

An Accumulation Rate Curve Estimator for Total Species

Supplementary Material

Konstantin Shestopaloff^{1,2,3*}

1. Big Data Institute, University of Oxford, Oxford, United Kingdom

2. Department of Statistics, University of Oxford, Oxford, United Kingdom

3. Dalla Lana School of Public Health, University of Toronto, Toronto, Canada

* Corresponding Author: konstantin.shestopaloff@utoronto.ca

List of Figures:

Figure S1A: Simulation Results - Bias and MSE by Scenario for Best Methods (ARC and MDCC)

Figure S1B: Simulation Results - Bias and MSE by Scenario for Best Methods (EM and Jackknife)

Figure S2: Simulation Results - Individual Curve Bias

Figure S3: Weighted Point Estimate Optimization - Weight Cutoffs

Figure S4: Applied Example 2 - MSE by Year

List of Tables:

Table S1: Simulation Results - Bias

Table S2: Simulation Results - MSE

Table S3: Simulation Results - CI Coverage by Scenario

Table S4: Simulation Results - CI Width by Scenario

Table S5: Absolute Bias and MSE for Point Estimate Regularization by Scenario

Table S6: MSE for Count Covariance by Scenario

Figures

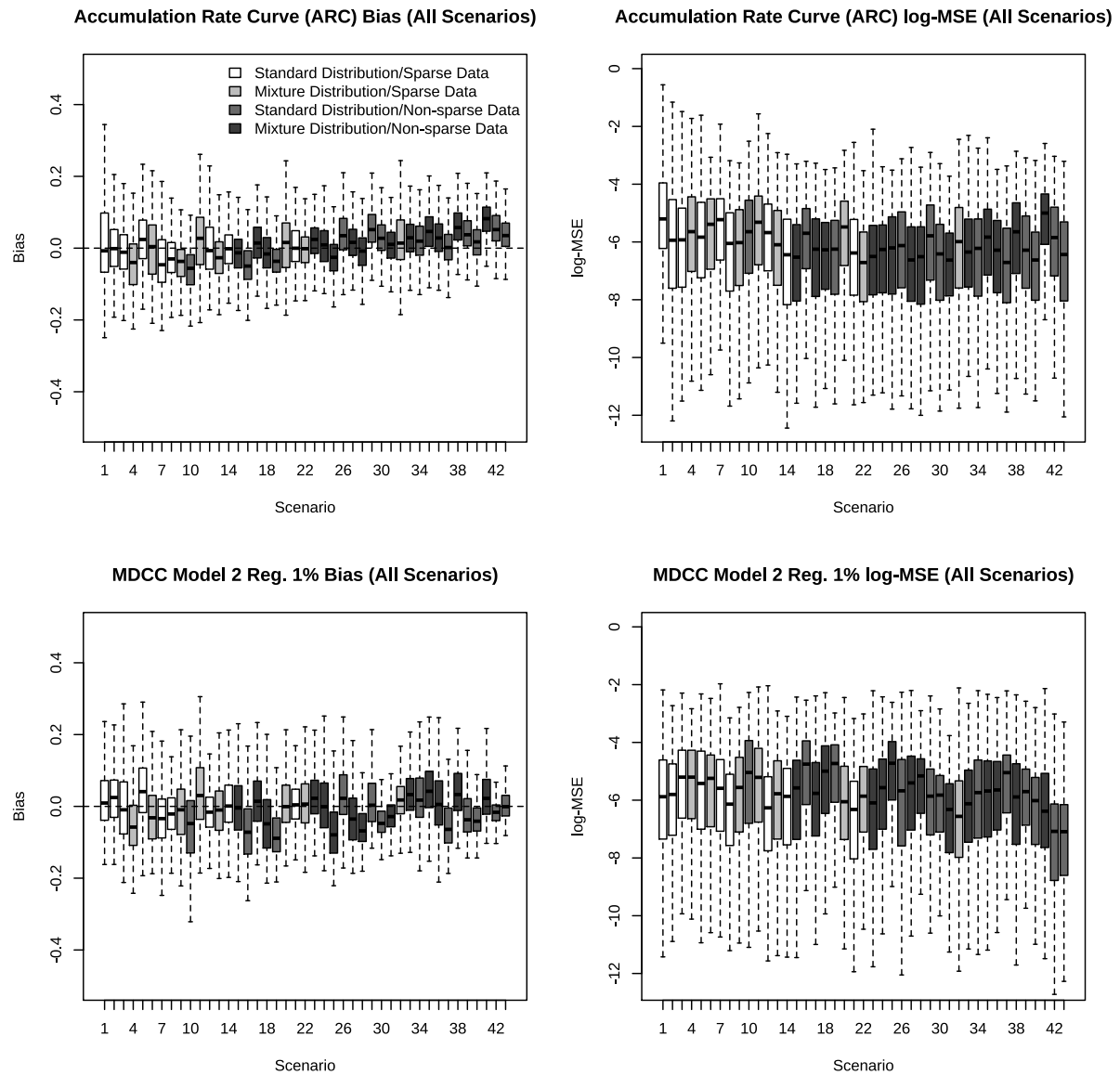


Figure S1A: Simulation Results - Bias and MSE by Scenario for Best Methods: The relative bias, $(\hat{S} - S)/S$, (left panels) and MSE, $[(\hat{S} - S)/S]^2$, (right panels), across all iterates within each of the scenarios, coloured by scenario group. Results are presented for the proposed ARC method (top) and MDCC (bottom). Results for exponential mixture and jackknife are presented in Figure S1B.

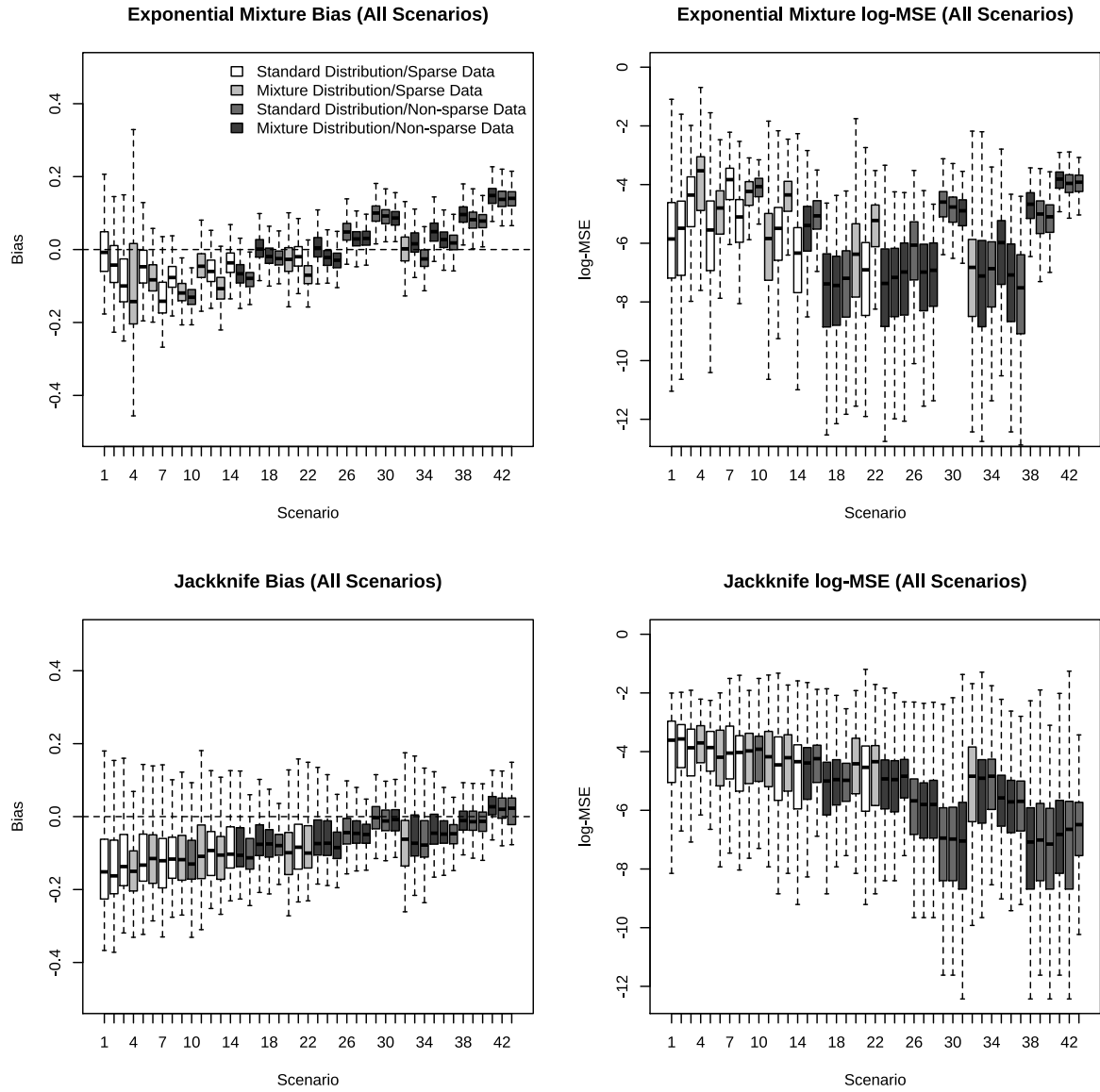


Figure S1B: Simulation Results - Bias and MSE by Scenario for Best Methods: The relative bias, $(\hat{S} - S)/S$, (left panels) and MSE, $[(\hat{S} - S)/S]^2$, (right panels), across all iterates within each of the scenarios, coloured by scenario group. Results are presented for the exponential mixture (top) and jackknife (bottom). Results for ARC and MDCC are presented in Figure S1A.

