Inflation Transmission Diagnostics via a Bayesian Graph Vector Autoregressive Model with Markov Switching

Jiali Fu  
Wenzhou University

Fengjing Cai (caifj7704@wzu.edu.cn)  
Wenzhou University  https://orcid.org/0000-0002-9518-7159

Jinran Wu  
Australian Catholic University  https://orcid.org/0000-0002-2388-3614

Shangrui Zhao  
Wuhan University of Technology  https://orcid.org/0000-0001-7272-046X

You-Gan Wang  
Australian Catholic University  https://orcid.org/0000-0003-0901-4671

Research Article

Keywords: Inflation transmission, Markov Switching, Contemporaneous causality, G7

Posted Date: August 3rd, 2023

DOI: https://doi.org/10.21203/rs.3.rs-3222276/v1

License: This work is licensed under a Creative Commons Attribution 4.0 International License. Read Full License

Additional Declarations:
Competing interests: The authors declare no competing interests.
Inflation Transmission Diagnostics via a Bayesian Graph Vector Autoregressive Model with Markov Switching

FU Jiali · CAI Fengjing · WU Jinran · ZHAO Shangrui · WANG You-Gan

DOI:
Received: x x 20xx / Revised: x x 20xx
©The Editorial Office of JSSC & Springer-Verlag GmbH Germany 2021

Abstract The transmission of inflation is a widespread occurrence, and managing inflationary pressures is a crucial macroeconomic challenge. Although inflation is a typical macroeconomic variable, its contemporaneous and lagged causal relationships have not been thoroughly investigated, which could result in missing important policy insights. The Bayesian graph vector autoregression (BGVAR) model can identify contemporaneous and lagged causal relationships among economic variables, but it lacks practical research on inflationary inflation. To account for the structural transformation in the inflation transmission process, we propose a Bayesian graph vector autoregressive model with Markov switching (MS-BGVAR), which considers both regime switching and contemporaneous causality among macroeconomic variables. Our study focuses on analyzing the dynamics of inflation transmission relationships among G7 countries under different regimes, as these countries represent developed nations. We use inflation data from 1971-2019, which shows two distinct inflation regimes within the sample period. We conduct simulation experiments to generate moderately dimensional simulated data for both regimes and indicators, demonstrating the theoretical reliability of our model in accurately identifying graph structures. Finally, we apply the proposed model to identify structural breaks and causal transmission relationships in the inflation transmission process of G7 economies, demonstrating that the proposed model has significant economic significance and good explanatory power in the selected target countries.

Keywords Inflation transmission, Markov Switching, Contemporaneous causality, G7.


1 Introduction

Inflation is a major issue involving political stability, economic development, and social harmony. Maintaining a low and stable inflation rate is an important issue for macroeconomic management in most countries. Since the outbreak of the oil crisis, it has been common for inflation to spread from one country to another country through international capital and trade channels. A country’s inflation management should not only take full account of the reality of domestic inflation but also pay close attention to the trend of global inflation. Therefore, clarifying the dynamic transmission mechanism of inflation is of great importance for central banks and monetary authorities.

1.1 Literature review

Many scholars have discussed and argued for an international transmission theory of inflation, which has now been applied to several economies. Many studies have looked at the Group of Seven (G7) countries, pointing to the existence of significant inflationary transmission relationships within the G7, and have found that the United States is a major exporter of inflation. Some studies focus on Eurozone countries and find that peripheral countries receive inflation from core zone countries while exporting inflation to them; in addition, a few Chinese scholars study the Greater China region and find that inflation in the Greater China region exists has a long-run equilibrium relationship. Most of the above studies are based on impulse correspondence functions and variance decomposition methods to determine the correlation of inflation across countries, and such methods face the challenge of limited or undetermined reliability.

In contrast to traditional methods, the graphical model approach is a probabilistic model that uses a graph structure to describe conditionally independent relationships among multiple random variables. Graphical models are powerful tools for studying multivariate time series. Bayesian Graphical Structural Vector Autoregression (BGVAR) as a Gaussian graphical model (GGM) and structural vector autoregression (SVAR) model set, the variable relationships were divided into contemporaneous causal and lagged causal structures. Also, the MCMC algorithm is adopted to estimate the graph structure, overcoming the limitation of traditional models that rely on residual terms for contemporaneous structure inference. In addition, the BGVAR model allows researchers to understand the relationship between variables without economic theory support. Currently, BGVAR has been used to conduct empirical studies in finance, such as house price linkage, financial risk transmission, crude oil and exchange rate transmission, and stock price transmission.

The BGVAR model has proven to be effective in dealing with model identification and selection for multivariate time series of moderate dimensionality. Inflation is an important macroeconomic variable, but no research on constructing dynamic inflation transmission relationships based on BGAVR exists. This paper intends to fill this academic gap. In fact, the transmission relationship of inflation between countries is nonlinear, which is highly correlated with the economic environment and monetary policy and evolves in different ways under different economic states. Markov Switching (MS) is the classical method for dealing
with time series analysis influenced by regimes. Depending on the number of regimes given, the MS approach can efficiently identify the structural change points and variable relationships under each regime\cite{24}. Thams \cite{25} and Burdekin and Tao \cite{26} have found evidence of structural breaks in the transmission process of inflation based on the MS model. However, it should be noted that they did not consider the contemporaneous causality of inflation under different regimes, which could reflect the impact and direction of policy actions\cite{13}.

1.2 The motivation

According to the above literature review, it is important to consider regime switching when studying the inflation transmission relationship. However, there are still some shortcomings in the existing studies: (1) In the analysis of regime-switching of inflation, researchers have only considered lagged causality and neglected to examine contemporaneous causality. (2) The BGVAR method can identify causal relationships among economic variables and has been used in empirical studies in the field of finance, but there is almost a gap in the research in the field of inflation. (3) Currently, no relevant studies are combining MS and BGVAR methods to deal with structural breaks in the inflation transmission process and contemporaneous causality.

1.3 The contribution

To fill this gap, this paper combines MS with BGVAR to construct a new model for identifying inflationary transmission relationships. The main contributions of this study are as follows:

- In this paper, we established a Markov Switching Bayesian Graphical Vector Autoregressive (MS-BGVAR) model that considers both contemporaneous causality and structural transformations of inflation. The MS-BGVAR model on the one hand avoided pre-specifying the timing of shifts and allowed regime changes to be determined by potentially unobservable stochastic processes. On the other hand, it provided insight into the simultaneous and dynamic dependence of inflation under different regimes through causal interpretation.

- Simulation experiments were conducted to compare the graph structure, graph coefficients, and state transfer probability matrix with the real values, and the results showed that our proposed method performs well. Our study extended the BGVAR model at the algorithmic level.

