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Overconstrained Cable-Driven Parallel Manipulators

Statics Analysis based on Simplified Static Cable Model

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Abstract

In recent years, cable-driven parallel manipulators (CDPM) become more and more interesting topics of robot researchers due to its outstanding advantages. Unlike traditional parallel robots, CDPMs use many flexible cables in order to connect the robot fixed frame and the moving platform instead of using conventional rigid links. Since cables used in CDPM is very light compared to rigid links, its workspace can be very large. Besides, CDPMs are often enhanced load capacity by adding redundant actuators. They also help to widen the singularity-free workspace of CDPM. On the other hand, the redundant actuators produce the underdetermined system i.e. the system has non-unique solutions. Moreover, the elasticity and bendability of flexible cable caused by self-weight and external forces act on it, resulting in the kinematic problem of CDPMs are no longer related to the geometric problem. Therefore, the system of CDPM become non-

linear when the deformation of cable is considered. In this study, we introduce the simplified static cable model and use it to linearize the static model of redundantly actuated CDPM. The algorithm to solve the force distribution problem is proposed in section 4. The static-workspace and the performance of those are analyzed in a numerical test.

Keywords: Cable-Driven Parallel Manipulator, CDPM, Static, Force distribution, Cable Robot, Parallel Robot

1. Introduction

Because of the difference in the coupling between the moving platform and robot fixed frame, the way to solve force distribution problem on cable-driven parallel manipulators (CDPM) is also very different from the conventional parallel robot. Based on recent scientific papers on them, there are many issues that are still leaving behind or not included generally. The main problem of CDPM is caused by the high elasticity and bendability of flexible cable. The force distribution problem becomes more complex since the mass of cable is also taken into account. Ottaviano, Erika, et al are introduced the forward kinematic model for a 4-CDPM in [1]. Based on this model, collision-free path-planning for the same configuration of CDPM is introduced by Lahouar, Samir, et al in [2]. All of them have practical applications but the positioning accuracy of this kind of model will quickly decrease if CDPM is applied to the larger workspace. Perfectly straightly stress the cable is impossible due to their self-weight and it's easy to see that the length of the

cable is closely dependent on the tension force caused by an actuator and load of moving platform. Therefore, the kinematic problem of CDPM doesn't make sense without the presence of static or dynamic problems. Yuan, Han, et al present the effect of sagging cable on the static and dynamic behavior of CDPM and evaluate the pose error of end-effector due to sagging effect in a static state in [3]. They also deal with the effect of cable stretching on system accuracy. Their other study [4] completely established the non-negligible mass cable model in order to provide a full kinematic model of CDPM with the presence of both sagging and elasticity. Merlet and Jean-Pierre determined the solution of the forward kinematic problem by using a heuristic interval analysis approach in a static state [5,6]. Furthermore, this method also takes care of the overconstrained system where the number of cables of CDPM greater than its degree of freedom. However, it produces large and unbalanced forces distribution that is illustrated in their experiment section.

Another problem that cannot be ignored is redundant actuators in CDPM. In order to increase the load capacity and completely restrain all degrees of freedom of the moving platform [7], many redundant actuators often present in the system, resulting in the overconstrained system. Furthermore, the force distribution problem of redundant resolution CDPM has a non-unique solution. Hence, there was more likely exist set of non-negative tension force and the valid workspace of these kinds of system are larger. However, solving these systems are usually expensive in computational time [8]. To the

best of our knowledge, numerical approaches are usually applied in order to extract the approximated solution from the undetermined system. Gosselin, Clément, and Martin Grenier addressed the force distribution problem in the redundantly actuated system. The unique solution introduced in [9] is obtained by using a non-iterative algorithm while minimizing the 4-norm of the relative force. Though, this method is not applicable to the systems that have more than one redundant actuator, in our case is 2. Linear programming employed to find safe tension distribution is presented in [10]. Li, Hui, et al propose the algorithm to determine the optimal force distribution based on quadratic programming in [11]. The closed-form algorithm based improved force distribution is introduced by Pott, Andreas in [12].

In this paper, the simplified static cable model is presented by linearizing the sagging cable model of Irvine [13] in terms of analysis. Thereby, the static problem of CDPM is coordinated with the new cable model to produce a simpler static model that can be solved in real-time. In section 4, we also introduced an algorithm used to obtain the optimal set of cable tension forces. The static-workspace is also analyzed in the simulation section with a given certain configuration. The validation of the proposed model is also considered.

2. Static analysis of cable driven parallel manipulators and sagging cable model

Our CDPM used in this study is illustrated in Figure 1. This figure shows the spatial CDPM contained two redundant actuators. Because the cable segment from an actuator to a

pulley is perfectly vertical and its length is almost constant, this study just takes care of the active segment of cable that traces from bear to the moving platform. One more assumption is that the friction on the bears is ignorable. Figure 2 illustrates the geometric problem of a general CDPM model that is also treated as the kinematic model without the presence of sagging and elasticity.

Let $\{W\}$ be the reference frame located in the robot fixed frame. From now on, any vector without frame denotation is considered in the reference frame. Let $\{R\} = (\mathbf{P}(t), \boldsymbol{\theta}(t))$ be the moving platform frame attached on its body where $\mathbf{P} = [x_P \ y_P \ z_P]^T$ and $\boldsymbol{\theta} = [\alpha_P \ \beta_P \ \gamma_P]^T$ are the position of the center of mass of moving platform and the its orientation respect to $\{W\}$.

In this study, the ideal static state of CDPM is taken into account which leads to the i^{th} cable of CDPM perfectly lies in the plane that defined by the cable frame $\{A\}$. This frame is also illustrated in Fig. 2. Let $\mathbf{A}_i$ and $\mathbf{B}_i$ be the point where i^{th} cable $\mathbf{b}_i^R$ attached to robot fixed frame and moving platform respectively. $\mathbf{b}_i^R$ is the constant vector demonstrates the position where the reaction force of tension $\mathbf{T}_i = [T_{x_i} \ T_{y_i} \ T_{z_i}]^T$ exerts on the end-effector respect to $\{R\}$. $\mathbf{b}_i$ can be determined by employing Eq. (1).

$$\mathbf{b}_i = \left({}^W\mathbf{R}_R \right)^T \cdot \mathbf{b}_i^R \quad (1)$$

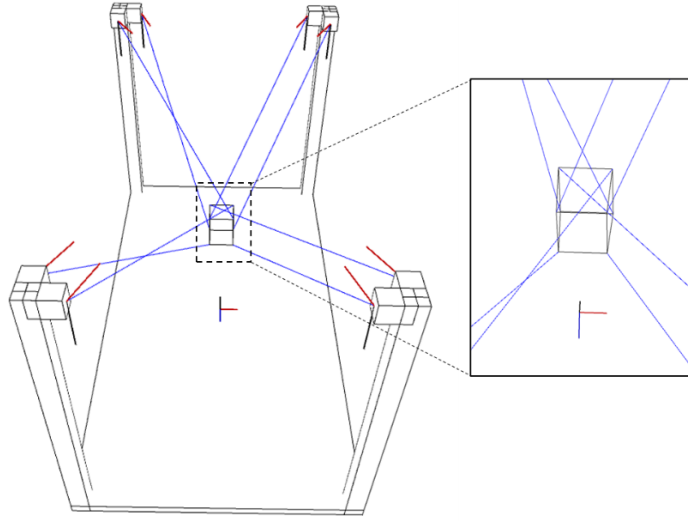


Figure 1. The simulated CDPM with 2 redundant actuators used for checking validation of proposed method.

