Appendix

$$E_{I(i-1)} = \frac{-\mu_R p_i E_{F(i-1)} \cos[q_{i-1}(d-x_{F(i-1)})] + \mu_R q_{i-1} E_{F(i-1)} \sin[q_{i-1}(d-x_{F(i-1)})]}{\mu_{(R-1)} 2p_i \exp(-p_i w)} \quad \textbf{(A1a)}$$

$$E_{I(i-1)} = \frac{-\mu_R p_i E_{F(i-1)} \sinh[Q_{i-1}(d-x_{F(i-1)})] - \mu_R Q_{i-1} E_{F(i-1)} \cosh[Q_{i-1}(d-x_{F(i-1)})]}{\mu_{(R-1)} 2p_i \exp(-p_i w)} \quad \textbf{(A1b)}$$

$$E_{I(i-2)} = \frac{-\mu_R p_i E_{F(i-1)} \cos[q_{i-1}(d-x_{F(i-1)})] - \mu_R q_{i-1} E_{F(i-1)} \sin[q_{i-1}(d-x_{F(i-1)})]}{\mu_{(R-1)} 2p_i} \quad \textbf{(A2a)}$$

$$E_{I(i-2)} = \frac{-\mu_R p_i E_{F(i-1)} \sinh[Q_{i-1}(d-x_{F(i-1)})] + \mu_R Q_{i-1} E_{F(i-1)} \cosh[Q_{i-1}(d-x_{F(i-1)})]}{\mu_{(R-1)} 2p_i} \quad \textbf{(A2b)}$$

$$x_{Fi} = \frac{1}{p_i} \tan^{-1}\left(\frac{\mu_{(i-1)} p_{(i-1)} (E_{I(i-3)} \exp(-p_{i-1} w) - E_{I(i-2)})}{\mu_i p_i (E_{I(i-3)} \exp(-p_{i-1} w) + E_{I(i-2)})}\right) \quad \textbf{(A3a)}$$

$$x_{Fi} = \frac{-1}{p_i} \tanh^{-1}\left(\frac{\mu_{(i-1)} p_i (E_{I(i-3)} \exp(-p_{i-1} w) + E_{I(i-2)})}{p_{i-1} \mu_i (E_{I(i-3)} \exp(-p_{i-1} w) - E_{I(i-2)})}\right) \quad \textbf{(A3b)}$$

$$E_{Fi} = \frac{-\mu_i E_{I(i-3)} \exp(-p_{i-1} w) - \mu_i E_{I(i-2)}}{\mu_{(i-1)} \cos(q_i x_{Fi})} \quad \textbf{(A4a)}$$

$$E_{Fi} = \frac{-\mu_i E_{I(i-3)} \exp(-p_{i-1} w) - \mu_i E_{I(i-2)}}{\mu_{(i-1)} \sinh(Q_i x_{Fi})} \quad \textbf{(A4b)}$$

$$x_{F2} = \frac{1}{p_2} \tan^{-1}\left(\frac{p_1}{q_2} \frac{\mu_{(i-1)}}{\mu_i}\right) \quad \textbf{(A5a)}$$

$$x_{F2} = \frac{-1}{Q_2} \tanh^{-1}\left(\frac{Q_2}{p_1} \frac{\mu_{(i-1)}}{\mu_i}\right) \quad \textbf{(A5b)}$$

$$E_{F2} = \frac{E_S}{\cos(q_2 x_{F2})} \quad \textbf{(A6a)}$$

$$E_{F2} = \frac{E_S}{\sinh(Q_2 x_{F2})} \quad \textbf{(A6b)}$$

In the localized arbitrary nonlinear guiding layer

$$a_R^2 = \frac{\sqrt{q_R^4 + 2\alpha_R k_0^2 K_R} + q_R^2}{\alpha_R k_0^2} \quad \textbf{(A7a)}$$

$$a_R'^2 = \frac{\sqrt{Q_R^4 + 2\alpha_R k_0^2 K_R} - Q_R^2}{\alpha_R k_0^2} \quad \textbf{(A7b)}$$

