

Supplementary Material for Do Physics-informed Neural Networks Satisfy Local and Global Mass Balance?

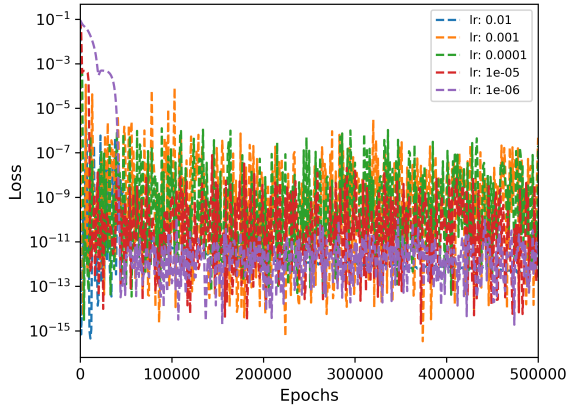
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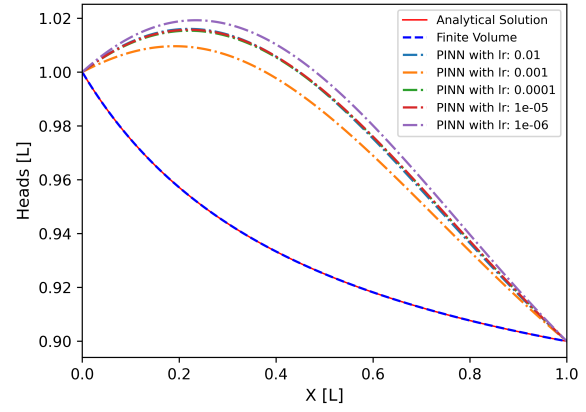
²Geology and Geological Engineering, University of Mississippi, University, MS 38677

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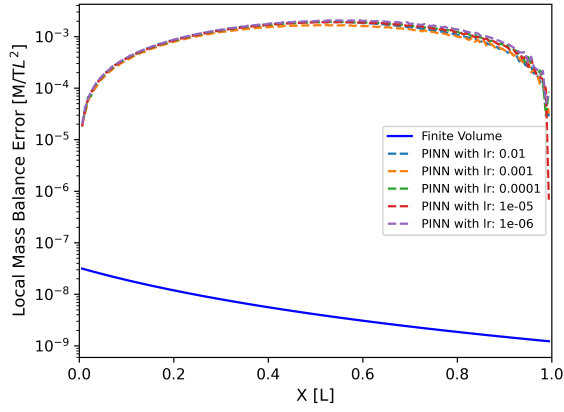
⁴Environmental Molecular Science Laboratory, Pacific Northwest National Laboratory, Richland, WA 99352