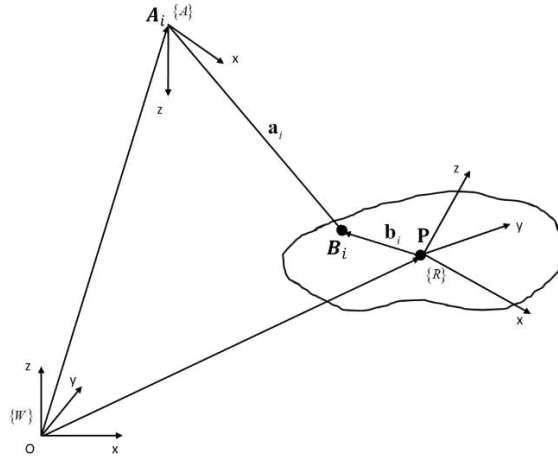


Figure 2. The geometric problem of general CDPM.

Where ${}^W\mathbf{R}_R$ is the rotational matrix that presents the transformation from $\{R\}$ to $\{W\}$ and can be expressed as Eq. (2). For convenience, we denote C_x and S_x are $\cos(x)$ and $\sin(x)$ respectively.

$${}^W\mathbf{R}_R = \begin{bmatrix} C_{\alpha_p}C_{\beta_p} & C_{\alpha_p}S_{\beta_p}S_{\gamma_p} - S_{\alpha_p}C_{\gamma_p} & C_{\alpha_p}S_{\beta_p}C_{\gamma_p} + S_{\alpha_p}S_{\gamma_p} \\ S_{\alpha_p}C_{\beta_p} & S_{\alpha_p}S_{\beta_p}S_{\gamma_p} + C_{\alpha_p}C_{\gamma_p} & S_{\alpha_p}S_{\beta_p}C_{\gamma_p} - C_{\alpha_p}S_{\gamma_p} \\ -S_{\beta_p} & C_{\beta_p}S_{\gamma_p} & C_{\beta_p}C_{\gamma_p} \end{bmatrix} \quad (2)$$

Vector $\mathbf{a}_i$ can be treated as the unique solution of inverse kinematic problem of conventional parallel robots. According to Figure 2, it can be given by Eq. (3).

$$\mathbf{a}_i = \mathbf{P} + \mathbf{b}_i - \mathbf{A}_i = \mathbf{B}_i - \mathbf{A}_i \quad (3)$$

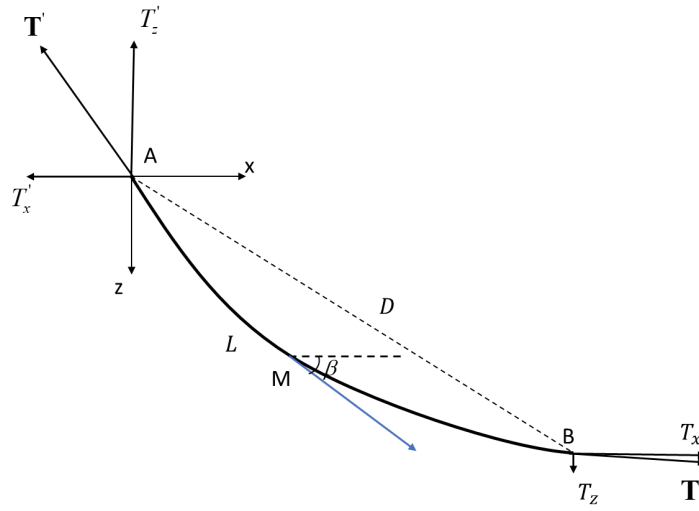
However, $\mathbf{a}_i$ cannot be considered as the solution of inverse kinematic problem of CDPM due to the flexible links that are used to connect between robot's fixed frame and moving platform. As we discussed, the length of cable just can be exactly determined if static or dynamic problems take part in. Meanwhile, the exact value of length of suspended cable is difficult to extract since its natural complexity. However, the approximated solution of Eq. (3) for CDPM can be determined via the realistic sagging cable model presented by Irvine in [13]. This model has been proven usability through several applications such as cable-stayed bridge, improving analysis of CDPM, ship anchor analysis, etc. cable's mass and elongation are also taken into account in Irvine's sagging cable model. Let consider a certain cable illustrated in Figure 3 has one extremity fixed to $\mathbf{A}$, the other one $\mathbf{B}$ is free and affected by the external force $\mathbf{T}^A = [T_x^A \ T_z^A]^T$. The reaction force at $\mathbf{A}$ is denoted as $\mathbf{T}^{A'} = [T_x^{A'} \ T_z^{A'}]^T$. Let L be the unstrained length of the cable. The ρ , E , A are the linear density, Young's modulus, and ordinary cross-section area of cable respectively. The Cartesian coordinate of $\mathbf{B} = [x_B^A \ z_B^A]^T$ respect to $\{\mathbf{A}\}$ can be given by Eqs. (4) and (5).

122

$$x_{B^A} = \frac{T_x^A L}{EA} + \frac{|T_x^A|}{\rho g} \left[\sinh^{-1} \left(\frac{T_z^A}{T_x^A} \right) - \sinh^{-1} \left(\frac{T_z^A - \rho g L}{T_x^A} \right) \right] \quad (4)$$

123

$$z_{B^A} = \frac{T_z^A L}{EA} - \frac{\rho g L^2}{2EA} + \frac{1}{\rho g} \left[\sqrt{(T_x^A)^2 + (T_z^A)^2} - \sqrt{(T_x^A)^2 + (T_z^A - \rho g L)^2} \right] \quad (5)$$



124

125 **Figure 3.** Static equilibrium of a certain cable with the presence of sagging.

126 It should be noted that the exact solution $\mathbf{T}^A$ cannot be extracted directly from (4) and (5).

127 Let ${}^W\mathbf{R}_{A_i}$ be the rotational matrix that presents transformation from $\{A_i\}$ to $\{W\}$ and can be

128 expressed as Eq. (6).

129

$${}^W\mathbf{R}_{A_i} = \begin{bmatrix} C_{\theta_i} & S_{\theta_i} & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad (6)$$

130 Where θ_i can be defined by Eq. (7).

131

$$\theta_i = a \tan 2(a_{y_i}, a_{x_i}) \quad (7)$$

132 If only the tensions of cables and load of the moving platform are treated as the external
 133 forces exerted on the moving platform then we have the static equilibrium Eqs. (8), (9).

$$\begin{aligned}
 \sum_i -T_{x_i}^{A_i} \cos(\theta_i) &= 0 & \sum_i -T_{x_i}^{A_i} \sin(\theta_i) &= 0 \\
 \sum_i T_{z_i}^{A_i} - mg &= 0
 \end{aligned} \tag{8}$$

$$\begin{aligned}
 \sum_i (b_{y_i} T_{z_i}^{A_i} + b_{z_i} T_{x_i}^{A_i} \sin(\theta_i)) &= 0 \\
 \sum_i (-b_{z_i} T_{x_i}^{A_i} \cos(\theta_i) - b_{x_i} T_{z_i}^{A_i}) &= 0 \\
 \sum_i (-b_{x_i} T_{x_i}^{A_i} \sin(\theta_i) + b_{y_i} T_{x_i}^{A_i} \cos(\theta_i)) &= 0
 \end{aligned} \tag{9}$$

136 Since the relationship between length of cable and a force exerts on it, the solution of
 137 static problem of general spatial CDPM can be expressed as in Eq. (10).

$$\left[\left\{ T_{x_i}^{A_i} \quad T_{z_i}^{A_i} \right\}_{i=1}^n \right]^T \tag{10}$$

