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% PSEUDOSPECTRAL METHOD FOR COUPLED HIROTA LIKE EQUATIONS
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clc
tic
set(0,'defaultaxesfontsize',43, 'defaultaxesfontWeight','bold',
'defaultaxesLinewidth', 1);
% GRIDS PARAMETERS

L = 100; % SPATIAL GRID LENGTH
N = 256; % NUMBER OF SPATIAL GRID POINTS
t0 = 0.0; % initial time
tu = 5.00e-17; % Time unite
dt = 5.00e-16; %2.33e-11 % dt = 0.02; % TIME STEP
tmax = 40.0e-12; %tmax = 1710*dt; % 1710*dt; % tmax = 40; % FINAL
TIME VALUE
nmax = 8000; %nmax = round(tmax/dt); % NUMBER OF TIMESTEPS
dx = L/N; % SPATIAL GRID RESOLUTION
x = [-L/2:dx:L/2-dx]'; % CREATION OF THE SPATIAL GRID
k = [0:N/2-1 -N/2:-1]'*2*pi/L; % CREATION OF THE WAVENUMBER MODES
GRID
k1 = 1i*k; k2 = -k.^2; k3 = -1i*k.^3;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% MODEL PARAMETERS %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

hbar = 1.054e-34;
J_0 = 3.28e-22;
E_0 = 1.55e-22;
W_0 = 6.72e-20;
m = 1.9e-25;
chi1 = 1.5e-11;
chi2 = 6.3e-11;
chi3 = 2.0e-11;
lambda = -0.1999; %lambda = -0.1995; lambda = -0.2006; lambda = -0.2006;
lambda_m = 1 - lambda; lambda_p = 1 + lambda;
epsi_0 = 20.0e-3; %epsi_0 = 20.0e-3; %epsi_0 = 2.0e-3; %epsi_0 = (4.5e-
10)^(1/3);
delta = 0.010982;
eta1 = 0.7e-11;
eta2 = 1.0e-11;
K1 = 19.36;
beta = 0.20; %beta = 0.22 ; % beta_critical = 0.202625;
v0 = epsi_0 *(K1/m)^(0.5);
v1 = v0*sqrt(1-(1/K1*beta));

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Expressions Omega_j (j=0,1,2,3) %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

Omega_1 = chi1 + chi2 + chi3;
Omega_2 = chi2-chi3;
Omega_3 = chi2 + chi3;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Expressions lambda j (j=0,1,2,3,4,5) %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
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lambda_0 = 2*beta*(chi1 - Omega_2);
lambda_1 = 2*beta*Omega_3;
lambda_2 = beta*(chi1 - 4*Omega_3)/3;
lambda_3 = beta*(chi1 - Omega_3)/3;
lambda_4 = beta*(chi1 - Omega_3);
lambda_5 = beta*( chi1 - Omega_3 );

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% termes en epsi_0^(0) %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

C0 = (1/li*hbar)*dt; %C0 = -li*hbar;
C1 = eta1^2*delta;
C2 = - 2*Omega_1*lambda_0;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% termes en epsi_0^(1) %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

C3 = - epsi_0*J_0*lambda_m;
C4 = -2*epsi_0*Omega_2*lambda_0;
C5 = -2*epsi_0*Omega_1*lambda_1;
C6 = 2*epsi_0*eta1*eta2*delta;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% termes en epsi_0^(2) %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

C7 = - epsi_0^2*J_0*lambda_m*0.5;
C8 = C7*4*Omega_1*lambda_2/(J_0*lambda_m);
C9 = C7*4*Omega_1*lambda_3/(J_0*lambda_m);
C10 = C7*2*lambda_0*(chi1 + 4*Omega_2)/(3*J_0*lambda_m);
C11 = C7*2*Omega_2*lambda_0/(J_0*lambda_m);
C12 = C7*4*Omega_2*lambda_1/(J_0*lambda_m);
C13 = -C7*8*eta2^2*delta/(J_0*lambda_m);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% termes en epsi_0^(3) %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

C14 = -epsi_0^3*J_0*lambda_m/6;
C15 = C14*2*lambda_1*(chi1 + 4*Omega_2)/(J_0*lambda_m);
C16 = C14*6*Omega_2*lambda_1/(J_0*lambda_m);
C17 = C14*12*Omega_2*(lambda_2 + lambda_4)/(J_0*lambda_m);
C18 = C14*12*Omega_2*lambda_3/(J_0*lambda_m);
C19 = C14*12*Omega_2*lambda_5/(J_0*lambda_m);
C20 = C14*12*Omega_2*lambda_2/(J_0*lambda_m);
C21 = -C14*2*eta1*eta2*delta/(J_0*lambda_m);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% LRI %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

r = 1.8000 ; %r = 1.8000; %r = 1.7995; %r = 1.7990; %r = 1.7985;
a1 = 0.0;
b1 = 0.0;
J = 15.47e-23;

for i=1:10
    a1=a1+1./(i^(r-2));
    b1=b1+1./(i^(r));
end

zeta = J*a1;
zeta0 = 2*J*b1;

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Omega_0 = E_0 + W - J_0*lambda_m -lambda*zeta0;

zeta1 = 1+(2*lambda*zeta)/(J_0*lambda_m);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% beta %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

beta1 = -(2*Omega_1*lambda_0)/(J_0*lambda_p);
beta2 = -(eta1^2*delta)/(J_0*lambda_p);
beta4 = -(24*Omega_2*lambda_0)/(J_0*lambda_m);
beta5 = -12*(Omega_2*lambda_0+Omega_1*lambda_1)/(J_0*lambda_p);
beta6 = -(12*eta1*eta2*delta)/(J_0*lambda_m);
beta11 = -(epsi_0*J_0*lambda_m)/(hbar);
beta12 = -((epsi_0^2*J_0*lambda_p)/(2*hbar))*zeta1;
beta13 = (epsi_0^2*J_0*lambda_p*beta1)/(hbar);
beta14 = -(epsi_0^3*J_0*lambda_m)/(6*hbar);
beta15 = -(epsi_0^2*J_0*lambda_p*beta2)/(hbar);
beta16 = -(epsi_0^3*J_0*lambda_m*beta6)/(6*hbar);
beta17 = -(epsi_0^3*J_0*lambda_m*beta4)/(6*hbar);
beta18 = -(epsi_0^3*J_0*lambda_p*beta5)/(6*hbar);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% INITIAL CONDITIONS %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

c = -08.00e+4;
K = 25;
P = 1.0/epsi_0;

% c = -10.00e-6;
% K = 200;
% P = 1.352/epsi_0;

% K = 25;
% c = 0.2;
% P = 1/epsi_0;

zeta_10 = 0.025;%zeta_10 = 0.025

A = 2*P*((beta11-4*beta14*P^2)/(c+beta12*P*K));

