Appendix A Experimental Setup and Plots

A.1 Dataset Generation

A.1.1 Aircraft Dynamics Modeling Experiment

Given the initial position of an aircraft, the aircraft model in [7] calculates the guidance behavior based on the current state and control commands, and outputs the next state of the aircraft. The control commands consist of six types of discrete instructions, each of which is composed of accelerations in the x,y, and z directions of the Cartesian coordinate system. These six instructions represent level flight at constant speed, horizontal right turn, horizontal left turn, climbing, diving, and horizontal acceleration. The data features include two parts: 1) the one-hot representation of the aircraft's control command index at the current moment; 2) the current state of the aircraft, including the x,y, and z positions in the Cartesian coordinate system, scalar velocity, pitch angle, and heading. The data labels only contain the aircraft state at the next moment (0.005 seconds later), also including the x,y,z positions, scalar velocity, pitch angle, and heading in the Cartesian coordinate system.

Each flight trajectory lasts 3,000 time steps, with the initial state of the aircraft randomly distributed in the x,y, and z directions of the Cartesian coordinate system within a range of 1,000 meters. The entire dataset consists of 200 random flight trajectories, approximately 600,000 samples, of which 80% are used for training and 20% for testing.

A.1.2 Missile Dynamics Modeling Experiment

We employ the missile model in [7] as the cloning or modeling object and adopt the proportional guidance method. Given the initial state of the missile and the target, the missile model calculates the corresponding guidance maneuvers based on its current state and the relative state of the target, and outputs the next state of the missile using the RK4 numerical integration method with adjustable time step based on the proportional guidance formula. The missile state comprises six dimensions: coordinates x,y,z in the inertial coordinate system, scalar velocity, heading, and pitch angle. Similarly, the state of the target aircraft includes these six dimensions, represented by x,y,z,v, ψ, and γ. After providing the initial state of the missile and the target aircraft, the target aircraft can freely execute various maneuvers. The missile adjusts its three accelerations according to the different states generated by the target and ultimately intercepts the target aircraft.

The data features include two parts: 1) the current state of the target aircraft, including the x,y,z positions in the Cartesian coordinate system, scalar velocity, pitch angle, and heading; 2) the current state of the missile, including the x,y,z positions in the Cartesian coordinate system, scalar velocity, pitch angle, and heading. The data labels only contain the missile state at the next moment (0.005 seconds later), also including the x,y,z positions, scalar velocity, pitch angle, and heading in the Cartesian coordinate system.

Since the time for the missile to attack the target aircraft is not fixed, each missile flight trajectory lasts between 2,000 and 4,000 time steps. Without loss of generality, we fix the missile launch coordinates at the origin, and the initial state of the aircraft is randomly distributed in the$x$,y,$z$directions of the Cartesian coordinate system within a range of 1,000 meters. The entire dataset consists of 200 random flight trajectories, approximately 600,000 samples, of which 80% are used for training and 20% for testing.

A.2 Experimental Details

The VHNN model constructs a residual module with a three-layer MLP neural network with 256 hidden units, connecting the encoder, decoder, and Hamiltonian network or energy network. Each network uses this residual module as the main body to ensure smooth gradient flow between different network structures. The encoder takes data features as input and outputs 32-dimensional generalized coordinates q and 32-dimensional generalized momenta p. The energy and Hamiltonian networks take q, p, and the target state as inputs and output a 1-dimensional abstract energy. The energy derivatives with respect to p and q are calculated using Hamilton's equations (**??**) and (**??**), and the next moment's generalized coordinates q' and momenta p' are obtained using Euler integration within the time interval Δt. The next moment's rigid body state is then decoded by the decoder.

We train four models using the same dataset: a three-layer MLP with 512 hidden units, VHNN, and HNN and HGN models with network structures and hyperparameters similar to the VHNN. It is worth noting that the original HGN paper mainly focuses on image sequences, but techniques related to image sequences are not necessary for the rigid body model cloning task. Therefore, only the core idea of HGN is retained for comparison. Additionally, as the original HNN and HGN algorithms do not include an input mechanism for external control variables, we incorporate the conditional input into the encoder module according to the experimental feature requirements, while the decoder still outputs the next moment's state.