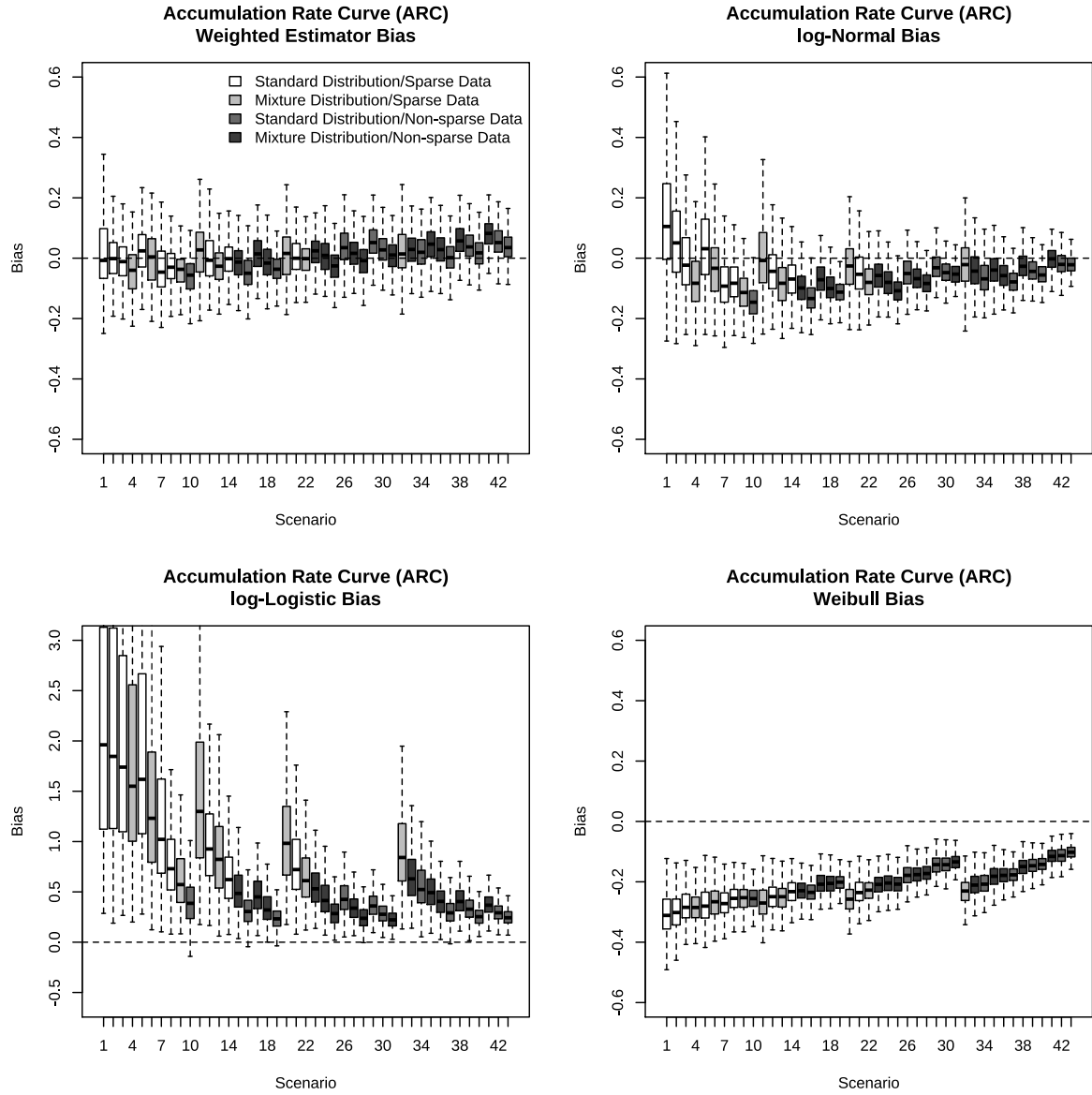


Figure S2: The relative bias, $(\hat{S} - S)/S$, of the weighted estimator and the individual log-normal, log-logistic and Weibull curves, across all iterates within each of the scenarios, coloured by scenario group.

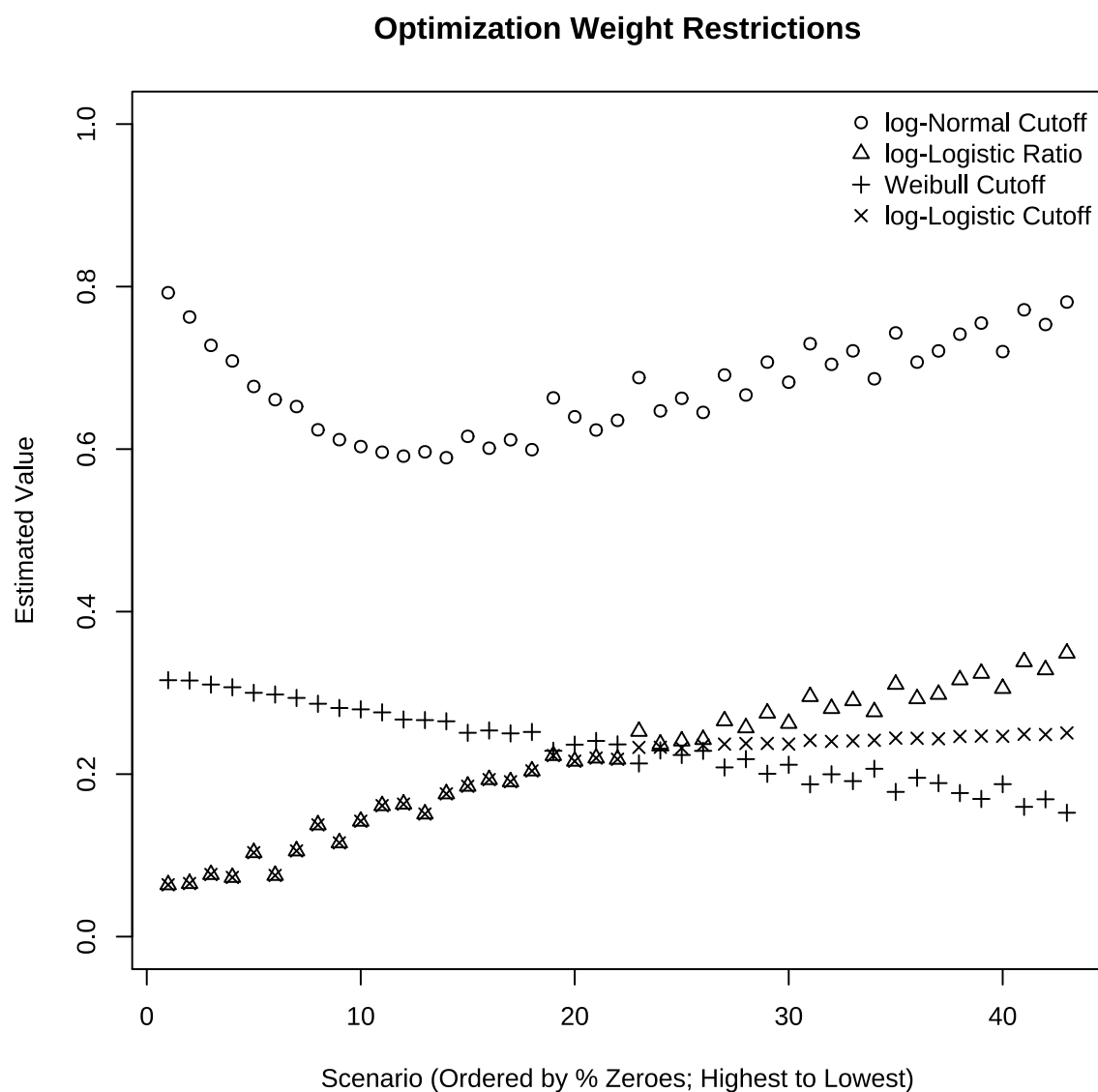


Figure S2: Weighted Point Estimate Optimization - Weight Cutoffs: The average upper bounds on the weight cutoffs for each of the parametric curves; log-normal (circle), log-logistic (cross) and Weibull (plus). The log-logistic ratio (triangle), prior to averaging with the Weibull to obtain the cutoff, is also included. The plot is ordered by the proportion of zeros in each scenario.