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B = -(6*P*beta18)/(c+beta12*P*K);
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C = -(3*P*beta13)/(K*(c+beta12*P*K));
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%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Solution(1) %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
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```
% % chi1 = 3.8e-11;
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```
% % chi2 = 2.0e-11;
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```
% % chi3 = 2.0e-11;
```

```
% a_0 = 0.20;
```

```
% a_1 = 0.15;
```

```
% a_2 = 0.00;
```

```
% b_1 = -(Omega_1*lambda_0/2*epsi_0*K)/(Omega_2*lambda_0+Omega_1*lambda_1) ;
```

```
% omega1 = -(K/3)*b_1^2;
```

```
% omega2 = (beta12*K+c/P)+(2/b_1)*(beta11-4*P^2*beta14);
```

```
% omega3 = P^2*beta12;
```

```
% omega = omega1*omega2+omega3;
```

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% D = -3*P*(omega -beta12*P^2)/(K*(c+beta12*P*K));
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% Delta = sqrt(A^2+4*D);
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% b_01 = -0.5*(A+Delta);
```

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% b_02 = -0.5*(A-Delta);
```

```
% b_0 = (b_1*a_0)/a_1;
```

```
% b_2 = 0.0;
```

```
% b_3 = 0.0;
```

```
% A_10 = a_1/b_1;
```

```
% A_20 = A_10;
```

```
% A_30 = A_10;
```

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%
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```
% %%%% variables
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```
% xi = b_1.*(k.*x + c.*t0- zeta_10);
```

```
% theta1 = P*x + omega*t0;
```

```
% theta2 = -(Omega_0/hbar)*t0;
```

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%
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```
% %e^[b_1(zeta-zeta_10)] and e^[2*b_1(zeta-zeta_10)]
```

```
% u_alpha11 = cosh(xi)+sinh(xi);
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```
% u_alpha22 = cosh(2*xi)+sinh(2*xi);
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```
% u_alpha33 = b_1./(1-b_3*u_alpha22);
```

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%
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```
% %q_alpha
```

```
% q_alpha1 = A_10.*u_alpha11.*exp(1i*theta1);
```

```
% q_alpha2 = A_20.*u_alpha11.*exp(1i*theta1);
```

```
% q_alpha3 = A_30.*u_alpha11.*exp(1i*theta1);
```

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%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Solution(2) %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
% % chi1 = 2.0e-11;
```

```
% % chi2 = 2.0e-11;
```

```
% % chi3 = 7.5e-11;
```

```
% a_1 = 0.05; % a_1 = 0.012;
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```
% a_2 = 0.0008; % a_2 = 0.0008;
```