The energy comparisons of each model in Appendices B and C are formed by scaling the Hamiltonian or VHNN energy to the mechanical energy of the true trajectory, resulting in a contrast of energy differences.

A.3 Aircraft Dynamics Modeling Experiment Plots

Deep Aircraft Dynamics Modeling Experiment Results Figures:A1,A2,A3,A4,A5

A.4 Missile Guidance Dynamics Modeling Experiment Plots

Deep modeling experimental results of missile guidance dynamics: A6,A7,A8,A9,A10

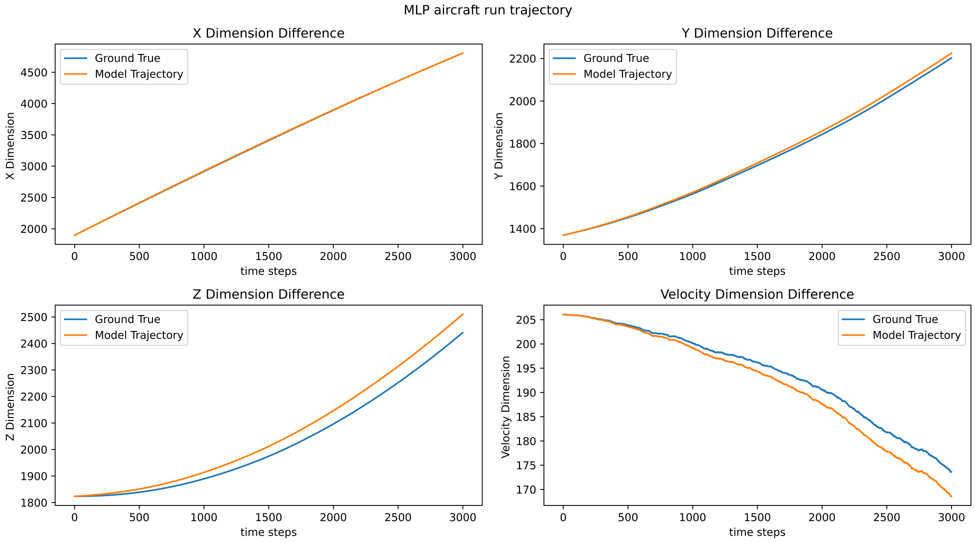

Fig. A1 Aircraft dynamic modeling experiment simulation trajectory of the MLP model, incorporating differences in the Cartesian coordinate system's three-axis coordinates and disparities in velocity magnitudes.

