Additional Supplementary Information for
Pathways to a Sustainable Food Future in Sub-Saharan Africa

Patrice Dumas[1], Liqing Peng[2]*, Deepak Ray[3], Stefan Wirsenius[4], Mario Herrero[5], Srishti Vishwakarma[6], Timothy D. Searchinger[2,7]*

[1] Centre de coopération internationale en recherche agronomique pour le développement (CIRAD), UMR CIRED, Montpellier, France.
[2] World Resources Institute, Washington, DC, USA.
[3] Institute on the Environment, University of Minnesota, Twin Cities, MN, USA.
[4] Chalmers University of Technology, Gothenburg, Sweden.
[5] Cornell University, Ithaca, NY, USA.
[6] Joint Global Change Research Institute, Pacific Northwest National Laboratory, College Park, MD, USA.
[7] Princeton University, Princeton, NJ, USA.

***Correspondence to:** Timothy D. Searchinger (tsearchi@princeton.edu); Liqing Peng (liqing.peng@wri.org)

Projection of requirement per head and per output

The method uses a panel regression on livestock systems[1] allowing to link efficiency per head to efficiency per output. We note I the input-output requirement ratio, in dry matter per unit of main product quantity in fresh matter. It is the inverse of the efficiency e used in the main methods, we use it here as it is easier for the formalization. We note O the output per head, the function is

$$I = A \cdot O^{-B} \tag{1}$$

As we only use ratios, the value of A is not directly used. Using only changes also allows to use heads directly instead of taking into account herd effects as long as the herd effect enters as a constant coefficient.

We also determined the coefficients with a different dataset[2], and the value of B are similar for the two data sources (see main text methods).

To determine the global target in term of input-output, we project the change in output per head at the world scale, using a linear regression on FAOSTAT production per head data aggregated at the world scale, on the 1961-2020 range. With b^O the time evolution coefficient of output per head:

$$I_l^W = I_{l,0}^W \frac{A \cdot \left(O_{l,0}^W + b^O(t - t_0)\right)^{-B}}{A \cdot O_{l,0}^{W\,-B}} = I_{l,0}^W \frac{\left(O_{l,0}^W + b^O(t - t_0)\right)^{-B}}{O_{l,0}^{W\,-B}}$$

We note $I_{r,l}^T$ the regional input output ratio, comprising all feed types and systems for the livestock type l.

Pasture yield past evolution and projection at the world scale

Past pasture yield increase can be obtained by a decomposition of pasture yield and grass feed:

$$Y_g = \frac{F_g}{A_g} = \frac{Q\ I\ \phi_g}{A_g}$$

with Y_g grass yield, F_g grass feed, A_g grass area, Q the quantity produced I the input output feed requirement coefficient and ϕ_g the share of grass in feed input.

We decompose an increase between two dates. Initial date variables are denoted with a hat accent. We note $r_X = X/\hat{X}$ the increase ratio. We have:

$$
\begin{aligned}
r_F &= \frac{F_g}{\hat{F}_g} = \frac{Q\ I\ \phi_g}{\hat{Q}\ \hat{I}\ \hat{\phi}_g} \\
r_F &= r_Q r_I \frac{\phi_g}{\hat{\phi}_g}
\end{aligned}
$$

We have $\phi_g + \phi_O + \phi_R + \phi_F = 1$ with ϕ_O occasional feeds share, ϕ_R crop residues feed share and ϕ_F concentrate feed share.

We consider different possibilities depending on the year and livestock sector. For dairy, we consider that the share of grass in livestock input is constant over time. Dairy systems, indeed have been integrated to crop systems for a long time, and although the share of the non grass feeds may have changed, we assume that grass share did not change. This probably leads to an underestimation of grass share in dairy systems in past years, but we consider that this hypothesis is both simple and accurate enough.