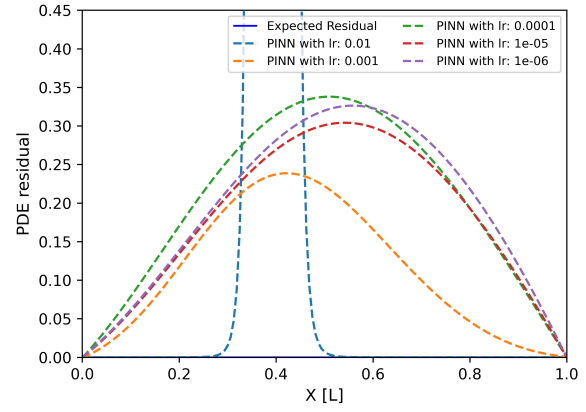
(a) Training PINN



(b) PINN solutions for hydraulic head

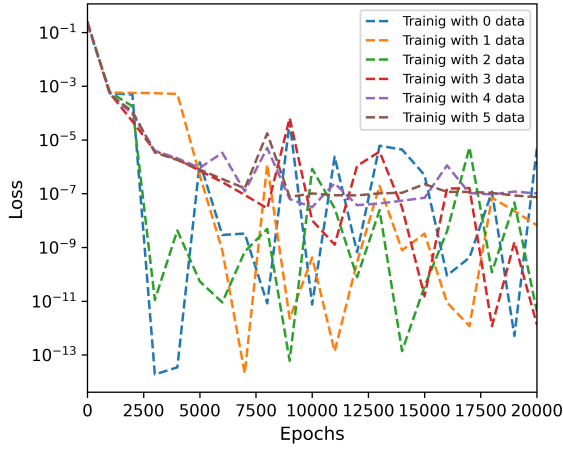


(c) Mass balance error of the PINN solutions

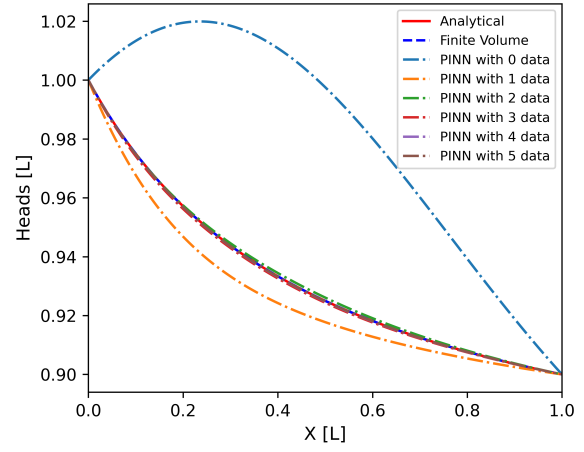


(d) PDE residual of the PINN solutions

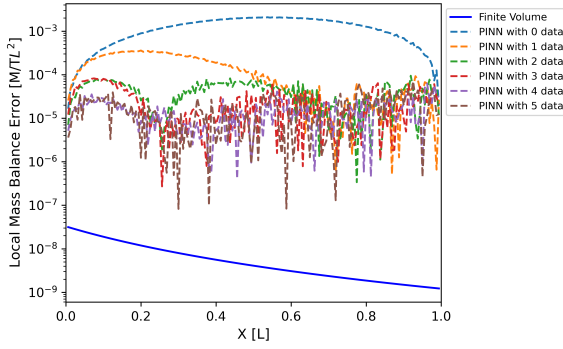
FIGURE S1. a) Training PINN varying learning rates with only boundary conditions and PDE, b) Analytical, finite volume, and PINN solutions varying learning rates for hydraulic head for the heterogeneous porous media, c) Mass balance error of the PINN solutions varying learning rate and finite volume solution, and d) PDE residual of the PINN solutions varying learning rates.