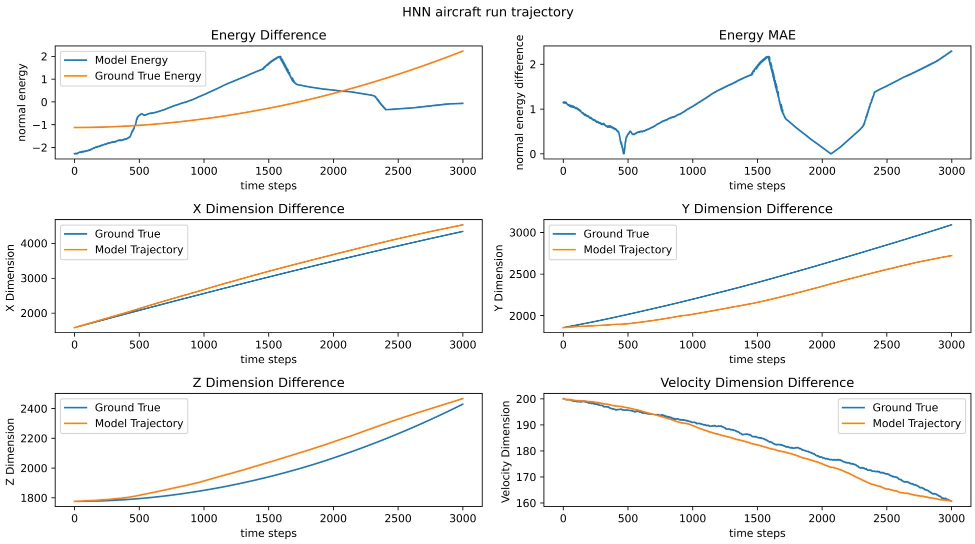

Fig. A2 Aircraft dynamic modeling experiment simulation trajectory of the HNN model, comprising normalized energy discrepancies, Mean Absolute Error (MAE) of normalized energies, distinctions in the Cartesian coordinate system's three-axis coordinates, and disparities in velocity magnitudes.

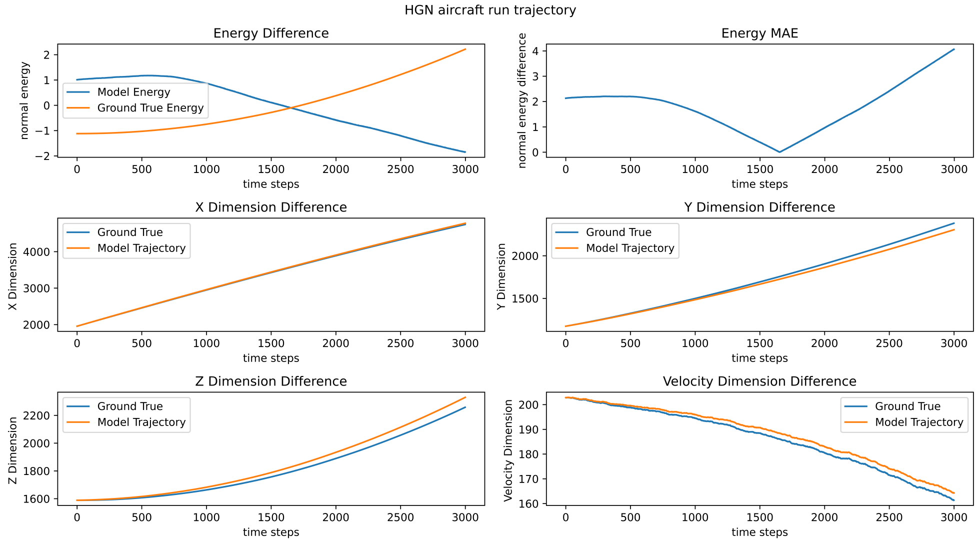

Fig. A3 Aircraft dynamic modeling experiment simulation trajectory of the HGN model, encompassing normalized energy discrepancies, Mean Absolute Error (MAE) of normalized energies, variations in the Cartesian coordinate system's three-axis coordinates, and disparities in velocity magnitudes.