For initial years in the 60', and meat production sectors, we consider that there is no concentrate feed in the initial year $\hat{\phi}_F = 0$. This could be an overestimation of grass used in some regions, but not not in many region, as change of other feed types may also have happened. For years after 1990, we consider that the share of concentrate is constant. We also assume that $\phi_O + \phi_R = \alpha^{o,g}\phi_g$, which leads to

$$\phi_g = 1 - (\phi_O + \phi_R + \phi_F) = \frac{1 - \phi_F}{1 + \alpha^{o,g}}$$

Assuming a constant $\alpha^{o,g}$, we obtain, for the meat producing sectors in the 60':

$$
\begin{aligned}
r_F &= r_Q r_I (1 - \phi_F) \\
\hat{F}_g &= \frac{F_g}{r_Q r_I (1 - \phi_F)}
\end{aligned}
$$

For the other cases, the formula is similar, but without the term in ϕ_F. I evolution per livestock type and region is given by the panel regression equation (1). The evolution of Q per type and per region is also known. We do not have r_I for small ruminant meat, we use beef sector input-output ratio change.

We only know A_g and F_g per type of livestock in a reference year, not for every year. The yield can be determined over a region with area $\hat{A}_g^T$, by summing over all livestock sectors, that we now index explicitly:

$$\begin{aligned}\hat{Y}_g^T &= \frac{\sum_l \hat{F}_{g,l}}{\hat{A}_g^T} \\ &= \frac{\sum_l \frac{F_{g,l}}{r_{Q,l} r_{I,l} (1-\phi_{F,l})}}{\hat{A}_g^T}\end{aligned}$$

World scale pasture yield evolution factor is obtained by using the yearly linear regression coefficient obtained by a regression on the grass yields. 25 years of regression (1985-2010) are used.

Technical change factor

To attain the global target, we change system shares within a region, change regional production shares, and apply a technical change factor that increases overall input efficiency. The reference and improved systems are based the same approach[1].

The switch between systems allows to take into account the change in input-output efficiency corresponding to change in livestock diets. Other management changes could also lead to a change in input-output efficiency at the herd level. Past evolutions have been described as being accompanied by important changes in livestock efficiency associated with herd management, for instance decrease in age of slaughter that takes place when animals are less used for draft or as assets and more used for production. Improvements in health, decreases in mortality and increases in natality should also be included in this efficiency change.

Without more information, we consider a relative change evenly spread over every systems and livestock types in regions. The relative increase in efficiency because of management changes not related to changes in livestock diets is noted $\mu_{l,r}$ and we have

$$\hat{I}_{r,l,f,s} = \frac{\hat{I}_{r,l,f,s,0}}{1 + \mu_{l,r}}$$

According to the improved systems data, most efficient regions cannot gain in efficiency by switching systems. Therefore, the change in input-output efficiency for the most efficient regions can be interpreted as $\mu_{l,r}$. For this parameter, we consider 5% increase between 2010 and 2050 for developping regions and 2.1% increase between 2010 and 2050 for developped regions.

The region leader is assumed to be the region with lowest input feed requirement in reference years. The leader therefore does not change with systems switchs even if some region may gain more than the reference year leader and become more efficient with system switches.

$$r_l^L = \mathrm{argmin}_r I_{r,l,0}^T \tag{2}$$

Regional efficiency target

We have a global target of feed efficiency:

$$\frac{\sum_r Q_{r,l}^T I_{r,l}^T}{\sum_r Q_{r,l}^T} = I_l^W \tag{3}$$

With $\sum_r Q_{r,l}^T = Q_l^W = D_l^W$ and $D_{r,l}^T$ known.

The target for each region is assumed to be the same relative change in efficiency with respect to the leader, the same efficiency gap relative closing. The remaining yield gap is noted η_l^W. $\eta_l^W = 0$ means no yield gap anymore in the future, $\eta_l^W = 1$ means an unchanged relative yield gap. $\forall r, l$

$$\frac{\frac{I_{r,l}^T}{I_{r_l^L,l}^T} - 1}{\frac{I_{r,l,0}^T}{I_{r_l^L,l,0}^T} - 1} = \eta_l^W \tag{4}$$

$$I_{r,l}^T = I_{r_l^L,l}^T \left(\eta_l^W \left(\frac{I_{r,l,0}^T}{I_{r_l^L,l,0}^T} - 1 \right) + 1 \right) \tag{5}$$

Note that I^T implicitely incorporates the change in “herd management” efficiency as described above.