```
% b_2 = 0.0;
```

```
% b_3 = 0.0;
```

```
% a_0 = (0.25*a_1^2)/(a_2);
```

```
% b_1 = -
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```
0.5*((Omega_1*lambda_0/2*epsi_0*K)/(Omega_2*lambda_0+Omega_1*lambda_1)) ;
```

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% omega1 = -( (4*K)/3)*(b_1^2);
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% omega2 = (beta12*K+c/P)+(1/b_1)*(beta11-4*P^2*beta14);
% omega3 = P^2*beta12;
% omega = omegal*omega2+omega3;
% D = -3*P*(omega -beta12*P^2)/(K*(c+beta12*P*K));
% Delta = 2*sqrt(A^2+4*D);
% b_01 = -0.125*(2*A+Delta);
% b_02 = -0.125*(2*A-Delta);
% b_0 = (b_1*a_1)/(2*a_2);
% A_10 = (a_2)/(b_1^2);
% A_20 = 0.0 ; %A_20 = A_10 ;
% A_30 = 0.0 ; %A_30 = A_10 ;
% %%%%% variables
% xi = b_1.*(k.*x + c.*t0- zeta_10);
% thetal = P.*x + omega.*t0;
% theta2 = -(Omega_0/hbar).*t0;
%
% % e^[b_1(zeta-zeta_10)] and e^[2*b_1(zeta-zeta_10)]
% u_alpha11 = cosh(xi)+sinh(xi);
% u_alpha22 = cosh(2*xi)+sinh(2*xi);
% u_alpha33 = u_alpha22+b_0^2;
%
% % q_alpha
% q_alpha1 = (A_10.*u_alpha33-a_0).*exp(1i*thetal);
% q_alpha2 = (A_20.*u_alpha33-a_0).*exp(1i*thetal);
% q_alpha3 = (A_30.*u_alpha33-a_0).*exp(1i*thetal);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Solution(3)%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% chi1 = 1.5e-11;
% chi2 = 6.3e-11;
% chi3 = 2.0e-11;

a_0 = 0.00;
a_1 =0.336; %a_1 = 0.336; a_1 =0.323;
a_2 = 0.0;
b_0 = 0.0;
omegal = -(K/3)*(beta12*K+c/P);
omega2 = 9*(C/B)^2 + A*(C/B);
omega3 = P^2*beta12;
omega = omegal*omega2+omega3;
D = -3*P*(omega -beta12*P^2)/(K*(c+beta12*P*K));
Delta = sqrt(A^2+36*D);
b_01 = -(1/6)*(A+Delta);
b_02 = -(1/6)*(A-Delta);
b_1 = -(1/3)*((B*D)/C);
b_2 = 0.0;
b_3 = -(1/3)*B*a_1^2;

A_10 = a_1; %A_10 = - a_1;
A_20 = a_1;
A_30 = a_1;

%%%%%% variables
xi = b_1.*(k.*x + c.*t0- zeta_10);
thetal = P*x+ omega*t0;
theta2 = -(Omega_0/hbar)*t0;
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% e^[b_1(zeta-zeta_10)] and e^[2*b_1(zeta-zeta_10)]
u_alpha11 = cosh(xi)+sinh(xi);
u_alpha22 = cosh(2*xi)+sinh(2*xi);
u_alpha33 = b_1./(1-b_3*u_alpha22);

% q_alpha
q_alpha1 = A_10.*(u_alpha11).*sqrt(u_alpha33).*exp(1i*theta1);
q_alpha2 = A_20.*(u_alpha11).*sqrt(u_alpha33)).*exp(1i*theta1);
q_alpha3 = A_30.*(u_alpha11).*sqrt(u_alpha33)).*exp(1i*theta1);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% a_j %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

u_1 = epsi_0.*q_alpha1.*exp(1i*theta2);
u_2 = epsi_0.*q_alpha2.*exp(1i*theta2);
u_3 = epsi_0.*q_alpha3.*exp(1i*theta2);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% perturbation condition %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

u1 = abs(u_1); u1max = max(u1), u1min = min (u1), Am1 = abs(u1max - u1min);
u2 = abs(u_2); u2max = max(u2), u2min = min (u2), Am2 = abs(u2max - u2min);
u3 = abs(u_3); u3max = max(u3), u3min = min (u3), Am3 = abs(u3max - u3min);

u_1data = u_1 + 0.01*Am1*(rand(N,1)-0.5);
u_2data = u_2 + 0.01*Am2*(rand(N,1)-0.5);
u_3data = u_3 + 0.01*Am3*(rand(N,1)-0.5);

t = t0; tdata = 0; d6 = 1/6; %d6 = dt/6

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% initialisation phase et energie %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% initialisation de l'energie dans chaque reseau %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

TabE1 = []; TabE2 = []; TabE3 = []; TabE = [];
E01 = 0; E02 = 0; E03 = 0; E0 = 0;

for n =1:N

    E01 = E01 + conj(u_1(n)).*u_1(n);
    E02 = E02 + conj(u_2(n)).*u_2(n);
    E03 = E03 + conj(u_3(n)).*u_3(n);
    E0 = E01 + E02 + E03 ;

end

TabE1 = [TabE1; E01]; TabE2 = [TabE2; E02];
TabE3 = [TabE3; E03]; TabE = [TabE; E0];

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% initialisation de la phase dans chaque reseau

Phidata1 = []; Phidata2 = []; Phidata3 = [];

Phidata10 = angle(u_1);
Phidata20 = angle(u_2);
Phidata30 = angle(u_3);

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Phidata1 = [Phidata1 Phidata10];
Phidata2 = [Phidata2 Phidata20];
Phidata3 = [Phidata3 Phidata30];

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% INTEGRATION LOOP %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

for nn=1:nmax

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% SQUARE MODULUS %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

    P_1 = conj(u_1).*u_1;
    P_2 = conj(u_2).*u_2;
    P_3 = conj(u_3).*u_3;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% COMPLEX CONJUGATES %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

    u_1b = conj(u_1);
    u_2b = conj(u_2);
    u_3b = conj(u_3);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% FUNCTIONS 1ST DERIVATIVES %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% fft = fast fourier transform : Y = fft(X); %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% ifft = Inverse fast fourier transform : X = ifft (Y) = ifft [fft(X)]%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

    u_1x = ifft( k1.*fft( u_1 ) ); u_1xb = conj(u_1x); P_1x = u_1xb.*u_1x;
    u_2x = ifft( k1.*fft( u_2 ) ); u_2xb = conj(u_2x); P_2x = u_2xb.*u_2x;
    u_3x = ifft( k1.*fft( u_3 ) ); u_3xb = conj(u_3x); P_3x = u_3xb.*u_3x;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% FUNCTIONS 2ND DERIVATIVES %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

    u_1xx = ifft( k2.*fft( u_1 ) ); u_1xxb = conj(u_1xx);
    u_2xx = ifft( k2.*fft( u_2 ) ); u_2xxb = conj(u_2xx);
    u_3xx = ifft( k2.*fft( u_3 ) ); u_3xxb = conj(u_3xx);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% FUNCTIONS 3RD DERIVATIVES %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

    u_1xxx = ifft( k3.*fft( u_1 ) ); u_1xxxb = conj(u_1xxx);
    u_2xxx = ifft( k3.*fft( u_2 ) ); u_2xxxb = conj(u_2xxx);
    u_3xxx = ifft( k3.*fft( u_3 ) ); u_3xxxb = conj(u_3xxx);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% 1ST RK4 FUNCTIONS %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% termes en epsi_0^(0)%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

    A0 = ( Omega_0 + C1*( P_2 + P_3 - 2*P_1 ) + C2*P_1 ).*u_1;
    B0 = ( Omega_0 + C1*( P_1 + P_3 - 2*P_2 ) + C2*P_2 ).*u_2;
    F0 = ( Omega_0 + C1*( P_1 + P_2 - 2*P_3 ) + C2*P_3 ).*u_3;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% termes en epsi_0^(1)%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

    A1 = C3*u_1x + C4*( u_1.*u_1xb + 2*u_1x.*u_1b ).*u_1 + C5*u_1.^2.*u_1xb;
    B1 = C3*u_2x + C4*( u_2.*u_2xb + 2*u_2x.*u_2b ).*u_2 + C5*u_2.^2.*u_2xb;
    F1 = C3*u_3x + C4*( u_3.*u_3xb + 2*u_3x.*u_3b ).*u_3 + C5*u_3.^2.*u_3xb;

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A2 = C6*( ( u_2b.*u_2x + u_3b.*u_3x - 2*u_1b.*u_1x ).*u_1 + ( P_2 + P_3 -
2*P_1 ).*u_1x );
B2 = C6*( ( u_1b.*u_1x + u_3b.*u_3x - 2*u_2b.*u_2x ).*u_2 + ( P_1 + P_3 -
2*P_2 ).*u_2x );
F2 = C6*( ( u_1b.*u_1x + u_2b.*u_2x - 2*u_3b.*u_3x ).*u_3 + ( P_1 + P_2 -
2*P_3 ).*u_3x );

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%% termes en ϵ_0^2 %%

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A3 = C7.*zeta1.*u_1xx + ( C8*u_1.*u_1xxb + C9*( u_1xx.*u_1b + 2*P_1x )
).*u_1 + C10*( 2*P_1x + u_1.*u_1xxb + u_1xx.*u_1b ).*u_1 + ...
C11*( P_1.*u_1xx + 2*( u_1.*u_1xb + u_1x.*u_1b ).*u_1x ) + C12*(
2*P_1x + u_1xxb.*u_1 ).*u_1 + ...
C13*( u_2b.*u_2x + u_3b.*u_3x - 2*u_1b.*u_1x ).*u_1x;
B3 = C7.*zeta1.*u_2xx + ( C8*u_2.*u_2xxb + C9*( u_2xx.*u_2b + 2*P_2x )
).*u_2 + C10*( 2*P_2x + u_2.*u_2xxb + u_2xx.*u_2b ).*u_2 + ...
C11*( P_2.*u_2xx + 2*( u_2.*u_2xb + u_2x.*u_2b ).*u_2x ) + C12*(
2*P_2x + u_2xxb.*u_1 ).*u_2 + ...
C13*( u_1b.*u_1x + u_3b.*u_3x - 2*u_2b.*u_2x ).*u_2x;
F3 = C7.*zeta1.*u_3xx + ( C8*u_3.*u_3xxb + C9*( u_3xx.*u_3b + 2*P_3x )
).*u_3 + C10*( 2*P_3x + u_3.*u_3xxb + u_3xx.*u_3b ).*u_3 + ...
C11*( P_3.*u_3xx + 2*( u_3.*u_3xb + u_3x.*u_3b ).*u_3x ) + C12*(
2*P_3x + u_3xxb.*u_1 ).*u_3 + ...
C13*( u_1b.*u_1x + u_2b.*u_2x - 2*u_3b.*u_3x ).*u_3x;