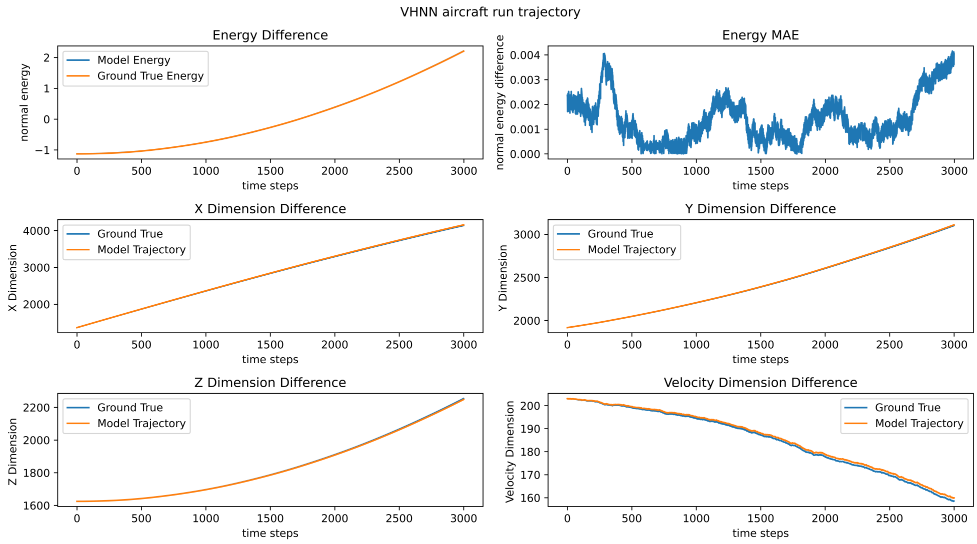

Fig. A4 Aircraft dynamic modeling experiment simulation trajectory of the VHNN model, including normalized energy discrepancies, Mean Absolute Error (MAE) of normalized energies, differences in the Cartesian coordinate system's three-axis coordinates, and disparities in velocity magnitudes.

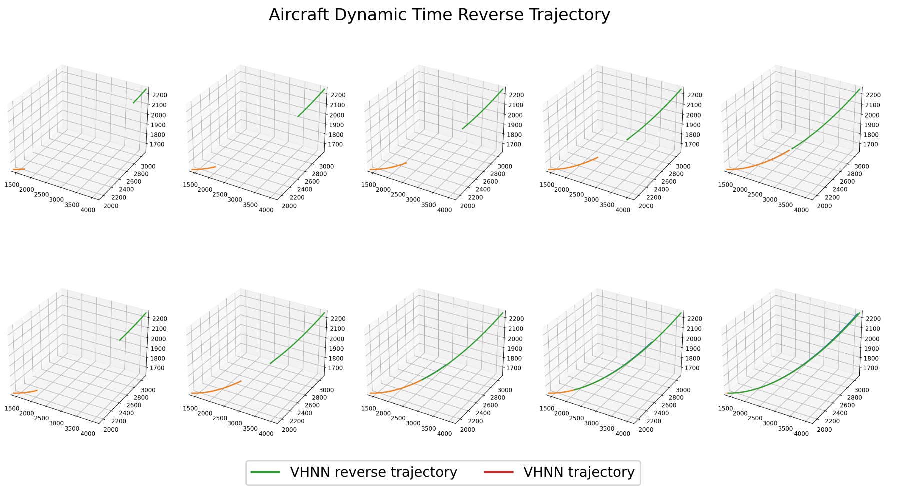

Fig. A5 This is a comparison diagram of forward and backward extrapolation over time in the aircraft modeling experiment using the VHNN model.

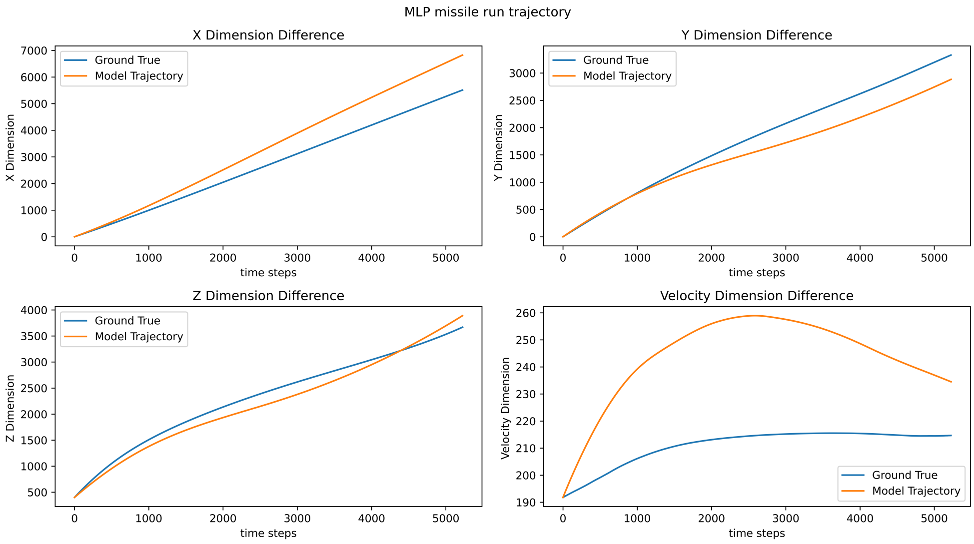

Fig. A6 Trajectory simulation of missile guidance dynamics modeling experiment using MLP model, including the differences in Cartesian coordinate system's three-axis coordinates and the differences in velocity scalar magnitudes.

