Supplementary Information

1 Model description

In our model individuals live for 2 periods of 20 years. They are either White (w) or Black (b) in terms of race, and can be either high-skilled (s) or low-skilled (u) depending on the education level assigned by their parents. The model runs for 80 years (4 generations) from year 2020 to year 2100. In the first period of their lives, the individuals are children that only consume parental wealth. In our model, a child of race i and skill level j consumes τ^j units of their parent's wealth which is different for children with different skill levels. In the second period of their life, all adults will work in the only economic sector of the model where they earn wages. The wealth of an individual is the combination of their wage and the capital they have inherited from their parents. Adults work, consume goods, and will spend a part of their time on raising children. The objective of each individual is to maximize lifetime utility of their consumption and their children's future well-being (measured by their wealth). Parents achieve this goal by making consumption, inheritance, fertility, and education decisions.

1.1 Preferences

The utility function of an adult of skill level i and race j has three components: immediate consumption, inheritance, and future children's wages. This

captures the altruistic attitude of parents:

$$v(c_t^{i,j}, H_{t+1}^{i,j}, n_t^{i,s}, n_t^{i,u}) = \gamma_1^i \; ln\big(\sum_{k\in[s,u]} n_t^{i,k} \; w_{t+1}^k \big) + \gamma_2 \; ln(H_{t+1}^{i,j}) + (1-\gamma_1-\gamma_2) \; ln(c_t^{i,j}) \tag{1}$$

where $n_t^{i,k}$ is the number of children with education level k of a parent with education level i, variable $c_t^{i,j}$ is consumption of goods, and $H_{t+1}^{i,j}$ is the intergenerational capital transfer. Future wage of children with education level k is denoted by w_{t+1}^k.

Parents have preferences over own consumption, their children's inheritance, and the children's expected lifetime earnings. We derive the budget constraint corresponding to (1) for every adult:

$$c_t^{i,j} + H_{t+1}^{i,j} = \big(1 - \sum_{k\in[s,u]} n_t^{i,k} \; \tau^k\big) \big[\underbrace{w_t^i + (1+\rho_t-\delta)k_t^j}_{\Omega_t^{i,j}} \big] \tag{2}$$

where τ^k is the portion of parental time spent on raising a child and providing them with the skill level k. Adults' wealth ($\Omega_t^{i,j}$) is composed of their wage w_t^i which only depends on their skill level i, and the level of capital they inherit from their parents k_t^j which only depends on their race j. The total available capital in each generation is used for production of a consumable good. Capital rent is ρ_t and δ accounts for capital depreciation. Maximization of equation (1) subject to equation (2) yields:

$$\sum_{k\in[s,u]} n_t^{i,k} \; \tau^k = \gamma_1^i \tag{3}$$

$$H_{t+1}^{i,j} = \gamma_2 \; \Omega_t^{i,j} \tag{4}$$

$$c_t^{i,j} = (1-\gamma_1^i-\gamma_2) \; \Omega_t^{i,j}. \tag{5}$$

which also results in

$$\frac{\tau^s}{\tau^u} = \frac{w^s}{w^u} = w^r \tag{6}$$

Equations (3)-(5) demonstrate several important characteristics of the model. First, equation (3) shows the number of children of a parent with skill level i depends on the child-rearing cost τ^k. Second, comparing equations (4) and (5), the following relationship holds between the optimal level of the current consumption and the capital transferred to the next generation:

$$\frac{c_t^{i,j}}{H_{t+1}^{i,j}} = \frac{1-\gamma_1-\gamma_2}{\gamma_2} \tag{7}$$

1.2 Capital and labor dynamics

The cumulative capital available in each generation can be obtained from

$$\bar{K}_{t+1} = \sum_{i\in[s,u]} \sum_{j\in[W,B]} H_{t+1}^{i,j}\ N_t^{i,j} \tag{8}$$

where $N_t^{i,j}$ is the adult population of skill level i and racial group j at time t, and $\bar{K}_{t+1}$ is the aggregated capital inherited from the parents of each generation. This aggregated capital will be used for the production of goods.