Substituting (5) in (3) leads to

$$I_l^W = \frac{\sum_r Q_{r,l}^T I_{r_l^L,l}^T \left(\eta_l^W \left(\frac{I_{r,l,0}^T}{I_{r_l^L,l,0}^T} - 1 \right) + 1 \right)}{Q_l^W}$$

To determine the share of production, we consider that two determinant play a role. First, we consider current production market shares reflects relative competitivity taking into account not only efficiency but also health, infrastructures and transformation constraints. We also assume that increase in efficiency allows to expand market share proportionaly. The new market share is therefore along:

$$\frac{I^T_{r,l,0}}{I^T_{r,l}}\frac{Q^T_{r,l,0}}{Q^W_{l,0}}$$

Rescaling such that the total production is the total demand leads to the production:

$$Q^{T,m}_{r,l} = \frac{\frac{I^T_{r,l,0}}{I^T_{r,l}}\frac{Q^T_{r,l,0}}{Q^W_{l,0}}}{\sum_{r'}\frac{I^T_{r',l,0}}{I^T_{r',l}}\frac{Q^T_{r',l,0}}{Q^W_{l,0}}} D^W_l$$

Production and demand of some livestock product also tend to be located in nearby places, for many reasons such as transportation costs, use of animals as an asset to be sold on local markets, or requirement of freshness. We consider that this is the case of dairy products.

We consider that the initial ratio of production over demand is an indicator of this constraint strength:

$$Q^{T,D}_{r,l} = \frac{Q^T_{r,l,0}}{D^T_{r,l,0}} D^T_{r,l}$$

We consider that this demand change can only increase the produced quantity so we combine with the previous “market share” based production using a max, before combining the processes. We assume that both processes play a similar role, so we simply average the two resulting production quantities. Therefore, for dairy, the production is the average of the increases given by the two processes, and follows the “market share” evolution if the demand do not follow the “market share” increase in the region.

$$Q^{T,m+D}_{r,l} = \frac{\max(Q^{T,m}_{r,l}, Q^{T,D}_{r,l}) + Q^{T,m}_{r,l}}{2}$$

We again need to rescale such that total production is equal to total demand:

$$Q^T_{r,l} = \frac{Q^{T,m+D}_{r,l}}{\sum_{r'} Q^{T,m+D}_{r',l}} D^W_l$$

initial system	possible transitions
LA	LA+conc, MA, MA+conc
MA	MA+conc
LX	LX+conc, LX+past, MX, MX+conc, MX+past
MX	MX+conc, MX+past
O	O+conc

Table 1: System improvement possible transitions. X corresponds to T or H. +conc means with concentrate and +past with improved pastures, otherwise the system is the initial system of the reference year. O corresponds to both other and urban systems.

Regional scale improvements

System shares

We note $S^C_{s,l}$ the share of system s (a system like MA for mixed arid, LT for pastoral temperate or urban) for livestock sector l in "climate" C (A, T or H, O, O stands for "Other and urban" systems). We note $\hat{S}^r_{C,l}$ the share of climate C systems in region r. We note $\mathcal{O}_l$ the set of initial systems, and $\hat{\mathcal{O}}_{l,C}$ the set of initial systems in climate C.

Improved system in a climatic zone

In a first step, the possibilities of systems improvements within a climatic zone are determined. A first rule is that mixed systems cannot become pastoral systems, other and urban systems remain the same too. We also assume that in arid and other and urban systems, switching to systems with pasture improvement is not possible. We do not allow change to added concentrates when it leads to a quantity per head above the maximum in the reference year (1000 kg dry matter of grain per TLU for beef and 3000 kg dry matter of grain per TLU for dairy). There are many systems with added concentrate, the one with the best input output ratio is selected. With those rules, the possible transitions are listed in table 1.

Last, we consider that the transition to the best system in term of input output ratio is selected which means that we keep only one possible improved system per initial system. We note $\mathcal{I}(s)$ the improved system associated to initial system s.

We assume that the share of the initial mixed, urban and other systems that becomes improved is the same all over the region, and noted $x_{r,l}$. We also assume that the percentage of the initial pastoral systems that becomes improved is also the same all over the region, and that it is linked to $x_{r,l}$ by a proportional factor ψ less than one and common to all regions and livestock sectors. Indeed, we consider that pastoral systems are more remote such that it is more difficult to bring in concentrates or sell products in order to invest in concentrates or pastures improvements. Without more evidence, we set ψ to 0.5. With this value, if 70% of mixed and other systems are improved, 35% of pastoral systems will be improved too.