```

%% termes en ϵ_0^3 %%
%%

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A4 = C14*u_1xxx + C15*( 2*u_1x.*u_1xxb + u_1.*u_1xxxb + u_1xx.*u_1xb
).*u_1;
B4 = C14*u_2xxx + C15*( 2*u_2x.*u_2xxb + u_2.*u_2xxxb + u_2xx.*u_2xb
).*u_2;
F4 = C14*u_3xxx + C15*( 2*u_3x.*u_3xxb + u_3.*u_3xxxb + u_3xx.*u_3xb
).*u_3;

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A5 = C16*( u_1.*u_1xx.*u_1xb + 2*( u_1.*u_1xxb + P_1x ).*u_1x ) +
C17*u_1.*u_1x.*u_1xxb + C18*( ( u_1xx.*u_1b + 2*P_1x ).*u_1x + P_1.*u_1xxx ) +
...
( C19*u_1xx.*u_1xb + C20*u_1.*u_1xxxb ).*u_1;
B5 = C16*( u_2.*u_2xx.*u_2xb + 2*( u_2.*u_2xxb + P_2x ).*u_2x ) +
C17*u_2.*u_2x.*u_2xxb + C18*( ( u_2xx.*u_2b + 2*P_2x ).*u_2x + P_2.*u_2xxx ) +
...
( C19*u_2xx.*u_2xb + C20*u_2.*u_2xxxb ).*u_2;
F5 = C16*( u_3.*u_3xx.*u_3xb + 2*( u_3.*u_3xxb + P_3x ).*u_3x ) +
C17*u_3.*u_3x.*u_3xxb + C18*( ( u_3xx.*u_3b + 2*P_3x ).*u_3x + P_3.*u_3xxx ) +
...
( C19*u_3xx.*u_3xb + C20*u_3.*u_3xxxb ).*u_3;

```

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A6 = C21*( ( u_2b.*u_2xxx + u_3b.*u_3xxx - 2*u_1b.*u_1xxx ).*u_1 + ( P_2 +
P_3 - 2*P_1 ).*u_1xxx );
B6 = C21*( ( u_1b.*u_1xxx + u_3b.*u_3xxx - 2*u_2b.*u_2xxx ).*u_2 + ( P_1 +
P_3 - 2*P_2 ).*u_2xxx );
F6 = C21*( ( u_1b.*u_1xxx + u_2b.*u_2xxx - 2*u_3b.*u_3xxx ).*u_2 + ( P_1 +
P_2 - 2*P_3 ).*u_3xxx );

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%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
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```
L11 = C0*( A0 + A1 + A2 + A3 + A4 + A5 + A6 );
L21 = C0*( B0 + B1 + B2 + B3 + B4 + B5 + B6 );
L31 = C0*( F0 + F1 + F2 + F3 + F4 + F5 + F6 );
```

```
u_1 = u_1 +0.5*L11;
u_2 = u_2 +0.5*L21;
u_3 = u_3 +0.5*L31;
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% SECOND RK4 FUNCTIONS %%%%%%%%%%
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%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% SQUARE MODULUS %%%%%%%%%%
```

```
P_1 = conj(u_1).*u_1;
P_2 = conj(u_2).*u_2;
P_3 = conj(u_3).*u_3;
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% COMPLEX CONJUGATES %%%%%%%%%%
```

```
u_1b = conj(u_1);
u_2b = conj(u_2);
u_3b = conj(u_3);
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% FUNCTIONS 1RST DERIVATIVES %%%%%%%%%%
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% fft = fast fourier transform : Y = fft(X); %%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% ifft = Inverse fast fourier transform : X = ifft (Y) = ifft [fft(X)]%%%
```

```
u_1x = ifft( k1.*fft( u_1 ) ); u_1xb = conj(u_1x); P_1x = u_1xb.*u_1x;
u_2x = ifft( k1.*fft( u_2 ) ); u_2xb = conj(u_2x); P_2x = u_2xb.*u_2x;
u_3x = ifft( k1.*fft( u_3 ) ); u_3xb = conj(u_3x); P_3x = u_3xb.*u_3x;
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% FUNCTIONS 2ND DERIVATIVES %%%%%%%%%%
```

```
u_1xx = ifft( k2.*fft( u_1 ) ); u_1xxb = conj(u_1xx);
u_2xx = ifft( k2.*fft( u_2 ) ); u_2xxb = conj(u_2xx);
u_3xx = ifft( k2.*fft( u_3 ) ); u_3xxb = conj(u_3xx);
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% FUNCTIONS 3RD DERIVATIVES %%%%%%%%%%
```

```
u_1xxx = ifft( k3.*fft( u_1 ) ); u_1xxxb = conj(u_1xxx);
u_2xxx = ifft( k3.*fft( u_2 ) ); u_2xxxb = conj(u_2xxx);
u_3xxx = ifft( k3.*fft( u_3 ) ); u_3xxxb = conj(u_3xxx);
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% 1RST RK4 FUNCTIONS %%%%%%%%%%
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% termes en epsi_0^(0) %%%%%%%%%%
```

```
A0 = ( Omega_0 + C1*( P_2 + P_3 - 2*P_1 ) + C2*P_1 ).*u_1;
B0 = ( Omega_0 + C1*( P_1 + P_3 - 2*P_2 ) + C2*P_2 ).*u_2;
F0 = ( Omega_0 + C1*( P_1 + P_2 - 2*P_3 ) + C2*P_3 ).*u_3;
```

%% termes en epsi_0^(1)%%

```

A1 = C3*u_1x + C4*( u_1.*u_1xb + 2*u_1x.*u_1b ).*u_1 + C5*u_1.^2.*u_1xb;
B1 = C3*u_2x + C4*( u_2.*u_2xb + 2*u_2x.*u_2b ).*u_2 + C5*u_2.^2.*u_2xb;
F1 = C3*u_3x + C4*( u_3.*u_3xb + 2*u_3x.*u_3b ).*u_3 + C5*u_3.^2.*u_3xb;

A2 = C6*( ( u_2b.*u_2x + u_3b.*u_3x - 2*u_1b.*u_1x ).*u_1 + ( P_2 + P_3 -
2*P_1 ).*u_1x );
B2 = C6*( ( u_1b.*u_1x + u_3b.*u_3x - 2*u_2b.*u_2x ).*u_2 + ( P_1 + P_3 -
2*P_2 ).*u_2x );
F2 = C6*( ( u_1b.*u_1x + u_2b.*u_2x - 2*u_3b.*u_3x ).*u_3 + ( P_1 + P_2 -
2*P_3 ).*u_3x );