For $m \geq 7$, $R \in \{4,6,8,\ldots,m-3\}$

$$K_R = k_0^2 E_{I(R-1)}^2 \left(\left(\frac{\mu_R}{\mu_{(R+1)}}\right)^2 \left(\beta^2 - \varepsilon_{(R+1)}\mu_{(R+1)}\right) + \left(\varepsilon_R \mu_R - \beta^2\right) + \frac{\alpha_R E_{I(R-1)}^2}{2} \right) + k_0^2 E_{IR}^2 \exp\left(-2p_{(R+1)}w\right)$$
$$\times \left[\left(\frac{\mu_R}{\mu_{(R+1)}}\right)^2 \left(\beta^2 - \varepsilon_{(R+1)}\mu_{(R+1)}\right) + \left(\varepsilon_R \mu_R - \beta^2\right) + \frac{\alpha_R E_{IR}^2}{2} \exp\left(-2p_{(R+1)}w\right) \right]$$
$$+2E_{I(R-1)}E_{IR}k_0^2 \exp\left(-p_{(R+1)}w\right) \times \left[\varepsilon_R \mu_R - 2\beta^2 + \varepsilon_{(R+1)}\mu_{(R+1)} + \frac{3}{2}\alpha_R \times E_{I(R-1)}E_{IR} \exp\left(-p_{(R+1)}w\right)\right]$$
$$+2\alpha_R E_{I(R-1)}E_{IR}k_0^2 \exp\left(-p_{(R+1)}w\right) \times \left[E_{I(R-1)}^2 + E_{IR}^2 \exp\left(-2p_{(R+1)}w\right)\right] \quad \textbf{(A8)}$$

$$x_{0R} = cn^{-1}\left[\frac{E_{I(R-3)}\exp\left(-p_{(R-1)}\right) + E_{I(R-2)}}{b_R}\left(\frac{-\mu_R}{\mu_{(R-1)}}\right)\right] \quad \textbf{(A9)}$$

$$E_{IR} = \frac{1}{2}\{-A_R \frac{\mu_R}{\mu_{(R-1)}}\left[\Delta_1 sn(A_R d)dn(A_R d)\left[1 - l_R\left(1 - \frac{\Delta_1^2}{b_R^2}\right)\right] - \Delta_1 l_R sn(A_R d)cn^2(A_R d)dn(A_R d)\left(1 - \frac{\Delta_1^2}{b_R^2}\right)\right.$$
$$\left. + \left(dn^2(A_R d)cn(A_R d) - l_R sn^2(A_R d)cn(A_R d)\frac{\Delta_1^2}{b_R^2}\right)\frac{\Delta_2 p_{(R-1)}}{A_R}\right] /$$
$$2p_{(R+1)}\left[1 - l_R sn^2(A_R d)\left(1 - \frac{\Delta_1^2}{b_R^2}\right)\right]^2 - \frac{\mu_R}{\mu_{(R-1)}}\left[cn(A_R d)\Delta_1 - sn(A_R d)dn(A_R d)\frac{\Delta_2 p_{(R-1)}}{A_R}\right] /$$
$$\left[1 - l_R sn^2(A_R d)\left(1 - \frac{\Delta_1^2}{b_R^2}\right)\right]\} \quad \textbf{(A10)}$$

$$E_{I(R-1)} = \frac{1}{2\exp\left(p_{(R+1)}w\right)}$$
$$\times \{-A_R \frac{\mu_R}{\mu_{(R-1)}}\left[\Delta_1 sn(A_R d)dn(A_R d)\left[1 - l_R\left(1 - \frac{\Delta_1^2}{b_R^2}\right)\right] - \Delta_1 l_R sn\,(A_R d)cn^2(A_R d)dn(A_R d)\left(1 - \frac{\Delta_1^2}{b_R^2}\right)\right.$$
$$\left. + \left[dn^2\,(A_R d)cn(A_R d) - l_R sn^2(A_R d)cn(A_R d)\frac{\Delta_1^2}{b_R^2}\right]\frac{\Delta_2 p_{(R-1)}}{A_R}\right] /$$
$$2p_{(R+1)}\left[1 - l_R sn^2(A_R d)\left(1 - \frac{\Delta_1^2}{b_R^2}\right)\right]^2 - \frac{\mu_R}{\mu_{(R-1)}}\left[cn(A_R d)\Delta_1 - sn(A_R d)dn(A_R d)\frac{\Delta_2 p_{(R-1)}}{A_R}\right] /$$
$$\left[1 - l_R sn^2(A_R d)\left(1 - \frac{\Delta_1^2}{b_R^2}\right)\right]\} \quad \textbf{(A11)}$$