To simplify notation, we redefine ψ as a function of the system with $\psi(s) = 1$ for mixed and other systems and $\psi(s) = \psi$ for patoral systems. The actual change $\tilde{x}_{r,l,s}$ is bounded by the initial share of the system:

$$\tilde{x}_{r,l,s} = \min(S^C_{s,l}, \psi(s)x_{r,l})$$

Changes in climatic zones shares

Changes in climatic zones shares are based on rules on areas. We assume that arid zones can only be used for livestock, no forest nor cropland can possibly be established in those areas[1]. Irrigated areas may expand in arid zones to replace pastures, but this is not represented.

We note $G_{r,l,s} \equiv \frac{\hat{I}_{r,l,g,s,0}}{Y^g_{r,\mathcal{G}(s)}} \frac{1}{1+\mu_{l,r}}$ the grassland area requirement.

We note $\tilde{I}^C_{r,l,C}$ the grass requirement per climate and $\tilde{G}^C_{r,l,C}$ the grassland requirement per climate.

$$\begin{aligned}
\tilde{I}^C_{r,l,C} &= \frac{1}{1+\mu_{l,r}} \sum_{s\in\hat{O}_{l,C}} (S^C_{s,l} - \tilde{x}_{r,l,s})\hat{I}_{r,l,g,s,0} + \tilde{x}_{r,l,s}\hat{I}_{r,l,g,\mathcal{I}(s),0} \\
\tilde{G}^C_{r,l,C} &= \frac{1}{1+\mu_{l,r}} \sum_{s\in\hat{O}_{l,C}} (S^C_{s,l} - \tilde{x}_{r,l,s})\frac{\hat{I}_{r,l,g,s,0}}{Y^g_{r,\mathcal{G}(s)}} + \tilde{x}_{r,l,s}\frac{\hat{I}_{r,l,g,\mathcal{I}(s),0}}{Y^g_{r,\mathcal{G}(s)}} \\
&= \sum_{s\in\hat{O}_{l,C}} (S^C_{s,l} - \tilde{x}_{r,l,s})G_{r,l,s} + \tilde{x}_{r,l,s}G_{r,l,\mathcal{I}(s)}
\end{aligned}$$

In the following, we drop the r, l indices for clarity, as all the computations and assumptions are per livestock type and per region.

[1]Livestock pressure implicitly determines whether those areas corresponds more to natural grasslands or pasture lands, though this is also true for other climate zones.

The arid areas are constant per livestock type and region, therefore shares of arid zones are determined using:

$$\begin{aligned} A_{\mathrm{A},0} &= Q^T \hat{S}_{\mathrm{A}} \sum_{s \in \hat{\mathcal{O}}_{\mathrm{A}}} (S_s^{\mathrm{A}} - \tilde{x}_s) G_s + \tilde{x}_s G_{\mathcal{I}(s)} \\ &= Q^T \hat{S}_{\mathrm{A}} \tilde{G}_{\mathrm{A}}^C \end{aligned} \quad (6)$$

This sets the arid areas share:

$$\hat{S}_{\mathrm{A}} = \frac{A_{\mathrm{A},0}}{Q^T \tilde{G}_{\mathrm{A}}^C}$$

Other and urban system areas remain the same, for each livestock type:

$$A_{\mathrm{O},0} = Q^T \hat{S}_{\mathrm{O}} \sum_{s \in \hat{\mathcal{O}}_{\mathrm{O}}} (S_s^{\mathrm{O}} - \tilde{x}_s G_s) + \tilde{x}_s G_{\mathcal{I}(s)} \quad (7)$$

We assume that the relative proportion of humid and temperate areas remain the same, for each livestock type. Therefore,

$$\frac{A_{\mathrm{H},0}}{A_{\mathrm{T},0}} = \frac{Q^T \hat{S}_{\mathrm{H}} \sum_{s \in \hat{\mathcal{O}}_{\mathrm{H}}} (S_s^{\mathrm{H}} - \tilde{x}_s) G_s + \tilde{x}_s G_{\mathcal{I}(s)}}{Q^T \hat{S}_{\mathrm{T}} \sum_{s \in \hat{\mathcal{O}}_{\mathrm{T}}} (S_s^{\mathrm{T}} - \tilde{x}_s) G_s + \tilde{x}_s G_{\mathcal{I}(s)}}$$

