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A Modified IPT-PLL Technology for Single-phase Grid-connected Inverters in Complex Power Grid Conditions

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Abstract: Aiming at the common problems of frequency variations and harmonics in complex power grids, an improved inverse Park transform phase locked loop (IPT-PLL) technology for single-phase converters suitable for micro grid systems is proposed. Firstly, in the phase detection of PLL, the α component of Park transformation is selected as the reference voltage, and its orthogonal component is constructed using a 1/4 fundamental period delay method. Secondly, Lagrange interpolation polynomials are introduced to approximate fractional delay to solve the problem of delay calculation errors caused by frequency changes. Thirdly, in order to compensate for the poor ability of traditional proportional integral (PI) regulators, multi resonant controllers are superimposed to suppress low order harmonic disturbances. Finally, the design method and system performance of the PI regulator and each resonant controller are analyzed theoretically. The experimental results show that the proposed improved IPT-PLL method has strong adaptability to complex power grids. It can significantly improve the tracking performance of power grid frequency, suppress the interference of low order harmonics and DC bias. And it has good dynamic and static performance.

Key words: phase-locked loop (PLL); complex power grid; frequency adaptivity; harmonic interference; resonant controller

1. Introduction

Since the popularization of electrified transportation, the energy consumption of transportation has become increasingly serious. The contradiction between transportation, energy, and environment has become prominent. As a result, the integration of renewable energy and low-carbon transportation has become a hot field in electrical engineering. And microgrids constructed from renewable energy such as photovoltaic and wind power generation and energy storage are favored in the field of electrical transportation [1,2]. Fig.1 shows the microgrid power supply system for railway electrification locomotives and electric multiple units (EMU). In addition to the AC power grid, each traction electrical substation is also equipped with photovoltaic, energy storage and other units of a certain capacity. The DC/AC grid-connected inverter (GCI) is the interface circuit between the photovoltaic, energy storage system and the AC power grid. In order to achieve safe and efficient operation of microgrids, several technical difficulties must be solved, among which grid synchronization technology is one of the important issues [3,4]. However, as the traction converter of EMU belongs to single-phase nonlinear load, current harmonics will flow into the harmonic impedance of the transformer in the traction electrical substation, resulting in voltage distortion. On the other hand, the traction power of EMU is large and changes rapidly, which is easy to cause the fluctuation of power supply voltage frequency. The voltage of the traction power grid often presents harmonic, amplitude mutation, DC bias, frequency change and other complex situations, which greatly affects the performance of synchronization algorithm, and even causes damage to the GCI [5-6]. So, how to accurately obtain the frequency, phase and even amplitude information of the fundamental component of the voltage in the complex grid is the premise of designing the control system of the GCI. And it is also the key to the safe and stable operation of the traction power supply system [7].
Phase lock loops (PLL) is a widely synchronization method used in GCIs. It mainly includes three parts which are phase detector (PD), loop filter (LF) and voltage control oscillator (VCO). Single-phase PLL can be divided into two types: the stationary coordinate system PLL and the synchronous reference frame PLL (SRF-PLL) according to different PDs.

For the stationary coordinate system PLL, sine signal multiplication is often used to achieve phase detection, which inevitably suffers from interference of twice the frequency and affects the control accuracy of PLL. In order to solve the problem of double frequency oscillation, [9] and [10] introduced high-order low-pass filters and notch filters after PD, respectively, to improve the steady-state performance of PLL by setting a lower filter bandwidth, but its dynamic performance will be greatly affected.

The SRF-PLL borrows from the three-phase voltage PLL principle and utilizes rotating coordinate transformation to realize phase detection. This method has gained widespread application due to its fast dynamic response and ease of software implementation. However, for single-phase AC microgrid systems, there is only one voltage vector and rotational coordinate transformation cannot be directly applied as in the three-phase SRF-PLL. Therefore, different methods have been devised to design a Quadrature Signal Generator (QSG), which are then used for PD through rotational coordinate transformation. Commonly used QSG methods include the Second Order Generalized Integrator (SOGI) and T/4 (where T is the fundamental period) delay. In order to address issues such as voltage harmonics, DC offset, and frequency deviation in the power grid, a DC offset prediction circuit was designed in [15] for the SOGI method. This circuit effectively suppresses the influence of DC offset in the power grid. In [16], an additional integral branch was added to the front-end SOGI to suppress DC offset, and a sliding average filter was used in the back-end PLL loop instead of the integral controller of the PLL. This modification enhances filtering performance while speeding up the dynamic response. However, it should be noted that this method is more complex to implement. For the T/4 delay method, in [17], an adaptive delay SPF-PLL algorithm was constructed based on the deviation between the estimated frequency and the nominal frequency of the power grid. However, it did not consider issues such as the DC bias and harmonics of the grid voltage. In [18], the T/4 delay + Inverse Park Transformation (IPT) method was used to construct orthogonal components, effectively suppressing the DC bias of the grid voltage. In [19] a multi-harmonic decoupling compensation network was established by performing Park transformation and its inverse transformation at different harmonic frequencies, the influence of grid harmonics on the PLL is eliminated. Due to the need for a large number of Park transforms, the calculation speed of PLL is affected. A high-pass filter was used to extract the harmonic components in the dq components after Park transformation in [20], and then the difference between the extracted components and the original dq components was taken to filter out the grid voltage harmonics. But the adverse effects caused by the phase lag of the high-pass filter on the harmonic filtering algorithm were not addressed.