$$\Delta_1 = -\mu_R E_{I(R-3)} \exp\left(-p_{(R-1)}w\right) - \mu_R E_{I(R-2)}$$

$$\Delta_2 = -\mu_R E_{I(R-3)} \exp\left(-p_{(R-1)}w\right) + \mu_R E_{I(R-2)}$$

$$E_{I(R-3)} = \frac{-\mu_R p_{(R-1)} E_{F(R-2)} \cos\left[q_{(R-2)}\left(d - x_{F(R-2)}\right)\right] + \mu_R q_{(R-2)} E_{F(R-2)} \sin\left[q_{(R-2)}\left(d - x_{F(R-2)}\right)\right]}{\mu_{(R-1)} 2p_{(R-1)} \exp\left(-p_{(R-1)}w\right)} \quad \textbf{(A12a)}$$

$$E_{I(R-3)} = \frac{-\mu_R p_{(R-1)} E_{F(R-2)} \sinh\left[Q_{(R-2)}\left(d - x_{F(R-2)}\right)\right] - \mu_R Q_{(R-2)} E_{F(R-2)} \cosh\left[Q_{(R-2)}\left(d - x_{F(R-2)}\right)\right]}{\mu_{(R-1)} 2p_{(R-1)} \exp\left(-p_{(R-1)}w\right)}$$

(A12b)

$$E_{I(R-2)} = \frac{-\mu_R p_{(R-1)} E_{F(R-2)} \cos\left[q_{(R-2)}\left(d - x_{F(R-2)}\right)\right] - \mu_R q_{(R-2)} E_{F(R-2)} \sin\left[q_{(R-2)}\left(d - x_{F(R-1)}\right)\right]}{\mu_{(R-1)} 2p_R} \quad \textbf{(A13a)}$$

$$E_{I(R-2)} = \frac{-\mu_R p_{(R-1)} E_{F(R-2)} \sinh[Q_{(R-2)}(d-x_{F(R-2)})] + \mu_R Q_{(R-2)} E_{F(R-2)} \cosh[Q_{(R-2)}(d-x_{F(R-1)})]}{\mu_{(R-1)} 2 p_R}$$

(A13b)

$$E_C = \frac{E_{F(m-1)}}{\cos[q_{(m-1)}(d-x_{F(m-1)})]} \quad \textbf{(A14a)}$$

$$E_C = \frac{E_{F(m-1)}}{\sinh[Q_{(m-1)}(d-x_{F(m-1)})]} \quad \textbf{(A14b)}$$

For m ≥ 7, R=2

$$K_2 = k_0^2 E_S^2 \left(\left(\frac{\mu_2}{\mu_1}\right)^2 (\beta^2 - \varepsilon_1 \mu_1) + (\varepsilon_2 \mu_2 - \beta^2) + \frac{\alpha_2 E_S^2}{2} \right) \quad \textbf{(A15)}$$

$$x_{02} = \frac{1}{A_2} cn^{-1}\left(\frac{E_S}{b_2}\frac{\mu_2}{\mu_1}\right) \quad \textbf{(A16)}$$

$$E_{I1} = \left\{ -\mu_1 A_2 E_S \left[1 - l_2\left(1 - \frac{E_S^2}{b_2^2}\right)\right] - l_2\left(1 - \frac{E_S^2}{b_2^2}\right) cn^2(A_2 d)\, sn(A_2 d)\, dn(A_2 d) - A_2 E_S p_1 cn(A_2 d) \left[dn^2(A_2 d) - l_2 \frac{E_S^2}{b_2^2} sn^2(A_2 d) \right] \right\} / 2 p_3 \mu_2 \left[1 - l_2 sn^2(A_2 d)\left(1 - \frac{E_S^2}{b_2^2}\right) \right]^2 - \mu_1 \left[E_s cn(A_2 d) + E_S p_1 sn(A_2 d) dn(A_2 d) \right] / 2\mu_2 \left[1 - l_2 sn^2(A_2 d)\left(1 - \frac{E_S^2}{b_2^2}\right) \right]$$

(A17)

$$E_{I2} = \frac{1}{\exp(-p_3 w)} \left(\left\{ -\mu_1 A_2 E_S \left[1 - l_2\left(1 - \frac{E_S^2}{b_2^2}\right)\right] - l_2\left(1 - \frac{E_S^2}{b_2^2}\right) cn^2(A_2 d)\, sn(A_2 d)\, dn(A_2 d) - A_2 E_S p_1 cn(A_2 d) \left[dn^2(A_2 d) - l_2 \frac{E_S^2}{b_2^2} sn^2(A_2 d) \right] \right\} / 2 p_3 \mu_2 \left[1 - l_2 sn^2(A_2 d)\left(1 - \frac{E_S^2}{b_2^2}\right) \right]^2 - \mu_1 \left[E_s cn(A_2 d) + E_S p_1 sn(A_2 d) dn(A_2 d) \right] / 2\mu_2 \left[1 - l_2 sn^2(A_2 d)\left(1 - \frac{E_S^2}{b_2^2}\right) \right] \right) \quad \textbf{(A18)}$$

$$E_C = \frac{E_{F(m-1)}}{\cos[q_{(m-1)}(d-x_{F(m-1)})]} \quad \textbf{(A19a)}$$

$$E_C = \frac{E_{F(m-1)}}{\sinh[Q_{(m-1)}(d-x_{F(m-1)})]} \quad \textbf{(A19b)}$$