- At the application level, G7 countries were selected, and real inflation data for these seven countries were used to verify the accuracy of the proposed model. We estimate a two-regime MS-BGVAR model in which the lagged causal structure reveals the linkage of inflation in different regimes and the contemporaneous causal structure reveals the impact of policy actions implemented in one country in different regimes on inflation in other countries. This study extended the research frameworks of the inflation transmission relationship and the application fields of the BGVAR model.
1.4 The structure of the paper

The rest of the paper is organized as follows. Section 2 introduces the basic theory of MS and BGVAR. Section 3 describes the definition of the MS-BGVAR model and the inference scheme. Section 4 conducts simulation experiments to evaluate the proposed model on a simulated data set. Section 5 conducts a case study to analyze the identification results on real data. Finally, Section 6 concludes the paper. In addition, some symbols of this paper are given in Table 1.

Table 1: Table of notation.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_t$</td>
<td>Response vector.</td>
</tr>
<tr>
<td>$Y$</td>
<td>$Y = (Y_1, \ldots, Y_T)$ is the observation data matrix.</td>
</tr>
<tr>
<td>$p$</td>
<td>Optimal lag order.</td>
</tr>
<tr>
<td>$B_0$</td>
<td>Coefficient matrix of contemporaneous dependencies.</td>
</tr>
<tr>
<td>$B_t$</td>
<td>Coefficient matrix of lagged dependencies, $1 \leq i \leq p$.</td>
</tr>
<tr>
<td>$G_0$</td>
<td>Contemporaneous causal structure.</td>
</tr>
<tr>
<td>$\Phi_s$</td>
<td>The coefficient matrix corresponding to the graph $G_s$.</td>
</tr>
<tr>
<td>$x_t$</td>
<td>The explanatory variables in MS model.</td>
</tr>
<tr>
<td>$p_{ij}$</td>
<td>The probability of transition from state $i$ to state $j$.</td>
</tr>
<tr>
<td>$\beta_{x_{ik}}$</td>
<td>The regression coefficients in MS model.</td>
</tr>
<tr>
<td>$G_0(k)$</td>
<td>Contemporaneous causal graph structure under regime $k$.</td>
</tr>
<tr>
<td>$A_+(k)$</td>
<td>Coefficient matrix in MSVAR model.</td>
</tr>
<tr>
<td>$G_+(k)$</td>
<td>The graph structure corresponding to $A_+(k)$.</td>
</tr>
<tr>
<td>$X(k)$</td>
<td>The lagged data matrix under regime $k$.</td>
</tr>
<tr>
<td>$Y(k)$</td>
<td>The matrix of observed data under regime $k$.</td>
</tr>
<tr>
<td>$\Sigma_u(k)$</td>
<td>The covariance matrix of $u_t$ under regime $k$.</td>
</tr>
<tr>
<td>$G_+(k)$</td>
<td>The lagged causal graph structure under regime $k$.</td>
</tr>
<tr>
<td>$U$</td>
<td>Uniform distribution.</td>
</tr>
<tr>
<td>$N$</td>
<td>Normal distribution.</td>
</tr>
<tr>
<td>$Dir$</td>
<td>Dirichlet distribution.</td>
</tr>
<tr>
<td>$Beta$</td>
<td>Beta Distribution.</td>
</tr>
</tbody>
</table>

2 The preliminaries

2.1 The Bayesian graphical structural vector autoregression

Bayesian Graphical Structural Vector Autoregression is an econometric model that combines Structural Vector Autoregression (SVAR) with a graph model and uses the MCMC algorithm to approximate the graph structure and parameters. Suppose $Y_t$ is a $d$-dimensional response vector, then the SVAR generation process is as follows:

$$Y_t = \sum_{s=0}^{p} B_s Y_{t-s} + \varepsilon_t, \quad t = 1, \ldots, T,$$

where $p$ is the lag order; $B_s, 0 \leq s \leq p$ is a $d \times d$ matrix of coefficients that represent the dependencies between variables. For $s = 0$, $B_0$ represents the contemporaneous dependence between the variables. For $1 \leq s \leq p$, $B_s$ represents the lagged dependence between variables; $\varepsilon_t$ is a $d$-dimensional vector of structural error terms, independent and identically normal, i.e., $\varepsilon_t \sim N(0, \Sigma)$. 
However, it is not possible to estimate the true value of $B_s$ directly from the data. Therefore, Ahelegbey et al. [15] proposed a method based on graphical models and Bayesian inference to identify the coefficient matrix of the SVAR model. Specifically, assume that there is a one-to-one correspondence between the coefficient matrix $B_s$ and a directed acyclic graph (DAG). The correspondence is expressed as follows:

$$Y_{j,t-s} \rightarrow Y_{i,t} \iff B_{s,ij} \neq 0, \quad 0 \leq s \leq p,$$

where $Y_{j,t-s}$ is the value of the $j$-th variable at $t - s$ moments and $Y_{i,t}$ is the value of the $i$-th variable at $t$ moments. $B_{s,ij}, 0 \leq s \leq p$ is the value of the $i$-th row and $j$-th column of the coefficient matrix $B_s$. For $s = 0$, $B_{0,ij} \neq 0$ indicates a contemporaneous causal relationship between $Y_{j,t}$ and $Y_{i,t}$. For $1 \leq s \leq p$, $B_{s,ij} \neq 0$ indicates a lagged causal relationship between $Y_{j,t-s}$ and $Y_{i,t}$.

Then, according to the representation of the Eq. (2), $B_s$ can be decomposed into a connection matrix and a coefficient matrix. The decomposition is as follows:

$$B_s = G_s \circ \Phi_s, \quad 0 \leq s \leq p,$$

where $G_s, \Phi_s, 0 \leq s \leq p$ are the connectivity matrix and coefficient matrix of $d \times d$, respectively. For $s = 0$, $G_s = G_0$ represents the contemporaneous causal structure; for $1 \leq s \leq p$, $G_s$ represents the lagged causal structure; the operator $\circ$ represents the Hadamard product (i.e., $B_{s,ij} = G_{s,ij} \Phi_{s,ij}$). When $G_s$ is known, $\Phi_s$ has a one-to-one correspondence with $B_s$ as follows:

$$B_{s,ij} = \begin{cases} 0, & \text{if } G_{s,ij} = 0 \rightarrow Y_{j,t-s} \not\rightarrow Y_{i,t} \\ \Phi_{s,ij} \in R, & \text{if } G_{i,j|s} = 1 \rightarrow Y_{j,t-s} \rightarrow Y_{i,t}. \end{cases}$$

Thus, the estimation of $B_s$ can be converted into an inference for $G_s$ and $\Phi_s$, then the BGVAR model is generated. The standard equation of the BGVAR model is given as:

$$Y_t = (G_0 \circ \Phi_0)Y_t + \sum_{i=1}^{p}(G_i \circ \Phi_i)Y_{t-i} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \Sigma_\varepsilon).$$