In addition, methods such as Discrete Fourier Transform (DFT), Hilbert Transform, Kalman Filtering, and Double Complex Coefficient Filtering have also been used for voltage signal orthogonal component
construction. However, these methods have various issues, such as requiring a large amount of controller memory or experiencing significant phase locking errors and low-frequency oscillations in complex power grid environments.

Based on the shortcomings of the existing single-phase PLL technology, this paper proposes an improved PLL technology on the basis of the traditional IPT-PLL, aiming at the synchronization control problem of the GCI under the complex grid with voltage harmonics, DC bias and frequency offset. Firstly, the α component of the IPT-PLL reverse Park transformation is used as the reference and the orthogonal components is constructed by the use of the T/4 delay method. Secondly, the Lagrange interpolation polynomial is used to estimate the fractional delay, which improves the accuracy of the T/4 delay algorithm and enhances the frequency adaptability of the PLL. Thirdly, a PI+ multi-resonant controller is used to construct the LF method, which helps to mitigate the impact of voltage harmonics in the complex power grid. Design methods of the PI regulator and multi resonant controllers of LF are analyzed. At last, good performances of the proposed improvement scheme are discussed.

2. Basic Principles of IPT-PLL

2.1 Principle of PD in IPT-PLL

IPT-PLL structure is shown in Fig.2 [18], where PD consists of three parts: Park transformation, low pass filter (LPF), and inverse Park transformation. The grid voltage \( u_s \) and the output signal \( \bar{u}_\beta \) of inverse Park transformation are transformed into dq components \( u_d \) and \( u_q \) through Park transformation. And the high-order harmonics of dq components are filtered out by LPF to obtain the DC form \( u'_d \) and \( u'_q \), which are then transformed into \( \bar{u}_d \) and \( \bar{u}_\beta \) through inverse Park transformation. The q-axis component \( u'_q \) from LPF is taken as the output of PD, the error angular frequency \( \Delta \omega \) is generated through the LPF and PI regulator of LF, and then it is summed with the nominal angular frequency \( \omega_N \) (\( \omega_N = 100 \pi \) rad/s) of the complex power grid’s fundamental voltage. And the PLL output \( \hat{\theta} \) and the power grid frequency observation signal \( \hat{f} \) are obtained from VCO.

According to the PD shown in Fig.2, the transfer function \( F(s) \) of the first-order LPF is defined as

\[
F(s) = \frac{\omega_c}{s + \omega_c}
\]  

(1)

where \( \omega_c \) represents the cutoff frequency of the LPF.

The time-domain relationship between the input of the Park transform in Fig.2 and the output of the inverse Park transform can be derived as

\[
\begin{bmatrix}
\bar{u}_d(t) \\
\bar{u}_\beta(t)
\end{bmatrix} = \begin{bmatrix}
\cos \hat{\theta} & -\sin \hat{\theta} \\
\sin \hat{\theta} & \cos \hat{\theta}
\end{bmatrix} \times \left\{ \begin{bmatrix}
0 \\
f(t)
\end{bmatrix} * \begin{bmatrix}
\cos \hat{\theta} & \sin \hat{\theta} \\
-\sin \hat{\theta} & \cos \hat{\theta}
\end{bmatrix} \times \begin{bmatrix}
u_s(t) \\
\bar{u}_\beta(t)
\end{bmatrix} \right\}
\]  

(2)

where * represents the convolution operator, \( f(t) \) is the unit impulse response of the LPF.

In terms of Euler’s formula, Park transformation matrix can be expressed as
\[
\begin{bmatrix}
\cos \hat{\theta} & \sin \hat{\theta} \\
-\sin \hat{\theta} & \cos \hat{\theta}
\end{bmatrix} = \frac{1}{2} \begin{bmatrix}
e^{j\hat{\omega}t} + e^{-j\hat{\omega}t} & j e^{j\hat{\omega}t} - je^{-j\hat{\omega}t} \\
-j e^{j\hat{\omega}t} + je^{-j\hat{\omega}t} & e^{j\hat{\omega}t} + e^{-j\hat{\omega}t}
\end{bmatrix}
\]

where \( \hat{\theta} = \hat{\omega}t \).