Applied Example 2: Catalogue Size - MSE by Year

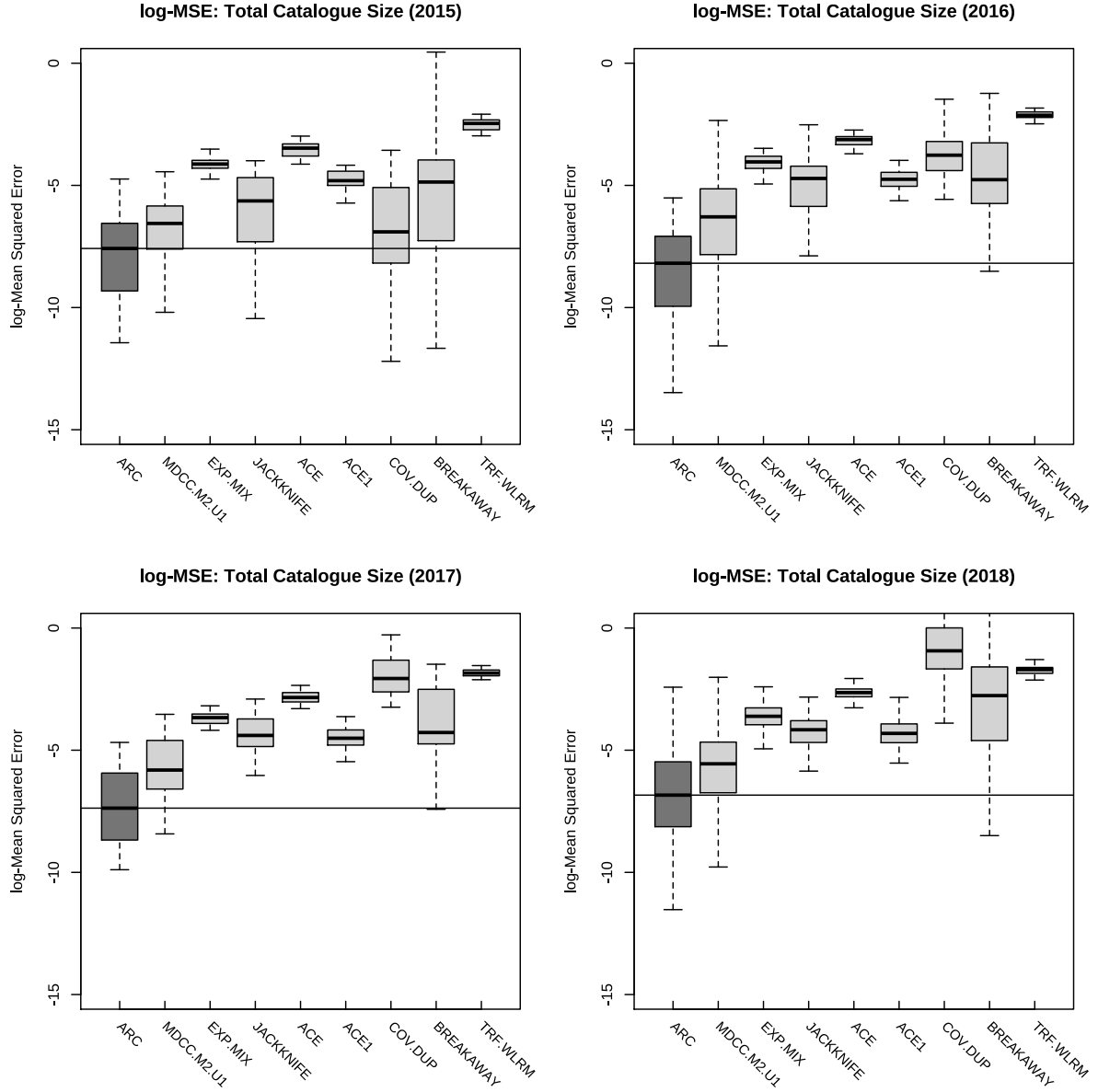


Figure S3: Applied Example 2 – log-MSE by Year: The log-MSE of catalogue size estimates, $\log[(\hat{S} - S)/S]^2$, across datasets in each of the four years. The proposed ARC method is dark grey and comparison methods are light grey. The solid line is the median MSE of the best method.