### 2.2 The Markov switching

The Markov transition model is an extension of the simple transition regime model, which is more suitable for regime analysis[27]. The basic idea of the MS is to capture the change from one regime or state to another through an unobserved Markov chain. The generation of Markov chains depends on the unobservable regime (state) variables and the state transition probability matrix. The Markov chain generation process is as follows:

$$\text{Prob}(s_t = j \mid s_{t-1} = i) = p_{ij}, i, j = 1, \cdots, K,$$

where $p_{ij}$ is the transfer probability of state $i$ to state $j$. For convenience, only the bivariate regression model for the two regimes is briefly presented here, and the other MS models are
based on this model\textsuperscript{[28] [29]}. The basic definition of the bivariate Markov switching model is as follows:

\[
y_t = \alpha_{s_t} + x_t \beta_{s_t} + v_t, \\
v_t \sim N \left(0, \sigma_{s_t}^2\right),
\]

with

\[
P = \begin{pmatrix}
p_{11} & p_{12} \\
p_{21} & p_{22}
\end{pmatrix},
\]

where \(y_t\) is the response variable; \(x_t\) is the explanatory variable with a state correlation coefficient of \(\beta_{s_t}\); \(\alpha_{s_t}\) is the regime-dependent intercept; \(v_t\) is an error term submitted to a normal distribution with a regime-dependent variance \(\sigma_{s_t}^2\). The duration of the regime \(k\) (\(k=1\) and \(2\)) is calculated as follows:

\[
D(k) = \frac{1}{1 - p_{kk}}.
\]

3 The proposed method

In this paper, we assume that there are no exogenous explanatory variables. Based on the theoretical knowledge in Section 2, the main idea of MS-BGVAR is to combine data under the same regime into new samples at each iteration and to identify causal relationships for each new sample until a predetermined number of iterations is reached.

3.1 The model formulation

\(Y_t\) is the \(d\)-dimensional response vector, and the form of the SVAR model with structural transformation is as follows:

\[
Y_t = B_0(s_t)Y_t + \sum_{i=1}^{p} B_i(s_t)Y_{t-i} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \Sigma_{\varepsilon}(s_t)), \quad t = 1, \cdots, T, \quad (3)
\]

where \(Y_t\) is the \(d\)-dimensional response vector; \(p\) is the optimal lag order; \(s_t\) is the regime (state) variable; \(B_0(s_t)\) is the regime-dependent structural coefficient matrix of \(d \times d\). \(B_i(s_t), 1 \leq i \leq p\) is the regime dependent lag coefficient matrix of \(d \times d\); \(\varepsilon_t\) is the \(d\)-dimensional structural error vector associated with the regime.

In the discussion of this paper, only two regimes (states) are considered, i.e. \(s_t = 1\) and \(2\), then the simplified form of Eq. (3) is as follows:

\[
Y_t = B_0(k)Y_t + \sum_{i=1}^{p} B_i(k)Y_{t-i} + \varepsilon_t,
\]

\[
B_0(k) = \sum_{k=1}^{2} B_0(s_t)I_{(k)}(s_t),
\]

\[
B_i(k) = \sum_{k=1}^{2} B_i(s_t)I_{(k)}(s_t),
\]

\[
\Sigma_{\varepsilon}(k) = \sum_{k=1}^{2} \Sigma_{\varepsilon}(s_t)I_{(k)}(s_t), \quad (4)
\]
where \( k = 1 \) and \( 2 \), and \( \mathbb{I}_k(s_t) \) is an indicator function that takes the value 1 when the state \( s_t \) takes the value \( k \) at time \( t \). Similar to the meaning and treatment of the variables in Eq. (1), \( B_s(k), s = 0, 1, \cdots, p \) is the coefficient matrix under regime \( k \) in Eq. (3), which can be decomposed as follows:

\[
B_s(k) = G_s(k) \circ \Phi_s(k), \quad 0 \leq s \leq p,
\]

where \( G_s(k) \) and \( \Phi_s(k) \) are the \( d \times d \) matrices that represent the causal graph structure and graph coefficient matrix between variables in regime \( k \), respectively. Conditional on \( G_s(k) \), the correspondence between \( \Phi_s(k) \) and \( B_s(k) \) is as follows:

\[
B_{s,ij}(k) = \begin{cases} 
0, & \text{if } G_{s,ij}(k) = 0 \rightarrow Y_{j,d_k-s}(k) \not\rightarrow Y_{i,d_k}(k) \\
\Phi_{s,ij}(s_t) \in R, & \text{if } G_{s,ij}(k) = 1 \rightarrow Y_{j,d_k-s}(k) \rightarrow Y_{i,d_k}(k),
\end{cases}
\]

where \( Y = (Y_1, \cdots, Y_T) \) is a \( d \)-dimensional observation data matrix of length \( T \), \( Y(k) \) is a \( d_k \times d \)-dimensional data matrix consisting of observations with state values equal to \( k \). After the above steps, the MS-BGVAR model is defined as follows:

\[
Y_t = (G_0(k) \circ \Phi_0(k))Y_t + \sum_{i=1}^{p} (G_i(k) \circ \Phi_i(k))Y_{t-i} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \Sigma_{e}(k)),
\]

where \( G_0(k) \) and \( G_s(k) = (G_1(k), \cdots, G_p(k) \) are the connection matrices of \( d \times d \) and \( d \times (dp) \), respectively, and represent the contemporaneous causal structure and lagged causal structure under the representative regime \( k \), respectively.

Finally, by shifting to Eq. (3), we can carry the reduced form of the regime-transformed SVAR model, which is the Markov switching vector autoregressive model (MSVAR). The MSVAR model is as follows:

\[
Y_t = \sum_{i=1}^{p} A_i(k)Y_{t-i} + u_t, \quad u_t \sim N(0, \Sigma_u(k)),
\]

where \( A_i(k), 1 \leq i \leq p \) is the coefficient matrix of the reduced form of the dependent regime and \( A_i(k) = (I_{n_y} - B_0(k))^{-1}B_i(k); u_t = (I_{n_y} - B_0(k))^{-1}\varepsilon_t, \Sigma_u = (I_{n_y} - B_0(k))^{-1}\Sigma_e(k) \left( (I_{n_y} - B_0(k))^{-1} \right)' \) and \( u_t \) is the error term in the reduced form associated with the regime. Such that \( A_+(k) = (A_1(k), \cdots, A_p(k)) \), and \( G_+(k) \) is the graph structure corresponding to \( A_+(k) \).

### 3.2 The reasoning scheme

Statistical inference of graph models is a challenging task, and this study uses Bayesian inference methods as the basis for parameter inference. In the Bayesian framework, since the marginal likelihood function of the graph structure can be obtained by integrating the graph coefficients, the sampling estimation of the graph structure can be achieved by directly relying on the observation data matrix divided by the state sequence. After estimating the graph structure, variable selection can be performed based on the conditional independence relationship between variables, and this practice can further improve the efficiency and accuracy of coefficient estimation. Bayesian inference consists of two steps: setting the prior distribution and MCMC sampling.
3.2.1 A priori and posterior distributions

The prior distribution of the lag order $p$ is discrete and homogeneous, and the optimal value of $p$ can be eventually determined by the Bayesian Information Criterion (BIC)\textsuperscript{[13]}. The specific form of the prior distribution of $p$ is as follows:

$$p \sim U(p_{\min}, p_{\max}),$$

where $p_{\min}$ and $p_{\max}$ are the minimum lag order and maximum lag order, respectively. $p$ can only take integer values between $p_{\min}$ and $p_{\max}$. In this study, we set $p_{\min} = 1$, $p_{\max} = 4$.