According to equation (3) and the property of frequency shift, the Laplace transform of equation (2) can be obtained as

\[
\begin{bmatrix}
\hat{u}_\alpha(s) \\
\hat{u}_\beta(s)
\end{bmatrix} = \frac{1}{2} \begin{bmatrix}
F(s + j\hat{\omega}) + H(s - j\hat{\omega}) & -jF(s + j\hat{\omega}) + jH(s - j\hat{\omega}) \\
-jF(s + j\hat{\omega}) + jH(s - j\hat{\omega}) & F(s + j\hat{\omega}) + H(s - j\hat{\omega})
\end{bmatrix} \cdot \begin{bmatrix}
u_s(s) \\
\hat{u}_\beta(s)
\end{bmatrix}
\]

(4)

The transfer functions \( \frac{\hat{u}_\alpha(s)}{u_s(s)} \) and \( \frac{\hat{u}_\beta(s)}{u_s(s)} \) derived by (4) can be expressed as (5) and (6) respectively.

\[
T_\alpha(s) = \frac{\hat{u}_\alpha(s)}{u_s(s)} = \frac{\omega_c}{s^2 + \omega_c s + \omega^2}
\]

(5)

\[
T_\beta(s) = \frac{\hat{u}_\beta(s)}{u_s(s)} = \frac{\omega_c}{s^2 + \omega_c s + \omega^2}
\]

(6)

The Bode plots of equations (5) and (6) are shown in Fig.3, where \( \omega_c = 2000\pi \) rad/s. It can be seen that \( T_\alpha(s) \) is a second-order band-pass filter with angular frequency centered on \( \hat{\omega} \), which can filter DC and high-frequency components of the complex power grid voltage. \( T_\beta(s) \) is a second-order LPF with an amplitude of 1 at a frequency of 50Hz and a phase shift of -90°. It can be inferred that \( \hat{u}_\beta(s) \) represents the quadrature component of the grid voltage \( u_s \).

![Fig.3 Bode plot of IPT transfer function](image)

2.2 Some Problems with IPT-PLL

For a single-phase complex power grid, it generally includes a DC bias, fundamental and harmonic voltage harmonics. The complex grid voltage \( u_s(t) \) can be written as

\[
u_s(t) = U_0 + U \cos(\omega t + \phi) + \sum_{h=3,5,7...} U_h \cos(h\omega t + \phi_h)
\]

(7)

where \( U_0, U \), and \( U_h \) represent the amplitudes of the DC component, fundamental component, and \( h^{th} \) harmonic component of the grid voltage respectively. \( \omega \) is the angular frequency of the fundamental voltage, while \( \phi \) and \( \phi_h \) represent the phases of the fundamental and \( h^{th} \) harmonic voltages, respectively. If PLL output angular frequency \( \hat{\omega} = \omega \), the steady-state components of \( \hat{u}_\alpha(t) \) and \( \hat{u}_\beta(t) \) are shown in (8) and (9) can be calculated from equations (5)-(7).

\[
\hat{u}_\alpha(t) = U \cos(\omega t + \phi) + \frac{1}{2} \sum_{h=3,5,7...} \left[ |T_\alpha(-j\omega h)| + |T_\alpha(j\omega h)| \right] U_h \cos \left[ h\omega t + \phi_h + \angle T_\alpha(j\omega h) \right]
\]

(8)

\[
\hat{u}_\beta(t) = U \sin(\omega t + \phi) + \frac{1}{2} \sum_{h=3,5,7...} \left[ |T_\beta(-j\omega h)| + |T_\beta(j\omega h)| \right] U_h \sin \left[ h\omega t + \phi_h + \angle T_\beta(j\omega h) \right]
\]

(9)

where \( |T_\alpha(j\omega h)|, |T_\alpha(-j\omega h)|, |T_\beta(j\omega h)| \) and \( |T_\beta(-j\omega h)| \) are moduli of \( T_\alpha(s) \) and \( T_\beta(s) \), \( \angle T_\alpha(j\omega h) \) and \( \angle T_\beta(j\omega h) \) are phase angles of \( T_\alpha(s) \) and \( T_\beta(s) \) when \( s = \pm j\omega h \) respectively.
By substituting (8) and (9) into the Park transformation matrix, the expression for \( u_q \) can be obtained as

\[
u_q(t) = U \sin(\theta - \hat{\theta}) + U_0 \cos(\hat{\theta}) + \sum_{h=3,5,7...} n[(h+1)\alpha t, (h-1)\alpha t]
\]

where

\[
n[(h+1)\alpha t, (h-1)\alpha t] = \frac{[|T_a(-j\alpha h\omega)| + |T_a(j\alpha h\omega)|]}{4} [\sin((h+1)\alpha t + \phi_h + \angle T_a(j\alpha h\omega)] - \sin((h-1)\alpha t + \phi_h + \angle T_a(j\alpha h\omega)]
\]

According to equation (10), in steady state, \( \sin(\theta - \hat{\theta}) \) is approach to \( \theta - \hat{\theta} \). The DC component of the grid voltage can be expressed as \( U_0 \cos(\hat{\theta}) \), which is an AC component with a frequency of \( \omega \). The \( h \)th voltage harmonic is transformed into \( (h\pm1) \)th harmonic component. Consider the DC and harmonic components of the grid voltage as disturbances \( N(s) \), which is the frequency domain expression of \( n[(h+1)\alpha t, (h-1)\alpha t] \) to the PLL.