139 3. Linearizing the relationship between $\mathbf{T}_x^A$ and $\mathbf{T}_z^A$

140 According to Irvine's sagging cable model, there exist a unique point BA that satisfies the
 141 given a certain set of parameters: L, T_x, T_z, ρ, E and A . If the parameter described material
 142 of cable, ρ, E and A is fixed then we can state that the Eq. (11) is also the bijective function.

$$f: [\mathbf{B}_d^A \quad \boldsymbol{\varsigma} \quad L]^T \mapsto [T_x^A \quad T_z^A]^T \tag{11}$$

144 Where $\boldsymbol{\varsigma} = [\rho \ E \ A]^T$ is the vector that denotes properties of material that used in cable and
 145 $\mathbf{B}_d^A$ is the desired position of the free extremity of considered cable. However, in the

inverse kinematic problem of CDPM, L is unknown which leads to the problem has non-unique solution. In this section, we will try to figure out the relationship between T_x^A and T_z^A with varies L in order to reduce the number of variables in our system. This relation is proposed by Irvine in [13] but this is just applicable for inextensible cable. However, Nguyen, Dinh Quan, et al. prove that the cable elastic has a small change in shape of cable as long as $T^A \ll EA$ [14]. In this study, we assume that $T^A \ll EA$. Besides, the nonlinear part of the relationship between T_x^A and T_z^A is located where T_x^A belong to the open interval $(0, T_{x\min})$. Thereby, this relation can be linearized if the condition $T_x^A \geq T_{x\min}$ was satisfied. Based on these assumptions, the relationship between T_x^A and T_z^A can be given by Eq. (12).

$$T_z^A = a_\pi T_x^A + b_\pi \quad (12)$$

According to Eq. (12), we know that a_π and b_π in equation Eq. (12) depend on $[\mathbf{B}_d^A \boldsymbol{\varsigma}]^T$. On the other hand, since $T^A \ll EA$, a and b now only depend on $[\mathbf{B}_d^A \boldsymbol{\rho}]^T$.

Let m and α be the magnitude and heading of tension force $\mathbf{T}^A$ acts on free extremity $\mathbf{B}^A$. θ is the heading of positioning vector $\mathbf{B}_d^A$ and D is its L2-norm respect to $\{A\}$. The heading of $\mathbf{T}^A$ can be expressed as Eq. (13).

$$\alpha = \tan^{-1} \left(\frac{T_z^A}{T_x^A} \right) \quad (13)$$

163 According to the relationship between T_x^A and T_z^A given by Eq. (12), the heading of $\mathbf{T}^A$ can
 164 be rewritten as Eq. (14).

$$165 \quad \alpha = \tan^{-1} \left(a_\pi + \frac{b_\pi}{T_x} \right) \quad (14)$$

166 Let consider the tension force $\mathbf{T}^A$ exert on free extremity $\mathbf{B}^A$ and its reaction force $\mathbf{T}^{A'}$
 167 exerted on the other one. We know that if the cable is perfectly straight then $\mathbf{T}^A + \mathbf{T}^{A'} = 0$.
 168 In reality, it's impossible due to weight of the cable itself, resulting in the change of z-axis
 169 component of tension along the cable. This explains the phenomenon of not being able
 170 to straightly stress the cable. Meanwhile, if T_z^A approach to infinity then weight of the
 171 cable can be ignored, resulting in that α approaches to β . This expression can be given by
 172 Eq. (15).

$$173 \quad \lim_{|T_z| \rightarrow \infty} \alpha = \beta \quad (15)$$

174 Based on the Eq. (12) and Eq. (14), we can state that if T_z^A approaches to infinity and a_π is
 175 non-zero then α approaches to $\tan^{-1}(a_\pi)$. This express can be given by Eq. (16).

$$176 \quad \lim_{|T_z| \rightarrow \infty} \alpha = \lim_{|T_x| \rightarrow \infty} \left(\tan^{-1} \left(a + \frac{b}{T_x} \right) \right) = \tan^{-1}(a) \quad (16)$$

177 From Eqs. (15) and (16), we can prove that if Eq. (15) is considered as the relationship
 178 between T_x^A and T_z^A then the parameter a_π in the Eq. (12) can be formulated by Eq. (17).

$$179 \quad a_\pi = \tan(\beta) \quad (17)$$

According to Fig. 3, Let **M** be the point located in cable where the tension force exerts on it is parallel to the chord. In statics, since the resultant force at each point along the cable is affected only by tension cable and gravitational force, the resultant force component along x-axis is not changed over the length of cable. The resultant force component along z-axis is combined between tension cable and its weight, and this component at **M** can be given by Eq. (18).

$$T_{z_M}^A = \tan(\beta) T_x^A \quad (18)$$

Thereby, the parameter b_π in Eq. (12) depends on the weight of cable segment surrounded by **M** and **B**. It can be stated in Eq. (19).

$$b = \rho g L_{MB} \quad (19)$$

Where L_{MB} is the length of cable segment **MB**. However, determining exactly L_{MB} is inefficient by its computational time. Furthermore, in CDPM problems, the presence of sagging based on adjusting the cable length not only reduces tension but also reduces cable stiffness if the distance between **M** and chord is large enough. Therefore, the sag of cable should not be large. In fact, the sag depends on **T**. If our assumption $T_x \geq T_{xmin}$ is satisfied and T_{xmin} is large enough, the shape of cable is approximated flat parabola then $L_{MB} \approx D/2$. Thus, the Eq. (12) is completely simplified to Eq. (20).

$$T_z = \tan(\beta) T_x + \frac{\rho g D}{2} \quad (20)$$

4. Force distribution on the overconstrained cable driven parallel manipulators

The couple of Eqs. (8), (9) present the force distribution of CDPM in a static state. Its solution is easily extracted by applying it for a fully constrained system which usually links the end-effector and robot fixed frame by 6 cables. However, as we discussed, this problem becomes more complex with the presence of redundant actuators. In this section, we introduced a procedure in order to obtain a feasible set of tension for overconstrained CDPM. By substituting Eq. (20) to Eqs. (8) and (9), the number of variables in the static problem is reduced and these equations can be expressed as Eq. (21).

$$\mathbf{W} \cdot \mathbf{t}_x + \mathbf{b} = \mathbf{0} \quad (21)$$

$$\mathbf{W} = \begin{bmatrix} -\cos(\theta_1) & \dots & -\cos(\theta_n) \\ -\sin(\theta_1) & \dots & -\sin(\theta_n) \\ \tan(\beta_1) & \dots & \tan(\beta_n) \\ b_{y_1} \tan(\beta_1) + b_{z_1} \sin(\theta_1) & \dots & b_{y_n} \tan(\beta_n) + b_{z_n} \sin(\theta_n) \\ -b_{z_1} \cos(\theta_1) - b_{x_1} \tan(\beta_1) & \dots & -b_{z_n} \cos(\theta_n) - b_{x_n} \tan(\beta_n) \\ -b_{x_1} \sin(\theta_1) + b_{y_1} \cos(\theta_1) & \dots & -b_{x_n} \sin(\theta_n) + b_{y_n} \cos(\theta_n) \end{bmatrix} \quad (22)$$

Where $\mathbf{W}$ presents the wrench matrix given by Eq. (22), $\mathbf{t}_x = [T_{x1}^A \ T_{x2}^A \ \dots \ T_{xn}^A]$ is a vector of cable horizontal force components represented in $\{A\}$, and $\mathbf{b}$ is a complement of the static model and it can be obtained by employing Eq. (23).