This leads to:

$$\begin{aligned} \frac{A_{\mathrm{H},0}}{A_{\mathrm{T},0}} &= \frac{\hat{S}_{\mathrm{H}} \tilde{G}_{\mathrm{H}}^C}{\hat{S}_{\mathrm{T}} \tilde{G}_{\mathrm{T}}^C} \\ \hat{S}_{\mathrm{H}} &= \frac{A_{\mathrm{H},0}}{A_{\mathrm{T},0}} \frac{\tilde{G}_{\mathrm{T}}^C}{\tilde{G}_{\mathrm{H}}^C} \hat{S}_{\mathrm{T}} = \phi \hat{S}_{\mathrm{T}} \end{aligned} \quad (8)$$

with $\phi = \frac{A_{\mathrm{H},0}}{A_{\mathrm{T},0}} \frac{\tilde{G}_{\mathrm{T}}^C}{\tilde{G}_{\mathrm{H}}^C}$.

If either humid or temperate (or both) do not need grassland areas, the humid and temperate climate zones shares cannot be related through their grassland requirements. In that case we assume that their relative shares remain the same, and $\phi = \frac{\hat{S}_{\mathrm{H}0}}{\hat{S}_{\mathrm{T}0}}$:

$$\hat{S}_{\mathrm{H}} = \frac{\hat{S}_{\mathrm{H}0}}{\hat{S}_{\mathrm{T}0}} \hat{S}_{\mathrm{T}} = \phi \hat{S}_{\mathrm{T}}$$

If, for some climate zones with fixed areas, there is no need for grassland areas, the share cannot be obtained based on areas. We note $\mathcal{F}$ the set of climates with fixed areas and a need for grasslands, only those climate shares can be determined with the areas constraints.

For the climate zones with fixed areas without need for grasslands, we assume that the share relative to the sum of all climate types that do not require grassland or have no constraint on areas (temperate and humid) remain the same. We note $\mathcal{U}$ the set of climate zones with fixed areas without need for grassland ($\tilde{G}_C^C = 0$). We note $\mathcal{C}$ the set of climate zones that either do not need area or have non fixed areas ($\mathcal{U}$ plus temperate and humid).

The assumption is, $\forall C \in \mathcal{U}$:

$$\frac{\hat{S}_C}{\sum_{Z\in\mathcal{C}} \hat{S}_Z} = \frac{\hat{S}_{C,0}}{\sum_{Z\in\mathcal{C}} \hat{S}_{Z,0}} \tag{9}$$

Note that systems with fixed areas and no need for grassland areas cannot reach their area target, and some fixed areas will end up being unused.

This implies

$$\begin{aligned}
\frac{\sum_{Z\in\mathcal{U}} \hat{S}_Z}{\sum_{Z\in\mathcal{C}} \hat{S}_Z} &= \frac{\sum_{Z\in\mathcal{U}} \hat{S}_{Z,0}}{\sum_{Z\in\mathcal{C}} \hat{S}_{Z,0}} \\
\frac{\sum_{Z\in\mathcal{C}} \hat{S}_Z}{\sum_{Z\in\mathcal{U}} \hat{S}_Z} &= \frac{\sum_{Z\in\mathcal{C}} \hat{S}_{Z,0}}{\sum_{Z\in\mathcal{U}} \hat{S}_{Z,0}} \\
\frac{\sum_{Z\in\mathcal{C}} \hat{S}_Z}{\sum_{Z\in\mathcal{U}} \hat{S}_Z} - 1 &= \frac{\sum_{Z\in\mathcal{C}} \hat{S}_{Z,0}}{\sum_{Z\in\mathcal{U}} \hat{S}_{Z,0}} - 1 \\
\frac{\sum_{Z\in\mathcal{C}\setminus\mathcal{U}} \hat{S}_Z}{\sum_{Z\in\mathcal{U}} \hat{S}_Z} &= \frac{\sum_{Z\in\mathcal{C}\setminus\mathcal{U}} \hat{S}_{Z,0}}{\sum_{Z\in\mathcal{U}} \hat{S}_{Z,0}} \\
\sum_{Z\in\mathcal{U}} \hat{S}_Z &= (\hat{S}_{\mathrm{T}} + \hat{S}_{\mathrm{H}}) \frac{\sum_{Z\in\mathcal{U}} \hat{S}_{Z,0}}{\sum_{Z\in\mathcal{C}\setminus\mathcal{U}} \hat{S}_{Z,0}} \\
&= \hat{S}_{\mathrm{T}}(1+\phi)\xi
\end{aligned}$$

with $\xi = \frac{\sum_{Z\in\mathcal{U}} \hat{S}_{Z,0}}{\sum_{Z\in\mathcal{C}\setminus\mathcal{U}} \hat{S}_{Z,0}}$.