The linear model block diagram of IPT-PLL is shown in Fig.4.

![Fig.4 Linear model of traditional IPT-PLL](image)

The transfer function of the PI controller and LPF1 of LF in Fig.4 are given as (11) and (12) respectively

\[
W_{PI}(s) = k_p + \frac{k_i}{s}
\]

\[
W_{LPF1}(s) = \frac{\omega_{c1}}{s + \omega_{c1}}
\]

where \( \omega_{c1} \) is the cutoff frequency of the LPF in LF of the PLL.

According to Fig.4, it can be seen that the closed-loop transfer function under the input signal \( \theta(s) \) and the disturbance signal \( N(s) \) is the same. It can be given by

\[
G_c(s) = \frac{(k_p s + k_i) \omega_{c1} k_{vco}}{s^2 + \omega_{c1} s^2 + k_p \omega_{c1} s + k_i \omega_{c1} \omega_{c1} k_{vco}}
\]

where \( k_{vco} \) is the gain of VCO, and \( k_{vco}-U \).

Through some simple theoretical derivations, the output \( \hat{\theta}(s) \) and the error \( E(s) \) in Fig.4 can be obtained as

\[
\begin{bmatrix}
\hat{\theta}(s) = G_c(s) [N(s) + \theta(s)] \\
E(s) = \frac{(k_p s + k_i) \omega_{c1} k_{vco} N(s)}{s^3 + \omega_{c1} s^2 + k_p \omega_{c1} s + k_i \omega_{c1} \omega_{c1} k_{vco}} - \frac{(k_p s + k_i) \omega_{c1} k_{vco} N(s)}{s^3 + \omega_{c1} s^2 + k_p \omega_{c1} s + k_i \omega_{c1} \omega_{c1} k_{vco}}
\end{bmatrix}
\]

From (14), it can be seen that the closed-loop phase error \( E(s) \) based on IPT contains the low-frequency component of \( N(s) \) is shown in Fig.13. This will cause phase fluctuation of PLL output, which in turn affecting the stable operation of the GCI. In addition, the traditional IPT-PLL has not solved the issue of phase-locked loop output double frequency ripple caused by grid frequency variations.

### 3. Improved IPT-PLL

This paper proposes an improved strategy for the shortcomings of the traditional IPT-PLL, as shown in Fig.5. Since \( T_a(s) \) is a bandpass filter, it can filter out the DC component and higher-order harmonics of the grid voltage, while preserving all the information of the fundamental frequency. In this paper, \( \hat{u}_a(s) \) is taken as the
research object, its delay T/4 is used to obtain the corresponding orthogonal component \( u_{\beta}'(s) \). And the fractional delay caused by grid frequency variations is estimated using Lagrange interpolation polynomials to solve the problem of computational accuracy. Then the q-axis component is obtained through the Park transformation, and PD is realized. Considering the influence of low-order harmonics in the grid voltage on the PLL, a multi-resonant (MR) controller is paralleled with the PI controller to improve the anti-interference performance of PLL against low order harmonics.

\[ D(z) = z^{-\frac{f_s}{4(\hat{f}+f)}} = z^{-(I+F)} \]  

where \( f_s \) is the switching frequency of the GCI, \( \hat{f} \) is the output frequency of the PLL. If the grid frequency varies, \( \frac{f_s}{\hat{f}} \) may not be an integer. It can be represented as the sum of its integer part \( I \), and its fractional part \( F \).

In order to improve the accuracy of T/4 delay calculation, the fractional order delay element can be approximated using Lagrange interpolation [25], \( z^{-F} \) can be approximated by

\[ z^{-F} \approx \sum_{k=0}^{N} d_k z^{-k} \]  

where \( N \) is the highest degree of the Lagrange interpolation polynomial, \( d_k = \prod_{i=k}^{N} \frac{F-i}{k-i}, \) \((k = 0,1,2...N)\) are polynomial coefficients.

Table 1 shows the calculation formulas for the corresponding interpolation polynomial coefficients as \( N \) varies from 1 to 4.

<table>
<thead>
<tr>
<th>( N )</th>
<th>( d(0) )</th>
<th>( d(1) )</th>
<th>( d(2) )</th>
<th>( d(3) )</th>
<th>( d(4) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(-F)</td>
<td>( F )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>((F-1)(F-2)/2)</td>
<td>(-F(F-2))</td>
<td>( F(F-1)/2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>((F-1)(F-2)(F-3)/6)</td>
<td>((F-1)(F-3)/2)</td>
<td>(-F(F-1)(F-3)/2)</td>
<td>( F(F-1)(F-2)/6)</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>((F-1)(F-2)(F-3)(F-4)/24)</td>
<td>((F-2)(F-3)(F-4)/6)</td>
<td>((F-1)(F-3)(F-4)/4)</td>
<td>(-F(F-1)(F-2)(F-4)/6)</td>
<td>( F(F-1)(F-2)(F-3)/24)</td>
</tr>
</tbody>
</table>