For m ≥ 7, R=m-1

$$K_{m-1} = k_0^2 E_c^2 \left(\left(\frac{\mu_{m-1}}{\mu_m}\right)^2 (\beta^2 - \varepsilon_m \mu_m) + (\varepsilon_{(m-1)} \mu_{(m-1)} - \beta^2) + \frac{\alpha_{(m-1)} E_c^2}{2} \right) \quad \textbf{(A20)}$$

$$X_{0(m-1)} = cn^{-1} \left[\frac{-\mu_{(m-3)} E_{I(m-4)} \exp(-p_{m-2}) - \mu_{(m-3)} E_{I(m-3)}}{\mu_{(m-2)} b_{m-1}} \right] \quad \textbf{(A21)}$$

$$E_C = \{-\mu_{(m-3)} cn(A_{m-1} d) \left[E_{I(m-4)} \exp(-p_{m-2} w) + E_{I(m-3)} \right] - sn(A_{m-1} d) dn(A_{m-1} d) \frac{E_{I(m-4)} \exp(-p_{m-2} w) - E_{I(m-3)}}{p_{m-2} A_{m-1}} \} / \mu_{(m-2)} \left\{ 1 - l_{m-1} sn^2(A_{m-1} d) \left[1 - \left(\frac{E_{I(m-4)} \exp(-p_{m-2} w) - E_{I(m-3)}}{b_{m-1}} \right)^2 \right] \right\} \quad \textbf{(A22)}$$

For m = 3, R=2

$$K_2 = k_0^2 E_S^2 \left(\left(\frac{\mu_2}{\mu_1}\right)^2 (\beta^2 - \varepsilon_1 \mu_1) + (\varepsilon_2 \mu_2 - \beta^2) + \frac{\alpha_2 E_S^2}{2} \right) \quad \textbf{(A23)}$$

$$x_{02} = \frac{1}{A_2} cn^{-1} \left(\frac{E_S}{b_2} \frac{\mu_2}{\mu_1} \right) \quad \textbf{(A24)}$$

$$E_C = \frac{cn(A_2 d) \frac{E_S}{b_2} + sn(A_2 d) dn(A_2 d) \frac{E_S q_S}{A_2 b_2}}{1 - l_2 sn^2(A_2 d) \left(1 - \frac{E_S^2}{b_2^2} \right)} \quad \textbf{(A25)}$$

For m = 5, R=2

$$K_2 = k_0^2 E_S^2 \left(\left(\frac{\mu_2}{\mu_1}\right)^2 (\beta^2 - \varepsilon_1 \mu_1) + (\varepsilon_2 \mu_2 - \beta^2) + \frac{\alpha_2 E_S^2}{2} \right) \quad \textbf{(A26)}$$

$$x_{02} = \frac{1}{A_2} \text{cn}^{-1} \left(\frac{E_s}{b_2} \frac{\mu_2}{\mu_1} \right) \quad \textbf{(A27)}$$

$$x_{F4} = \frac{1}{p_4} \tan^{-1} \left(\frac{\mu_3 p_3 (E_{I1} \exp(-p_3 w) - E_{I2})}{\mu_4 p_4 (E_{I1} \exp(-p_3 w) + E_{I2})} \right) \quad \textbf{(A28a)}$$

$$x_{F4} = \frac{-1}{p_4} \tanh^{-1} \left(\frac{\mu_3 p_4 (E_{I1} \exp(-p_3 w) + E_{I2})}{\mu_4 p_3 (E_{I1} \exp(-p_3 w) - E_{I2})} \right) \quad \textbf{(A28b)}$$

$$E_{F4} = \frac{-\mu_3 E_{I1} \exp(-p_3) - \mu_3 E_{I2}}{\mu_4 \cos(q_4 x_{F4})} \quad \textbf{(A29a)}$$

$$E_{F4} = \frac{-\mu_3 E_{I1} \exp(-p_3) - \mu_3 E_{I2}}{\mu_4 \sinh(Q_4 x_{F4})} \quad \textbf{(A29b)}$$