```

%% termes en epsi_0^(2)%%

```

A3 = C7.*zeta1.*u_1xx + ( C8*u_1.*u_1xxb + C9*( u_1xx.*u_1b + 2*P_1x )
).*u_1 + C10*( 2*P_1x + u_1.*u_1xxb + u_1xx.*u_1b ).*u_1 + ...
C11*( P_1.*u_1xx + 2*( u_1.*u_1xb + u_1x.*u_1b ).*u_1x ) + C12*(
2*P_1x + u_1xxb.*u_1 ).*u_1 + ...
C13*( u_2b.*u_2x + u_3b.*u_3x - 2*u_1b.*u_1x ).*u_1x;
B3 = C7.*zeta1.*u_2xx + ( C8*u_2.*u_2xxb + C9*( u_2xx.*u_2b + 2*P_2x )
).*u_2 + C10*( 2*P_2x + u_2.*u_2xxb + u_2xx.*u_2b ).*u_2 + ...
C11*( P_2.*u_2xx + 2*( u_2.*u_2xb + u_2x.*u_2b ).*u_2x ) + C12*(
2*P_2x + u_2xxb.*u_1 ).*u_2 + ...
C13*( u_1b.*u_1x + u_3b.*u_3x - 2*u_2b.*u_2x ).*u_2x;
F3 = C7.*zeta1.*u_3xx + ( C8*u_3.*u_3xxb + C9*( u_3xx.*u_3b + 2*P_3x )
).*u_3 + C10*( 2*P_3x + u_3.*u_3xxb + u_3xx.*u_3b ).*u_3 + ...
C11*( P_3.*u_3xx + 2*( u_3.*u_3xb + u_3x.*u_3b ).*u_3x ) + C12*(
2*P_3x + u_3xxb.*u_1 ).*u_3 + ...
C13*( u_1b.*u_1x + u_2b.*u_2x - 2*u_3b.*u_3x ).*u_3x;

```

%% termes en
epsi_0^(3)%%
%%

```

A4 = C14*u_1xxx + C15*( 2*u_1x.*u_1xxb + u_1.*u_1xxxb + u_1xx.*u_1xb
).*u_1;
B4 = C14*u_2xxx + C15*( 2*u_2x.*u_2xxb + u_2.*u_2xxxb + u_2xx.*u_2xb
).*u_2;
F4 = C14*u_3xxx + C15*( 2*u_3x.*u_3xxb + u_3.*u_3xxxb + u_3xx.*u_3xb
).*u_3;

A5 = C16*( u_1.*u_1xx.*u_1xb + 2*( u_1.*u_1xxb + P_1x ).*u_1x ) +
C17*u_1.*u_1x.*u_1xxb + C18*( ( u_1xx.*u_1b + 2*P_1x ).*u_1x + P_1.*u_1xxx ) +
...
( C19*u_1xx.*u_1xb + C20*u_1.*u_1xxxb ).*u_1;
B5 = C16*( u_2.*u_2xx.*u_2xb + 2*( u_2.*u_2xxb + P_2x ).*u_2x ) +
C17*u_2.*u_2x.*u_2xxb + C18*( ( u_2xx.*u_2b + 2*P_2x ).*u_2x + P_2.*u_2xxx ) +
...
( C19*u_2xx.*u_2xb + C20*u_2.*u_2xxxb ).*u_2;
F5 = C16*( u_3.*u_3xx.*u_3xb + 2*( u_3.*u_3xxb + P_3x ).*u_3x ) +
C17*u_3.*u_3x.*u_3xxb + C18*( ( u_3xx.*u_3b + 2*P_3x ).*u_3x + P_3.*u_3xxx ) +
...
( C19*u_3xx.*u_3xb + C20*u_3.*u_3xxxb ).*u_3;

```

```

    A6 = C21*( ( u_2b.*u_2xxx + u_3b.*u_3xxx - 2*u_1b.*u_1xxx ).*u_1 + ( P_2 +
P_3 -2*P_1 ).*u_1xxx );
    B6 = C21*( ( u_1b.*u_1xxx + u_3b.*u_3xxx - 2*u_2b.*u_2xxx ).*u_2 + ( P_1 +
P_3 -2*P_2 ).*u_2xxx );
    F6 = C21*( ( u_1b.*u_1xxx + u_2b.*u_2xxx - 2*u_3b.*u_3xxx ).*u_2 + ( P_1 +
P_2 -2*P_3 ).*u_3xxx );

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

```

```

    L12 = C0*( A0 + A1 + A2 + A3 + A4 + A5 + A6 );
    L22 = C0*( B0 + B1 + B2 + B3 + B4 + B5 + B6 );
    L32 = C0*( F0 + F1 + F2 + F3 + F4 + F5 + F6 );

```

```

    u_1 = u_1 +0.5*L12;
    u_2 = u_2 +0.5*L22;
    u_3 = u_3 +0.5*L32;

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% THIRD RK4 FUNCTIONS %%%%%%%%%%

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% SQUARE MODULUS %%%%%%%%%%

```

```

    P_1 = conj(u_1).*u_1;
    P_2 = conj(u_2).*u_2;
    P_3 = conj(u_3).*u_3;

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% COMPLEX CONJUGATES %%%%%%%%%%

```

```

    u_1b = conj(u_1);
    u_2b = conj(u_2);
    u_3b = conj(u_3);

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% FUNCTIONS 1RST DERIVATIVES %%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% fft = fast fourier transform : Y = fft(X); %%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% ifft = Inverse fast fourier transform : X = ifft (Y) = ifft [fft(X)]%

```

```

    u_1x = ifft( k1.*fft( u_1 ) ); u_1xb = conj(u_1x); P_1x = u_1xb.*u_1x;
    u_2x = ifft( k1.*fft( u_2 ) ); u_2xb = conj(u_2x); P_2x = u_2xb.*u_2x;
    u_3x = ifft( k1.*fft( u_3 ) ); u_3xb = conj(u_3x); P_3x = u_3xb.*u_3x;

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% FUNCTIONS 2ND DERIVATIVES %%%%%%%%%%

```

```

    u_1xx = ifft( k2.*fft( u_1 ) ); u_1xxb = conj(u_1xx);
    u_2xx = ifft( k2.*fft( u_2 ) ); u_2xxb = conj(u_2xx);
    u_3xx = ifft( k2.*fft( u_3 ) ); u_3xxb = conj(u_3xx);

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% FUNCTIONS 3RD DERIVATIVES %%%%%%%%%%

```

```

    u_1xxx = ifft( k3.*fft( u_1 ) ); u_1xxxb = conj(u_1xxx);
    u_2xxx = ifft( k3.*fft( u_2 ) ); u_2xxxb = conj(u_2xxx);
    u_3xxx = ifft( k3.*fft( u_3 ) ); u_3xxxb = conj(u_3xxx);

