SUPPLEMENTARY MATERIAL

Accelerated Drift in Age Structured Populations: Implications for Mutation Accumulation

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In this Supplementary Material we give details of the numerical determination of the numbers of individuals in all age classes, when they have their equilibrium expected values. That is, these numbers have no fluctuations/time dependence. We shall write the equilibrium expected number of individuals in age class x simply as n_x (with $x = 1, 2, ..., w$). Henceforth, we shall refer to n_x just as the *number* of individuals in age class x.

Determination of age structure

We consider an age structured population, where there are K age classes, labelled $x = 1, 2, ..., K$. We use the abbreviation AC_x for age class x.

Time is denoted by t and takes the values $t = 0, 1, 2, ...$

We use

1. $n_x(t)$ to denote the number of individuals in AC_x at time t;
2. m_x to denote the number of offspring produced by each individual in AC_x
3. l_x to denote the probability an individual in AC_x survives to AC_{x+1}

The $n_x(t)$ are censused in the population after reproduction has occurred. We adopt the life cycle.

Description	**Process**	**Numbers**
individuals at time t		$n_x(t)$
	promotion to next age	$\downarrow$
post promotion individuals		$n_x^*(t)$
	reproduction	$\downarrow$
individuals at time $t+1$		$n_x(t+1)$

We use $\mathbf{n}(t)$ to denote a column vector containing the numbers in each age class at time t:

$$\mathbf{n}(t) = \begin{pmatrix} n_1(t) \\ n_2(t) \\ \vdots \\ n_K(t) \end{pmatrix} \tag{1}$$

and $\mathbf{n}(0)$ contains the initial numbers in each age class.

After promotion to the next age class, the number of individuals in AC_{x+1} is given by $l_x n_x(t)$. We write this as

$$n_{x+1}^*(t) = l_x n_x(t) \tag{2}$$

for $x = 1, 2, ..., K-1$. That is

$$\mathbf{n}^*(t) = \begin{pmatrix} 0 \\ l_1 n_1(t) \\ l_2 n_2(t) \\ \vdots \\ l_{K-1} n_{K-1}(t) \end{pmatrix}. \tag{3}$$

We write Eq. (3) as

$$\mathbf{n}^*(t) = \mathbf{B}\mathbf{n}(t) \tag{4}$$

where

$$\mathbf{B} = \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 \\ l_1 & 0 & & & \\ & l_2 & 0 & \ddots & \\ & & \ddots & 0 & \\ & & & l_{K-1} & 0 \end{pmatrix}. \tag{5}$$

Next, individuals reproduce and contribute to age class 1, where m_x is the number of offspring that each individual in age class x contributes to age class 1. We have

$$n_1(t+1) = \sum_{x=2}^{K} m_x n_x^*(t). \tag{6}$$

This is the number, after reproduction, in age class 1, at time $t+1$. For all other age classes ($x = 2, 3, .., K$) $n_x(t+1)$ is identical to $n_x^*(t)$. We thus have

$$\mathbf{n}(t+1) = \begin{pmatrix} \sum_{x=2}^{K} m_x n_x^*(t) \\ n_2^*(t) \\ \vdots \\ n_K^*(t) \end{pmatrix} \tag{7}$$

and we write this as

$$\mathbf{n}(t+1) = \mathbf{A}\mathbf{n}^*(t) \tag{8}$$

where

$$\mathbf{A} = \begin{pmatrix} 0 & m_2 & m_3 & \cdots & m_K \\ & 1 & & & \\ & & 1 & & \\ & & & \ddots & \\ & & & & 1 \end{pmatrix}. \tag{9}$$

We can then write the full dynamics as

$$\mathbf{n}(t+1) = \mathbf{L}\mathbf{n}(t) \tag{10}$$

with

$$\mathbf{L} = \mathbf{A}\mathbf{B}. \tag{11}$$

To numerically implement the behaviour in Eq. (10), we start with an arbitrary $\mathbf{n}(0)$, with $\sum_{x=1}^{K} n_x(0) = 1$. We iterate Eq. (10) according to

$$\mathbf{n}(t+1) = \frac{\mathbf{L}\mathbf{n}(t)}{\sum_{x=1}^{K} [\mathbf{L}\mathbf{n}(t)]_x} \tag{12}$$

to obtain a solution that has the property $\sum_{x=1}^{K} n_x(t) = 1$ for all t. Once $\mathbf{n}(t)$ converges to $\mathbf{n}(\infty)$, we take the equilibrium solution to be

$$\hat{\mathbf{n}} = \mathbf{n}(\infty) \times N \tag{13}$$

where N is the total equilibrium population size.