The constraint on shares of climate types reads, for all livestock type and region:

$$\sum_{C\in\{\mathrm{A,H,T,O}\}} \hat{S}_C = 1 \tag{10}$$

which can be solved in $\hat{S}_{\mathrm{T}}$ as

$$
\begin{aligned}
1 &= \sum_{F\in\mathcal{F}} \frac{A_{F,0}}{Q^T \tilde{G}^C_F} + \xi \hat{S}_{\mathrm{T}}(1+\phi) + \hat{S}_{\mathrm{T}}(1+\phi) \\
\hat{S}_{\mathrm{T}} &= \frac{1 - \frac{1}{Q^T}\sum_{F\in\mathcal{F}} \frac{A_{F,0}}{\tilde{G}^C_F}}{(1+\phi)(1+\xi)}
\end{aligned}
$$

Changes in climatic zones shares with a regional pasture yield target

The shares can be determined by solving the preceding equations in S when assuming predetermined pasture yields. However, we want also to find a solution for which the change in pasture yield is specified, by combining what comes from system switches and a pasture yield change coefficient applied on the whole region and sector. Past world scale yearly pasture yield changes are used to determine the regional yield change. Each region pasture yield is assumed to increase by the same relative yearly linear factor. We note $Y^{g,T}_{r,l}$ the target pasture yield over the region.

We note $\eta_{r,l}$ the pasture yield efficiency change factor applied over all the systems. In the following, we again drop the r and l indices for more clarity.

The pasture yield target reads

$$
Y^{g,T} = \frac{Q^T \sum_{C\in\{\mathrm{A,H,T,O}\}} \hat{S}_C \tilde{I}^C_C}{Q^T \eta \sum_{C\in\{\mathrm{A,H,T,O}\}} \hat{S}_C \tilde{G}^C_C} \tag{11}
$$

(6) and (7) become, $\forall C \in \mathcal{F}$:

$$
A_{C,0} = Q^T \hat{S}_C \eta \tilde{G}^C_C
$$

(8) is not really changed:

$$
\frac{A_{\mathrm{H},0}}{A_{\mathrm{T},0}} = \frac{\eta \hat{S}_{\mathrm{H}} \tilde{G}^C_{\mathrm{H}}}{\eta \hat{S}_{\mathrm{T}} \tilde{G}^C_{\mathrm{T}}} \tag{12}
$$

In the following, we also drop the 0 index for fixed areas. Substituting in (10) and (12) leads to

$$
\begin{aligned}
1 &= \sum_{\mathcal{F}} \frac{A_C}{Q^T \eta \tilde{G}_C^C} + \hat{S}_{\mathrm{T}}(1+\phi)(1+\xi) \\
\hat{S}_{\mathrm{T}} &= \frac{1 - \frac{1}{\eta Q^T} \sum_{\mathcal{F}} \frac{A_C}{\tilde{G}_C^C}}{(1+\phi)(1+\xi)} \\
\hat{S}_{\mathrm{H}} &= \frac{1 - \frac{1}{\eta Q^T} \sum_{\mathcal{F}} \frac{A_C}{\tilde{G}_C^C}}{(1+\phi)(1+\xi)} \phi
\end{aligned}
$$

Substituting in (11) leads to:

$$
Y^{g,T} = \frac{\sum_{\mathcal{F}} \frac{A_C \tilde{I}_C^C}{\eta \tilde{G}_C^C} + Q^T \left(1 - \frac{1}{\eta Q^T} \sum_{\mathcal{F}} \frac{A_C}{\tilde{G}_C^C}\right) \frac{\tilde{I}_{\mathrm{T}}^C + \tilde{I}_{\mathrm{H}}^C \phi}{(1+\phi)(1+\xi)}}{\sum_{\mathcal{F}} A_C + \eta Q^T \frac{\tilde{G}_{\mathrm{T}}^C + \tilde{G}_{\mathrm{H}}^C \phi}{(1+\phi)(1+\xi)} \left(1 - \frac{1}{\eta Q^T} \sum_{\mathcal{F}} \frac{A_C}{\tilde{G}_C^C}\right)} \tag{13}
$$

