How synchronized human networks escape local minima: supplemental materials

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ABSTRACT

We studied the dynamics of complex human networks and observed that humans have different methods to avoid local minima than other networks. Humans can change the coupling strength between them or change their tempo. This leads to different dynamics than other networks and makes human networks more robust and better resilient against perturbations. We observed high-order vortex states, oscillation death, amplitude death, and phase discontinuity, due to the unique dynamics of the network. In this document, we present the results on different network sizes and the detailed analytical derivations of our models.

1 Analytical models

1.1 Derivation of the tempo slowing down

We model the tempo slowing down due to the delay from the Kuramoto model. To analyze the player dynamics, we start with the Kuramoto model for N coupled oscillators with delayed coupling. We denote each oscillator's phase as a time function as $\varphi_n(t)$. Its dynamic is determent by:

$$\frac{\partial \varphi_n(t)}{\partial t} = \omega_n + \kappa \sin(\varphi_{n+1}(t-\Delta t) - \varphi_n(t)), \tag{1}$$

where the frequency of the oscillator is ω_n, the coupling strength is κ, and we assume periodic boundary conditions, as $\varphi_{N+1} = \varphi_1$. We evaluate the phase difference between each adjacent oscillator as

$$\Delta\varphi_n(t) = \varphi_{n+1}(t) - \varphi_n(t) \tag{2}$$

so:

$$\sum_{n=1}^{N} \Delta\varphi_n = 0. \tag{3}$$

We assume that all the oscillators are equally separated in the phase-space, so $\Delta\varphi_n = \Delta\varphi$, for all $n < N$, and $\Delta\varphi_N = -(N-1)\Delta\varphi$. From the equation for $\Delta\varphi_{N-1}$, which is the end of the ring, we obtain:

$$\frac{\partial \Delta\varphi_{N-1}(t)}{\partial t} = \frac{\partial \Delta\varphi(t)}{\partial t} = \Omega_{N-1} + \kappa \sin(\varphi_1(t-\Delta t) - \varphi_N(t)) - \kappa \sin(\varphi_N(t-\Delta t) - \varphi_{N-1}(t)), \tag{4}$$

where $\Omega_n = \omega_{n+1} - \omega_n$. All the players are starting to play in-phase and therefore as long as the delay satisfies $\Delta t < T/N$, we assume $\varphi_n(t-\Delta t) \approx \varphi_n(t) - \Delta t \partial\varphi_n/\partial t$. Since we are close to phase locking, we approximate $\sin(\alpha) \approx \alpha$. We evaluate the phase difference as a function of time for $\Delta\varphi_{N-1}$, according to Eq. 8, to:

$$\frac{\partial \Delta\varphi(t)}{\partial t} = \frac{\Omega - N\kappa\Delta\varphi}{1-(N-1)\kappa\Delta t}, \tag{5}$$

From Eq. 5, we obtain that the phase-locking condition does not change when introducing a small coupling delay between the players, so, the system remains phase-locked. However, when considering the average phase as a function of time $\varphi = \sum \varphi_n / N$, we obtain:

$$\frac{\partial \varphi(t)}{\partial t} = \frac{\omega}{1 + N\kappa\Delta t}, \tag{6}$$

where $\omega = \sum \omega_n / N$. Thus, the tempo of the coupled oscillators slows down as long as the players stay phase locked. This tempo slowing down insure that the condition of $\Delta t < T/N$ is satisfied indicating that our assumptions are valid.

1.2 Derivation of the effective potential

We calculate the effective potential of the system. Here we assume, a large phase difference, and that when the system reaches a solution, the phases of all the oscillators are changing linearly in time with about the same tempo, so all the phases follow:

$$\varphi_n(t) \approx \phi_n + \omega t. \tag{7}$$

where ϕ_n is the relative phase between the players. Placing Eq. 7 into Eq. 4 gives:

$$\frac{\partial \Delta\varphi(t)}{\partial t} = \Omega_n + \kappa \sin(-(N-1)\Delta\varphi - \omega\Delta t) - \kappa \sin(-\Delta\varphi - \omega\Delta t), \tag{8}$$

For solving the dynamic of the system, we define an effective potential V, as:

$$\frac{\partial \Delta\varphi(t)}{\partial t} = -\frac{\partial V(\Delta\varphi)}{\partial \Delta\varphi}. \tag{9}$$

where:

$$V(\Delta\varphi) = -\Omega_n \Delta\varphi - \frac{\kappa}{N-1}\cos((N-1)\Delta\varphi + \omega\Delta t) + \kappa \cos(\Delta\varphi + \omega\Delta t), \tag{10}$$

Calculating the potential reveals that for $\Delta t = 0$ there is a global minimum when $\Delta\varphi = 0$, so all the oscillators have the same frequency and phase, namely, the in-phase state of synchronization. However, when the delay increases beyond $\omega\Delta t > \pi/(2N)$, the $\Delta\varphi = 0$ becomes a local minimum and a new global minimum appears in $\Delta\varphi = -\pi/2$, namely, the vortex state of synchronization.