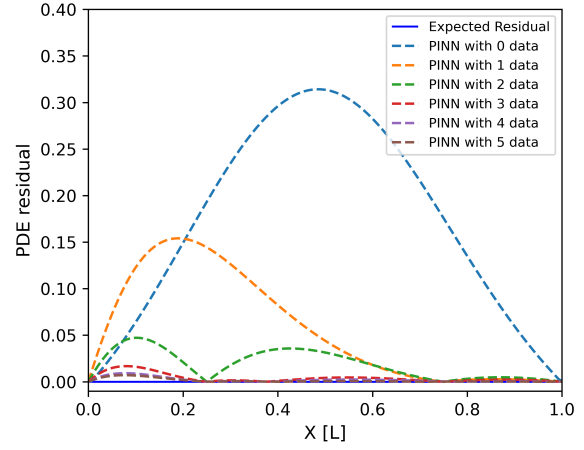
(a) Training PINN



(b) PINN solutions for hydraulic head

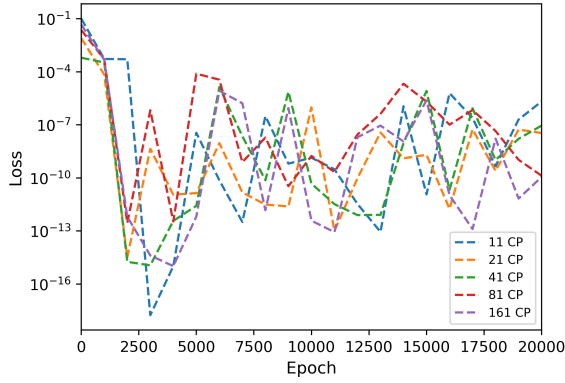


(c) Mass balance error of the PINN solutions

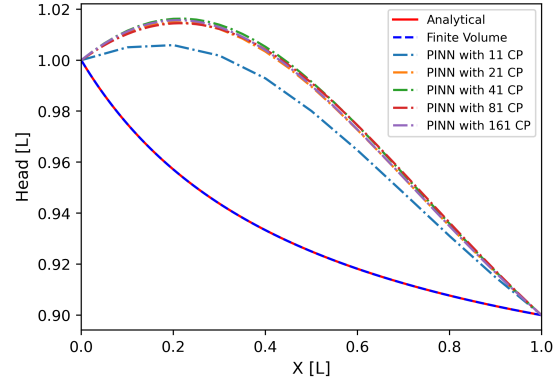


(d) PDE residual of the PINN solutions

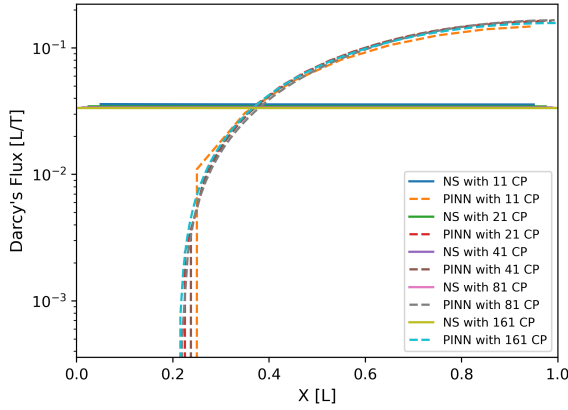
FIGURE S2. a) Training PINN varying training data with boundary conditions and PDE, b) Analytical, finite volume, and PINN solutions varying training data for hydraulic head for the heterogeneous porous media, c) Mass balance error of the PINN solutions varying training data and finite volume solution, and d) PDE residual of the PINN solutions varying training data.