Fig.6 shows the frequency response curves of different-order Lagrange interpolation polynomials approximating the fractional-order delay element at \( F=0.2, 0.5, \) and \( 0.8. \) Here, \( N=0 \) represents the original fractional-order delay element. It can be observed that when \( N \) is set to 1–3, the calculated interpolation polynomials can approximate the original delay element within a certain frequency range. However, when \( N \) is set to 4 or higher, the obtained interpolation polynomials exhibit significant differences in both magnitude and phase compared to the original element. Taking into account the magnitude-frequency and phase-frequency
characteristics, polynomial with \(N=3\) is chosen to approximate the fractional-order delay element.

Based on the previous analysis, the implementation method of T/4 delay with frequency adaptivity can be obtained as shown in Fig.7.

3.2 Harmonic Issues and Solutions in IPT-PLL
3.2.1 Harmonic Issues in IPT-PLL

By using the improved phase estimation method for IPT proposed in this paper, the complex power grid voltage expression (7) and the bandpass filter characteristics of \(T_\alpha(s)\) can be used to obtain the result shown in Fig.5.

By using the improved IPT phase detection method proposed in this paper, the expression of \(u'_\alpha\), as shown in Fig.5, can be derived from the complex power grid voltage expression (7) and the bandpass filter characteristics of \(T_\alpha(s)\).

\[
 u'_\alpha = U \cos(\omega t + \phi) + \sum_{h=3,5,7...} U'_h \cos(h \omega t + \phi_h) \tag{17}
\]

where \(U'_h\) represents the amplitude of the harmonic voltage contained in \(u'_\alpha\), which has a certain attenuation relative to \(U_h\) due to the effect of filter \(T_\alpha(s)\).

Under the effect of T/4 delay, the orthogonal signal \(u'_\beta\) corresponding to \(u'_\alpha\) can be written as

\[
 u'_\beta = U \cos(\omega(t - \frac{T}{4}) + \phi) + \sum_{h=3,5,7...} U'_h \cos(h \omega(t - \frac{T}{4}) + \phi_h) \tag{18}
\]

After some trigonometric transformations, equations (17) and (18) can be rewritten as

\[
\begin{bmatrix} u'_\alpha \\ u'_\beta \end{bmatrix} = \begin{bmatrix} U \cos(\omega t + \phi) \\ U \sin(\omega t + \phi) \end{bmatrix} + \sum_{h=3,5,7...} U'_h \begin{bmatrix} \cos(-h \omega t - \phi_h) \\ \sin(-h \omega t - \phi_h) \end{bmatrix} + \sum_{h=5,9,13...} U'_h \begin{bmatrix} \cos(h \omega t + \phi_h) \\ \sin(h \omega t + \phi_h) \end{bmatrix} \tag{19}
\]
From equation (19), it can be seen that $u'_\alpha$ and $u'_\beta$ constructed by the T/4 delay module include fundamental components, negative sequence harmonic components such as the 3rd, 7th, and 11th harmonics, as well as positive sequence harmonic components such as the 5th, 9th, and 13th harmonics. By performing a Park transformation on $u'_\alpha$ and $u'_\beta$, the q-axis component $u''_q$ is given by

$$u''_q(t) = U\sin(\theta - \bar{\theta}) + \sum_{n=3,7,11} U'_n \sin((-h - 1)\omega t - \phi_h) + \sum_{h=5,9,13} U'_h \sin((h - 1)\omega t + \phi_h)$$

(20)

According to equation (20), it can be seen that the q-axis component based on the improved IPT phase detection contains harmonic components such as ±4, ±8, and so on, which have an impact on the PLL.

### 3.2.2 The Solution to the Harmonic Problem

The harmonic components of $u''_q$ can be regarded as disturbance signals to the PLL. Due to the limited gain of the PI controller at harmonic frequencies, it is unable to eliminate the influence of low-order harmonic signals on the PLL. It is well known that resonant controllers include an internal model of the AC signal, which can achieve infinite gain at the AC frequency \[26-27\]. This paper proposes parallel resonant controllers at each harmonic frequency on the traditional PLL’s PI controller to suppress harmonic disturbance signals.

The transfer function of the MR controller is given as follows

$$G_{MR}(s) = \sum_{h=4,8} \frac{2k_{rh}\omega_{ch}s}{s^2 + 2\omega_{ch}s + (h_0^2)}$$

(21)

where $k_{rh}$ and $\omega_{ch}$ are the resonant gain and cutoff frequency of the resonant controller at $h^{th}$ harmonic frequencies, respectively. Considering the suppression effect of bandpass filter $T_d(s)$ on high-order harmonics of the grid voltage, it can be assumed that the orthogonal components $u'_\beta$ contain harmonics from the 3rd to the 9th order. Therefore, resonant controllers at the 4th and 8th harmonic frequencies can be added to the PI controller in the LF of the PLL. Fig.8 shows the Bode plot of the MR controller ($h=4, 8$) with $k_{rh}=50$ and $\omega_{ch}=4\text{rad/s}$. From the figure, it can be seen that the resonant controller at the $h^{th}$ harmonic frequency has infinite gain at the ±$h^{th}$ harmonic frequencies, which enables the suppression of disturbance signals at the ±$h^{th}$ harmonic frequencies.