The size of the labor force of race j and skill level k in the next time step will be calculated from multiplying the current adult population $N_t^{i,j}$ with their number of children. Furthermore, the total number of adults of skill level k in the next time step is the sum of the adults of skill level k from both races:

$$N_{t+1}^{j,k} = \sum_{i\in[s,u]} N_t^{i,j}\ n_t^{i,k} \tag{9}$$

$$N_{t+1}^{k} = \sum_{j \in [W,B]} N_{t+1}^{j,k} \tag{10}$$

Furthermore, The inherited capital will be divided equally among the children of each parent of race j:

$$k_{t+1}^{j} = \frac{H_{t+1}^{i,j}}{\sum_{k \in [s,u]} n_{t}^{i,j,k}} \tag{11}$$

1.3 Production

We adopt a standard two-factor Cobb-Douglas production function with capital and labor as inputs of the production. The profit maximization can be written as

$$Max \left\{ Y_t - \sum_{i \in [s,u]} w_t^i \, N_t^i \right\} \tag{12}$$

s.t.

$$Y_t = A_t \, L_t^{\alpha} \, \bar{K}_t^{1-\alpha} \tag{13}$$

$$L_t = \left[\sum_{i \in [s,u]} \beta_t^i \, (N_t^i)^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}} \tag{14}$$

$$\sum_{i \in [s,u]} \beta_i = 1 \tag{15}$$

where Y_t is the output, L_t is the aggregated labor, and $\bar{K}_t$ is the aggregated capital from equation (8). Labor is aggregated using a constant elasticity of substitution (CES) function with skill shares β_t^i. Finally, total factor productivity A_t is optimized at each step in order to have the total output growth follow the SSP growth of GDP.

Wages of labor of different races can be calculated by solving maximization problems in equation (12):

$$w_t^i = \alpha \ \beta^i \ Y_t \ L_t^{\frac{1-\epsilon}{\epsilon}} \ (N_t^i)^{\frac{-1}{\epsilon}} \tag{16}$$

Similarly, capital rent can be calculated as

$$\rho_t = (1-\alpha) \ \frac{Y_t}{\bar{K}_t} \tag{17}$$

1.4 Racial and skill-based wealth gap

The racial ($\Delta w_t^{W,B}$) and skill-based wealth gap ($\Delta w_t^{s,u}$) can be defined and calculated in each time period as an indicator of inequality

$$\overline{\Omega}_t^{W,B} = 1 - \frac{\overline{\Omega}_t^B}{\overline{\Omega}_t^W} \tag{18}$$

where $\overline{\Omega}_t^B$ and $\overline{\Omega}_t^W$ are the average wealth of the Black and the White sub-populations at time t. Similarly, for the skill-based wealth gap we have

$$\overline{\Omega}_t^{s,u} = 1 - \frac{\overline{\Omega}_t^u}{\overline{\Omega}_t^s} \tag{19}$$

where $\overline{\Omega}_t^u$ and $\overline{\Omega}_t^s$ are the average wealth of the low-skilled and the high-skilled sub-populations at time t.

1.5 Reparation and Wealth tax

We can investigate the role of reparation in closing wealth inequality gap between racial and skill groups by considering a one-time flat-rate (5%) wealth

tax which is collected from White individuals either at the beginning the 2020-2040 period as an "inheritance tax" on the capital they have inherited from the previous generation, or at the end of this period as an "estate tax" on the capital they will leave to the next generation. As a result, we expect the inheritance tax to affect inequality of the 2020-2040 generation and beyond while the impact of the estate tax will start to be felt from 2040-2060 period onward. In either case, the total amount of collected tax will be distributed equally among the Black generation that live concurrently with the generation of White tax payers. As a result, different amount of total tax is collected and distributed to different number of Black population under each scheme depending on the tax type and timing.

1.6 Model calibration

We calibrate the model for two races of *White (W)* and *Black (B)* in the U.S. based on historical labor data from year 2000 to 2020.