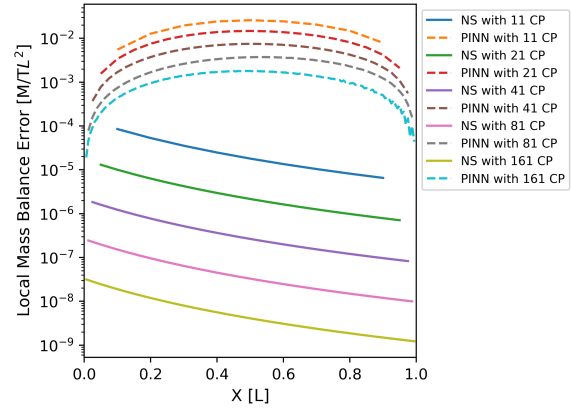
(a) Training PINN



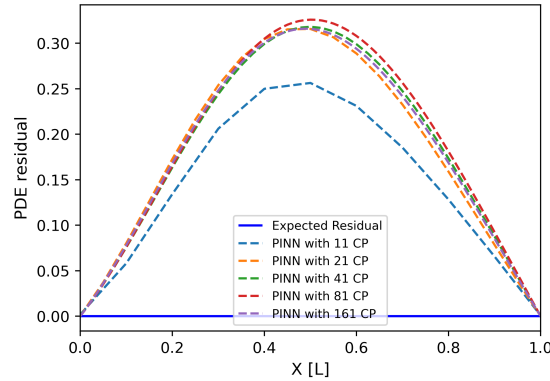
(b) PINN solutions for hydraulic head



(c) Darcy's Flux of the PINN solutions

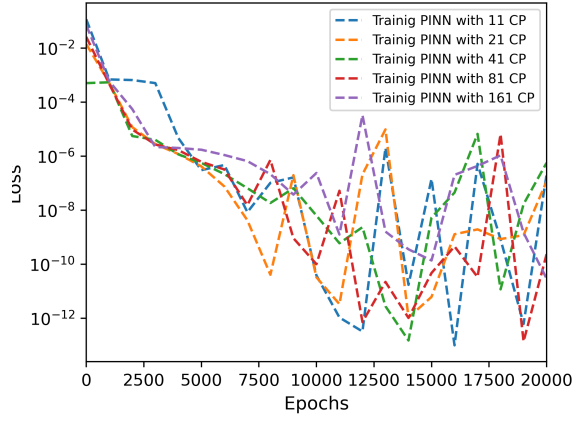


(d) Mass balance error of the PINN solutions

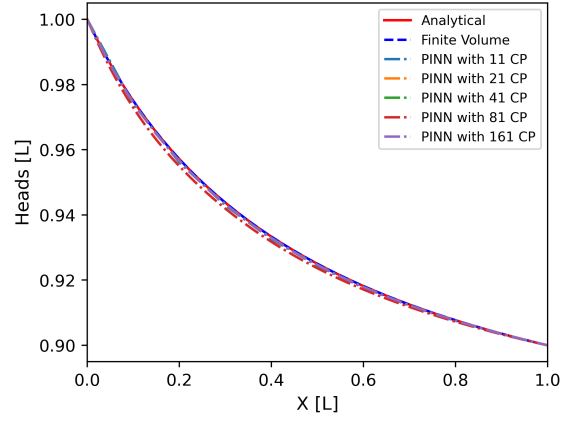


(e) PDE residual of the PINN solutions

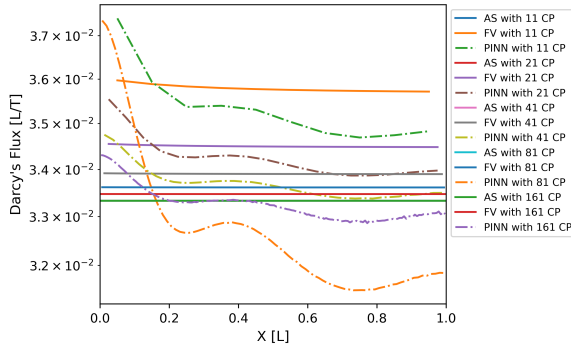
FIGURE S3. a) Training PINN varying collocation points without using training data, b) Analytical, finite volume, and PINN solutions varying collocation points without using training data for hydraulic head for the heterogeneous porous media, c) Mass balance error of the PINN solutions varying collocation points without using training data and finite volume solution, and d) PDE residual of the PINN solutions varying collocation points without using training data.



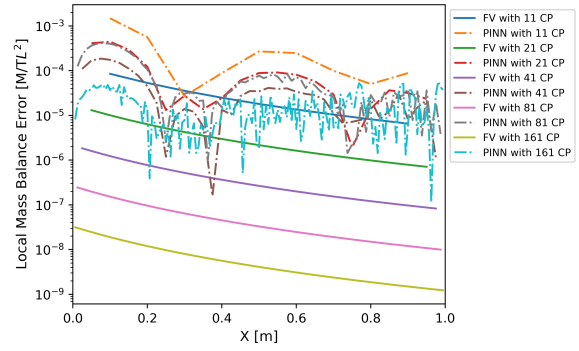
(a) Training PINN



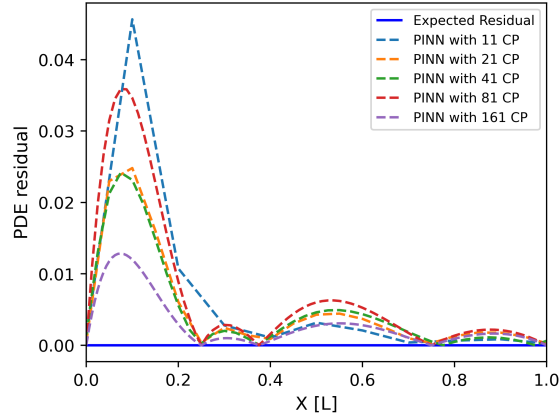
(b) PINN solutions for hydraulic head



(c) Darcy's Flux of the PINN solutions



(d) Mass balance error of the PINN solutions



(e) PDE residual of the PINN solutions

FIGURE S4. a) Training PINN varying collocation points with 3 Training data, b) Analytical, finite volume, and PINN solutions varying collocation points with 3 Training data for hydraulic head for the heterogeneous porous media, c) Mass balance error of the PINN solutions varying collocation points with 3 Training data and finite volume solution, and d) PDE residual of the PINN solutions varying collocation points with 3 Training data.