Extending the limits of the abatement cost

Appendix

A Proof of Proposition 5 and 6

We use the notation, introduced in Proposition 5:

$$A(\mu) = \int_0^T e^{-\mu t} a_t dt,$$

and the presentation is further alleviated with the notation for the "annualized" abatement

$$\bar{a}(\mu) = \Big[\mu A(\mu) + e^{-\mu T} E\Big]$$

The launch time maximizes the cost

$$\Gamma(s) = P_0 \frac{E}{\mu}(1 - e^{-\mu(s+T)}) - P_0 e^{-\mu s} A(\mu) + e^{-(i+\mu)s} I(\mu) \tag{1}$$

$$= P_0 \frac{E}{\mu} - P_0 \frac{e^{-\mu s}}{\mu} \bar{a} + e^{-(i+\mu)s} I(\mu) \tag{2}$$

canceling the derivative gives:

$$(i+\mu) e^{-\mu s} I(\mu) = P_0 \bar{a}$$

Proposition 5 follows.

The total discounted cost with the optimal launch time is

$$\Gamma = P_0 \frac{E}{\mu} - P_0 \Big[\frac{1}{\mu} - \frac{1}{i+\mu}\Big] \bar{a} e^{-\mu s} \qquad \text{replacing } I \text{ with eq. (18)}$$

$$= P_0 \frac{E}{\mu} - P_0 \Big[\frac{1}{\mu} - \frac{1}{i+\mu}\Big] \bar{a} \left[\frac{1}{i+\mu} \frac{\bar{a} P_0}{I}\right]^{\frac{\mu}{i}} \qquad \text{replacing } e^{-s} \text{ with eq. (18)}$$

From the first equation above, when comparing two projects one should compare the $e^{-\mu s}\bar{a}$ which is equivalent to comparing their total expected emissions.

Then, from the second equation, getting rid of commons terms and factors, the project associated with the lowest cost among two projects is the one with the lowest:

$$\bar{a}^{-(1+\frac{\mu}{i})} I^{\frac{\mu}{i}}$$

Proposition 6 follows.

B Proof of Proposition 7

With $P_t = P_0 e^{\gamma t}$ the project abatements are worth (from today point of view):

$$\int_s^{s+T} e^{-it} P_t a_{t-s} dt = \int_s^{s+T} e^{-\beta t} P_0 a_{t-s} dt = P_0 e^{-\beta s} \int_0^T e^{-\beta t} a_t dt = P_0 e^{-\beta s} A(\beta)$$

The total discounted cost (2) is then

$$\begin{aligned}\Gamma(s) &= \int_0^{s+T} e^{-\beta t} P_0 E dt - P_0 e^{-\beta s} A(\beta) + e^{-is} I \\ &= P_0 \frac{E}{\beta} - P_0 \frac{e^{-\beta s}}{\beta} \bar{a}(\beta) + e^{-is} I\end{aligned}$$

Then canceling the derivative with respect to the starting date s gives equation (21). The second order condition is satisfied since the second order derivative with respect to s is $(i-\beta) P_0 e^{-\beta s} \bar{a} > 0$ at s^*.

For comparing competing projects, similar calculations as before gives

$$\Gamma(s^*) = P_0 \frac{E}{\beta} - P_0 \Big[\frac{1}{\beta} - \frac{1}{i}\Big] \bar{a}(\beta) \Big[\frac{P_0 \bar{a}}{iI}\Big]^{\frac{\beta}{\gamma}}$$

The best project is the one with the largest second term, and so is the one with the lowest (beware that β might be negative):

$$I \bar{a}^{-|\frac{i}{\beta}|}$$

If β is negative ($\gamma > i$) then $1/\beta - 1/i$ is negative and the best project is the one with the lowest $\bar{a}^{1+\beta/\gamma} I^{-\beta/\gamma} = \bar{a}^{i/\gamma} I^{-\beta/\gamma}$, raised to the power $-\gamma/\beta$ gives the above expression.