Table 1: Non-civilian workforce population by race and education level in 1999 (Numbers in thousands)
Source: https://www.bls.gov/cps/aa1999/CPSAAT7.PDF

Group	**variable**	**White**	**Black**
Less than highschool		9,815	1,684
Highschool, no college		31,145	4,944
Less than a bachelour		26,621	4,219
College degree		30,444	2,693
High-skilled	N^s_{2000}	30,444	2,693
Low-skilled	N^u_{2000}	67,581	10,847
Total	N_{2000}	98,025	13,540
Skill ratio	h_{2000}	0.45	0.25

Historical adult labor population of each race can be obtained from the U.S. Bureau of Labor Statistics (BLS). According to these data, the population of civilian labor force of each race by education level is given in Table 1. Having the population by education level and race, we can then calculate the skill

ratio of the combined workforce. We assume that only people with a college degree are considered high-skilled and the rest of the workforce is low-skilled.

$$h_{2000} = \frac{\sum_{j\in[W,B]} N^{s,j}_{2000}}{\sum_{j\in[W,B]} N^{u,j}_{2000}} = 0.42 \tag{20}$$

Next we obtain the wage ratio between the high-skilled and low-skilled labor[1] $w^r_{2000} = \frac{12\times4520}{12\times1960} = 2.31$. Applying Equation 16 to each skill group will yield

$$w^r = \frac{\beta^s_{2000}}{\beta^u_{2000}} (h_{2000})^{\frac{-1}{\epsilon}} \tag{21}$$

From Equation 15, we have $\beta^s_{2000} + \beta^u_{2000} = 1$ which allows us to calculate the labor shares as $\beta^s_{2000} = 0.56$ and $\beta^u_{2000} = 0.44$. From Equation 14, we can calculate total labor $L_{2000} = 49707$. The GDP of the USA in 2000 was about $Y_{2000} = 11167.70$ (2005 billion USD)[2]. Having the wages, GDP, and number of adults in 2000 for each skill group, allows us to calculate the share of labor (α) in the production function from Equation 16:

$$\alpha = \frac{w^s_{2000}}{\beta^s_{2000}\ Y_{2000}\ L_{2000}^{\frac{1-\epsilon}{\epsilon}}\ (N^s_{2000})^{\frac{-1}{\epsilon}}} = 0.33 \tag{22}$$

Total available capital in 2000 is given by $\overline{K}_{2000} = \gamma_2 \times Y_{1980} = 1214.23$ (2005 billion USD). Total factor productivity (TFP) in 2000 is obtained from Equation 13 and equals to $A_{2000} = 248.10$. From Equation 17, capital rent in 2000 is calculated as $\rho_{2000} = 2.85$.

In order to calculate the average wealth of each skill and race group, first we asses the White to Black wealth ratio in 2000 from the Survey of Consumer

[1] Accessible at https://tradingeconomics.com/united-states/wages-high-skilled
[2] Accessible at https://climate-adapt.eea.europa.eu/en/metadata/portals/shared-socioeconomic-pathways-ssps-database

Finances (SCF) conducted by the U.S. Federal Reserve[3]. We use the net worth data reported for the year 2001 which gives the wealth ratio as $\Omega^r_{2000} = \frac{\Omega^W_{2000}}{\Omega^B_{2000}} = \frac{706.19}{103.49} = 6.82$. The Gini coefficient of the USA in 2000 was about $Gini_{2000} = 40\%$.[4] To calculate the wealth of each group, we calculate the share of low-skilled group's wealth as $\frac{\overline{\Omega}^u_{2000}}{\overline{\Omega}_{2000}} = \frac{N^u_{2000}}{N_{2000}} - Gini_{2000} = 0.30$. The total wealth of all labor in 2000 can be estimated as

$$\begin{aligned}\overline{\Omega}_{2000} &= \overline{\Omega}^s_{2000} + \overline{\Omega}^u_{2000} \\ &= w^s_{2000} \times N^s_{2000} + w^u_{2000} \times N^u_{2000} + (1 + \rho_{2000} - \delta) \times \overline{K}_{2000} \\ &= 8,069.03 \text{ (2005 billion USD)}\end{aligned}$$