Fig.8 Bode plot of the MR controller

### 4. Tuning Procedure of the Improved IPT-PLL

In the improved IPT-PLL shown in Fig.5, assuming the system is in steady state, the grid voltage contains low-order harmonics such as the 3rd to 9th harmonics. Equation (20) can be approximated as follows

$$u''_q(t) \approx U\sin(\theta - \bar{\theta}) + \sum_{n=3,7} U'_n \sin((-h - 1)\omega t - \phi_h) + \sum_{h=5,9} U'_h \sin((h - 1)\omega t + \phi_h)$$

(22)

The improved IPT-PLL linear model structure diagram can be obtained as shown in Fig.9.
The corresponding open-loop transfer function $G_o(s)$ is

$$G_o(s) = \frac{K(1+s/\omega_z)}{s^2(1+s/\omega_p)/\omega_z}$$

(24)

where $\omega_z = k_i/k_p$, $\omega_p = \omega_{c1}$, $K = k_p U_s$.

The crossover frequency $\omega_{co}$, where the open loop gain is unity, can be solved by $|G_o(j\omega_{co})| = 1$.

$$\omega_{co} = \frac{K\cos(\phi_p)}{\sin(\phi_z)}$$

(25)

where $\phi_z = \arctan(\omega_{co}/\omega_z)$, $\phi_p = \arctan(\omega_{co}/\omega_p)$.

The PM expressed in (26) of the open-loop system can be obtained from equation (24).

$$PM = \arctan\left(\frac{\omega_{co}}{\omega_z}\right) - \arctan\left(\frac{\omega_{co}}{\omega_p}\right)$$

(26)

Differentiating equation (26) with respect to $\omega_{co}$ and setting it to zero. Then $\omega_{co}$ can be calculated as

$$\omega_{co} = \sqrt{\omega_p \omega_z} = \sqrt{\frac{k_i\omega_{c1}}{k_p}}$$

(27)

Hence the phase margin (PM) of the PLL control loop is maximized, if the control system loop bandwidth $\omega_{co}$ satisfies equation (27). In the meantime, $\sin(\phi_z) = \cos(\phi_p)$, $\omega_{co} = K$.

Define the parameter $b = \frac{\omega_p}{\omega_z}$, and PM can be calculated as

$$PM = \arctan\frac{b-1}{2\sqrt{b}}$$

(28)

It can be seen that the maximum PM is exclusively determined by $b$.

In this paper, the value of $\omega_{c1}$ for the LPF in LF is set to 2000$\pi$ rad/s. According to (27) and (28), Fig.10 illustrates the PLL PM and $\omega_{co}$ for different $b$ values from 1 to 13. It shows that, if the PM is set to 50°, the value of $b$ can be chosen as 3 and $\omega_{co}$=3626 rad/s which are derived by (27) and (28). From (25) and (27), the parameters of PI controller can be computed as $k_p=12.3$, $k_i=19400$. 

Fig.9 Linear model of improved IPT-PLL

The corresponding open-loop transfer function $G_o(s)$ is

$$G_o(s) = \left[k_p + k_i + \sum_{h=4,8} \frac{2R\omega \omega_{ch}k_h}{s^2+2\omega_{ch}s+(\omega_h)^2}\right] \frac{U\omega_{c1}}{s(\omega_{c1})}$$

(23)

According to the conclusion obtained from [26], the PI controller determines the bandwidth and DC gain of the open-loop system, while the resonant controller determines the gain at the resonant frequency of the open-loop system. It can be considered that the PI controller and the resonant controller are decoupled, and the phase margin (PM) of the control system can be used as a constraint to separately design each controller.

4.1 Design of PI Controller

The open-loop transfer function $G_{o1}(s)$ of the IPT-PLL linear model, as shown in Fig.9, can be obtained by ignoring the influence of the MR controller.

$$G_{o1}(s) = \frac{K(1+s/\omega_z)}{s^2(1+s/\omega_p)/\omega_z}$$

(24)

where $\omega_z = k_i/k_p$, $\omega_p = \omega_{c1}$, $K = k_p U_s$.

The PM expressed in (26) of the open-loop system can be obtained from equation (24).

$$PM = \arctan\left(\frac{\omega_{co}}{\omega_z}\right) - \arctan\left(\frac{\omega_{co}}{\omega_p}\right)$$

(26)

Differentiating equation (26) with respect to $\omega_{co}$ and setting it to zero. Then $\omega_{co}$ can be calculated as

$$\omega_{co} = \sqrt{\omega_p \omega_z} = \sqrt{\frac{k_i\omega_{c1}}{k_p}}$$

(27)

Hence the phase margin (PM) of the PLL control loop is maximized, if the control system loop bandwidth $\omega_{co}$ satisfies equation (27). In the meantime, $\sin(\phi_z) = \cos(\phi_p)$, $\omega_{co} = K$.