Tables

Scn.	% Zero	Data	Density	G(2,1)	G(1,1)	G(1,2)	G(1,3)	Bias (SD): ARC	Bias (SD): MDCC.M2.U1	Bias (SD): EXP.MIX	Bias (SD): JACKKNIFE
1	0.67	Sparse	Standard	1.0	0.0	0.0	0.0	0.04 (0.17)	0.0207 (0.089)	-0.0284 (0.145)	-0.1343 (0.122)
2	0.63	Sparse	Standard	0.8	0.2	0.0	0.0	0.0193 (0.117)	0.0233 (0.076)	-0.0327 (0.102)	-0.1258 (0.123)
3	0.58	Sparse	Standard	0.6	0.4	0.0	0.0	-0.0058 (0.096)	-0.0013 (0.103)	-0.07 (0.113)	-0.1107 (0.122)
4	0.56	Sparse	Mixture	0.8	0.0	0.2	0.0	-0.0381 (0.088)	-0.0489 (0.084)	-0.0635 (0.372)	-0.1271 (0.121)
5	0.60	Sparse	Standard	0.4	0.6	0.0	0.0	0.0294 (0.097)	0.046 (0.09)	-0.0357 (0.095)	-0.0982 (0.126)
6	0.55	Sparse	Mixture	0.8	0.0	0.0	0.2	-0.0039 (0.086)	-0.0203 (0.091)	-0.0609 (0.088)	-0.0957 (0.12)
7	0.53	Sparse	Standard	0.6	0.2	0.2	0.0	-0.0324 (0.086)	-0.0302 (0.083)	-0.1036 (0.142)	-0.1062 (0.123)
8	0.50	Sparse	Standard	0.2	0.8	0.0	0.0	-0.0215 (0.068)	-0.0212 (0.064)	-0.0711 (0.054)	-0.0969 (0.11)
9	0.47	Sparse	Mixture	0.6	0.2	0.0	0.2	-0.0386 (0.061)	-0.0143 (0.086)	-0.1083 (0.061)	-0.0889 (0.122)
10	0.45	Dense	Standard	0.4	0.4	0.2	0.0	-0.061 (0.06)	-0.0515 (0.1)	-0.1295 (0.033)	-0.0996 (0.114)
11	0.57	Sparse	Mixture	0.6	0.0	0.4	0.0	0.0206 (0.093)	0.0378 (0.099)	-0.0381 (0.07)	-0.0825 (0.132)
12	0.52	Sparse	Standard	0.0	1.0	0.0	0.0	0.0043 (0.083)	-0.012 (0.066)	-0.0521 (0.065)	-0.0789 (0.12)
13	0.49	Sparse	Mixture	0.4	0.4	0.0	0.2	-0.0208 (0.073)	-0.0099 (0.083)	-0.09 (0.087)	-0.0938 (0.11)
14	0.47	Sparse	Standard	0.2	0.6	0.2	0.0	-0.0013 (0.063)	0.0043 (0.072)	-0.033 (0.05)	-0.0702 (0.112)
15	0.44	Dense	Mixture	0.6	0.0	0.2	0.2	-0.0147 (0.058)	-0.0027 (0.089)	-0.0638 (0.046)	-0.0747 (0.103)
16	0.42	Dense	Standard	0.4	0.2	0.4	0.0	-0.0461 (0.06)	-0.0617 (0.098)	-0.0804 (0.032)	-0.0823 (0.108)
17	0.42	Dense	Mixture	0.2	0.6	0.0	0.2	0.0155 (0.061)	0.0174 (0.081)	0.0024 (0.037)	-0.0528 (0.088)
18	0.39	Dense	Mixture	0.6	0.0	0.0	0.4	-0.0121 (0.058)	-0.0398 (0.095)	-0.0173 (0.031)	-0.0529 (0.099)
19	0.37	Dense	Standard	0.0	0.8	0.2	0.0	-0.0339 (0.051)	-0.0732 (0.072)	-0.0234 (0.029)	-0.0596 (0.092)
20	0.53	Sparse	Mixture	0.4	0.2	0.2	0.2	0.0125 (0.086)	0.0131 (0.083)	-0.0217 (0.06)	-0.0832 (0.111)
21	0.48	Sparse	Standard	0.2	0.4	0.4	0.0	0.0093 (0.071)	0.0079 (0.06)	-0.0193 (0.042)	-0.0661 (0.106)
22	0.46	Sparse	Mixture	0.0	0.8	0.0	0.2	-0.0025 (0.053)	0.0095 (0.075)	-0.0642 (0.049)	-0.0664 (0.111)
23	0.43	Dense	Mixture	0.4	0.2	0.0	0.4	0.0218 (0.057)	0.0271 (0.071)	0.0074 (0.04)	-0.0425 (0.109)
24	0.41	Dense	Mixture	0.4	0.0	0.6	0.0	0.0076 (0.057)	0.0084 (0.091)	-0.0221 (0.034)	-0.044 (0.105)
25	0.38	Dense	Mixture	0.2	0.4	0.2	0.2	-0.0236 (0.056)	-0.0625 (0.087)	-0.0293 (0.03)	-0.0626 (0.094)
26	0.38	Dense	Standard	0.0	0.6	0.4	0.0	0.0396 (0.061)	0.0348 (0.084)	0.0494 (0.034)	-0.0231 (0.09)
27	0.36	Dense	Mixture	0.4	0.0	0.4	0.2	0.0187 (0.056)	-0.0243 (0.087)	0.0299 (0.03)	-0.0229 (0.097)
28	0.33	Dense	Mixture	0.2	0.4	0.0	0.4	-0.0083 (0.055)	-0.0531 (0.066)	0.0302 (0.028)	-0.0333 (0.09)
29	0.33	Dense	Standard	0.2	0.2	0.6	0.0	0.0553 (0.055)	0.0175 (0.077)	0.0972 (0.04)	0.0072 (0.079)
30	0.31	Dense	Standard	0.0	0.6	0.2	0.2	0.0293 (0.052)	-0.037 (0.053)	0.0906 (0.032)	-0.0026 (0.072)
31	0.28	Dense	Mixture	0.4	0.0	0.2	0.4	0.0101 (0.049)	-0.0227 (0.05)	0.0859 (0.03)	7e-04 (0.089)
32	0.50	Sparse	Mixture	0.2	0.2	0.4	0.2	0.0272 (0.084)	0.0228 (0.07)	0.0045 (0.054)	-0.055 (0.109)
33	0.45	Dense	Mixture	0.0	0.6	0.0	0.4	0.0299 (0.059)	0.0345 (0.062)	0.0196 (0.044)	-0.0431 (0.103)
34	0.43	Dense	Standard	0.0	0.4	0.6	0.0	0.0197 (0.06)	0.0238 (0.082)	-0.022 (0.039)	-0.0461 (0.105)
35	0.40	Dense	Mixture	0.2	0.0	0.8	0.0	0.0473 (0.061)	0.0471 (0.076)	0.0494 (0.038)	-0.017 (0.1)
36	0.38	Dense	Mixture	0.2	0.2	0.2	0.4	0.0299 (0.057)	0.0108 (0.089)	0.0272 (0.032)	-0.0266 (0.09)
37	0.35	Dense	Standard	0.0	0.4	0.4	0.2	0.0045 (0.055)	-0.0445 (0.08)	0.0184 (0.03)	-0.028 (0.1)
38	0.35	Dense	Mixture	0.2	0.0	0.6	0.2	0.062 (0.056)	0.0433 (0.076)	0.0944 (0.047)	0.002 (0.077)
39	0.33	Dense	Standard	0.0	0.2	0.8	0.0	0.0415 (0.053)	-0.0173 (0.074)	0.0813 (0.032)	-6e-04 (0.082)
40	0.30	Dense	Standard	0.0	0.4	0.2	0.4	0.0177 (0.05)	-0.0309 (0.055)	0.0784 (0.028)	-0.0101 (0.064)
41	0.30	Dense	Mixture	0.2	0.0	0.4	0.4	0.0814 (0.052)	0.0319 (0.071)	0.1467 (0.039)	0.0379 (0.077)
42	0.28	Dense	Standard	0.0	0.2	0.6	0.2	0.0542 (0.052)	-0.0114 (0.043)	0.1367 (0.039)	0.0268 (0.071)
43	0.25	Dense	Standard	0.0	0.2	0.4	0.4	0.0382 (0.046)	0.0038 (0.044)	0.1385 (0.034)	0.0256 (0.076)

Table S1: Simulation Results - Bias: The average bias (SD) by scenario for the proposed ARC method and three best comparison methods; MDCC, exponential mixture and jackknife.