```

%% 1RST RK4 FUNCTIONS %%%

%% termes en epsi_0^(0)%%

```
A0 = ( Omega_0 + C1*( P_2 + P_3 - 2*P_1 ) + C2*P_1 ).*u_1;  
B0 = ( Omega_0 + C1*( P_1 + P_3 - 2*P_2 ) + C2*P_2 ).*u_2;  
F0 = ( Omega_0 + C1*( P_1 + P_2 - 2*P_3 ) + C2*P_3 ).*u_3;
```

%% termes en epsi_0^(1)%%

```
A1 = C3*u_1x + C4*( u_1.*u_1xb + 2*u_1x.*u_1b ).*u_1 + C5*u_1.^2.*u_1xb;  
B1 = C3*u_2x + C4*( u_2.*u_2xb + 2*u_2x.*u_2b ).*u_2 + C5*u_2.^2.*u_2xb;  
F1 = C3*u_3x + C4*( u_3.*u_3xb + 2*u_3x.*u_3b ).*u_3 + C5*u_3.^2.*u_3xb;  
  
A2 = C6*( ( u_2b.*u_2x + u_3b.*u_3x - 2*u_1b.*u_1x ).*u_1 + ( P_2 + P_3 -  
2*P_1 ).*u_1x );  
B2 = C6*( ( u_1b.*u_1x + u_3b.*u_3x - 2*u_2b.*u_2x ).*u_2 + ( P_1 + P_3 -  
2*P_2 ).*u_2x );  
F2 = C6*( ( u_1b.*u_1x + u_2b.*u_2x - 2*u_3b.*u_3x ).*u_3 + ( P_1 + P_2 -  
2*P_3 ).*u_3x );
```

%% termes en epsi_0^(2)%%

```
A3 = C7.*zeta1.*u_1xx + ( C8*u_1.*u_1xxb + C9*( u_1xx.*u_1b + 2*P_1x )  
) .*u_1 + C10*( 2*P_1x + u_1.*u_1xxb + u_1xx.*u_1b ).*u_1 + ...  
C11*( P_1.*u_1xx + 2*( u_1.*u_1xb + u_1x.*u_1b ).*u_1x ) + C12*(  
2*P_1x + u_1xxb.*u_1 ).*u_1 + ...  
C13*( u_2b.*u_2x + u_3b.*u_3x - 2*u_1b.*u_1x ).*u_1x;  
B3 = C7.*zeta1.*u_2xx + ( C8*u_2.*u_2xxb + C9*( u_2xx.*u_2b + 2*P_2x )  
) .*u_2 + C10*( 2*P_2x + u_2.*u_2xxb + u_2xx.*u_2b ).*u_2 + ...  
C11*( P_2.*u_2xx + 2*( u_2.*u_2xb + u_2x.*u_2b ).*u_2x ) + C12*(  
2*P_2x + u_2xxb.*u_1 ).*u_2 + ...  
C13*( u_1b.*u_1x + u_3b.*u_3x - 2*u_2b.*u_2x ).*u_2x;  
F3 = C7.*zeta1.*u_3xx + ( C8*u_3.*u_3xxb + C9*( u_3xx.*u_3b + 2*P_3x )  
) .*u_3 + C10*( 2*P_3x + u_3.*u_3xxb + u_3xx.*u_3b ).*u_3 + ...  
C11*( P_3.*u_3xx + 2*( u_3.*u_3xb + u_3x.*u_3b ).*u_3x ) + C12*(  
2*P_3x + u_3xxb.*u_1 ).*u_3 + ...  
C13*( u_1b.*u_1x + u_2b.*u_2x - 2*u_3b.*u_3x ).*u_3x;
```

%% termes en
epsi_0^(3)%%
%%

```
A4 = C14*u_1xxx + C15*( 2*u_1x.*u_1xxb + u_1.*u_1xxx + u_1xx.*u_1xb  
) .*u_1;  
B4 = C14*u_2xxx + C15*( 2*u_2x.*u_2xxb + u_2.*u_2xxx + u_2xx.*u_2xb  
) .*u_2;  
F4 = C14*u_3xxx + C15*( 2*u_3x.*u_3xxb + u_3.*u_3xxx + u_3xx.*u_3xb  
) .*u_3;  
  
A5 = C16*( u_1.*u_1xx.*u_1xb + 2*( u_1.*u_1xxb + P_1x ).*u_1x ) +  
C17*u_1.*u_1x.*u_1xxb + C18*( ( u_1xx.*u_1b + 2*P_1x ).*u_1x + P_1.*u_1xxx ) +  
...  
( C19*u_1xx.*u_1xb + C20*u_1.*u_1xxx ).*u_1;
```

```

B5 = C16*( u_2.*u_2xx.*u_2xb + 2*( u_2.*u_2xxb + P_2x ).*u_2x ) +
C17*u_2.*u_2x.*u_2xxb + C18*( ( u_2xx.*u_2b + 2*P_2x ).*u_2x + P_2.*u_2xxx ) +
...
( C19*u_2xx.*u_2xb + C20*u_2.*u_2xxxb ).*u_2;
F5 = C16*( u_3.*u_3xx.*u_3xb + 2*( u_3.*u_3xxb + P_3x ).*u_3x ) +
C17*u_3.*u_3x.*u_3xxb + C18*( ( u_3xx.*u_3b + 2*P_3x ).*u_3x + P_3.*u_3xxx ) +
...
( C19*u_3xx.*u_3xb + C20*u_3.*u_3xxxb ).*u_3;

A6 = C21*( ( u_2b.*u_2xxx + u_3b.*u_3xxx - 2*u_1b.*u_1xxx ).*u_1 + ( P_2 +
P_3 -2*P_1 ).*u_1xxx );
B6 = C21*( ( u_1b.*u_1xxx + u_3b.*u_3xxx - 2*u_2b.*u_2xxx ).*u_2 + ( P_1 +
P_3 -2*P_2 ).*u_2xxx );
F6 = C21*( ( u_1b.*u_1xxx + u_2b.*u_2xxx - 2*u_3b.*u_3xxx ).*u_2 + ( P_1 +
P_2 -2*P_3 ).*u_3xxx );

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

```

```

L13 = C0*( A0 + A1 + A2 + A3 + A4 + A5 + A6 );
L23 = C0*( B0 + B1 + B2 + B3 + B4 + B5 + B6 );
L33 = C0*( F0 + F1 + F2 + F3 + F4 + F5 + F6 );

```

```

u_1 = u_1 + L13;
u_2 = u_2 + L23;
u_3 = u_3 + L33;

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% FOURTH RK4 FUNCTIONS %%%%%%%%%

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% SQUARE MODULUS %%%%%%%%%

```

```

P_1 = conj(u_1).*u_1;
P_2 = conj(u_2).*u_2;
P_3 = conj(u_3).*u_3;

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% COMPLEX CONJUGATES %%%%%%%%%