Define the parameter $b = \frac{\omega_p}{\omega_z}$, and PM can be calculated as

$$PM = \arctan\frac{b-1}{2\sqrt{b}}$$

(28)

It can be seen that the maximum PM is exclusively determined by $b$. 

In this paper, the value of $\omega_{c1}$ for the LPF in LF is set to 2000$\pi$ rad/s. According to (27) and (28), Fig.10 illustrates the PLL PM and $\omega_{co}$ for different $b$ values from 1 to 13. It shows that, if the PM is set to 50°, the value of $b$ can be chosen as 3 and $\omega_{co}$=3626 rad/s which are derived by (27) and (28). From (25) and (27), the parameters of PI controller can be computed as $k_p=12.3$, $k_i=19400$. 

Fig.9 Linear model of improved IPT-PLL
4.2 Design of MR

The MR controller shown in Fig.9 includes two sets of parameters: resonant gain and open-loop cutoff frequency $\omega_{ch}$. Considering frequency selectivity, it is possible to choose the same open-loop cutoff frequency for each resonance controller. In this paper, $\omega_{ch}$ is set to 4 rad/s. And then the resonant gains $k_{r4}$ and $k_{r8}$, which are 4 times and 8 times the fundamental frequency resonant controllers respectively, are calculated based on the phase constraints. Taking the example of a PI controller with a resonant controller that is 4 times the fundamental frequency ($4\hat{\omega}$), the range of values for $k_{r4}$ is discussed in this paper. The open-loop transfer function $G_{02}(s)$ of IPT-PLL with PI and a resonant controller under $4\hat{\omega}$ is written as

$$G_{02}(s) = \left[ k_p + \frac{k_i}{s} + \frac{2k_{r4}\omega_{c4}s}{s^2+2\omega_{c4}s+(4\hat{\omega})^2} \right] \frac{U\omega_{c1}}{s(s+\omega_{c1})} \tag{29}$$

In order to ensure system stability, the phase of system at $4\hat{\omega}$ must satisfy the following inequality

$$180^\circ + \left\{ \arg[G_{02}(s)] \right\}_{s=j4\hat{\omega}} \times \frac{180^\circ}{\pi} > 0^\circ \tag{30}$$

Fig.11(a) shows the Nyquist plot of $G_{02}(s)$ for different values of $k_{r4}$. It can be observed that as $k_{r4}$ increases, the phase at the resonant frequency approaches -180°. For $k_{r4}<400$, equation (30) holds true. In this paper, $k_{r4}$ is chosen to be 300. And the phase at the resonant frequency is -160°, providing a phase margin of 20° at the resonant frequency. Similarly, the open-loop transfer function $G_{03}(s)$ of the PI controller with an 8 times resonant frequency resonant controller of IPT-PLL is given by

$$G_{03}(s) = \left[ k_p + \frac{k_i}{s} + \frac{2k_{r8}\omega_{c8}s}{s^2+2\omega_{c8}s+(8\hat{\omega})^2} \right] \frac{U\omega_{c1}}{s(s+\omega_{c1})} \tag{31}$$

According to the Nyquist plot shown in Fig.11(b), when $k_{r8}$ is set to 200, the phase at the resonant frequency is -165°, and the open-loop system has a phase margin of 15° at the resonant frequency.
According to the calculated parameters of the PI controller and the MR controller in the improved IPT-PLL scheme, the corresponding Bode plot of the open-loop transfer function (23) is shown in Fig.12. It can be observed that the resonant controller does not affect the stability of the original system with the PI controller, and each resonant controller operates independently. The system exhibits a significant gain increase at the resonant frequency, ensuring effective suppression of harmonic interference at the resonant frequency in the PLL control loop.

For the open-loop transfer function (23), the frequency domain expression of the error $E_c(s)$ in the improved IPT-PLL scheme shown in Fig.9 can be obtained as

$$E_c(s) = \frac{\theta(s)}{1+G_o(s)} - \frac{G_o(s)N(s)}{1+G_o(s)}$$ \hspace{1cm} (32)

Comparing the expression of the error signal $E(s)$ for the traditional IPT-PLL in (13), the amplitude-frequency characteristics of the PLL error signal for both schemes are plotted in Fig.13. It can be observed that the improved IPT-PLL effectively suppresses the influence of low-order harmonics from the grid on the PLL.
5. Experimental Results

To validate the correctness and effectiveness of the improved IPT-PLL proposed in this paper, the method was applied to the GCI of a traction substation at a railway station of the National Railway Group shown in Fig. 14. The phase-locked loop algorithm was implemented using TI's DSP (TMS320F28377) as the core controller. All the parameters of the implemented experimental setup are listed in Table 2.