Scn.	% Zero	Data	Density	G(2,1)	G(1,1)	G(1,2)	G(1,3)	MSE (SD): ARC	MSE (SD): MDCC.M2.U1	MSE (SD): EXP.MIX	MSE (SD): JACKKNIFE
1	0.67	Sparse	Standard	1.0	0.0	0.0	0.0	0.0304 (0.099)	0.0083 (0.015)	0.0218 (0.053)	0.033 (0.029)
2	0.63	Sparse	Standard	0.8	0.2	0.0	0.0	0.0141 (0.034)	0.0064 (0.009)	0.0114 (0.042)	0.0309 (0.024)
3	0.58	Sparse	Standard	0.6	0.4	0.0	0.0	0.0093 (0.022)	0.0106 (0.014)	0.0176 (0.019)	0.0271 (0.024)
4	0.56	Sparse	Mixture	0.8	0.0	0.2	0.0	0.0092 (0.016)	0.0094 (0.011)	0.142 (2.206)	0.0307 (0.024)
5	0.60	Sparse	Standard	0.4	0.6	0.0	0.0	0.0103 (0.021)	0.0103 (0.014)	0.0103 (0.023)	0.0256 (0.022)
6	0.55	Sparse	Mixture	0.8	0.0	0.0	0.2	0.0074 (0.008)	0.0087 (0.011)	0.0114 (0.016)	0.0236 (0.023)
7	0.53	Sparse	Standard	0.6	0.2	0.2	0.0	0.0085 (0.016)	0.0078 (0.011)	0.031 (0.111)	0.0265 (0.026)
8	0.50	Sparse	Standard	0.2	0.8	0.0	0.0	0.0051 (0.007)	0.0046 (0.006)	0.0079 (0.008)	0.0214 (0.023)
9	0.47	Sparse	Mixture	0.6	0.2	0.0	0.2	0.0052 (0.007)	0.0076 (0.01)	0.0155 (0.009)	0.0229 (0.021)
10	0.45	Dense	Standard	0.4	0.4	0.2	0.0	0.0073 (0.009)	0.0126 (0.016)	0.0179 (0.008)	0.0229 (0.024)
11	0.57	Sparse	Mixture	0.6	0.0	0.4	0.0	0.009 (0.014)	0.0111 (0.016)	0.0064 (0.013)	0.0242 (0.025)
12	0.52	Sparse	Standard	0.0	1.0	0.0	0.0	0.0068 (0.01)	0.0045 (0.009)	0.007 (0.013)	0.0207 (0.027)
13	0.49	Sparse	Mixture	0.4	0.4	0.0	0.2	0.0057 (0.008)	0.0069 (0.01)	0.0157 (0.015)	0.0209 (0.022)
14	0.47	Sparse	Standard	0.2	0.6	0.2	0.0	0.004 (0.006)	0.0052 (0.007)	0.0036 (0.008)	0.0176 (0.024)
15	0.44	Dense	Mixture	0.6	0.0	0.2	0.2	0.0035 (0.005)	0.0079 (0.011)	0.0062 (0.008)	0.0163 (0.026)
16	0.42	Dense	Standard	0.4	0.2	0.4	0.0	0.0057 (0.007)	0.0133 (0.014)	0.0075 (0.005)	0.0183 (0.024)
17	0.42	Dense	Mixture	0.2	0.6	0.0	0.2	0.004 (0.005)	0.0069 (0.01)	0.0014 (0.005)	0.0106 (0.019)
18	0.39	Dense	Mixture	0.6	0.0	0.0	0.4	0.0035 (0.004)	0.0106 (0.012)	0.0013 (0.002)	0.0125 (0.024)
19	0.37	Dense	Standard	0.0	0.8	0.2	0.0	0.0037 (0.005)	0.0105 (0.009)	0.0014 (0.002)	0.0119 (0.024)
20	0.53	Sparse	Mixture	0.4	0.2	0.2	0.2	0.0076 (0.01)	0.0071 (0.012)	0.0041 (0.01)	0.0192 (0.021)
21	0.48	Sparse	Standard	0.2	0.4	0.4	0.0	0.0051 (0.009)	0.0037 (0.005)	0.0022 (0.004)	0.0157 (0.024)
22	0.46	Sparse	Mixture	0.0	0.8	0.0	0.2	0.0028 (0.004)	0.0057 (0.007)	0.0065 (0.007)	0.0167 (0.021)
23	0.43	Dense	Mixture	0.4	0.2	0.0	0.4	0.0037 (0.007)	0.0058 (0.01)	0.0017 (0.003)	0.0136 (0.029)
24	0.41	Dense	Mixture	0.4	0.0	0.6	0.0	0.0033 (0.004)	0.0083 (0.012)	0.0016 (0.004)	0.0129 (0.026)
25	0.38	Dense	Mixture	0.2	0.4	0.2	0.2	0.0037 (0.004)	0.0115 (0.011)	0.0017 (0.002)	0.0127 (0.025)
26	0.38	Dense	Standard	0.0	0.6	0.4	0.0	0.0052 (0.007)	0.0083 (0.013)	0.0036 (0.004)	0.0086 (0.027)
27	0.36	Dense	Mixture	0.4	0.0	0.4	0.2	0.0035 (0.006)	0.0081 (0.011)	0.0018 (0.002)	0.01 (0.036)
28	0.33	Dense	Mixture	0.2	0.4	0.0	0.4	0.0031 (0.004)	0.0071 (0.007)	0.0017 (0.002)	0.0092 (0.029)
29	0.33	Dense	Standard	0.2	0.2	0.6	0.0	0.0061 (0.008)	0.0062 (0.01)	0.011 (0.007)	0.0063 (0.029)
30	0.31	Dense	Standard	0.0	0.6	0.2	0.2	0.0035 (0.005)	0.0042 (0.005)	0.0092 (0.006)	0.0052 (0.022)
31	0.28	Dense	Mixture	0.4	0.0	0.2	0.4	0.0025 (0.003)	0.003 (0.004)	0.0083 (0.005)	0.0079 (0.036)
32	0.50	Sparse	Mixture	0.2	0.2	0.4	0.2	0.0078 (0.013)	0.0054 (0.012)	0.0029 (0.009)	0.0149 (0.021)
33	0.45	Dense	Mixture	0.0	0.6	0.0	0.4	0.0044 (0.008)	0.0051 (0.007)	0.0023 (0.006)	0.0123 (0.025)
34	0.43	Dense	Standard	0.0	0.4	0.6	0.0	0.004 (0.006)	0.0073 (0.011)	0.002 (0.003)	0.0131 (0.024)
35	0.40	Dense	Mixture	0.2	0.0	0.8	0.0	0.006 (0.008)	0.008 (0.012)	0.0039 (0.005)	0.0103 (0.028)
36	0.38	Dense	Mixture	0.2	0.2	0.2	0.4	0.0041 (0.006)	0.0079 (0.011)	0.0017 (0.002)	0.0089 (0.027)
37	0.35	Dense	Standard	0.0	0.4	0.4	0.2	0.0031 (0.005)	0.0084 (0.01)	0.0012 (0.002)	0.0108 (0.033)
38	0.35	Dense	Mixture	0.2	0.0	0.6	0.2	0.007 (0.009)	0.0076 (0.012)	0.0111 (0.008)	0.0059 (0.028)
39	0.33	Dense	Standard	0.0	0.2	0.8	0.0	0.0046 (0.007)	0.0058 (0.009)	0.0076 (0.005)	0.0067 (0.027)
40	0.30	Dense	Standard	0.0	0.4	0.2	0.4	0.0028 (0.004)	0.0039 (0.005)	0.0069 (0.005)	0.0042 (0.019)
41	0.30	Dense	Mixture	0.2	0.0	0.4	0.4	0.0093 (0.01)	0.006 (0.012)	0.023 (0.009)	0.0073 (0.033)
42	0.28	Dense	Standard	0.0	0.2	0.6	0.2	0.0056 (0.007)	0.0019 (0.004)	0.0202 (0.009)	0.0058 (0.027)
43	0.25	Dense	Standard	0.0	0.2	0.4	0.4	0.0035 (0.005)	0.0019 (0.004)	0.0203 (0.008)	0.0065 (0.031)

Table S2: Simulation Results - MSE: The average MSE (SD) by scenario for the proposed ARC method and three best comparison methods; MDCC, exponential mixture and jackknife.