```

```

u_1b = conj(u_1);
u_2b = conj(u_2);
u_3b = conj(u_3);

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% FUNCTIONS 1RST DERIVATIVES %%%%%%%%%

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% fft = fast fourier transform : Y = fft(X); %%%%%%%%%
%% ifft = Inverse fast fourier transform : X = ifft (Y) = ifft [fft(X)]%%

```

```

u_1x = ifft( k1.*fft( u_1 ) ); u_1xb = conj(u_1x); P_1x = u_1xb.*u_1x;
u_2x = ifft( k1.*fft( u_2 ) ); u_2xb = conj(u_2x); P_2x = u_2xb.*u_2x;
u_3x = ifft( k1.*fft( u_3 ) ); u_3xb = conj(u_3x); P_3x = u_3xb.*u_3x;

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% FUNCTIONS 2ND DERIVATIVES %%%%%%%%%

```

```

u_1xx = ifft( k2.*fft( u_1 ) ); u_1xxb = conj(u_1xx);
u_2xx = ifft( k2.*fft( u_2 ) ); u_2xxb = conj(u_2xx);

```

```

u_3xx = ifft( k2.*fft( u_3 ) ); u_3xxb = conj(u_3xx);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% FUNCTIONS 3RD DERIVATIVES %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

u_1xxx = ifft( k3.*fft( u_1 ) ); u_1xxxb = conj(u_1xxx);
u_2xxx = ifft( k3.*fft( u_2 ) ); u_2xxxb = conj(u_2xxx);
u_3xxx = ifft( k3.*fft( u_3 ) ); u_3xxxb = conj(u_3xxx);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% 1RST RK4 FUNCTIONS %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% termes en epsi_0^(0)%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

A0 = ( Omega_0 + C1*( P_2 + P_3 - 2*P_1 ) + C2*P_1 ).*u_1;
B0 = ( Omega_0 + C1*( P_1 + P_3 - 2*P_2 ) + C2*P_2 ).*u_2;
F0 = ( Omega_0 + C1*( P_1 + P_2 - 2*P_3 ) + C2*P_3 ).*u_3;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% termes en epsi_0^(1)%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

A1 = C3*u_1x + C4*( u_1.*u_1xb + 2*u_1x.*u_1b ).*u_1 + C5*u_1.^2.*u_1xb;
B1 = C3*u_2x + C4*( u_2.*u_2xb + 2*u_2x.*u_2b ).*u_2 + C5*u_2.^2.*u_2xb;
F1 = C3*u_3x + C4*( u_3.*u_3xb + 2*u_3x.*u_3b ).*u_3 + C5*u_3.^2.*u_3xb;

A2 = C6*( ( u_2b.*u_2x + u_3b.*u_3x - 2*u_1b.*u_1x ).*u_1 + ( P_2 + P_3 - 2*P_1 ).*u_1x );
B2 = C6*( ( u_1b.*u_1x + u_3b.*u_3x - 2*u_2b.*u_2x ).*u_2 + ( P_1 + P_3 - 2*P_2 ).*u_2x );
F2 = C6*( ( u_1b.*u_1x + u_2b.*u_2x - 2*u_3b.*u_3x ).*u_3 + ( P_1 + P_2 - 2*P_3 ).*u_3x );

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% termes en epsi_0^(2)%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

A3 = C7.*zeta1.*u_1xx + ( C8*u_1.*u_1xxb + C9*( u_1xx.*u_1b + 2*P_1x ) ).*u_1 + C10*( 2*P_1x + u_1.*u_1xxb + u_1xx.*u_1b ).*u_1 + ...
C11*( P_1.*u_1xx + 2*( u_1.*u_1xb + u_1x.*u_1b ).*u_1x ) + C12*( 2*P_1x + u_1xxb.*u_1 ).*u_1 + ...
C13*( u_2b.*u_2x + u_3b.*u_3x - 2*u_1b.*u_1x ).*u_1x;
B3 = C7.*zeta1.*u_2xx + ( C8*u_2.*u_2xxb + C9*( u_2xx.*u_2b + 2*P_2x ) ).*u_2 + C10*( 2*P_2x + u_2.*u_2xxb + u_2xx.*u_2b ).*u_2 + ...
C11*( P_2.*u_2xx + 2*( u_2.*u_2xb + u_2x.*u_2b ).*u_2x ) + C12*( 2*P_2x + u_2xxb.*u_1 ).*u_2 + ...
C13*( u_1b.*u_1x + u_3b.*u_3x - 2*u_2b.*u_2x ).*u_2x;
F3 = C7.*zeta1.*u_3xx + ( C8*u_3.*u_3xxb + C9*( u_3xx.*u_3b + 2*P_3x ) ).*u_3 + C10*( 2*P_3x + u_3.*u_3xxb + u_3xx.*u_3b ).*u_3 + ...
C11*( P_3.*u_3xx + 2*( u_3.*u_3xb + u_3x.*u_3b ).*u_3x ) + C12*( 2*P_3x + u_3xxb.*u_1 ).*u_3 + ...
C13*( u_1b.*u_1x + u_2b.*u_2x - 2*u_3b.*u_3x ).*u_3x;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% termes en
epsi_0^(3)%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

A4 = C14*u_1xxx + C15*( 2*u_1x.*u_1xxb + u_1.*u_1xxxb + u_1xx.*u_1xb ).*u_1;
B4 = C14*u_2xxx + C15*( 2*u_2x.*u_2xxb + u_2.*u_2xxxb + u_2xx.*u_2xb ).*u_2;