If there are only fixed areas, the target yield cannot be achieved while at the same time using exactly the fixed areas. The value of η and $\hat{S}_C$ (sum not equal to 1) that give the target yield are computed as:

$$
\begin{aligned}
Y^{g,T} &= \frac{\sum_{\mathcal{F}} \frac{A_C \tilde{I}_C^C}{\eta \tilde{G}_C^C}}{\sum_{\mathcal{F}} A_C} \\
\eta &= \frac{\sum_{\mathcal{F}} \frac{A_C \tilde{I}_C^C}{\tilde{G}_C^C}}{Y^{g,T} \sum_{\mathcal{F}} A_C} \\
\hat{S}_C &= \frac{1}{\eta Q^T} \frac{A_C}{\tilde{G}_C^C}
\end{aligned}
$$

The value, noted $\hat{\eta}$, that satisfies the constraint on the areas when there are only fixed areas is:

$$
\hat{\eta} = \frac{1}{Q^T} \sum_{\mathcal{F}} \frac{A_C}{\tilde{G}_C^C}
$$

If there are only fixed areas systems, we use $\hat{\eta}$ and thus cannot stick to the yield target. When there are non fixed areas, $\hat{\eta}$ is the inferior limit, for this value there are only fixed areas, and η cannot be lower.

If there are non fixed areas, the yield target (13) leads to a quadratic equation for η, with coefficients

$$\begin{array}{ll}
0 & -\sum_{\mathcal{F}} \frac{A_C \tilde{I}_C^C}{\tilde{G}_C^C} + \frac{\tilde{I}_{\mathrm{T}}^C + \tilde{I}_{\mathrm{H}}^C \phi}{(1+\phi)(1+\xi)} \sum_{\mathcal{F}} \frac{A_C}{\tilde{G}_C^C} \\
1 & Y^{g,T} \sum_{\mathcal{F}} A_C - Y^{g,T} \frac{\tilde{G}_{\mathrm{T}}^C + \tilde{G}_{\mathrm{H}}^C \phi}{(1+\phi)(1+\xi)} \sum_{\mathcal{F}} \frac{A_C}{\tilde{G}_C^C} - Q^T \frac{\tilde{I}_{\mathrm{T}}^C + \tilde{I}_{\mathrm{H}}^C \phi}{(1+\phi)(1+\xi)} \\
2 & Y^{g,T} Q^T \frac{\tilde{G}_{\mathrm{T}}^C + \tilde{G}_{\mathrm{H}}^C \phi}{(1+\phi)(1+\xi)}
\end{array}$$

If there is only one type of pasture among T and H:

$$\begin{aligned}
\hat{S}_{\mathrm{T}} &= \frac{1 - \frac{1}{\eta Q^T} \sum_{\mathcal{F}} \frac{A_C}{\tilde{G}_C^C}}{1+\xi} \\
Y^{g,T} &= \frac{\sum_{\mathcal{F}} \frac{A_C \tilde{I}_C^C}{\eta \tilde{G}_C^C} + Q^T \left(1 - \frac{1}{\eta Q^T} \sum_{\mathcal{F}} \frac{A_C}{\tilde{G}_C^C}\right) \frac{\tilde{I}_{\mathrm{T}}^C}{1+\xi}}{\sum_{\mathcal{F}} A_C + \eta Q^T \left(1 - \frac{1}{\eta Q} \sum_{\mathcal{F}} \frac{A_C}{\tilde{G}_C^C}\right) \frac{\tilde{G}_{\mathrm{T}}^C}{1+\xi}}
\end{aligned} \tag{14}$$

This case is the same as above with ϕ set to 0.

If there are two positive solutions, the smallest is used. This problem may not lead to a valid solution, if there is no solution or only negative solution. It may also lead to a solution with negative shares for humid or temperate zones if the solution is $< \hat{\eta}$.