A graph $G$ consisting of $n$ nodes has a total of $\frac{n(n-1)}{2}$ possible edges, so a graph $G$ containing $n$ nodes has $2^{\frac{n(n-1)}{2}}$ possibilities in the structure. It is assumed that each graph in the graph space has the same probability of being selected, that is, the prior distribution of the graph $G$ is uniformly distributed. The specific form of this distribution is as follows:

$$P(G) = 2^{-\frac{n(n-1)}{2}},$$

with:

$$G_{ij} \sim Ber(n_{ij}),$$

where $n_{ij}$ is the probability of the existence of a directed edge ($G_{ij} = 1$) between node $i$ and node $j$. The probability of each edge existing is generally considered to be 0.5, i.e., $n_{ij} = 0.5$.

Since the estimation of the state sequence $s_{1:T} = (s_1, \cdots, s_T)$ relies on the Kalman filter, the inverse matrix of the covariance matrix, also known as the accuracy matrix $\Sigma_u^{-1}(k)$, is the key to the Kalman filter. Therefore, this study translates the estimation process of $B_0(k), B_i(k), 1 \leq i \leq p$ and $\Sigma_u^{-1}(k)$ into statistical inference on $A_+(k)$ and $\Sigma_u(k)$. In addition, since this paper is concerned with contemporaneous causality, the algorithm design process is based on $G_0(k), G_i(k), 1 \leq i \leq p$ to infer $G_+(k)$.

To evaluate $A_+(k)$ and $\Sigma_u^{-1}(k)$, we considered two typical prior distributions widely used in the Bayesian VAR literature, Minnesota (MP) and the normal-Wishart (NW) prior. Due to space constraints, we describe only the Normal-Wishart prior, and for Minnesota prior, we show the estimation results in numerical simulations. According to the Normal-Wishart prior, $A_+(k)$ and $\Sigma_u^{-1}(k)$ prior distributions are as follows\textsuperscript{[13]}:

$$A_+(k) \sim \mathcal{N}(\overline{A}(k), \overline{V}(k)), \Sigma_u^{-1}(k) \sim \mathcal{W}(\nu(k), S^{-1}(k)),$$

where $\overline{A}(k)$ and $\overline{V}(k)$ are the prior expectation and prior covariance of $A_+(k)$, respectively. $S(k)$ is the prior sum of squares, $\nu(k)$ is the associated degree of freedom.

Since the prior distribution of $A_+(k)$ is conjugate to the posterior distribution, the posterior expectation and posterior covariance of $A_i(k)$ conditional on $G_+(k)$ are expressed as follows:

$$\overline{A}_i(k) = \overline{V}_i(k) \left( \overline{V}_i^{-1}(k) \overline{A}_i(k) + \overline{\sigma}_i^{-2}(k) W_i^t(k) Y^i(k) \right),$$

$$\overline{V}_i(k) = \left( \overline{V}_i^{-1}(k) + \overline{\sigma}_i^{-2}(k) W_i^t(k) W_i(k) \right)^{-1},$$

where $W_i(k), i = 1, \cdots, d$ are the explanatory variables of $Y^i(k)$ and $Y^i(k)$ is the $i$-th variable in $Y(k)$. $\overline{\sigma}_i^2(k), i = 1, \cdots, d$ are the residuals of the posterior of $\Sigma_u(k)$. The posterior distribution
of $\Sigma_k^{-1}(k)$ is also the Wishart distribution, and its corresponding posterior parameters are as follows:

$$
\mathcal{S}(k) = \mathcal{S}(k) + \left( Y'(k) - X(k)\bar{A}(k) \right)' \left( Y(k) - X(k)\bar{A}(k) \right),
$$

$$
\mathcal{V}(k) = \mathcal{V}(k) + (d_k - p),
$$

where $X(k) = (Y_{p+1:s}(k), \cdots, Y_{d_k-s}(k))'$, $1 \leq s \leq p$ is the $(d_k - p) \times dp$ lagged data matrix. $\bar{A}(k)$ is the posterior mean of $A_i(k)$ with dimension $d \times d$ such that the elements of the relevant variables in $\bar{A}(k)$ store the corresponding elements in $A_i(k)$, $i = 1, \cdots, d$ and the rest (representing the unrelated variables) are restricted to 0. In addition, define $\pi_k = (p_{i1}, \cdots, p_{ij})'$ as the variable consisting of the elements of the $k$-th row of the transfer matrix $P$, and the prior distribution of $\pi_k$ as follows:

$$
\pi_k \sim Dir(c_{k1}, \cdots, c_{kK}),
$$

where $c_{ki}, i = 1, \cdots, K$ is the hyperparameter of the Dirichlet distribution. Since the focus of this study is on two regimes, the Dirichlet distribution degenerates to a Beta distribution, and the posterior distribution of $\pi_k$ is as follows:

$$
[\pi_k \mid s_{1:T}] \sim Beta(c_{k1} + count_{k1}, \cdots, c_{k2} + count_{k2}),
$$

where $count_{k1}$ is the number of observations filtered from state $k$ to state 1, $count_{k2}$ is the number of observations filtered from state $k$ to state 2, and this study sets $c_{11} = 1, c_{12} = 1, c_{21} = 1, and c_{22} = 1$.

### 3.2.2 The MCMC Sampling

The previous subsection sets up the parameters to be estimated and the prior distribution of the graph. This subsection combines Bayesian inference with the sampling algorithm to solve the estimation problem of the parameters of interest. The joint posterior distribution of the parameters is

$$
f \left( G_0, G_s, A_s, s_{1:T}, \Sigma_u, P \mid Y \right) \propto f \left( G_0, G_s, A_s, s_{1:T}, \Sigma_u, P, Y \right)
$$

$$
= f \left( Y \mid G_0, G_s, A_s, s_{1:T}, \Sigma_u, P \right) f \left( G_0, G_s, A_s, s_{1:T}, \Sigma_u, P \right),
$$

(5)

where $f \left( G_0, G_s, A_s, s_{1:T}, \Sigma_u, P \right) = f \left( s_{1:T} \right) f \left( G_0, G_s \mid s_{1:T} \right) f \left( A_s \mid G_0, G_s, s_{1:T} \right)$ $f \left( \Sigma_u \mid G_0, G_s, s_{1:T}, A_s \right) : f \left( P \mid G_0, G_s, s_{1:T}, A_s, \Sigma_u \right)$. To obtain the joint posterior distribution of $(G_0, G_s, A_s, s_{1:T}, \Sigma_u, P)$, one needs to compute Eq. (5), but solving Eq. (5) is very complicated, therefore, this paper uses the MCMC method to give its posterior distribution. The principle is to sample Monte Carlo simulation to obtain a set of samples from $f \left( G_0, G_s, A_s, s_{1:T}, \Sigma_u, P \mid Y \right)$, and to give the joint posterior distribution of $(G_0, G_s, A_s, s_{1:T}, \Sigma_u, P)$ by large sample simulation. We use the MH iterative algorithm and Gibbs sampling to model its posterior distribution. The sampling steps of the MS-BGVAR model can be seen in Figure 1.
Algorithm: MSBGVAR based on MCMC sample

