Supplementary Information

# Design of Metacontinua in the Aeroacoustic Spacetime

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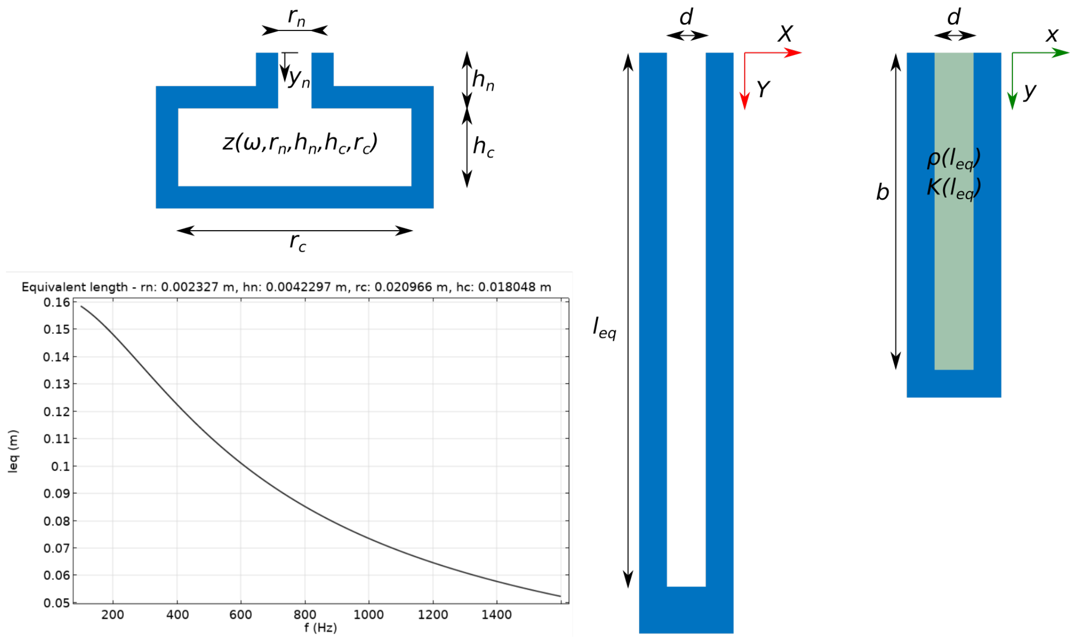


**Figure 1.** The design of the Helmholtz resonator defines its surface impedance, from which an equivalent length, function of frequency, for a straight duct with the same impedance can be derived. The properties of the metafluid are then evaluated arbitrarily setting its domain thickness.

## A metafluid model for phase gradient metasurfaces

### Acoustic impedance model for a Helmholtz resonator

In the equation of motion of the mass-spring system, representing the resonator in Fig.1, the displacement of the mass element $y_n$ can be written as a function of the pressure perturbation on the neck section

$$m\frac{d^2y_n}{dt^2} = (p+\Delta p)r_n, \qquad m = \rho_0 r_n \hat{h}_n \tag{S1}$$

Within the limit of small variations for the involved variables, we can write

$$m\frac{d^2y_n}{dt^2} = pr_n + \frac{dp}{d\rho}(-\rho_0^2 dv)r_n \qquad d\rho = -\rho_0^2 dv \tag{S2}$$

where $\frac{dp}{d\rho} = c_0$ and $dv = \frac{r_n y_n}{\rho_0 V}$, with $V = h_c r_c$. Hence, substituting in the previous, the governing equation for the displacement $y_n$ becomes

$$m\frac{d^2y_n}{dt^2} + \frac{c_0^2\rho_0 y_n r_n^2}{V} = pr_n \tag{S3}$$

Considering harmonic variations of the quantities $p = \tilde{p}_n e^{i\omega t}$ $y = \tilde{y}_n e^{i\omega t}$, Eq.S3, and dropping the time exponential terms, one obtains

$$-\omega^2 m\tilde{y}_n + \frac{c_0^2\rho_0 r_n^2}{V}\tilde{y}_n = \tilde{p}r_n \tag{S4}$$

Recalling that the acoustic particle velocity is $\tilde{u} = i\omega\tilde{y}_n$, the expression for the specific acoustic impedance is retrieved

$$z = \frac{1}{i\omega}(-\omega^2\rho_0\hat{h}_n + \frac{c_0^2\rho_0 r_n}{h_c r_c}) \tag{S5}$$

The term $\hat{h}_n$ is the length of the neck of the resonator, accounting for the end corrections as described by Bies & Calton for each side of the neck[?,?]

$$\hat{h}_n = h_n + 2\left(0.85r_n\left(1 - 1.33\frac{r_n}{r_c}\right)\right) \tag{S6}$$

## Design variables for PGMS cells

| | #1 | #2 | #3 | #4 | #5 | #6 | #7 | #8 |
|---|---|---|---|---|---|---|---|---|
| $r_n$ (mm) | 0 | 15.0 | 14.819 | 7.976 | 7.2855 | 5.9505 | 5.0 | 5.0 |
| $r_c$ (mm) | 24.912 | 46.5 | 46.482 | 45.798 | 45.729 | 45.595 | 45.5 | 45.5 |
| $h_n$ (mm) | 0 | 3.032 | 4.2043 | 3.8733 | 4.243 | 4.5 | 8.0613 | 22.5 |
| $h_c$ (mm) | 5.0 | 27.288 | 37.838 | 34.859 | 38.187 | 40.5 | 36.939 | 22.5 |

**Table 1.** Optimized cells design parameters for HR-based PGMS metasurface, nominal $\lambda^* = 0.19$m.

| | #1 | #2 | #3 | #4 | #5 | #6 | #7 | #8 |
|---|---|---|---|---|---|---|---|---|
| $a$ (mm) | 16.447 | 13.863 | 15.935 | 17.276 | 19.947 | 19.828 | 19.632 | 14.567 |
| $b$ (mm) | 23.747 | 35.985 | 41.557 | 41.557 | 39.762 | 33.646 | 23.992 | 23.747 |
| $d$ (mm) | 2.0581 | 5.2973 | 8.4895 | 11.953 | 17.981 | 14.923 | 10.096 | 9.9737 |
| $t$ (mm) | 3.6502 | 4.9418 | 3.9057 m | 3.2355 | 1.8998 | 1.9594 | 2.0576 | 4.59 |
| $n_w$ | 6 | 5 | 4 | 3 | 2 | 2 | 2 | 2 |
| $w$ (mm) | 1.8998 | 1.8998 | 1.8998 | 1.8998 | 1.8998 | 1.8998 | 1.8998 | 1.8998 |

**Table 2.** Optimized cells design parameters for SC-based PGMS metasurface, nominal $\lambda^* = 0.19$m.