To be able to obtain a solution in those cases, it is possible to relax the constraint that the pasture efficiency change factor is the same for all the systems. We still consider that fixed area systems have the same factor, η while the non fixed area systems have are characterised by a different factor, η'. Solving η' as a function of η, in order to reach the target yield gives:

$$\eta' = \frac{\sum_{\mathcal{F}} \frac{A_C \tilde{I}_C^C}{\eta \tilde{G}_C^C} + Q^T \left(1 - \frac{1}{\eta Q^T} \sum_{\mathcal{F}} \frac{A_C}{\tilde{G}_C^C}\right) \frac{\tilde{I}_{\mathrm{T}}^C + \tilde{I}_{\mathrm{H}}^C \phi}{(1+\phi)(1+\xi)} - Y^{g,T} \sum_{\mathcal{F}} A_C}{Y^{g,T} Q^T \frac{\tilde{G}_{\mathrm{T}}^C + \tilde{G}_{\mathrm{H}}^C \phi}{(1+\phi)(1+\xi)} \left(1 - \frac{1}{\eta Q^T} \sum_{\mathcal{F}} \frac{A_C}{\tilde{G}_C^C}\right)}$$

Based on this relationship, we try to minimize the ratio between η and $\eta'(\eta)$:

$$\min_{\eta} \frac{\max(\eta, \eta'(\eta))}{\min(\eta, \eta'(\eta))} - 1$$

If the ratio of the two factors is > 4, or if even with $\eta \to \infty$ and $\eta' = 0$ we do not reach the yield target, we set η to $2\hat{\eta}$ (equivalently fixed/non fixed systems have half of the production shares) and $\eta' = 4\eta$.

The situation in which the solution can be found that way is actually quite rare. In general not having a solution for the problem in the first place corresponds to a case in which fixed areas systems can satisfy all the demand, but with a lower yield than the target. Adding non fixed areas in general tend to decrease even more the grass need and therefore the yield. It is only when grass demand increases with η that the computations above can help finding a solution with the target yield reached.

In the remaining cases, the quadratic error between the recomputed yield and the target yield is minimized to determine η. To avoid negative shares, $\hat{\eta}$ is used as a lower bound for η.

This approach allows to specify regional pasture apparent yields. Even if all the regions pasture yield increase is the same, however, the global yield change may be different, as the change in production regional shares could favor regions that are more or less efficient to begin with and this composition effect could go in any direction.

Constraint on overall efficiency in the region

The constraint on overall efficiency in the region is, $\forall l$:

$$I^T_{r,l} = \frac{1}{1+\mu_{l,r}} \sum_{C \in \{\mathrm{A,H,T,O}\}} \hat{S}^r_{C,l} \sum_{s \in \hat{\mathcal{O}}_{l,C}} (S^C_{s,l} - \tilde{x}_{r,l,s}) \sum_f \hat{I}_{r,l,f,s,0} + \tilde{x}_{r,l,s} \sum_f \hat{I}_{r,l,f,\mathcal{I}(s),0} \tag{15}$$

Non specified sectors

There are no improved systems nor efficiency targets for the small ruminant meat sector. We use the beef sector changes to determine the small ruminant efficiency target and efficiency change. We consider the same efficiency target as beef, and apply the same changes in efficiency per climate type as what is obtained for beef. The regional yield targets and quantities produced are determined as for other sectors.

For each livestock sector non specified, there is a specified livestock sector $K(l)$, in practice beef for small ruminant meat. We assume

$$\tilde{G}^C_{C,l} = \frac{\tilde{I}^C_{C,K(l)}}{\tilde{I}^C_{C,K(l),0}}\tilde{G}^C_{C,l,0}$$

$$\tilde{I}^C_{C,l} = \frac{\tilde{I}^C_{C,K(l)}}{\tilde{I}^C_{C,K(l),0}}\tilde{I}^C_{C,l,0}$$

We also use the area shares rules to determine the change in shares.

The final efficiency change has no reason to be equal to the target, though it could be similar given all the efficiency changes come from the same sector. Since the efficiency target is only used to determine the regional production in that case this is not problematic.

Solving the system

The share of climate types are determined by (7) and (6) for climates types with fixed areas. The two remaining constraints on shares (8) and (10) allow to determine the share of the two other climate types. $x_{r,l}$ can then be determined by the constraint on overall efficiency (15).

This program may not have a solution, in case switching completly systems do not allow to reach the target. In that case, the resolution is restarted without the constrained regions, the constrained regions are set to their maximum efficiency change with complete switching.

References

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