$$E_C = \frac{E_{F4}}{\cos[q_4(d-x_{F4})]} \quad \textbf{(A30a)}$$

$$E_C = \frac{E_{F4}}{\sinh[Q_4(d-x_{F4})]} \quad \textbf{(A30b)}$$

$$E_{I1} = \left\{-A_2E_S\frac{\mu_2}{\mu_1}\left[1-l_2\left(1-\frac{E_S^2}{b_2^2}\right)\right]-l_2\left(1-\frac{E_S^2}{b_2^2}\right)cn^2(A_2d)\,sn(A_2d)\,dn(A_2d)-A_2E_Sp_1cn(A_2d)\left[dn^2(A_2d)-l_2\frac{E_S^2}{b_2^2}sn^2(A_2d)\right]\right\}/2p_3\left[1-l_2sn^2(A_2d)\left(1-\frac{E_S^2}{b_2^2}\right)\right]^2-\frac{\mu_2}{\mu_1}\left\{[E_scn(A_2d)+E_Sp_1sn(A_2d)dn(A_2d)]/2\left[1-l_2sn^2(A_2d)\left(1-\frac{E_s^2}{b_2^2}\right)\right]\right\} \quad \textbf{(A31a)}$$

$$E_{I2} = \frac{1}{\exp(-p_3w)}\left(\left\{-A_2E_S\frac{\mu_2}{\mu_1}\left[1-l_2\left(1-\frac{E_S^2}{b_2^2}\right)\right]-l_2\left(1-\frac{E_S^2}{b_2^2}\right)cn^2(A_2d)\,sn(A_2d)\,dn(A_2d)-A_2E_Sp_1cn(A_2d)\left[dn^2(A_2d)-l_2\frac{E_S^2}{b_2^2}sn^2(A_2d)\right]\right\}/2p_3\left[1-l_2sn^2(A_2d)\left(1-\frac{E_S^2}{b_2^2}\right)\right]^2-\frac{\mu_2}{\mu_1}[E_scn(A_2d)+E_Sp_1sn(A_2d)dn(A_2d)]/2\left[1-l_2sn^2(A_2d)\left(1-\frac{E_s^2}{b_2^2}\right)\right]\right) \quad \textbf{(A31b)}$$

For m =5, R=4

$$K_4 = k_0^2E_c^2\left(\left(\frac{\mu_4}{\mu_3}\right)^2(\beta^2-\varepsilon_3\mu_3)+(\varepsilon_4\mu_4-\beta^2)+\frac{\alpha_2E_c^2}{2}\right) \quad \textbf{(A32)}$$

$$X_{04} = cn^{-1}\left[\frac{-\mu_4E_{I1}\exp(-p_3)-\mu_4E_{I2}}{\mu_5b_4}\right] \quad \textbf{(A33)}$$

$$E_C = \frac{-\mu_4\left\{cn(A_4d)[E_{I1}\exp(-p_3w)+E_{I2}]-sn(A_4d)dn(A_{41}d)\frac{E_{I1}\exp(-p_{31}w)-E_{I2}}{p_4A_4}\right\}}{\mu_5\left\{1-l_4sn^2(A_4d)\left[1-\left(\frac{E_{I1}\exp(-p_3w)-E_{I2}}{b_4}\right)^2\right]\right\}} \quad \textbf{(A34)}$$

$$E_{I2} = \frac{-\mu_4p_3E_{F2}\cos[q_2(d-x_{F2})]+\mu_4q_2E_{F2}\sin[q_2(d-x_{F2})]}{\mu_32p_3\exp(-p_3w)} \quad \textbf{(A35a)}$$

$$E_{I2} = \frac{-\mu_4p_3E_{F2}\sinh[Q_2(d-x_{F2})]-\mu_4Q_2E_{F2}\cosh[Q_2(d-x_{F2})]}{\mu_32p_3\exp(-p_3w)} \quad \textbf{(A35b)}$$

$$E_{I1} = \frac{-\mu_4p_3E_{F2}\cos[q_2(d-x_{F2})]-\mu_4q_2E_{F2}\sin[q_2(d-x_{F2})]}{\mu_32p_3} \quad \textbf{(A36a)}$$

$$E_{I1} = \frac{-\mu_4p_3E_{F2}\sinh[Q_2(d-x_{F2})]+\mu_4Q_2E_{F2}\cosh[Q_2(d-x_{F2})]}{\mu_32p_3} \quad \textbf{(A36b)}$$