$$\mathbf{b} = \begin{bmatrix} 0 & 0 & \frac{\rho g}{2} \sum_{i=1}^n D_i - mg & \frac{\rho g}{2} \sum_{i=1}^n D_i b_{y_i} & -\frac{\rho g}{2} \sum_{i=1}^n D_i b_{x_i} & 0 \end{bmatrix}^T \quad (23)$$

213 The particular solution $\mathbf{t}_x$ of Eq. (21) may determine by finding the pseudo-inverse matrix
 214 of $\mathbf{W}$, but this may violate the previous assumption $T_{xi} \geq T_{ximin}$ and the worst case is $T_{xi} < 0$.
 215 The stiffness matrix is taken into account to tackle this problem in many research [3,4].
 216 However, a disadvantage of this method is high computational time. Therefore, in the
 217 very next step, the general solution of Eq. (21) will be determined first and a simple
 218 searching technique is employed to determine a feasible particular solution. Firstly, an
 219 arbitrary particular solution $\mathbf{t}_{x0} = [T_{x10}^A T_{x20}^A \dots T_{xn0}^A]$ is obtained by using the least-squares
 220 method. The general solution of Eq. (21) is given by Eq. (24).

$$221 \quad \mathbf{t}_x = \mathbf{t}_{x0} + \mathbf{N}\boldsymbol{\lambda} \quad (24)$$

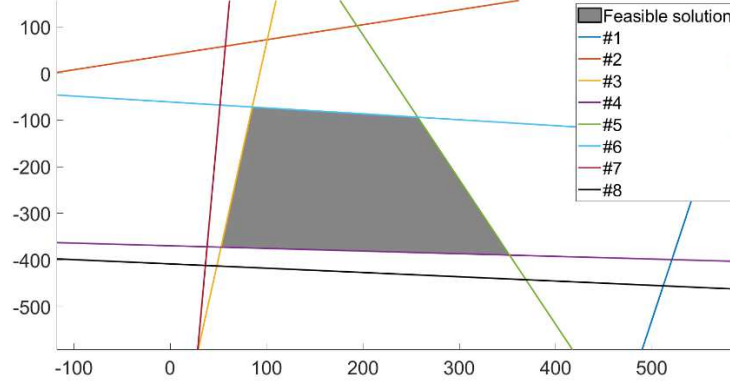
222 Where $\mathbf{N}$ is the $6 \times n$ matrix that presents the orthonormal basis for the null space of $\mathbf{H}$ and
 223 value of $\mathbf{t}_x$ depends on a vector $\boldsymbol{\lambda} = [\lambda_1 \lambda_2 \dots \lambda_{n-6}]$. To ensure that cable tensions force is
 224 positive, the necessary condition is $T_z^A \geq 0$ that must be satisfied. Thus, we also have
 225 condition defined by Eq. (25).

$$226 \quad T_x^A \geq -\rho g D / 2 \tan(\beta) \quad (25)$$

227 However, it does not guarantee that Eq. (22) is valid. By this reason, the sufficient
 228 condition is introduced and given by Eq. (26).

$$229 \quad \mathbf{t}_x > \mathbf{t}_{x_{min}}, T_{x_{imin}} = \frac{-\rho g D_i}{\tan(\beta_i)} + T_{x_i adj} \quad (26)$$

230 Where $\mathbf{t}_{x\min} = [T_{x1\min}^A \ T_{x2\min}^A \ \dots \ T_{xn\min}^A]$ is the allowable minimum horizontal component
 231 cable tension. $T_{xi/adj}$ is the adjusting force to maintain the accuracy of Eq. (22). By
 232 employing the Eq. (26) and Eq. (24), the feasible solution set Θ_λ is founded and defined
 233 by n-hypercube in $\mathbb{R}^{n-6}$.



235 **Figure 4.** Feasible solution for the certain case of 2-redundant actuators CDPM.

236 To be able to easily illustrate, we apply this procedure to our simulated system illustrated
 237 in Fig. 1 with a given certain configuration. Since our system contains 2 redundant
 238 actuators, Θ_λ in this case is a polygon related to $\mathbb{R}^2$ and that is illustrated in Fig. 4. In figure
 239 4, each line represents the equation (26) with #i represents i^{th} cable.

240 According to Fig. 4, the polygonal boundary of Θ_λ is determined by the vertex set $\Omega_\pi = \{\lambda_1,$
 241 $\lambda_2, \dots, \lambda_h\}$ where h is the number of vertices. In the case illustrated in Fig. 4, $h = 4$. However,
 242 we cannot directly determine a feasible particular solution $\mathbf{t}_x$ from Θ_λ since this set is
 243 defined as preimage of feasible tension force set Θ_t . Thus, we need to transform Θ_λ to Θ_t
 244 via Eq. (24) in order to determine feasible particular solution $\mathbf{t}_x$ later. Since Eq. (24) is an

affine transform and Θ_λ is a hyper-polyhedron in $\mathbb{R}^{n-6}$ then Θ_t is also a hyper-polyhedron in $\mathbb{R}^n$. In the example, Θ_λ is a polygon in $\mathbb{R}^2$ and Θ_t is a polygon in $\mathbb{R}^8$. Since the static problem is considered, the feasible particular solution $\mathbf{t}_x$ can take arbitrary in Θ_t . However, $\mathbf{t}_x$ with minimum L2-norm can be considered as optimum solution of our study. It can be easily obtained by finding the projection of origin on boundary of Θ_t . According to this procedure, we can take all of feasible particular solution $\mathbf{t}_x$ in the workspace of CDPM. In the next section, the static-workspace and tension force set in a certain case is considered.

5. Simulation and results

In order to prove the usability of the proposed method, we have applied it to our simulated system. This system is a spatial CDPM illustrated in Figure 1 has two redundant actuators. Thereby, our CDPMs has 8 cables that are used to link 4 poles of robot fixed frame and the moving platform. For convenience, we also noted that the first cable and the second one is fixed at the bottom right pole according to Figure 4 and they are numbered clockwise. Their configuration and cable' properties are specified in table 1.

Table 1. The configuration of simulated CDPM.

Workspace	6m x 3m x 3m
End-effector size	0.3m x 0.3m x 0.5m
End-effector load	50 kg

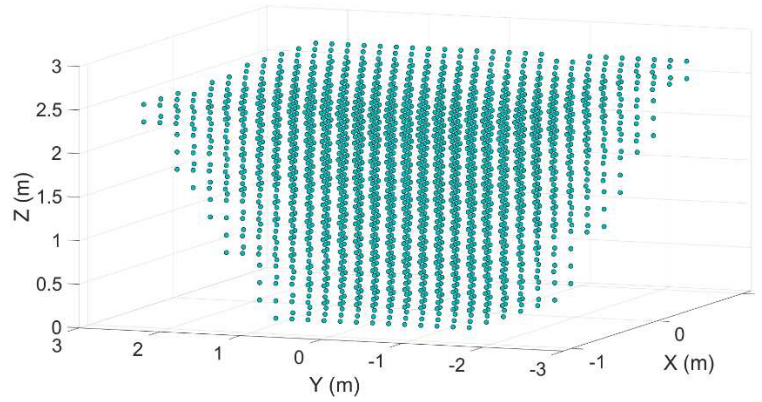
Cable's density	0.09 kg/m
Young's modulus	3.5 GPa
Cross-section area of cable	4 mm ²

261

262 Firstly, the static-workspace of the mentioned CDPM is established based on the
263 simplified static cable model. Static-workspace is the set of position and orientation of
264 the moving platform in that the static equilibrium Eq. (21) is satisfied. Orientation
265 constraint of $\{R\}$ is $\theta = [0 \ 0 \ 0]^T$. Based on numerical simulation, the moving platform is
266 experienced in all workspace of it, the valid positions located inside the polyhedron block
267 defined by our simulated data. This is illustrated in Fig. 5. Since the cable model is
268 simplified and does not fully correct that lead to smaller valid static-workspace. However,
269 it also has a trade-off in computational time, which can perform in real-time. We should
270 also pay attention to the path planning or obstacle avoidance problem in CDPM, real-time
271 workspace analysis is required. According to the CDPM configuration in table 1 and static-
272 workspace result in Fig. 5, the static-workspace can be defined by a frustum of a
273 rectangular pyramid with Oz as the axis of symmetry. The large base whose size is
274 approximated $1 \times 2.4 \text{ m}^2$, while $2.2 \times 4.6 \text{ m}^2$ is area of the small base. The distance
275 between two bases is 2.6 m where the small base located at $z=0$.