(1): Initialize $s_{1:T}^{(0)}$, $G_0^{(0)}(k)$, $G_0^{(0)}(k)$, $A_+^{(0)}(k)$, $\Sigma_u^{(0)}(k)$ and $P^{(0)}$, $k = 1$ and 2

- $s_{1:T/5}^{(0)} = 1$ and $s_{T/5:T}^{(0)} = 2$, obtain $Y^0(k)$
- $G_0^{(0)}(k)$ be $d \times d$ zero matrix
- $G_0^{(0)}(1)$ be $d \times dp$ zero matrix, $G_0^{(0)}(2)$ be $d \times dp$ matrix with all elements being 1
- $A_+^{(0)}(k)$ be $d \times dp$ zero matrix
- $\Sigma_u^{(0)}(k)$ be the residual covariance matrices corresponding to ordinary least squares

Sample $p_{11}, p_{22} \sim \text{Beta}(1,1)$, $p_{12} = 1 - p_{11}$, $p_{21} = 1 - p_{22}$

(2): For $l = 1$ to total iterations do

a. Update $s_{1:T}^{(l)}$ and $P^{(l)}$

   - Sample $s_{1:T}^{(l)} \sim f(s_{1:T}^{(l-1)}, \Sigma_u^{(l-1)}, Y^{l-1}(k))$, update $Y^l(k)$
   - Sample $P^{(l)} \sim f(P^{(l)}|s_{1:T}^{(l)})$

b. Update graph structure

   - Sample $G_0^{(l)}(k) \sim f(G_0^{(l)}(k)|Y^l(k))$
   - Sample $G_0^{(l)}(k) \sim f(G_0^{(l)}(k)|Y^l(k))$

b. Update graph structure

   - Sample $A_+^{(l)}(k)$ and $\Sigma_u^{(l)}$

   - Sample $A_+^{(l)}(k) \sim f(A_+^{(l)}(k)|G_0^{(l)}(k), G_0^{(l)}(k), \Sigma_u^{(l-1)}(k), Y^l(k))$
   - Sample $\Sigma_u^{(l)} \sim f(\Sigma_u^{(l)}|A_+^{(l)}(k), G_0^{(l)}(k), G_0^{(l)}(k), Y^l(k))$

Figure 1: The specific inference process of the MS-BGVAR model.

The above procedure is a combination of Gibbs sampling and the MH algorithm. The MH algorithm is applied to update the graph structure, i.e., a new graph $G^{\text{new}}$ is generated in each iteration based on the proposed distribution $Q$. The acceptance probability of $G^{\text{new}}$ is calculated as follows:

$$\tilde{P}(G^{\text{new}} \mid G) = \min \left\{ \frac{P(Y \mid G^{\text{new}})P(G^{\text{new}})Q(G \mid G^{\text{new}})}{P(Y \mid G)P(G)Q(G^{\text{new}} \mid G)}, 1 \right\},$$

where $P(Y \mid G)$ is the likelihood function, i.e., how well the graph $G$ matches the observation $Y$, $P(G)$ is the prior distribution of the graph $G$, and $Q(G^{\text{new}} \mid G)$ is the proposed distribution. Assuming that all graph structures $G$ in the graph space $G$ exist with equal probability, $P(G) = P(G^{\text{new}})$, while taking the proposed distribution satisfying symmetry, i.e., $Q(G^{\text{new}} \mid G) = Q(G \mid G^{\text{new}})$, Eq. (6) can be simplified as

$$\tilde{P}(G^{\text{new}} \mid G) = \min \left\{ \frac{P(Y \mid G^{\text{new}})}{P(Y \mid G)}, 1 \right\}.$$
Inflation transmission diagnostics

In this paper, we illustrate the performance of our model on a simulated data set, which is generated by the following processes:

\[ Y_t = \sum_{s=0}^{p} B_s(s_t) Y_{t-s} + \varepsilon_t, \varepsilon_t \sim N(0, \Sigma_{\varepsilon}(s_t)), \quad t = 1, \ldots, T, \]

where \( Y_t \) is an \( d \)-dimensional vector. \( p \) is the lag order, \( s_t \) is a state variable and \( s_t = 1 \) or \( 2 \). \( B_s(s_t), s = 0, \ldots, p \) is the sequence of \( d \times d \) coefficient matrices. In this study, we set \( T = 600, d = 5, p = 1, \) and the settings of \( B_s(s_t) \) and \( \Sigma_{\varepsilon,ij}(s_t) \) are shown in Table 2. The state transition probability matrix between the two regimes is \( P = \begin{pmatrix} 0.95 & 0.05 \\ 0.05 & 0.95 \end{pmatrix} \). Moreover, due to the \( A_1(s_t) = (I - B_0(s_t))^{-1}B_1(s_t) \), we calculate \( A_+ = A_1 \) from \( B_0, B_1 \) as shown in Tables 5 and 6.

| \( B_0 \) | \( B_0(1) = \begin{pmatrix} 0 & 0 & -0.8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \) | \( B_0(2) = \begin{pmatrix} 0.8 & 0 & 0 & -0.9 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \) |
| \( B_1 \) | \( B_1(1) = \begin{pmatrix} -0.8 & 0 & 0 & 0 & 0 \\ 0.6 & 0.5 & 0 & 0 & 0 \\ 0.7 & -0.5 & 0 & 0 \end{pmatrix} \) | \( B_1(2) = \begin{pmatrix} 0.6 & 0 & 0 & 0.8 & 0 \\ 0.9 & 0 & 0 & 0 \end{pmatrix} \) |
| \( \Sigma_{\varepsilon,ij} \) | \( \Sigma_{\varepsilon,ij}(1) \sim N(0,1) \) | \( \Sigma_{\varepsilon,ij}(2) \sim N(0,100) \) |

Based on the above settings, a simulated dataset was generated. According to Figure 1 of the sampling steps, 10,000 iterations were performed in this study, and the latter 5,000 iterations were taken for estimation.
Table 3: Accuracy of graph structure inference (MP).

<table>
<thead>
<tr>
<th></th>
<th>Regime 1</th>
<th>Regime 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_0$</td>
<td>85%</td>
<td>90%</td>
</tr>
<tr>
<td>$G_1$</td>
<td>60%</td>
<td>60%</td>
</tr>
</tbody>
</table>

We verified the robustness of the proposed model using the two previously mentioned prior distributions, Minnesota prior (MP) and Normal-Wishart prior (NW). The recognition results of the simulated data are shown in Tables 5 and 6.

First, the inference method proposed in this paper can effectively identify the periods when Markov transformations occur in the simulated data. The estimated values of the transition probabilities are less different from the true transfer probabilities in Table 7.