![Energy storage converter device](image)

**Table 2. Parameters for the Experimental Setup**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal conditions</td>
<td>$U=311$V, $f_s=50$Hz</td>
</tr>
<tr>
<td>Switching frequency</td>
<td>10kHz</td>
</tr>
<tr>
<td>Parameters of PI controller</td>
<td>$k_p=12.3$, $k_i=19400$</td>
</tr>
<tr>
<td>Lagrange interpolation polynomial order</td>
<td>$N=3$</td>
</tr>
<tr>
<td>Parameters of MR controller</td>
<td>$k_{kd}=300$, $k_{kr}=200$, $\omega_{ch}=4$ rad/s</td>
</tr>
<tr>
<td>Cutoff frequency of the LPF</td>
<td>$\omega_c=\omega_{cl}=2000\pi$ rad/s</td>
</tr>
<tr>
<td>The gain of VCO</td>
<td>$k_v=311$</td>
</tr>
</tbody>
</table>

The waveforms of the PLL output frequency $\hat{f}$ and phase $\hat{\theta}$ are shown in Fig. 15(a) and (b) respectively, when the grid voltage amplitude abruptly drops from 311 V to 70 V and abruptly rises from 70 V to 311 V. It can be observed that when the grid voltage amplitude undergoes a sudden change, the PLL control system stabilizes within 10ms. Fig. 15(c) and (d) show the waveforms of the PLL output frequency $\hat{f}$ and phase $\hat{\theta}$ when the grid voltage frequency drops from 50 Hz to 47 Hz and increases from 50 Hz to 53 Hz, respectively. It can be observed that the PLL is able to track the grid frequency accurately within 20ms. The improved IPT-PLL scheme enables accurate tracking of the grid frequency.
To verify the advancement of the improved IPT-PLL, experimental comparisons were conducted with the traditional IPT-PLL. Fig.16 shows the comparative experiment with a grid voltage containing an 8 V DC bias. Figure 16(a) displays the output waveforms of the traditional IPT-PLL, indicating that the DC bias in the grid voltage results in disturbances at twice the fundamental frequency, therefore affecting the output phase of the PLL. Fig.16(b) shows the experimental waveforms of the improved IPT-PLL. The proposed method effectively utilizes the characteristics of the band-pass filter, allowing for the suppression of the DC component in the grid voltage and eliminating the influence of the DC bias on the phase-locked loop.
Fig. 16 Comparative experiment with DC bias

Fig. 17 shows a comparative experiment of the grid voltage with low-order harmonics. The harmonics mainly include the 3rd to 9th harmonics, with the 3rd harmonic content at 3%, 5th harmonic content at 7.5%, 7th harmonic content at 5.0%, and 9th harmonic content at 2.0%. Fig. 17(a) represents the experimental waveform of the traditional IPT-PLL. It can be observed that the output frequency of the PLL contains low-order harmonic components such as the 4th and 8th harmonics, which affect the output phase of the PLL. Fig. 17(b) represents the experimental waveform of the improved IPT-PLL. Due to the introduction of the multi-harmonic resonant controller, the output frequency of the PLL has minimal low-order harmonic fluctuations, effectively eliminating the influence of low-order harmonics in the complex power grid voltage on the PLL.

6. Conclusions

This paper proposes an improved IPT-PLL technique suitable for single-phase GCIs under complex grid conditions. The goal of this improvement is to reliably obtain fundamental voltage phase and frequency
information under conditions with DC bias, low-order harmonics, and variable frequency, ensuring the smooth grid connection and safe operation of the microgrid.

This paper analyzes the drawbacks of the traditional IPT-PLL, including poor frequency adaptability, weak ability to resist DC bias and low-order harmonics. The output of the reverse Park transformation of the IPT is used as the reference grid voltage, delaying it by T/4 to construct orthogonal components, and proposing a PI+MR controller to build the improved PLL. Secondly, a fractional delay approximation of T/4 delay using Lagrange interpolation polynomial is employed to further enhance the frequency adaptability of the phase-locked loop. Next, the design methods for the PI and MR controllers in the improved IPT-PLL are discussed in detail, and the performance of the improved IPT-PLL is analyzed to evaluate its effectiveness. Finally, experimental validation of the proposed technique and design methods is conducted in the GCI, which is used in a traction substation at a railway station of a national railway group. Theoretical and experimental results demonstrate that the improved IPT-PLL can effectively solve the problems of DC bias and low-order harmonics in the complex power grid, and exhibit good dynamic and steady state performance with strong adaptability to complex power grid conditions.

Data availability

All data generated or analyzed during this study are included in this published article [and its supplementary information files]. The datasets used and/or analyzed during the current study are available from the corresponding author on reasonable request.

References


Author contributions
Zhangtao and Dong Dezhi wrote the main manuscript text. All authors reviewed the manuscript.

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Supplementary Files

This is a list of supplementary files associated with this preprint. Click to download.

- AppendixA.docx