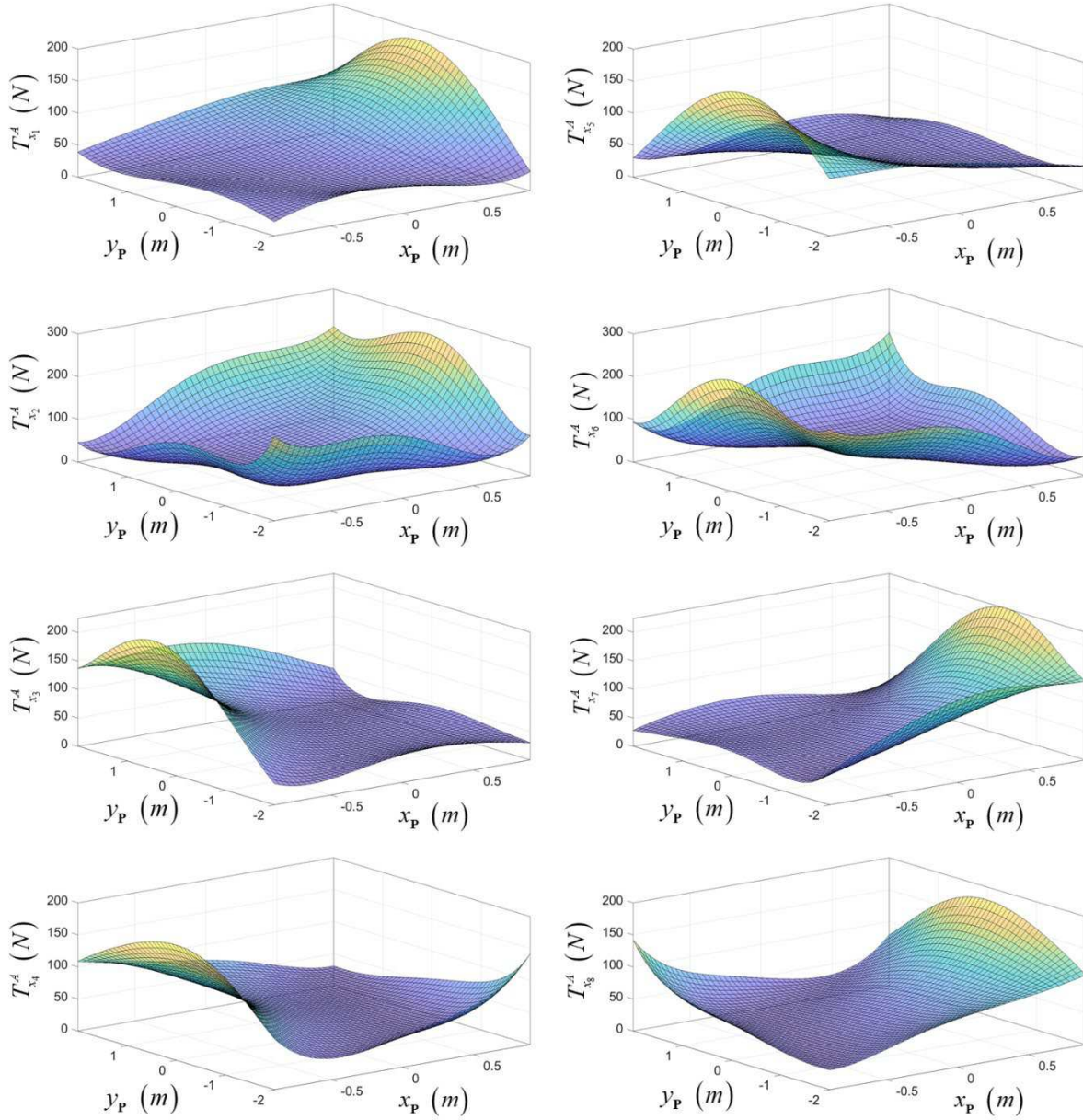
276 Based on current constraints, all feasible tension sets in obtained static-workspace are
277 easily constructed by applying Eqs. (24), (26). However, because of high dimensional

278 space, the illustration of them is limited. Thus, the minimum tension force in each feasible
279 tension set is considered and obtaining it by applying full procedure in section 4.



280

281 **Figure 5.** The static-workspace of proposed CDPM with $\theta = [0 \ 0 \ 0]^T$.



282

283 **Figure 6.** Force distribution along the x-axis respect to {A} in static-workspace with $z_P =$

284 1.8 mm.

285 After archiving the minimum tension force at this layer by using a numerical approach,

286 these data are continousized and smoothed by employing the non-linear least-square

287 fitting algorithm. Thereby, Fig. 6 indicates the minimum tension force of each cable at

each valid position with $z_p = 1.8$ mm in static-workspace. According to the result in Fig. 6, the layer with $z_p = 1.8$ mm has an approximated area of 1.8×3.8 m². In fact, tension force of a particular cable is high when the end-effector approach to the pole where it is fixed to and vice versa. Meanwhile, because of non-symmetric workspace and the limitation of simplified cable model, the maximum of minimum tension forces in the mentioned layer do not approach the pole where it is fixed to, they just approach to near them. Besides, the symmetric in force distribution is still reserved. In order to perform in all of these positions, $T_{x\max}$ should not be smaller than 250N.

6. Conclusion

In this work, the simplified static cable model was introduced in terms of analysis. Based on that result, the force distribution problem of spatial redundantly actuated CDPM in an ideal static state was established. By fully applying the proposed method in section 4, the minimum set of tension force with a certain configuration is evaluated. The static-workspace is also considered and experimented in a certain case. Our introduced cable model and proposed method to solve the distribution force problem is low computational time and can perform in real-time. However, due to the limitation caused by constraint from Eq. (25), the valid static-workspace is restricted and smaller than in reality. That inspired us for future works:

- Establish the relationship between L_{MB} and T_x instead of approximating $L_{MB} \approx D/2$, resulting in the valid static-workspace nearly approach to the real one.

308 • Quasi-static with low-speed trajectory will be considered and that will produce
309 new challenges in force distribution problems. Besides having to minimize tension force,
310 we also consider to the force distribution over time for continuous and smooth.

311 **Declarations**

312 **Availability of data and materials**

313 Not applicable.

314 **Competing interests**

315 The authors declare no competing financial interests.

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318 **Authors' contributions**

319 In this work, Nguyen Truong Thinh conceptualized and supervised the process of project.
320 Phan Gia Luan analyzed, investigated and proposed the method to obtain the simplified
321 cable model. The original draft was written and visualized by Phan Gia Luan. Nguyen
322 Truong Thinh reviewed, edited and provided the final version. All authors have read and
323 agreed to the published version of the manuscript.