Second, since the graph structure is a data matrix with elements consisting of only zeros and ones, the accuracy can be used to measure the recognition of the graph. The formula for the accuracy rate is as follows:

\[
\text{Acc} = \frac{\text{True positive} + \text{True negative}}{\text{True positive} + \text{True negative} + \text{False positive} + \text{False negative}}
\]

(7)

For the simulated data, we find that the average recognition accuracy of the model corresponding to the NW prior is higher than the accuracy corresponding to the use of the MP, in Tables 5 and 6.

When using the NW prior, our proposed model identifies the causal graphs well under both regimes and has higher accuracy for lagged causal graphs than for contemporaneous causal graphs. Under Regime 1, the estimation accuracy of the lagged causal graph and the contemporaneous causal graph are 100% and 95%, respectively. Under Regime 2, the estimation accuracy of lagged causal graphs and contemporaneous causal graphs are 96% and 85%, respectively.

When using the MP, the recognition accuracy of contemporaneous causality graphs under both regimes is high, 95% and 85%, respectively, while the recognition accuracy of lagged causality is only 60%. We find that the NW prior is more accurate than the MP in identifying $G_1$ because the idea of the MP deviates from the relationship between the variables in the simulated data.

Additionally, our model has a good estimation of the regression coefficients under NW prior, due to the small difference between the estimated and true values of the coefficients as reported in Table 6. This good coefficient estimation is reflected in two ways: on one hand, the elements that are zero in the true coefficient matrix are also essentially zero in the estimated coefficient matrix. And on the other hand, the specific values of the non-zero elements of the estimated coefficient matrix are close to the values taken by the non-zero elements in the true coefficient matrix.
### Table 4: Accuracy of graph structure inference (NW prior).

<table>
<thead>
<tr>
<th></th>
<th>Regime 1</th>
<th>Regime 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_0$</td>
<td>95%</td>
<td>85%</td>
</tr>
<tr>
<td>$G_1$</td>
<td>100%</td>
<td>96%</td>
</tr>
</tbody>
</table>

### Table 5: Inference results for relevant parameters (MP).

<table>
<thead>
<tr>
<th>Actual setup</th>
<th>Estimated results</th>
<th>Regime 1</th>
<th>Regime 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_0$</td>
<td>$G_0(1)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$G_0(1) = \begin{pmatrix} 0 &amp; 1 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 \ 1 &amp; 0 &amp; 1 &amp; 0 &amp; 0 \end{pmatrix}$</td>
<td>$G_0(1) = \begin{pmatrix} 0 &amp; 0 &amp; 0 &amp; 0 &amp; 1 \ 0 &amp; 1 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 \ 1 &amp; 0 &amp; 1 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 1 &amp; 0 &amp; 1 \end{pmatrix}$</td>
<td></td>
</tr>
<tr>
<td>$A_+ A_+ (1)$</td>
<td>$A_+ (1) = \begin{pmatrix} -0.36 &amp; 0.00 &amp; 0.40 &amp; 0.00 &amp; 0.00 \ 0.60 &amp; 0.00 &amp; 0.50 &amp; 0.00 &amp; 0.00 \end{pmatrix}$</td>
<td>$A_+ (1) = \begin{pmatrix} 0.53 &amp; -0.01 &amp; 0.10 &amp; 0.70 &amp; -0.01 \ -0.80 &amp; -0.02 &amp; 0.00 &amp; 0.10 &amp; 0.02 \end{pmatrix}$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$G_1$</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$G_1(1) = \begin{pmatrix} 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 \ 1 &amp; 0 &amp; 1 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 1 &amp; 1 &amp; 0 \ 0 &amp; 1 &amp; 0 &amp; 0 &amp; 1 \end{pmatrix}$</td>
<td>$G_1(1) = \begin{pmatrix} 0 &amp; 0 &amp; 1 &amp; 0 &amp; 1 \ 0 &amp; 1 &amp; 0 &amp; 0 &amp; 1 \end{pmatrix}$</td>
<td></td>
</tr>
<tr>
<td>$A_+ A_+ (2)$</td>
<td>$A_+ (2) = \begin{pmatrix} -0.36 &amp; 0.00 &amp; 0.40 &amp; 0.00 &amp; 0.00 \ 0.60 &amp; 0.00 &amp; 0.50 &amp; 0.00 &amp; 0.00 \end{pmatrix}$</td>
<td>$A_+ (2) = \begin{pmatrix} 0.53 &amp; -0.01 &amp; 0.10 &amp; 0.70 &amp; -0.01 \ -0.80 &amp; -0.02 &amp; 0.00 &amp; 0.10 &amp; 0.02 \end{pmatrix}$</td>
<td></td>
</tr>
</tbody>
</table>

### Inflation transmission diagnostics

13
Table 6: Inference results for relevant parameters (NW prior).

<table>
<thead>
<tr>
<th>Actual setup</th>
<th>Estimated results</th>
</tr>
</thead>
</table>
| $G_0$        | $G_0(1) =$ \[
\begin{pmatrix}
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0
\end{pmatrix}
\]        | $G_0(1) =$ \[
\begin{pmatrix}
0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0
\end{pmatrix}
\]        |
| $G_1$        | $G_1(1) =$ \[
\begin{pmatrix}
1 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 \\
0 & 1 & 0 & 0 & 1
\end{pmatrix}
\]        | $G_1(1) =$ \[
\begin{pmatrix}
1 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 \\
0 & 1 & 0 & 0 & 1
\end{pmatrix}
\]        |
| $A_+$        | $A_+(1) =$ \[
\begin{pmatrix}
-0.36 & 0.00 & 0.40 & 0.00 & 0.00 \\
0.60 & 0.00 & 0.50 & 0.00 & 0.00 \\
0.70 & 0.00 & -0.50 & 0.00 & 0.00 \\
0.30 & 0.00 & 0.75 & 0.70 & 0.00 \\
0.00 & -0.60 & 0.00 & 0.00 & 0.60
\end{pmatrix}
\]        | $A_+(1) =$ \[
\begin{pmatrix}
-0.80 & 0.00 & -0.05 & 0.00 & 0.00 \\
0.62 & 0.00 & 0.51 & 0.00 & 0.00 \\
0.67 & 0.00 & -0.49 & 0.00 & 0.00 \\
-0.02 & 0.00 & 0.48 & 0.67 & 0.00 \\
0.00 & -0.50 & 0.00 & 0.00 & 0.56
\end{pmatrix}
\]        |

Table 7: Estimates of the transfer probability of simulated data.

<table>
<thead>
<tr>
<th></th>
<th>Regime 1</th>
<th>Regime 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regime 1</td>
<td>0.9353</td>
<td>0.0647</td>
</tr>
<tr>
<td>Regime 2</td>
<td>0.0456</td>
<td>0.9544</td>
</tr>
</tbody>
</table>
Finally, the estimated effect of this paper is valid for the graph structure. To assess the convergence of the graph structure, two indicators are used, namely the potential scale reduction factor (PSRF) and the multivariate PSRF (MPSRF), which are formulated in Gelman and Rubin [30] and Brooks and Gelman [31], respectively. If the value of either PSRF or MPSRF is 1.2 or less, then the Markov chain has achieved convergence, indicating that the graph structure has converged. Tables 8 and 9 reveal that the values of PSRF and MPSRF in both regimes are less than 1.20, and the convergence of the graph structures corresponding to the two prior distributions is confirmed [15]. In other words, the model proposed in this paper is robust under both prior distributions, MP and NW. However, the NW prior is better than the MP prior, therefore, in the experimental stage, we use the NW prior to estimating the parameters.