Data								Method									
Scn.	% Zero	Data	Density	G(2,1)	G(1,1)	G(1,2)	G(1,3)	Accumulation Rate Curve	MDCC Mdl 2 Reg. 1%	Exponential Mixture	Jackknife	ACE	ACE1	Coverage Duplication	Breakaway	Transformed WLRM	WLRM
1	0.67	Sparse	Standard	1.0	0.0	0.0	0.0	0.95	0.85	0.87	0.49	0.05	0.37	0.91	1.00	1.00	0.67
2	0.63	Sparse	Standard	0.8	0.2	0.0	0.0	0.97	0.91	0.88	0.38	0.02	0.48	0.97	1.00	1.00	0.54
3	0.58	Sparse	Standard	0.6	0.4	0.0	0.0	0.94	0.83	0.52	0.40	0.01	0.45	0.99	1.00	1.00	0.38
4	0.56	Sparse	Mixture	0.8	0.0	0.2	0.0	0.94	0.76	0.45	0.32	0.00	0.25	0.98	1.00	1.00	0.47
5	0.60	Sparse	Standard	0.4	0.6	0.0	0.0	0.95	0.88	0.84	0.42	0.02	0.53	0.98	1.00	1.00	0.52
6	0.55	Sparse	Mixture	0.8	0.0	0.0	0.2	0.96	0.72	0.59	0.51	0.00	0.41	0.97	1.00	1.00	0.37
7	0.53	Sparse	Standard	0.6	0.2	0.2	0.0	0.92	0.87	0.27	0.44	0.00	0.19	0.93	1.00	1.00	0.39
8	0.50	Sparse	Standard	0.2	0.8	0.0	0.0	0.92	0.94	0.54	0.46	0.00	0.18	0.90	1.00	1.00	0.35
9	0.47	Sparse	Mixture	0.6	0.2	0.0	0.2	0.88	0.90	0.13	0.43	0.00	0.07	0.71	1.00	1.00	0.40
10	0.45	Dense	Standard	0.4	0.4	0.2	0.0	0.78	0.71	0.02	0.36	0.00	0.01	0.38	1.00	1.00	0.49
11	0.57	Sparse	Mixture	0.6	0.0	0.4	0.0	0.95	0.87	0.88	0.53	0.00	0.42	0.95	1.00	1.00	0.43
12	0.52	Sparse	Standard	0.0	1.0	0.0	0.0	0.94	0.90	0.73	0.56	0.00	0.33	0.93	1.00	1.00	0.36
13	0.49	Sparse	Mixture	0.4	0.4	0.0	0.2	0.90	0.91	0.25	0.51	0.00	0.19	0.88	1.00	1.00	0.40
14	0.47	Sparse	Standard	0.2	0.6	0.2	0.0	0.93	0.95	0.84	0.41	0.00	0.20	0.84	1.00	1.00	0.31
15	0.44	Dense	Mixture	0.6	0.0	0.2	0.2	0.93	0.91	0.52	0.38	0.00	0.07	0.67	1.00	1.00	0.35
16	0.42	Dense	Standard	0.4	0.2	0.4	0.0	0.86	0.67	0.24	0.30	0.00	0.02	0.29	1.00	1.00	0.44
17	0.42	Dense	Mixture	0.2	0.6	0.0	0.2	0.92	0.93	0.96	0.47	0.00	0.16	0.71	1.00	1.00	0.30
18	0.39	Dense	Mixture	0.6	0.0	0.0	0.4	0.93	0.70	0.92	0.46	0.00	0.04	0.43	1.00	1.00	0.34
19	0.37	Dense	Standard	0.0	0.8	0.2	0.0	0.89	0.53	0.89	0.40	0.00	0.01	0.19	1.00	1.00	0.32
20	0.53	Sparse	Mixture	0.4	0.2	0.2	0.2	0.95	0.85	0.92	0.57	0.00	0.34	0.94	1.00	1.00	0.39
21	0.48	Sparse	Standard	0.2	0.4	0.4	0.0	0.95	0.92	0.93	0.51	0.00	0.30	0.91	1.00	1.00	0.36
22	0.46	Sparse	Mixture	0.0	0.8	0.0	0.2	0.97	0.96	0.51	0.43	0.00	0.17	0.87	1.00	1.00	0.35
23	0.43	Dense	Mixture	0.4	0.2	0.0	0.4	0.94	0.93	0.94	0.49	0.00	0.20	0.83	1.00	1.00	0.32
24	0.41	Dense	Mixture	0.4	0.0	0.6	0.0	0.95	0.90	0.92	0.48	0.00	0.11	0.66	1.00	1.00	0.35
25	0.38	Dense	Mixture	0.2	0.4	0.2	0.2	0.91	0.59	0.84	0.39	0.00	0.03	0.31	1.00	1.00	0.38
26	0.38	Dense	Standard	0.0	0.6	0.4	0.0	0.89	0.90	0.68	0.71	0.00	0.25	0.72	1.00	1.00	0.28
27	0.36	Dense	Mixture	0.4	0.0	0.4	0.2	0.94	0.71	0.83	0.71	0.00	0.12	0.58	1.00	1.00	0.34
28	0.33	Dense	Mixture	0.2	0.4	0.0	0.4	0.92	0.61	0.80	0.69	0.00	0.05	0.29	1.00	1.00	0.29
29	0.33	Dense	Standard	0.2	0.2	0.6	0.0	0.82	0.87	0.10	0.92	0.02	0.33	0.73	1.00	1.00	0.31
30	0.31	Dense	Standard	0.0	0.6	0.2	0.2	0.91	0.66	0.11	0.87	0.00	0.19	0.51	1.00	1.00	0.34
31	0.28	Dense	Mixture	0.4	0.0	0.2	0.4	0.95	0.88	0.08	0.76	0.00	0.09	0.35	1.00	1.00	0.27
32	0.50	Sparse	Mixture	0.2	0.2	0.4	0.2	0.93	0.88	0.95	0.60	0.01	0.38	0.92	1.00	1.00	0.38
33	0.45	Dense	Mixture	0.0	0.6	0.0	0.4	0.93	0.92	0.93	0.51	0.01	0.32	0.89	1.00	1.00	0.35
34	0.43	Dense	Standard	0.0	0.4	0.6	0.0	0.94	0.94	0.90	0.47	0.00	0.22	0.81	1.00	1.00	0.32
35	0.40	Dense	Mixture	0.2	0.0	0.8	0.0	0.86	0.89	0.70	0.71	0.00	0.30	0.83	1.00	1.00	0.32
36	0.38	Dense	Mixture	0.2	0.2	0.2	0.4	0.92	0.87	0.86	0.73	0.00	0.19	0.70	1.00	1.00	0.31
37	0.35	Dense	Standard	0.0	0.4	0.4	0.2	0.94	0.59	0.91	0.68	0.00	0.08	0.42	1.00	1.00	0.34
38	0.35	Dense	Mixture	0.2	0.0	0.6	0.2	0.80	0.90	0.13	0.91	0.02	0.38	0.80	1.00	1.00	0.36
39	0.33	Dense	Standard	0.0	0.2	0.8	0.0	0.88	0.71	0.23	0.88	0.01	0.23	0.65	1.00	1.00	0.32
40	0.30	Dense	Standard	0.0	0.4	0.2	0.4	0.94	0.81	0.17	0.86	0.00	0.11	0.41	1.00	1.00	0.33
41	0.30	Dense	Mixture	0.2	0.0	0.4	0.4	0.67	0.87	0.00	0.86	0.07	0.54	0.83	1.00	1.00	0.41
42	0.28	Dense	Standard	0.0	0.2	0.6	0.2	0.80	0.91	0.00	0.81	0.03	0.36	0.68	1.00	1.00	0.37
43	0.25	Dense	Standard	0.0	0.2	0.4	0.4	0.88	0.95	0.00	0.77	0.02	0.32	0.59	1.00	1.00	0.35

Table S3: Simulation Results - CI Coverage by Scenario: The 95% CI coverage for each of the methods, by scenario. Coverage is averaged across the 500 iterates in each scenario.

Data								Method									
Scn.	%							Accumulation Rate Curve	MDCC Mdl 2 Reg. 1%	Exponential Mixture	Jackknife	ACE	ACE1	Coverage Duplication	Breakaway	Transformed WLRM	WLRM
Zero	Data	Density	G(2,1)	G(1,1)	G(1,2)	G(1,3)											
1	0.67	Sparse	Standard	1.0	0.0	0.0	0.0	661	220	303	317	242	353	1,251	24,174	14,685	460
2	0.63	Sparse	Standard	0.8	0.2	0.0	0.0	496	264	250	228	221	352	1,563	19,696	11,724	358
3	0.58	Sparse	Standard	0.6	0.4	0.0	0.0	390	290	187	230	184	301	1,201	24,748	8,489	278
4	0.56	Sparse	Mixture	0.8	0.0	0.2	0.0	316	246	156	224	154	250	784	26,499	5,844	310
5	0.60	Sparse	Standard	0.4	0.6	0.0	0.0	413	310	219	232	204	326	1,247	21,537	9,973	329
6	0.55	Sparse	Mixture	0.8	0.0	0.0	0.2	314	205	172	234	169	264	758	16,818	6,837	252
7	0.53	Sparse	Standard	0.6	0.2	0.2	0.0	291	259	137	228	144	223	539	22,051	5,269	233
8	0.50	Sparse	Standard	0.2	0.8	0.0	0.0	289	238	149	228	143	213	445	10,564	4,971	211
9	0.47	Sparse	Mixture	0.6	0.2	0.0	0.2	235	303	125	148	123	180	323	12,688	4,091	225
10	0.45	Dense	Standard	0.4	0.4	0.2	0.0	213	279	111	142	107	151	230	9,255	3,144	268
11	0.57	Sparse	Mixture	0.6	0.0	0.4	0.0	360	308	197	236	184	284	847	16,691	8,128	277
12	0.52	Sparse	Standard	0.0	1.0	0.0	0.0	303	196	163	236	156	238	557	13,949	6,151	226
13	0.49	Sparse	Mixture	0.4	0.4	0.0	0.2	282	278	134	230	137	208	434	17,318	4,649	225
14	0.47	Sparse	Standard	0.2	0.6	0.2	0.0	252	260	147	150	134	194	348	8,088	4,417	197
15	0.44	Dense	Mixture	0.6	0.0	0.2	0.2	223	310	127	146	118	168	271	7,516	3,697	183
16	0.42	Dense	Standard	0.4	0.2	0.4	0.0	212	251	113	142	102	139	199	4,970	2,806	203
17	0.42	Dense	Mixture	0.2	0.6	0.0	0.2	225	301	138	150	120	167	261	5,935	3,556	160
18	0.39	Dense	Mixture	0.6	0.0	0.0	0.4	215	248	123	145	104	141	197	4,755	2,804	152
19	0.37	Dense	Standard	0.0	0.8	0.2	0.0	204	178	112	140	93	122	157	3,175	2,275	144
20	0.53	Sparse	Mixture	0.4	0.2	0.2	0.2	325	216	184	238	167	250	598	12,985	6,719	251
21	0.48	Sparse	Standard	0.2	0.4	0.4	0.0	292	218	160	234	146	213	424	11,267	5,284	211
22	0.46	Sparse	Mixture	0.0	0.8	0.0	0.2	239	299	133	150	128	187	334	9,736	4,178	191
23	0.43	Dense	Mixture	0.4	0.2	0.0	0.4	231	275	147	152	128	181	297	6,794	4,051	178
24	0.41	Dense	Mixture	0.4	0.0	0.6	0.0	221	305	127	148	114	158	238	5,861	3,244	170
25	0.38	Dense	Mixture	0.2	0.4	0.2	0.2	212	206	116	142	99	133	181	4,309	2,508	159
26	0.38	Dense	Standard	0.0	0.6	0.4	0.0	224	300	139	150	115	158	232	5,869	3,310	151
27	0.36	Dense	Mixture	0.4	0.0	0.4	0.2	216	226	124	146	104	139	188	4,108	2,730	142
28	0.33	Dense	Mixture	0.2	0.4	0.0	0.4	197	162	116	142	91	117	147	3,448	2,153	125
29	0.33	Dense	Standard	0.2	0.2	0.6	0.0	218	244	134	150	106	140	186	4,440	2,750	135
30	0.31	Dense	Standard	0.0	0.6	0.2	0.2	201	125	124	146	95	122	153	5,840	2,196	122
31	0.28	Dense	Mixture	0.4	0.0	0.2	0.4	184	157	115	142	85	107	128	4,556	1,828	107
32	0.50	Sparse	Mixture	0.2	0.2	0.4	0.2	312	189	177	242	157	233	483	12,115	6,313	223
33	0.45	Dense	Mixture	0.0	0.6	0.0	0.4	254	242	157	154	138	199	352	7,885	4,548	195
34	0.43	Dense	Standard	0.0	0.4	0.6	0.0	227	306	134	150	124	177	291	7,390	3,702	174
35	0.40	Dense	Mixture	0.2	0.0	0.8	0.0	228	280	146	152	123	171	262	6,165	3,681	162
36	0.38	Dense	Mixture	0.2	0.2	0.2	0.4	222	290	129	148	111	151	217	5,194	2,994	148
37	0.35	Dense	Standard	0.0	0.4	0.4	0.2	211	178	118	144	98	129	171	3,671	2,428	142
38	0.35	Dense	Mixture	0.2	0.0	0.6	0.2	226	283	140	152	112	151	211	5,679	3,011	145
39	0.33	Dense	Standard	0.0	0.2	0.8	0.0	211	165	127	148	101	132	172	4,040	2,465	128
40	0.30	Dense	Standard	0.0	0.4	0.2	0.4	192	148	118	144	90	114	140	3,853	1,970	115
41	0.30	Dense	Mixture	0.2	0.0	0.4	0.4	214	204	136	152	104	135	174	7,435	2,572	127
42	0.28	Dense	Standard	0.0	0.2	0.6	0.2	196	120	125	148	93	118	144	4,893	2,095	113
43	0.25	Dense	Standard	0.0	0.2	0.4	0.4	180	151	118	144	84	105	123	9,203	1,739	102