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Figures

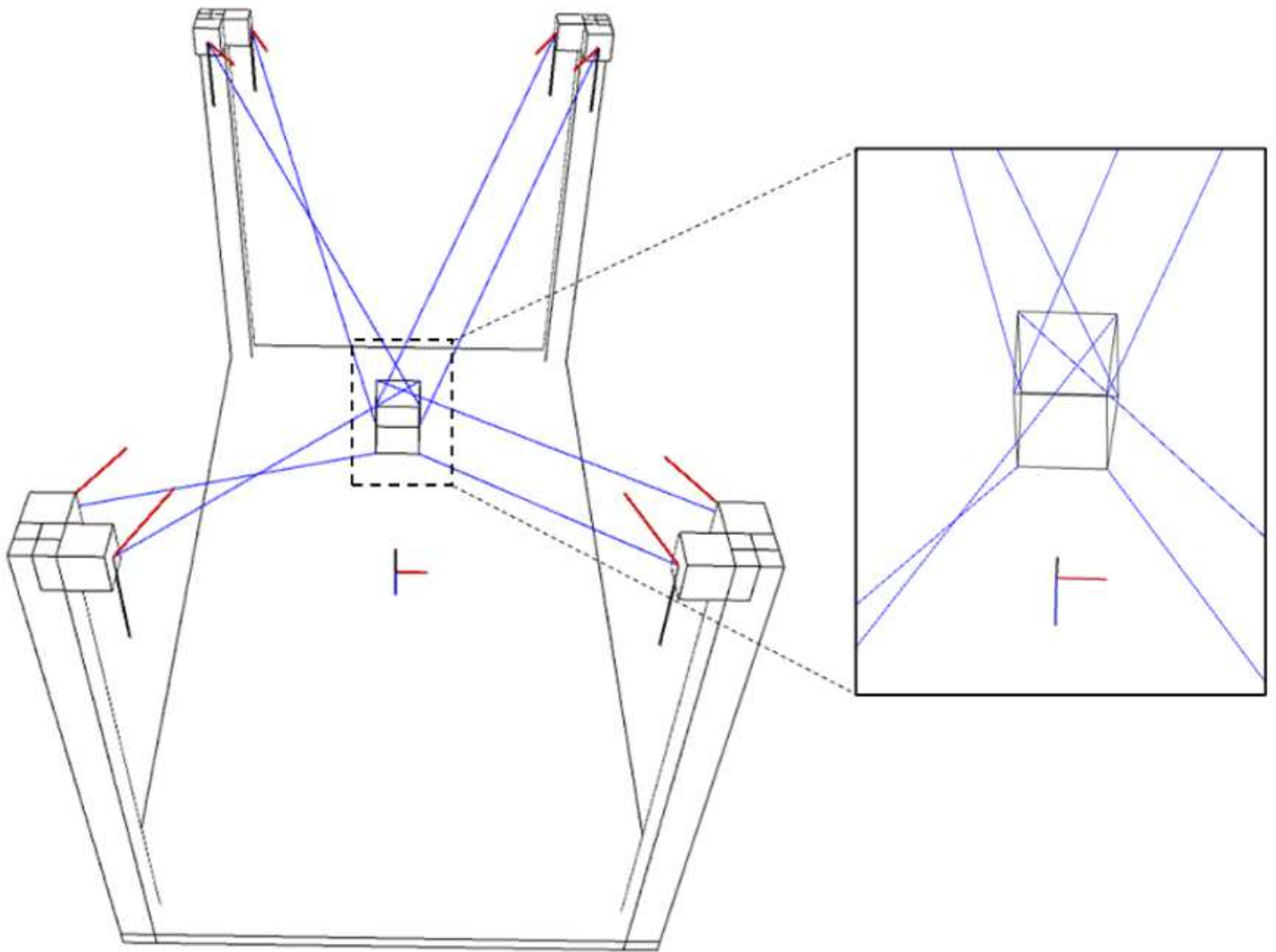


Figure 1

The simulated CDPM with 2 redundant actuators used for checking validation of proposed method.

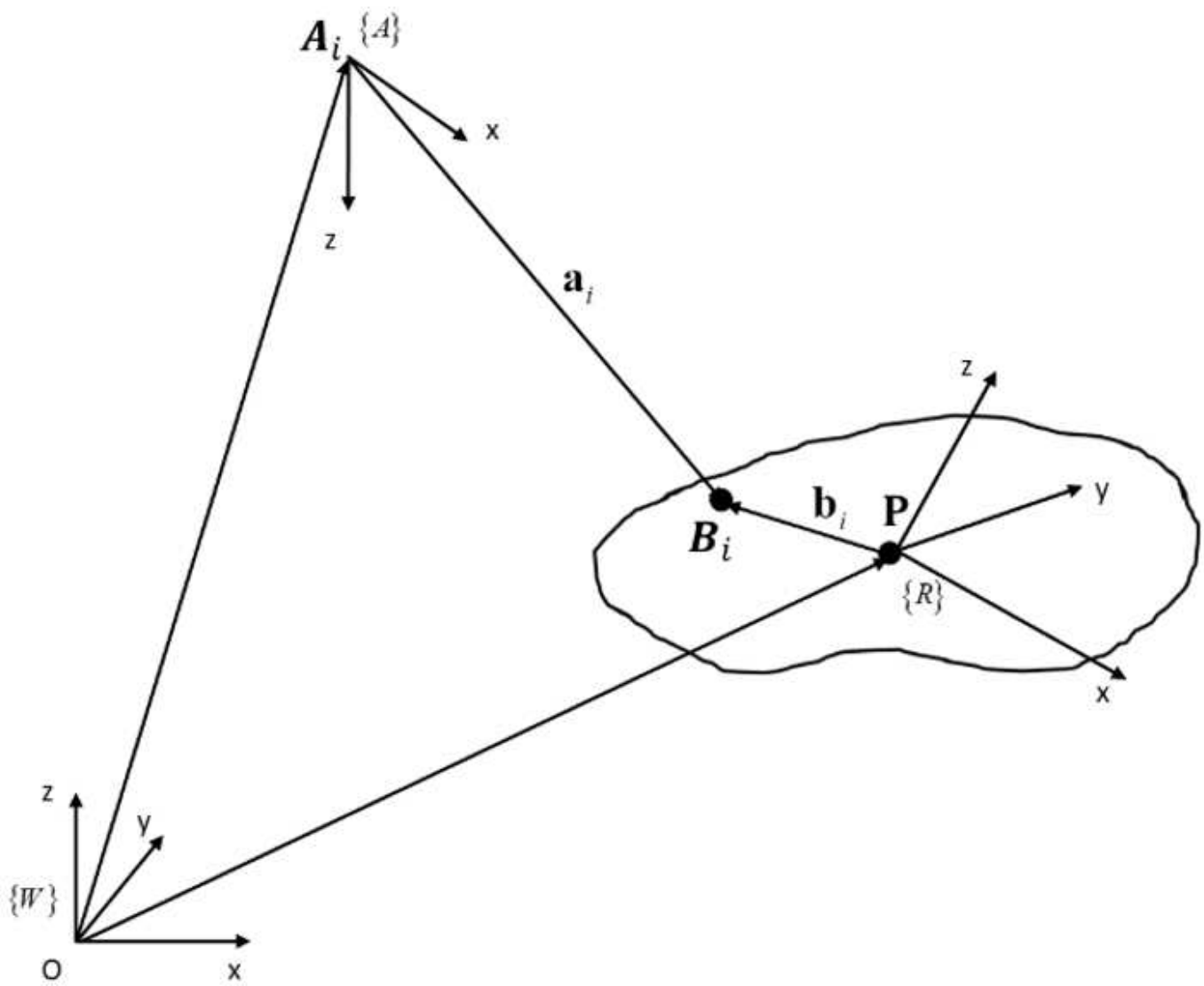


Figure 2

The geometric problem of general CDPM.