5 The case study

5.1 The data

The Group of Seven (G7) is the representative of the developed countries and is composed of the seven most developed industrialized countries, the United States (US), Japan (JAP), Germany (GE), the United Kingdom (UK), France (FR), Italy (IT) and Canada (CAN). As of 2020, the G7’s share of world GDP remains close to 50%. The G7, which plays a pivotal role in the world economy, has been closely linked in various political, economic, and cultural aspects since its establishment in 1976. However, it is this close connection that makes inflation within the G7 transmit to each other [9]. Standing in the long history, G7 economies have experienced not only periods of hyperinflation but also periods of low and stable normal inflation. Therefore, in this study, G7 is chosen as the target country to test the validity of the proposed model.

Since the consumer price index (CPI) is an important indicator of inflation, this paper uses CPI to represent the change in inflation (Inflation). However, there is a certain seasonality in CPI data. Then, in order to avoid the possible experimental results from this seasonality factor,
the data chosen in this paper is the month-on-month CPI data of G7 economies. Moreover, due to the availability of data, the time range of the selected sample is January 1971 to December 2019. Data are obtained from the IFIND database.

![Graph of monthly inflation in G7 economies](image)

**Figure 2:** Trend graph of monthly inflation in G7 economies.

The range and magnitude of changes in inflation in the G7 countries within the sample are large. From a statistical point of view, there may be a regime shift in the level of inflation in these seven countries. Therefore, in the empirical analysis reported in the next subsection, we use the MS-BGVAR to explore the inflation transmission relationship in the G7 economies. Figure 2 and Table 10 show the time series plots and statistical indicators for these seven countries, respectively.

<table>
<thead>
<tr>
<th>Country</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>Std.dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>-2.100</td>
<td>14.760</td>
<td>3.963</td>
<td>2.976</td>
</tr>
<tr>
<td>JAP</td>
<td>-2.500</td>
<td>24.900</td>
<td>2.510</td>
<td>4.393</td>
</tr>
<tr>
<td>UK</td>
<td>0.200</td>
<td>26.870</td>
<td>5.472</td>
<td>2.800</td>
</tr>
<tr>
<td>GE</td>
<td>-1.000</td>
<td>7.920</td>
<td>2.636</td>
<td>1.922</td>
</tr>
<tr>
<td>FR</td>
<td>-0.730</td>
<td>15.160</td>
<td>4.169</td>
<td>4.063</td>
</tr>
<tr>
<td>IT</td>
<td>-0.560</td>
<td>25.240</td>
<td>6.254</td>
<td>6.027</td>
</tr>
<tr>
<td>CAN</td>
<td>-4.550</td>
<td>23.380</td>
<td>4.327</td>
<td>4.872</td>
</tr>
</tbody>
</table>

**Table 10:** Statistical indicators of inflation data for seven countries.
5.2 The regime switching

In this paper, the MS-BGVAR model is applied to the inflation data of G7 economies. We first determine the lag order $p = 1$ according to the BIC criterion, in other words, only the effect of past period inflation on only current period inflation needs to be considered. Based on the relevant theoretical basis\cite{26}, we name the two regimes as “high inflation regime” and “low inflation regime”. The experiment was conducted with 10,000 samples and the statistical inference was made based on the remaining 5,000 samples. It should be noted that, in order to eliminate the error caused by the trend, we take a first-order differencing of the data in this paper before formal modeling.

The dates of regime switches identified in this study are highly correlated with the economic environment. The period 1971-1983 was extremely unstable for the world economy, and inflation levels were high during this period. At the same time, between 1972 and 1982, the inflation transmission relationship in the G7 countries was in Regime 2 (high inflation regime in Figure 3(a)).

While from 1990 onwards, some countries made maintaining price stability the primary goal of macroeconomic regulation. Since then, most countries have entered a period of declining inflation levels, remaining at a more reasonable level of inflation until 2019. Meanwhile, after 1990, the inflation transmission relationship in G7 countries has been in Regime 1 (low inflation regime in Figure 3(a)) for a long time.

Moreover, the stability and duration of the high inflation regime in this experiment are lower than those of the low inflation regime, which is consistent with Thams \cite{25}. The duration of the low and high inflation regimes is 52 months and 11 months, respectively. If the duration is higher than that of the low-inflation regime, there will be serious negative political, economic, and social impacts. The transfer probability matrices for the two inflation regimes are shown in Table 11 and their posterior probability densities are shown in Figure 4.

![Figure 3: Probability of filtering between two inflation regimes.](image-url)
Table 11: The posterior transition probability of inflation environment.

<table>
<thead>
<tr>
<th></th>
<th>High inflation regime</th>
<th>Low inflation regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>High inflation regime</td>
<td>0.9772</td>
<td>0.0228</td>
</tr>
<tr>
<td>Low inflation regime</td>
<td>0.1134</td>
<td>0.8866</td>
</tr>
</tbody>
</table>

Figure 4: Posterior density plot of the transition matrix $P$ of the inflation transmission relationship.

5.3 The causality relationship

In the previous subsection, we identified regime shifts in the transmission of inflation in the G7 economies. Then, this subsection specifically identifies the causal relationships between inflation in each country under both regimes, including contemporaneous and lagged causality, as shown in Figures 8 and 9. Meanwhile, our node characteristics calculated based on the complex network analysis method are shown in Table 12. Also, the posterior probability density of the coefficients is shown in Figure 5.
Inflation transmission diagnostics

Based on the proposed model, it is found that the existence and strength of contemporaneous and lagged causality varies across regimes. This also reveals that the transmission relationship between inflation in G7 countries is regime dependent.

The values in Figures 6 and 7 show the probability of the existence of directed edges between countries, and the red ones are the values with a probability greater than 0.5. If the probability of the existence of a directed edge is greater than 0.5, then it means that there is a directed edge between nodes, i.e., there is a causal relationship between countries. In Figures 8 and 9, we visualize the causality between countries.

(a) The strength of lagged causality

(b) The strength of contemporaneous causality

Figure 6: The strength of causality among G7 countries under high-inflation regime
From the perspective of lagged causality, network linkages are higher in the high inflation regime than in the low inflation regime, and lagged inflation in the U.S. has a strong spillover according to Table 12. In the high-inflation regime, lagged inflation in the U.S. points to Japan and France. Lagged inflation in Japan points to the U.S., Italy, France, and Canada. Lagged inflation in the U.K. points to Germany and Canada. Lagged inflation in Italy points to Japan. Neither France nor Canada issues a directional edge, suggesting that their lagged inflation does not affect other countries as illustrated in Figure 8(a). In contrast, only the U.S. and France export directed edges in the low-inflation regime. The US points to Japan and the UK, while France points to Italy and Germany from Figure 9(a).
From the perspective of contemporaneous causality, the relevant policies adopted by one country may have an immediate impact on the inflation levels of other countries, and this immediate impact will be more pronounced in the low inflation regime. Under a high inflation regime, policies or actions implemented in Canada and France immediately affect the level of inflation in the United States as shown in Table 12. In contrast, under a low-inflation regime, the United States is the country that exports the most contemporaneous inflation according to Figure 9(b). The level of inflation in Japan, the U.K., Germany, and Italy are all affected by U.S. policy. At the same time, inflation in the U.K., Germany, and Italy reacts to French economic policy as shown in Figure 9(b).