2 Experimental results

we study the synchronization dynamics of human networks with closed-ring topology and unidirectional time-delayed coupling. Such a network has a complex potential landscape with well-defined local and global minima that we can tune and control in real time. We prepare the system in a global minimum (fully synchronized) state and then adiabatically transform the potential landscape by tuning the coupling delay time such that this state becomes a local minimum. We then study in detail how the system escapes this local minimum into the new fully synchronized global minimum by measuring the amplitude, tempo, and phase of each node and identifying which node is following which. In this study, we set the players' network as a unidirectional ring, so that each player hears only a single neighbor with periodic boundary conditions.

2.1 Spreading of the phase

The first dynamic we present is when the players are spreading their phases to escape from the in-phase local minimum into a vortex-phase state global minimum. This dynamic is enabled by the players' ability to reduce their coupling until they find a stable state. The phase results as a function of time for networks with $N = 4, 5, 7,$ and 8 players are shown in Fig. 1. As shown, in all network sizes, we observe a phase locking in the beginning, where all the players have the same phase at the same time, so all the curves coincide, or at least parallel to each other. Then, at least one curve is leaving all the other curves. Then, all the different curves spread, until they spread over 2π.

2.2 Slowing down of the tempo and oscillations death

The second dynamic we observe is the slowing down of the players' tempo as an alternative strategy to maintain a stable in-phase synchronization state in the presence of delayed coupling. The measured phase results as a function of time for networks with $N = 3$ to 8 coupled players are shown in Fig. 2. From these results, we evaluate the players' tempo by calculating the average derivative of the phase. When the tempo slows too much, the players get stuck in a state of oscillation death, namely, all the players are playing the same note indefinitely, thereby maintaining a degenerate form of synchronization.