Therefore, the average wealth of a low-skilled and a high-skilled labor in 2000 would be

$$\begin{aligned}\Omega^u_{2000} &= \frac{\overline{\Omega}_{2000} \times 0.3}{N^u_{2000}} = 31,172 \text{ (2005 USD)} \\ \Omega^s_{2000} &= \frac{\overline{\Omega}_{2000} \times 0.7}{N^s_{2000}} = 16,9728 \text{ (2005 USD)}\end{aligned}$$

The average wealth of a Black and a White labor in 2000 would be

$$\Omega^B_{2000} = \frac{\overline{\Omega}_{2000}}{N^B_{2000} + \Omega^r_{2000} \times N^W_{2000}} = 11,824 \text{ (2005 USD)}$$

$$\Omega^W_{2000} = \Omega^r_{2000} \times \Omega^B_{2000} = 80,683 \text{ (2005 USD)}$$

We further assume that the wealth ratio of White to Black labor (Ω^r_{2000}) also holds as the wealth ratio of White to Black among high-skilled labor: $\frac{\Omega^{s,W}_{2000}}{\Omega^{s,B}_{2000}} =$

[3] Accessible at https://www.federalreserve.gov/econres/scf/dataviz/scf/chart/
[4] Accessible at https://data.worldbank.org/indicator/SI.POV.GINI?locations=US

Ω^r_{2000}. As a result, we can calculate the average wealth of each sub-population based on both skill level and race as follows

$$\Omega^{s,B}_{2000} = \frac{\overline{\Omega}_{2000} \times 0.7}{N^{s,B}_{2000} + \Omega^r_{2000} \times N^{s,W}_{2000}} = 28,825 \text{ (2005 USD)}$$

$$\Omega^{s,W}_{2000} = \Omega^r_{2000} \times \Omega^{s,B}_{2000} = 182,192 \text{ (2005 USD)}$$

$$\Omega^{u,B}_{2000} = \frac{\Omega^B_{2000} \times N^B_{2000} - \Omega^{s,B}_{2000} \times N^{s,B}_{2000}}{N^{u,B}_{2000}} = 8,753 \text{ (2005 USD)}$$

$$\Omega^{u,W}_{2000} = \frac{\Omega^W_{2000} \times N^W_{2000} - \Omega^{s,W}_{2000} \times N^{s,W}_{2000}}{N^{u,W}_{2000}} = 34,770 \text{ (2005 USD)}$$

Having the average wealth and wages we can calculate the average inherited capital for each sub-group as follows

$$k^{u,B}_{2000} = \frac{\Omega^{u,B}_{2000} - w^u_{2000}}{1 + \rho_{2000} - \delta} = -4,050 \text{ (2005 USD)}$$

$$k^{u,W}_{2000} = \frac{\Omega^{u,W}_{2000} - w^u_{2000}}{1 + \rho_{2000} - \delta} = 3,086 \text{ (2005 USD)}$$

$$k^{s,B}_{2000} = \frac{\Omega^{s,B}_{2000} - w^s_{2000}}{1 + \rho_{2000} - \delta} = -6,971 \text{ (2005 USD)}$$

$$k^{s,W}_{2000} = \frac{\Omega^{s,W}_{2000} - w^s_{2000}}{1 + \rho_{2000} - \delta} = 35,094 \text{ (2005 USD)}$$

The negative numbers here indicate the average debt that a Black adult inherit from their parents.

Next, we calculate the portion of parental time spent on raising children of each skill level. To do this, we first estimate the skill ratio of the combined

workforce in 2020 following the trajectory of the general population's skill ratio[5]:

$$h_{2020} = h_{2000} \times \frac{\overline{h}_{2020}}{\overline{h}_{2000}} = 0.55 \tag{23}$$

where $\overline{h}_{2000}$ and $\overline{h}_{2020}$ are skill ratio in the general population of the USA in 2000 and 2020, respectively. The fertility rate can be calculated as $n_{2000} = \frac{Pop_{2020}}{Pop_{2000}} = 1.19$ where Pop_{2000} and Pop_{2020} are total population in the USA in 2000 and 2020, respectively. Therefore, the population of high-skilled and low-skilled labor in 2020 can be calculated as follows

$$N^u_{2020} = \frac{n_{2000} \times N_{2000}}{1 + h_{2020}} = 86,039 \text{ (thousands)}$$

