g}$$

$$\gamma_j^{k,*} \geq 0, \forall c_j \in \mathbb{C}, k = 4, 5 \tag{2h}$$

$$\tag{2i}$$

$L$ is the lagrange function of the optimization problem $P1$, $\mu_i^{k,*}$ and $\gamma_j^{k,*}$ are lagrange multipliers of inequality constraints $H^k(\mathbf{s}) > 0$ and $\mathbf{d}$ is an auxiliary vector. Because Sub-formulas (2a) and (2b) are difficult to understand, we will proceed to analyze these two sub-formulas, and we will not further analyze others that are easy to understand. The lagrange function is

$$L = SU^t\left(s_{ij}^t\right) + \sum_{k=1}^{3}\sum_{i=1}^{I}\mu_i^k H_i^k(\mathbf{s}) + \sum_{k=4}^{5}\sum_{j=1}^{J}\mu_j^k H_j^k(\mathbf{s}), \tag{3}$$

According to (3), the first derivative of $L$ with respect to $\mathbf{s}$ is obtained as follows:

$$\frac{\partial L}{\partial \mathbf{s}} = g_1(\Delta)L1 + \sum_{i=1}^{I}\mu_i^1 - \sum_{i=1}^{I}\mu_i^2 + \sum_{i=1}^{I}\mu_i^3 + \sum_{j=1}^{J}\gamma_j^1\rho - \sum_{j=1}^{J}\gamma_j^2\rho, \tag{4}$$

where

$$g_1(\Delta) = \begin{cases} -\alpha\left(U_j^t\left(s_{ij}^t\right) - U_j^t(pre)\right)^{\alpha-1}, c_j \in \mathbb{P}_1 \\ -\lambda\beta\left(U_j^t(pre) - U_j^t\left(s_{ij}^t\right)\right)^{\beta-1}, c_j \in \mathbb{P}_2. \end{cases} \tag{5}$$

and

$$L1 = \frac{w_j^t\rho}{\sum_{j=1}^{J} d_{ij}^t - d_j^{\min t} + 1} - r_{ij}^t + \rho_j r_u - |tr_i - tr_j| \tag{6}$$

(2a) **implies that when $\mathbf{s} = \mathbf{s}^*$ the determinant of the matrix with (4) as elements is 0.** The second derivative of $L$ with respect to $\mathbf{s}$ is obtained as follows:

$$\frac{\partial^2 L}{\partial \mathbf{s}^2} = g_2(\Delta) L_1^2 + g_1(\Delta) \left( \frac{w_j^t \rho^2}{(\sum_{j=1}^{J} d_{ij}^t - d_j^{\min t} + 1)^2} \right), \tag{7}$$

where

$$g_2(\Delta) = \begin{cases} -\alpha(\alpha-1)\left(U_j^t\left(s_{ij}^t\right) - U_j^t(pre)\right)^{\alpha-2}, c_j \in \mathbb{P}_1 \\ -\lambda\beta(\beta-1)\left(U_j^t(pre) - U_j^t\left(s_{ij}^t\right)\right)^{\beta-2}, c_j \in \mathbb{P}_2, \end{cases} \tag{8}$$

(2b) **implies that when $\mathbf{s} = \mathbf{s}^*$ the matrix with (7) as elements is positive definite.** Finally, we conclude that $\mathbf{s}^*$ is a local optimum of $P1$ if and only if every subformulas in (2) holds.