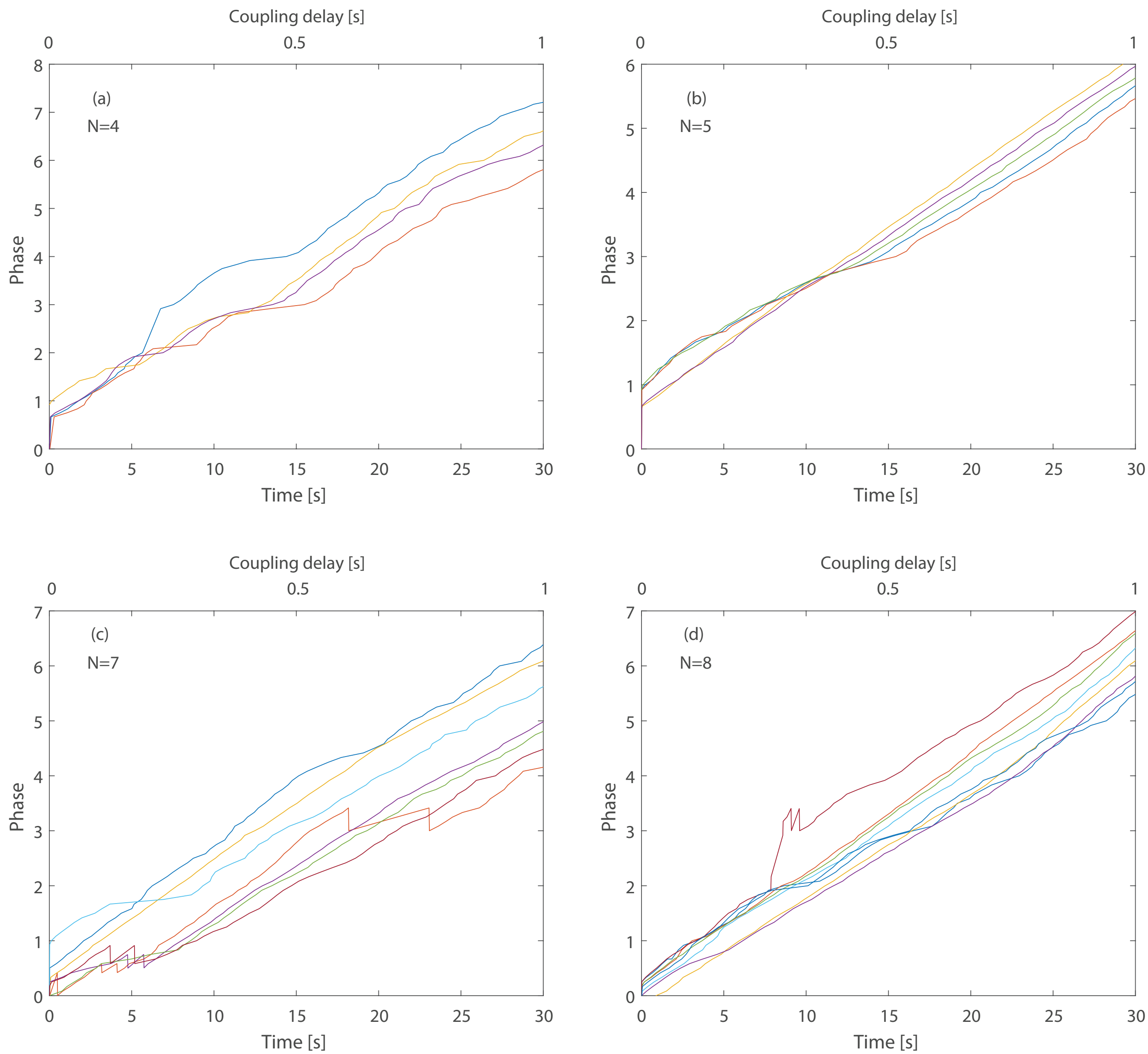

Figure 1. Coupled violin players situated on a ring with unidirectional coupling showing how the phase between the players is spread when at least one of the players reduces its coupling compared to the others.

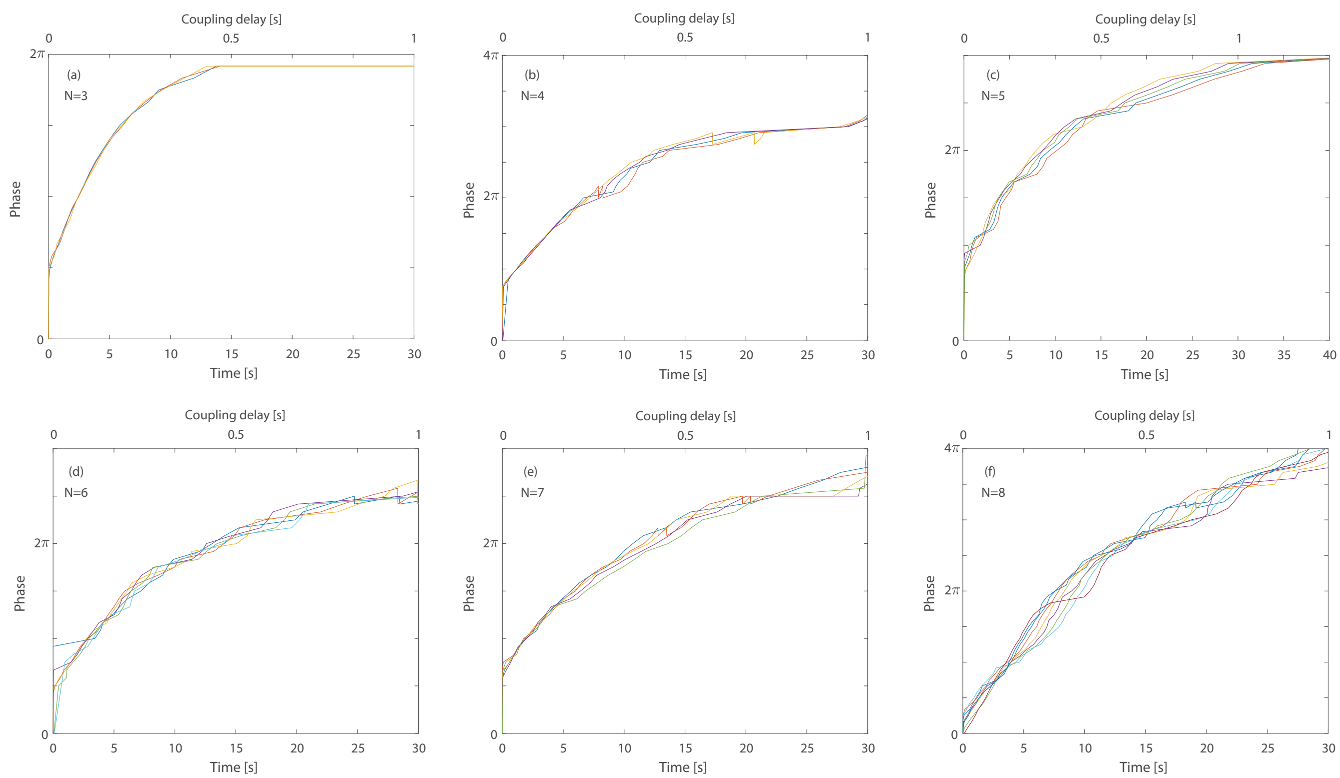

Figure 2. Coupled violin players situated on a ring with unidirectional coupling showing how the tempo slows down until reaching a state of oscillation death where all the players are playing the same note indefinitely.