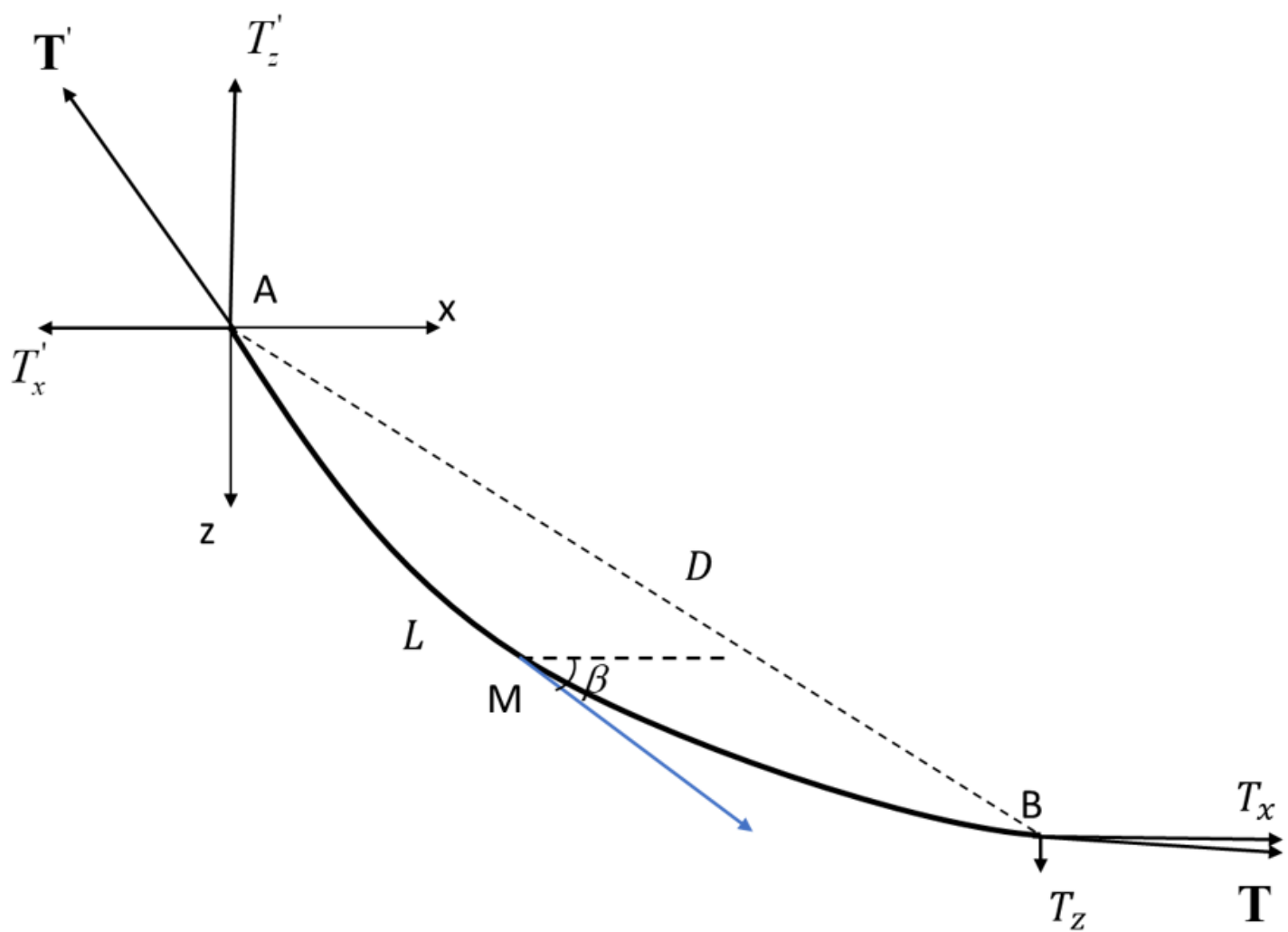


Figure 3

Static equilibrium of a certain cable with the presence of sagging.

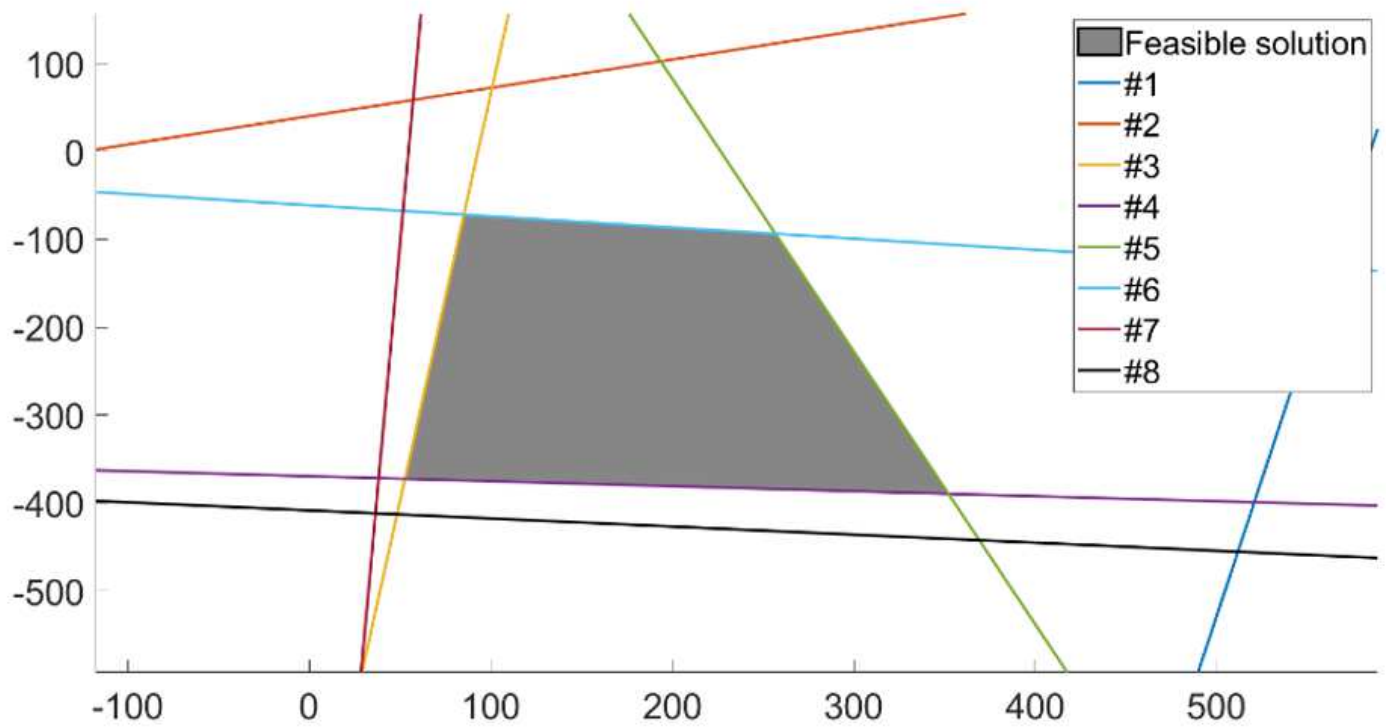


Figure 4

Feasible solution for the certain case of 2-redundant actuators CDPM.