$$N^s_{2020} = N^u_{2000} \times h_{2020} = 46,998 \text{ (thousands)}$$

This allows us to calculate the fertility rate within each skill group:

$$n^u_{2000} = \frac{N^u_{2020}}{N^u_{2000}} = 0.77$$

$$n^s_{2000} = \frac{N^s_{2020}}{N^s_{2000}} = 0.42$$

The child-rearing costs as a portion of time allocated to raising children of each type can be calculated using Equation 3 and Equation 6

$$\tau^u = \frac{0.5 \times (\gamma^s_1 + \gamma^u_1)}{n^u_{2000} + w^r \times n^s_{2000}} = 0.05$$

[5]Fertility rate and skill ratios can be constructed using population and school attainment data from SSP database available at https://tntcat.iiasa.ac.at/SspDb/dsd?Action=htmlpage&page=10

$$\tau^s = w^r \times \tau^u_{2000} = 0.12$$

where $\gamma^s_1 = 0.10$ and $\gamma^u_1 = 0.08$ are the average percentage of time of high-skilled and low-skilled parents allocated to child-rearing activities calibrated for year 2000 and averaged over both genders from (Dotti Sani and Treas, 2016).

1.7 Model dynamics

To run the model we use the parameter values specified in table (2). At each time step, the optimal number and education level of children of high-skilled parents $(n^{s,s}_t, n^{s,u}_t)$ and low-skilled parents $(n^{u,s}_t, n^{u,u}_t)$ are determined depending on the parent's skill level to meet the following criteria

$$n^{s,s}_t \times \tau_s + n^{s,u}_t \times \tau^u = \gamma^s_1$$

$$n^{u,s}_t \times \tau_s + n^{u,u}_t \times \tau^u = \gamma^u_1$$

$$\frac{n^{s,s}_t \times N^s_t + n^{u,s}_t \times N^u_t}{n^{s,u}_t \times N^s_t + n^{u,u}_t \times N^u_t} = h_{2000} \times \frac{\overline{h}^{SSP}_{t+1}}{\overline{h}_{2000}}$$

$$\frac{(n^{s,s}_t \times N^{s,W}_t + n^{u,s}_t \times N^{u,W}_t + n^{s,u}_t \times N^{s,W}_t + n^{u,u}_t \times N^{u,W}_t)/N^W_t}{(n^{s,s}_t \times N^{s,B}_t + n^{u,s}_t \times N^{u,B}_t + n^{s,u}_t \times N^{s,B}_t + n^{u,u}_t \times N^{u,B}_t)/N^B_t} = \frac{n^W_{2000}}{n^B_{2000}}$$

where $\overline{h}^{SSP}_{t+1}$ is the general population's skill ratio projection under each SSP scenario. The population growth rates of White and Black workforce from 2000

to 2020 is derived from BLS data[6].

$$n^{W}_{2000} = \frac{N^{W}_{2020}}{N^{W}_{2000}} = \frac{124.431}{98.025} = 1.27$$
$$n^{B}_{2000} = \frac{N^{B}_{2020}}{N^{B}_{2000}} = \frac{20.177}{13.540} = 1.49$$

Finally, the growth rate of TFP (Ag_t) is also optimazed and set to ensure GDP growth matches each SSP trajectory. Table 2 provides the list of parameters used in the model.

Table 2: Key parameters of the OLG model

Name	**Description**	**Value**
α	Output elasticity of labor	0.4
ϵ	Labor elasticity of substitution	1.5
δ	20-year capital depreciation rate	20%
γ_1^s	Utility coefficient of high-skilled child-rearing	0.10
γ_1^u	Utility coefficient of low-skilled child-rearing	0.08
γ_2	Utility coefficient of inheritance	0.24[a]

[a]Gross capital formation (% of GDP) in 2000 from the World Bank Database: https://data.worldbank.org/indicator/NE.GDI.TOTL.ZS?locations=US

[6]https://www.bls.gov/cps/tables.htm

References

Dotti Sani GM, Treas J (2016) Educational gradients in parents' child-care time across countries, 1965–2012. Journal of Marriage and Family 78(4):1083–1096