Table S4: Simulation Results - CI Width by Scenario: The average CI width for each of the methods, by scenario. Width is averaged across the 500 iterates in each scenario.

Scn.	Data								MAE		MSE		
	%	Data	Density	G(2,1)	G(1,1)	G(1,2)	G(1,3)	Unadj.	Adjusted	%	Unadj.	Adjusted	%
	Zero									Difference		Difference	Difference
1	0.67	Sparse	Standard	1.0	0.0	0.0	0.0	0.1153	0.1080	-0.063	0.0304	0.0236	-0.225
2	0.63	Sparse	Standard	0.8	0.2	0.0	0.0	0.0798	0.0609	-0.237	0.0141	0.0070	-0.505
3	0.58	Sparse	Standard	0.6	0.4	0.0	0.0	0.0687	0.0673	-0.020	0.0093	0.0069	-0.259
4	0.56	Sparse	Mixture	0.8	0.0	0.2	0.0	0.0747	0.0772	0.034	0.0092	0.0084	-0.087
5	0.60	Sparse	Standard	0.4	0.6	0.0	0.0	0.0740	0.0679	-0.082	0.0103	0.0070	-0.320
6	0.55	Sparse	Mixture	0.8	0.0	0.0	0.2	0.0721	0.0752	0.043	0.0074	0.0082	0.104
7	0.53	Sparse	Standard	0.6	0.2	0.2	0.0	0.0759	0.0663	-0.127	0.0085	0.0064	-0.244
8	0.50	Sparse	Standard	0.2	0.8	0.0	0.0	0.0572	0.0511	-0.108	0.0051	0.0042	-0.180
9	0.47	Sparse	Mixture	0.6	0.2	0.0	0.2	0.0578	0.0538	-0.070	0.0052	0.0045	-0.121
10	0.45	Dense	Standard	0.4	0.4	0.2	0.0	0.0695	0.0678	-0.023	0.0073	0.0069	-0.049
11	0.57	Sparse	Mixture	0.6	0.0	0.4	0.0	0.0772	0.0826	0.070	0.0090	0.0099	0.096
12	0.52	Sparse	Standard	0.0	1.0	0.0	0.0	0.0673	0.0603	-0.104	0.0068	0.0059	-0.140
13	0.49	Sparse	Mixture	0.4	0.4	0.0	0.2	0.0593	0.0503	-0.152	0.0057	0.0041	-0.279
14	0.47	Sparse	Standard	0.2	0.6	0.2	0.0	0.0489	0.0476	-0.026	0.0040	0.0036	-0.100
15	0.44	Dense	Mixture	0.6	0.0	0.2	0.2	0.0471	0.0477	0.014	0.0035	0.0035	0.003
16	0.42	Dense	Standard	0.4	0.2	0.4	0.0	0.0633	0.0616	-0.026	0.0057	0.0054	-0.045
17	0.42	Dense	Mixture	0.2	0.6	0.0	0.2	0.0508	0.0518	0.019	0.0040	0.0042	0.047
18	0.39	Dense	Mixture	0.6	0.0	0.0	0.4	0.0484	0.0485	0.002	0.0035	0.0035	0.015
19	0.37	Dense	Standard	0.0	0.8	0.2	0.0	0.0498	0.0486	-0.024	0.0037	0.0036	-0.032
20	0.53	Sparse	Mixture	0.4	0.2	0.2	0.2	0.0723	0.0721	-0.003	0.0076	0.0082	0.082
21	0.48	Sparse	Standard	0.2	0.4	0.4	0.0	0.0536	0.0506	-0.056	0.0051	0.0042	-0.166
22	0.46	Sparse	Mixture	0.0	0.8	0.0	0.2	0.0420	0.0419	-0.003	0.0028	0.0026	-0.063
23	0.43	Dense	Mixture	0.4	0.2	0.0	0.4	0.0479	0.0484	0.010	0.0037	0.0037	-0.005
24	0.41	Dense	Mixture	0.4	0.0	0.6	0.0	0.0469	0.0478	0.021	0.0033	0.0035	0.061
25	0.38	Dense	Mixture	0.2	0.4	0.2	0.2	0.0497	0.0483	-0.028	0.0037	0.0036	-0.033
26	0.38	Dense	Standard	0.0	0.6	0.4	0.0	0.0573	0.0597	0.043	0.0052	0.0056	0.071
27	0.36	Dense	Mixture	0.4	0.0	0.4	0.2	0.0460	0.0481	0.047	0.0035	0.0038	0.080
28	0.33	Dense	Mixture	0.2	0.4	0.0	0.4	0.0445	0.0452	0.015	0.0031	0.0032	0.029
29	0.33	Dense	Standard	0.2	0.2	0.6	0.0	0.0631	0.0651	0.031	0.0061	0.0065	0.065
30	0.31	Dense	Standard	0.0	0.6	0.2	0.2	0.0471	0.0489	0.037	0.0035	0.0038	0.077
31	0.28	Dense	Mixture	0.4	0.0	0.2	0.4	0.0413	0.0438	0.062	0.0025	0.0028	0.124
32	0.50	Sparse	Mixture	0.2	0.2	0.4	0.2	0.0665	0.0670	0.008	0.0078	0.0078	0.005
33	0.45	Dense	Mixture	0.0	0.6	0.0	0.4	0.0518	0.0524	0.011	0.0044	0.0044	-0.010
34	0.43	Dense	Standard	0.0	0.4	0.6	0.0	0.0505	0.0521	0.031	0.0040	0.0042	0.053
35	0.40	Dense	Mixture	0.2	0.0	0.8	0.0	0.0630	0.0648	0.028	0.0060	0.0062	0.048
36	0.38	Dense	Mixture	0.2	0.2	0.2	0.4	0.0512	0.0537	0.048	0.0041	0.0045	0.087
37	0.35	Dense	Standard	0.0	0.4	0.4	0.2	0.0432	0.0444	0.028	0.0031	0.0032	0.045
38	0.35	Dense	Mixture	0.2	0.0	0.6	0.2	0.0679	0.0706	0.040	0.0070	0.0075	0.070
39	0.33	Dense	Standard	0.0	0.2	0.8	0.0	0.0538	0.0561	0.043	0.0046	0.0050	0.087
40	0.30	Dense	Standard	0.0	0.4	0.2	0.4	0.0423	0.0440	0.039	0.0028	0.0030	0.074
41	0.30	Dense	Mixture	0.2	0.0	0.4	0.4	0.0840	0.0846	0.008	0.0093	0.0096	0.032
42	0.28	Dense	Standard	0.0	0.2	0.6	0.2	0.0617	0.0635	0.029	0.0056	0.0059	0.054
43	0.25	Dense	Standard	0.0	0.2	0.4	0.4	0.0475	0.0495	0.042	0.0035	0.0038	0.066
Average								0.0594	0.0585	-0.014	0.0062	0.0057	-0.081

Table S5: Absolute Bias and MSE for Point Estimate Regularization by Scenario: The average absolute bias and MSE of the unadjusted and adjusted point estimates in each of the scenarios and the percentage change from applying regularization.