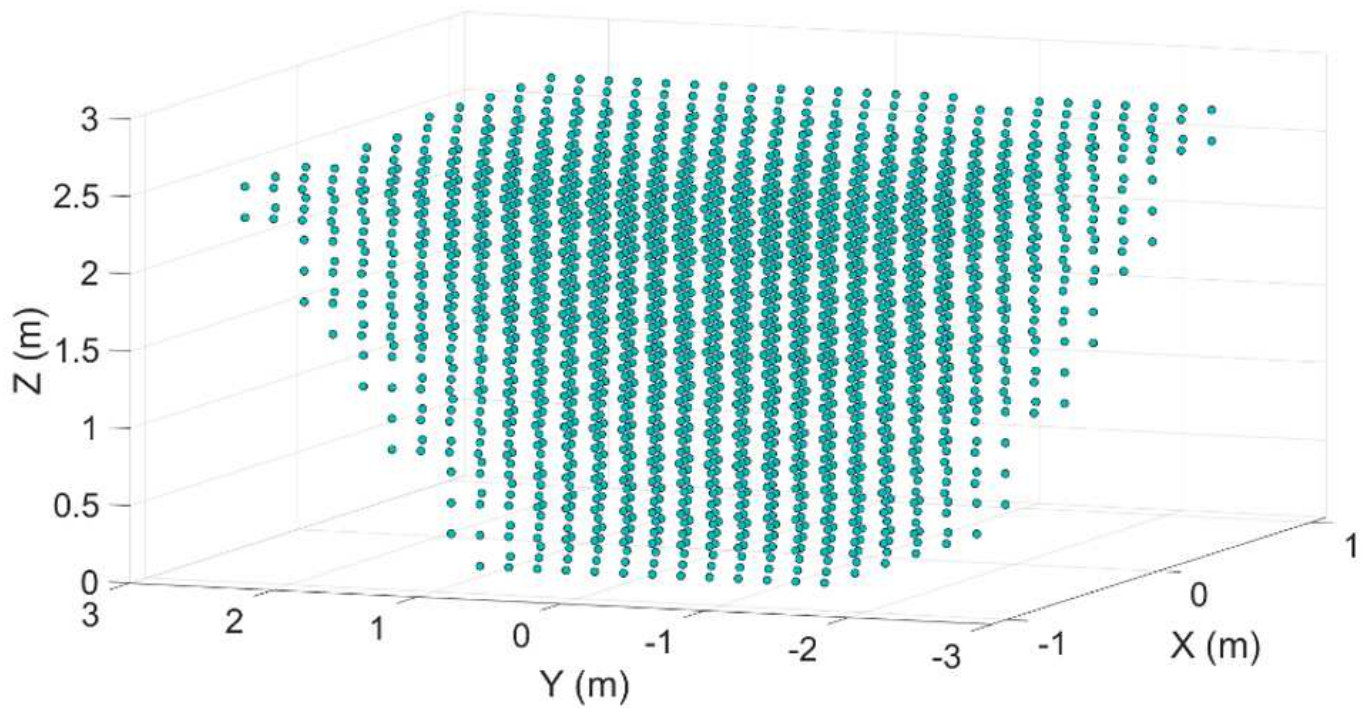


Figure 5

The static-workspace of proposed CDPM with $\theta = [0 \ 0 \ 0]^T$.

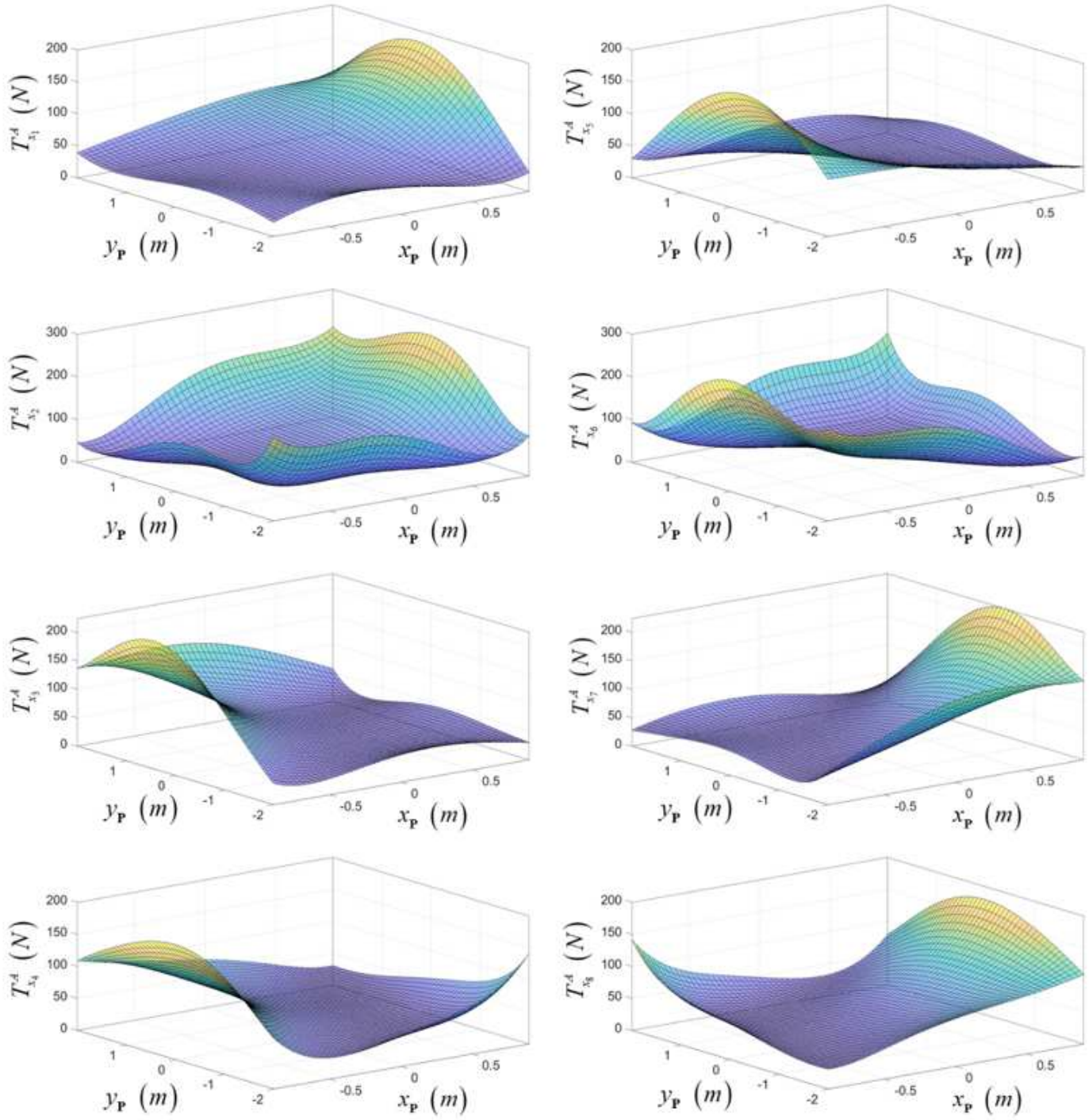


Figure 6

Force distribution along the x-axis respect to {A} in static-workspace with $z_P = 1.8$ mm.