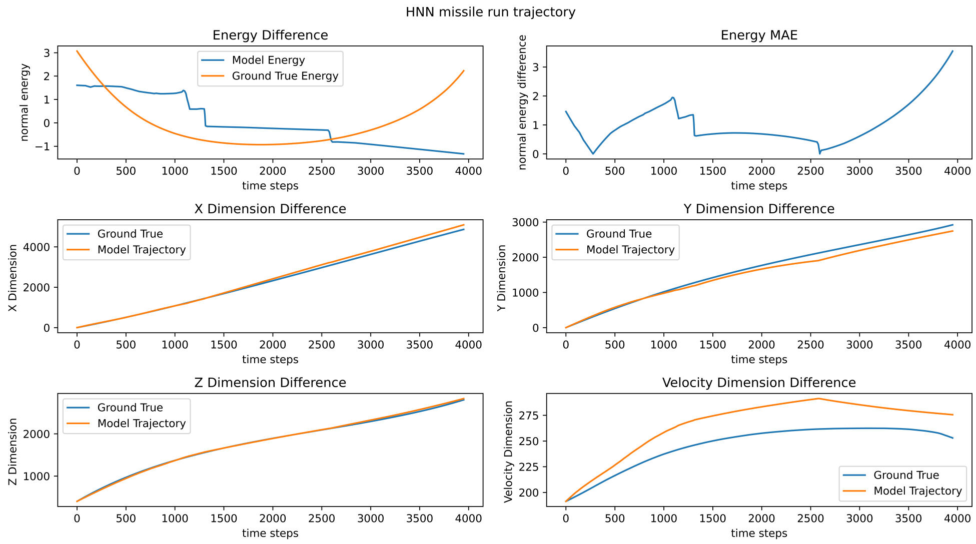

Fig. A7 Trajectory simulation of missile guidance dynamics modeling experiment employing HNN model, encompassing the disparities in normalized energy, normalized energy's mean absolute error (MAE), deviations in Cartesian coordinate system's three-axis coordinates, and the dissimilarities in velocity scalar magnitudes.

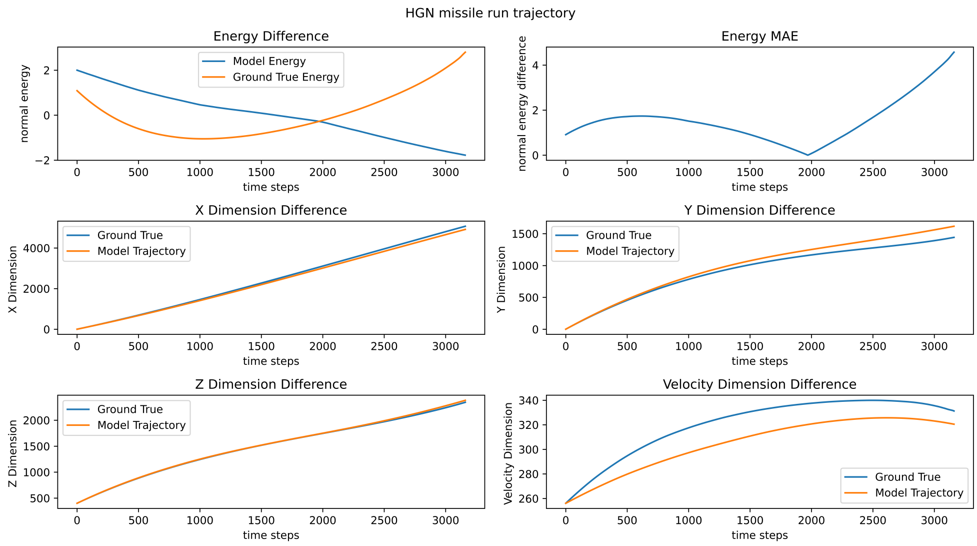

Fig. A8 Trajectory simulation of missile guidance dynamics modeling experiment using HGN model, comprising the discrepancies in normalized energy, normalized energy's mean absolute error (MAE), variations in Cartesian coordinate system's three-axis coordinates, and the distinctions in velocity scalar magnitudes.