Data								MSE				% Change		
	%							Observed Data	No Reg.	Min. Reg.	Bootstrap	Bootstrap vs. Observed	Bootstrap vs. No Reg.	Bootstrap vs. Min. Reg.
Scn.	Zero	Data	Density	G(2,1)	G(1,1)	G(1,2)	G(1,3)							
1	0.67	Sparse	Standard	1.0	0.0	0.0	0.0	20.4228	3.2174	0.6764	0.6160	-0.97	-0.81	-0.09
2	0.63	Sparse	Standard	0.8	0.2	0.0	0.0	17.4988	3.1025	0.6974	0.6019	-0.97	-0.81	-0.14
3	0.58	Sparse	Standard	0.6	0.4	0.0	0.0	12.4765	3.3594	0.9155	0.7001	-0.94	-0.79	-0.24
4	0.56	Sparse	Mixture	0.8	0.0	0.2	0.0	8.6973	3.1045	0.8729	0.6295	-0.93	-0.80	-0.28
5	0.6	Sparse	Standard	0.4	0.6	0.0	0.0	15.2959	3.4452	0.7825	0.5855	-0.96	-0.83	-0.25
6	0.55	Sparse	Mixture	0.8	0.0	0.0	0.2	11.1785	3.7425	0.9686	0.7038	-0.94	-0.81	-0.27
7	0.53	Sparse	Standard	0.6	0.2	0.2	0.0	8.0447	3.4232	0.9839	0.6614	-0.92	-0.81	-0.33
8	0.50	Sparse	Standard	0.2	0.8	0.0	0.0	8.5911	3.0754	0.9512	0.6090	-0.93	-0.80	-0.36
9	0.47	Sparse	Mixture	0.6	0.2	0.0	0.2	6.4722	3.6188	1.0907	0.6464	-0.90	-0.82	-0.41
10	0.45	Dense	Standard	0.4	0.4	0.2	0.0	5.1696	3.5345	1.0758	0.6799	-0.87	-0.81	-0.37
11	0.57	Sparse	Mixture	0.6	0.0	0.4	0.0	13.5500	3.5585	0.9529	0.7078	-0.95	-0.80	-0.26
12	0.52	Sparse	Standard	0.0	1.0	0.0	0.0	10.1411	3.6057	1.0118	0.6970	-0.93	-0.81	-0.31
13	0.49	Sparse	Mixture	0.4	0.4	0.0	0.2	7.3872	3.2147	1.0328	0.6648	-0.91	-0.79	-0.36
14	0.47	Sparse	Standard	0.2	0.6	0.2	0.0	7.9739	3.5989	1.0922	0.6261	-0.92	-0.83	-0.43
15	0.44	Dense	Mixture	0.6	0.0	0.2	0.2	6.0823	3.8699	1.0819	0.6690	-0.89	-0.83	-0.38
16	0.42	Dense	Standard	0.4	0.2	0.4	0.0	4.9885	3.2279	1.1158	0.6986	-0.86	-0.78	-0.37
17	0.42	Dense	Mixture	0.2	0.6	0.0	0.2	6.4628	3.4495	1.1347	0.6711	-0.90	-0.81	-0.41
18	0.39	Dense	Mixture	0.6	0.0	0.0	0.4	5.1978	3.4920	1.2412	0.7192	-0.86	-0.79	-0.42
19	0.37	Dense	Standard	0.0	0.8	0.2	0.0	4.3891	3.0437	1.1535	0.6215	-0.86	-0.80	-0.46
20	0.53	Sparse	Mixture	0.4	0.2	0.2	0.2	12.1873	3.8000	0.9361	0.6480	-0.95	-0.83	-0.31
21	0.48	Sparse	Standard	0.2	0.4	0.4	0.0	9.2992	3.2181	1.0292	0.6628	-0.93	-0.79	-0.36
22	0.46	Sparse	Mixture	0.0	0.8	0.0	0.2	6.9675	3.6068	1.0598	0.5542	-0.92	-0.85	-0.48
23	0.43	Dense	Mixture	0.4	0.2	0.0	0.4	7.4190	3.5745	1.1545	0.7439	-0.90	-0.79	-0.36
24	0.41	Dense	Mixture	0.4	0.0	0.6	0.0	5.7721	3.3557	1.1596	0.6542	-0.89	-0.81	-0.44
25	0.38	Dense	Mixture	0.2	0.4	0.2	0.2	4.7614	3.3568	1.1335	0.6355	-0.87	-0.81	-0.44
26	0.38	Dense	Standard	0.0	0.6	0.4	0.0	6.1467	3.6598	1.1814	0.6470	-0.89	-0.82	-0.45
27	0.36	Dense	Mixture	0.4	0.0	0.4	0.2	5.0048	3.4991	1.2188	0.6435	-0.87	-0.82	-0.47
28	0.33	Dense	Mixture	0.2	0.4	0.0	0.4	4.2801	3.2146	1.2393	0.6958	-0.84	-0.78	-0.44
29	0.33	Dense	Standard	0.2	0.2	0.6	0.0	5.2893	3.5738	1.2613	0.7289	-0.86	-0.80	-0.42
30	0.31	Dense	Standard	0.0	0.6	0.2	0.2	4.4325	3.1038	1.2608	0.6772	-0.85	-0.78	-0.46
31	0.28	Dense	Mixture	0.4	0.0	0.2	0.4	3.9593	3.0157	1.2998	0.7379	-0.81	-0.76	-0.43
32	0.50	Sparse	Mixture	0.2	0.2	0.4	0.2	11.0006	3.9271	1.0461	0.6715	-0.94	-0.83	-0.36
33	0.45	Dense	Mixture	0.0	0.6	0.0	0.4	8.5701	3.6659	1.0501	0.6470	-0.92	-0.82	-0.38
34	0.43	Dense	Standard	0.0	0.4	0.6	0.0	6.4996	3.8811	1.1237	0.6727	-0.90	-0.83	-0.40
35	0.40	Dense	Mixture	0.2	0.0	0.8	0.0	7.0047	3.9188	1.1934	0.6647	-0.91	-0.83	-0.44
36	0.38	Dense	Mixture	0.2	0.2	0.2	0.4	5.5380	3.3433	1.1649	0.6163	-0.89	-0.82	-0.47
37	0.35	Dense	Standard	0.0	0.4	0.4	0.2	4.6455	3.6568	1.2605	0.6697	-0.86	-0.82	-0.47
38	0.35	Dense	Mixture	0.2	0.0	0.6	0.2	5.8355	4.0758	1.2311	0.7210	-0.88	-0.82	-0.41
39	0.33	Dense	Standard	0.0	0.2	0.8	0.0	4.8173	3.4134	1.2959	0.7139	-0.85	-0.79	-0.45
40	0.30	Dense	Standard	0.0	0.4	0.2	0.4	4.2205	3.3720	1.1864	0.6069	-0.86	-0.82	-0.49
41	0.30	Dense	Mixture	0.2	0.0	0.4	0.4	5.1127	3.7091	1.3111	0.7136	-0.86	-0.81	-0.46
42	0.28	Dense	Standard	0.0	0.2	0.6	0.2	4.3360	3.1834	1.2334	0.7217	-0.83	-0.77	-0.41
43	0.25	Dense	Standard	0.0	0.2	0.4	0.4	3.9279	2.9253	1.2707	0.6974	-0.82	-0.76	-0.45
Average								7.6058	3.4590	1.0961	0.6664	-0.91	-0.81	-0.39

Table S6: MSE for Count Covariance by Scenario: The covariance MSE, $\sum_{i,j}(\hat{\sigma}_{ij} - \sigma_{ij})^2 / (K + 1)^2$, averaged across all iterates in each of the scenarios. Changes in MSE when using the bootstrap relative to the other methods are given in the last three columns.