```

```

F4 = C14*u_3xxx + C15*( 2*u_3x.*u_3xxb + u_3.*u_3xxx + u_3xx.*u_3xb
).*u_3;

A5 = C16*( u_1.*u_1xx.*u_1xb + 2*( u_1.*u_1xxb + P_1x ).*u_1x ) +
C17*u_1.*u_1x.*u_1xxb + C18*( ( u_1xx.*u_1b + 2*P_1x ).*u_1x + P_1.*u_1xxx ) +
...
( C19*u_1xx.*u_1xb + C20*u_1.*u_1xxx ).*u_1;
B5 = C16*( u_2.*u_2xx.*u_2xb + 2*( u_2.*u_2xxb + P_2x ).*u_2x ) +
C17*u_2.*u_2x.*u_2xxb + C18*( ( u_2xx.*u_2b + 2*P_2x ).*u_2x + P_2.*u_2xxx ) +
...
( C19*u_2xx.*u_2xb + C20*u_2.*u_2xxx ).*u_2;
F5 = C16*( u_3.*u_3xx.*u_3xb + 2*( u_3.*u_3xxb + P_3x ).*u_3x ) +
C17*u_3.*u_3x.*u_3xxb + C18*( ( u_3xx.*u_3b + 2*P_3x ).*u_3x + P_3.*u_3xxx ) +
...
( C19*u_3xx.*u_3xb + C20*u_3.*u_3xxx ).*u_3;

A6 = C21*( ( u_2b.*u_2xxx + u_3b.*u_3xxx - 2*u_1b.*u_1xxx ).*u_1 + ( P_2 +
P_3 -2*P_1 ).*u_1xxx );
B6 = C21*( ( u_1b.*u_1xxx + u_3b.*u_3xxx - 2*u_2b.*u_2xxx ).*u_2 + ( P_1 +
P_3 -2*P_2 ).*u_2xxx );
F6 = C21*( ( u_1b.*u_1xxx + u_2b.*u_2xxx - 2*u_3b.*u_3xxx ).*u_2 + ( P_1 +
P_2 -2*P_3 ).*u_3xxx );

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

L14 = C0*( A0 + A1 + A2 + A3 + A4 + A5 + A6 );
L24 = C0*( B0 + B1 + B2 + B3 + B4 + B5 + B6 );
L34 = C0*( F0 + F1 + F2 + F3 + F4 + F5 + F6 );

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% FIFTH RK4 FUNCTIONS %%%%%%%%%%

% ADVANCING SOLUTIONS IN TIME
u_1 = u_1 + (L11 + 2*L12 + 2*L13 + L14)*d6;
u_2 = u_2 + (L21 + 2*L22 + 2*L23 + L24)*d6;
u_3 = u_3 + (L31 + 2*L32 + 2*L33 + L34)*d6;

%%% phase
Phase1 = angle(u_1);
Phase2 = angle(u_2);
Phase3 = angle(u_3);

Et1 = 0; Et2 = 0 ; Et3 = 0;

%%% energie
for n = 1 : N
    Et1 = Et1 + conj(u_1(n)).*u_1(n);
    Et2 = Et2 + conj(u_2(n)).*u_2(n);
    Et3 = Et3 + conj(u_3(n)).*u_3(n);
    Et = Et1 + Et2 + Et3 ;
end

if mod(nn,round(nmax/25)) == 0
    u_1data = [u_1data u_1];

```

```

u_2data = [u_2data u_2];
u_3data = [u_3data u_3];

tdata    = [tdata nn*dt];

Phidata1 = [Phidata1 angle(u_1)];
Phidata2 = [Phidata2 angle(u_2)];
Phidata3 = [Phidata3 angle(u_3)];

TabE1 =[TabE1 Et1];
TabE2 =[TabE2 Et2];
TabE3 =[TabE3 Et3];
TabE   =[TabE Et];
end
end % integration en

% figure(1)
% surf(x,tdata*1.0e13,abs(u_1data').^2), shading interp, camlight left,
colormap(hsv),
% grid off
% xlabel(['x (0.1 ' Ang ')'])
% ylabel('t (ps)')
% zlabel('|a_{3j} (x,t)|^2')
% set(gca,'fontsize',30, 'FontWeight','bold')
% rotate_labels(gca);
% call_when_rotates = @(~,~,hax)(rotate_labels(hax));
% hrot = rotate3d;
% set(hrot,'ActionPreCallBack',...
% @(hfig,event_data)(set(hfig,'WindowButtonMotionFcn',{call_when_rotates
event_data.Axes})));
%
set(hrot,'ActionPostCallBack',@(hfig,~)(set(hfig,'WindowButtonMotionFcn','')))
;

figure(2)
leg = plot(x,abs(u_1data(:,10')).^2,'-','linewidth',8.0); Ang = char(197);
xlabel(['x (0.1 ' Ang ')']),
ylabel('|a_{3j} (x, t = 10 ps)|^2')
% hold on;
% % set (leg, 'Box', 'off')
% % legend ('Data', 'location','northwest', 'Box', 'off')

% plot(x,abs(u_2data(:,10')).^2,'-g', 'linewidth',3.0); xlabel('x'),
ylabel('|a_{3j}|^2'),
% set(gca,'fontsize',25, 'FontWeight','bold'), hold on,
% plot(x,abs(u_3data(:,10')).^2,'--k', 'linewidth',3.5); xlabel('x'),
ylabel('|a_{3j}|^2'),
% set(gca,'fontsize',25, 'FontWeight','bold');
%
% figure(3)
% mesh(tdata*1.0e11,x,Phidata1,
'FaceColor','interp','EdgeColor','none','FaceLighting','phong');

```



```

% % mesh(tdata,x,Phidata1,
'FaceColor','interp','EdgeColor','none','FaceLighting','phong');
% camlight left
% grid off
% axis tight
% set(gca,'fontsize',25, 'FontWeight','bold')
% xlabel('t');
% ylabel('x');
% zlabel('\Phi(x,t)');
% % axis([0 2000 -20 20 0 2.8]);
% %rotate_labels(gca);
% call_when_rotates = @(~,~,hax)(rotate_labels(hax));
% hrot = rotate3d;
% set(hrot,'ActionPreCallBack',...
% @(hfig,event_data)(set(hfig,'WindowButtonMotionFcn',{call_when_rotates
event_data.Axes})));
%
set(hrot,'ActionPostCallBack',@(hfig,~)(set(hfig,'WindowButtonMotionFcn','')))
;

% figure(4)
% plot(tdata,TabE1, '-r','linewidth',3.0); xlabel('t'), ylabel('E_{1}'),
% set(gca,'fontsize',25, 'FontWeight','bold'), hold on,
% plot(tdata,TabE2,'--g', 'linewidth',2.0); xlabel('t'), ylabel('E_{2}'),
% set(gca,'fontsize',25, 'FontWeight','bold'), hold on,
% plot(tdata,TabE3,'k', 'linewidth',1.5); xlabel('t'), ylabel('E_{3}'),
% set(gca,'fontsize',25, 'FontWeight','bold');
% plot(tdata,TabE, 'linewidth',1.5); xlabel('t'), ylabel('E'),
% set(gca,'fontsize',25, 'FontWeight','bold'), hold on
%
% figure(5)
% plot(tdata*1.0e11,TabE,'linewidth',2.0);
% % plot(tdata,TabE,'linewidth',2.0);
% camlight left
% grid off
% axis tight
% set(gca,'fontsize',25, 'FontWeight','bold')
% xlabel('t');
% ylabel('Energy');hold on

% figure(6)
% plot(tdata*1.0e11,TabE1,'linewidth',6.0); xlabel('t'), ylabel('E_{1}'),
% set(gca,'fontsize',25, 'FontWeight','bold'), hold on,
% plot(tdata*1.0e11,TabE2,'--r', 'linewidth',4.0); xlabel('t'),
ylabel('E_{2}'),
% set(gca,'fontsize',25, 'FontWeight','bold'), hold on,
% plot(tdata*1.0e11,TabE3,'c', 'linewidth',2.5); xlabel('t'),
ylabel('E_{3}'),
% set(gca,'fontsize',25, 'FontWeight','bold');

```