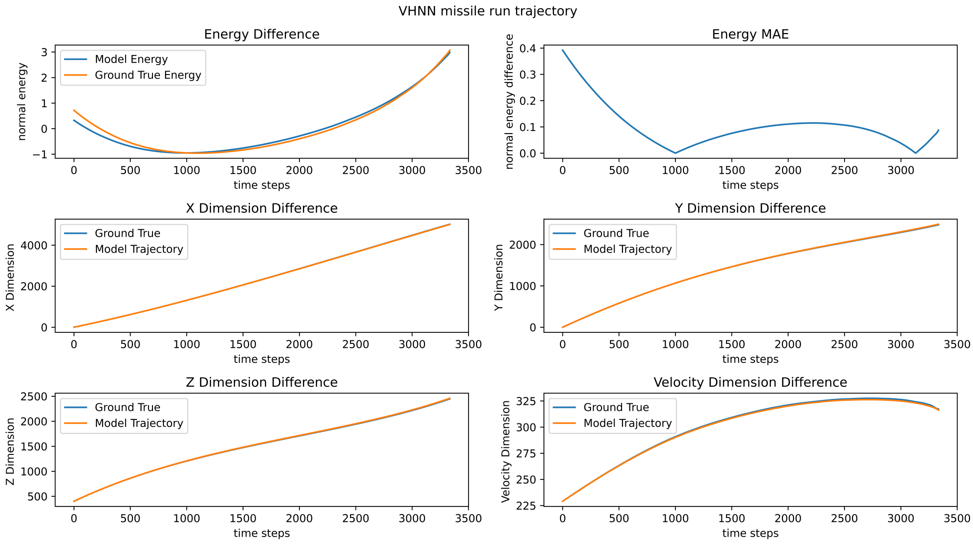

Fig. A9 Trajectory simulation of missile guidance dynamics modeling experiment conducted with VHNN model, incorporating the differences in normalized energy, normalized energy's mean absolute error (MAE), fluctuations in Cartesian coordinate system's three-axis coordinates, and the discrepancies in velocity scalar magnitudes.

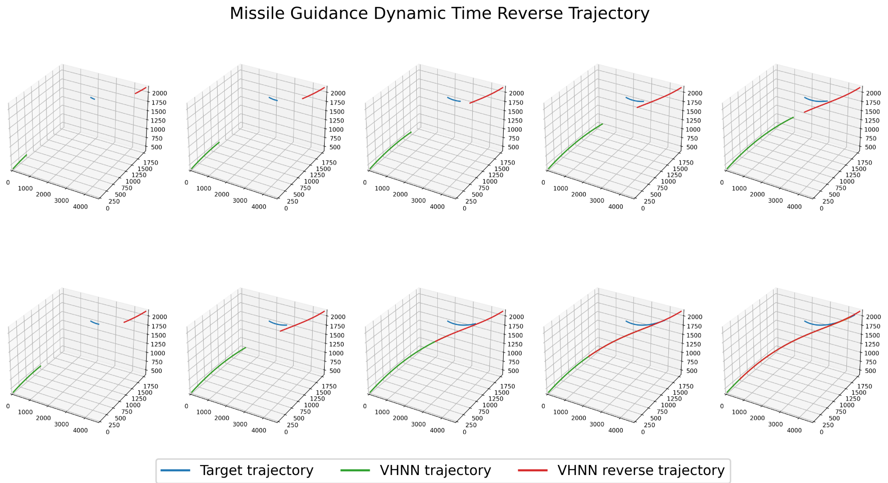

Fig. A10 This illustration depicts the comparison between forward and backward extrapolation over time in the VHNN model during the missile guidance modeling experiment.