In addition, according to Figures 6 and 7, we also observe information on the strength of the causal relationship between countries. On the one hand, we find that regardless of whether it is a high or low inflation regime, a country’s own level of inflation at the previous moment almost always has a strong effect on the level of inflation in that country at that moment, and this can be found in Figures 6(a) and 7(a), where the values represent the probability of the existence of a directional edge between countries, i.e., the strength of the causal relationship. At the same time, we also find that the lagged level of inflation in the U.S. can have a large impact on the current level of inflation in Japan. On the other hand, the number of causal relationships between the current level of inflation in one country and the current level of inflation in other countries is small and the probability of an impact is low according to Figures 6(b) and 7(b).

Last but not least, based on the causality analysis above, we find that the U.S., Japan, the U.K., and France play an important role in the inflation transmission network, and this performance is related to their economic status. Also, this experiment obtains evidence of weaker homogeneity of Germany in the lagged inflation transmission network, which is consistent with Thams [25]. Regardless of the inflation regime, Germany is only affected by lagged inflation in one country.
Table 12: Nodes Characteristics of Lagged Inflation Transmission Networks.

<table>
<thead>
<tr>
<th>Country</th>
<th>high inflation regime</th>
<th>low inflation regime</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Out-degree</td>
<td>In-degree</td>
</tr>
<tr>
<td>US</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>JAP</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>UK</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>GE</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>FR</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>IT</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>CAN</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Total</td>
<td>9</td>
<td>9</td>
</tr>
</tbody>
</table>

The degree is calculated without taking into account its own lagged inflation.

Out-degree: the amount of national inflation exported to other countries.

In-degree: the amount of inflation that the country receives from other countries.

Total-degrees: the number of countries directly connected to the country.

5.4 General discussion

The above experiments show that our proposed model has good explanatory power and that the conclusions drawn are of academic and economic interest. On one hand, the period of regime switching identified in this paper is similar to the study by Thams [25], which supports their findings to some extent. On the other hand, the study finds a large lag in the inflation linkage across countries in the high inflation regime, with nine directed edges in the lagged inflation transmission network as displayed in Figure 8. In contrast, the direct impact of a country’s policies or actions on inflation in other countries is not evident in the high inflation regime, with only three directed edges in the contemporaneous inflation transmission network in Figure 8(b). Interestingly, U.S. actions in the low-inflation regime have an immediate effect on inflation in other countries. There are a total of 7 directed edges in the contemporaneous causal diagram for the low inflation regime, 4 of which come from the U.S. according to Figure 9(b).

Besides analyzing the regime shift in the inflation transmission process and specifying the specific direction of transmission, the proposed model can also help the central banks and monetary authorities of the sample countries to develop feasible inflation control measures. Since inflation is closely correlated across countries, we suggest that countries should consider not only their own actual situation but also the inflation situation of the countries related to them when controlling inflation. Otherwise, their own efforts may be undermined by others. In further, there is a strong contemporaneous causality due to the low inflation regime. Therefore, we recommend that countries should intensify their research on the past policies of other countries in order to develop corresponding inflation control methods more quickly and systematically.

These recommendations apply to inflation management not only in the countries discussed in this paper but also in other countries.
6 Conclusions

This study proposes a novel model for analyzing the inflation transmission relationship based on regime transformation and causality, defined as the MS-BGVAR model. The model combines Markov switching and Bayesian graph vector autoregression methods to solve the problem of estimating multivariate causality under different regimes. In addition, through simulation experiments, the proposed model can accurately identify contemporaneous and lagged causal relationships between simulated data with regime transitions and passes the convergence diagnosis. The G7 economies are selected as the target countries, and the proposed model is applied to the analysis of inflation transmission relationships in these seven countries to construct lagged inflation transmission diagrams and contemporaneous inflation leads under different regimes, and the proposed model is found to have better economic explanatory power on the inflation data. After obtaining the direction of inflation transmission, the central banks and monetary authorities of each country can accordingly formulate relevant initiatives in line with this pattern. In addition, this paper finds that the U.S. and Japan are the main exporters in the inflation transmission network and that inflation in the U.S. has the greatest causal impact on the level of inflation in other countries, especially the stronger causality between the U.S. and Japan. The relevant economic initiatives in the U.S. are more likely to affect the current inflation level in other countries under the low inflation regime. The main findings of this study are as follows.

- The proposed MS-BGVAR model considers both regime switching and contemporaneous causality among economic variables, realizing a theoretical extension of the existing approach.

- The constructed model was validated by simulation experiments and performed well in the identification of contemporaneous and lagged causal structures under both regimes.

- Applying the proposed model to the G7 economies, it is found that there is a significant regime shift in the inflation transmission relationship for these seven countries within the sample and that inflation linkages are higher in the high inflation regime than in the low inflation regime.

In fact, there are three possible inflation regimes, i.e., a high inflation regime, a medium inflation regime, and a low inflation regime. Therefore, future studies should give due consideration to increasing the number of regimes rather than just two regimes. Due to the availability of data, developing countries such as Brazil, Russia, India, China, and South Africa are not considered in this paper. In future work, the proposed model can be considered for application to developing countries. Furthermore, the experiments in this paper are based on month-on-month data for G7 economies from 1971-2019, and the empirical results obtained may only be applicable to the sample countries. In the study of the inflation transmission relationship, the proposed model is validated by simulated data and is effective at the algorithmic level. Moreover, the proposed model can be applied to other financial areas, such as stock market linkage.
first, financial riskiness, and exchange rate linkage, and may have better economic explanatory power for the available data, thus extending the research framework in the corresponding areas.

Only two regimes are included in the inflation transmission relationship among the G7 countries discussed in this paper, the high inflation regime and the low inflation regime. Therefore, in designing the program, we only code the MSBGVAR model for both regimes. However, among other economic problems, there may be more than two states, and our model can only identify the causal relationships between the families of economic variables under both regimes, which is a limitation of this paper. It should be noted that the number of regimes ex-ante is unknown which may cause a misspecification problem if we incorrectly assume the number of regimes, and we can determine the number of states using a restriction on state identification based on weighted feature vector centrality before applying the MS-BGAVR model. If the state number is more than 2, we will extend our work to study three or four regimes in the future as follows: As the number of regimes increases, the dimensionality of the transfer probability matrix also increases. The elements of the rows in the transfer matrix follow the prior distribution of the Dirichlet distribution (when multidimensional, the Dirichlet distribution cannot be regarded as a Beta distribution), however, the remaining steps remain the same as those for the two states.

References


