Supplementary Information

1. Additional discussion on temperature dependence of relaxation time as observed in Extended data Figs. 4a and 4b.

At an elevated temperature, the 2DES at $\nu = 1$ becomes demagnetized. As a result, vacancies are created in the spin-up states in the lower Landau level. Therefore, we considered two possible decay channels for spin-up electrons in the $N = 1$ LL. First, the spin-up electrons can decay to the spin-down states in the $N = 0$ LL through a spin-flip relaxation process: $\tau^{spin\text{-}flip} = \alpha \times (1 - \rho^{spin\text{-}up})$, where $\rho^{spin\text{-}up}$ is the number of vacancies in the spin-up states in the $N = 0$ LL. Second, the spin-up electrons can decay to vacancies in the spin-up states in the $N = 0$ LL through a charge relaxation process that does not require the spin-flip: $\tau^{charge} = \beta \times \rho^{spin\text{-}up}$. The number of vacancies is proportional to 1 - $M(T)$, where $M(T) = (1 - e^{-\Delta_0/(k_B T)})$ is the temperature dependent magnetization of a 2DES[1] (Δ_0, α, and β are the fitting parameters). The black dashed line in Fig. S1 shows the total relaxation time, which is the reciprocal sum of the two relaxation times $\tau^{total} = [1/(\tau^{charge}) + 1/(\tau^{spin\text{-}flip})]^{-1}$.

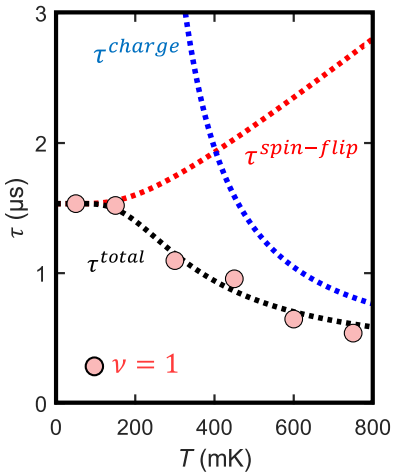

Fig. S1. Two-channel model for relaxation of pumped electrons at $\nu = 1$. Red circles show the temperature dependence of τ at $\nu = 1$ and $B_\perp = 6.0$ T. Blue, red, and black dashed lines show charge (τ^{charge}), spin-flip ($\tau^{spin\text{-}flip}$), and total decay ($\tau^{total} = [1/(\tau^{charge}) + 1/(\tau^{spin\text{-}flip})]^{-1}$) curves that give a best fit to the data at $\nu = 1$.

2. Spectral function of the $\nu = 1+$ state

We consider the spectral function characterizing the injection of an electron from the source quantum well into the second Landau level N=1 of the target quantum well while the first N=0 Landau level is occupied with electrons. In the symmetric gauge, the single-particle energies are those of a two-dimensional harmonic oscillator, with energies $E(N,m,\sigma) = \hbar\omega_c(N+\frac{1}{2}) + \frac{1}{2}g\mu_B B\sigma$, where N enumerates the Landau levels (LLs), m are intra-Landau level quantum numbers and $\sigma = \pm 1$ is the electron spin. The single-particle states are characterized by the angular momentum $l = m - N$ and the cyclotron energy $\hbar\omega_c = eB/m^*$, with $\hbar$ being the reduced Planck's constant, $e > 0$ - the elementary charge, B – the magnetic field, and m^* – the electron effective mass. Further, g is the electron Landé factor (negative for GaAs), and μ_B is the Bohr magneton.

Defining $c^+_{i,\sigma}(c_{i,\sigma})$ as the operator describing the creation (annihilation) of an electron in orbital i with spin σ, the many-body Hamiltonian describing n electrons in the lowest Landau level is

$$\hat{H} = \Sigma_{i,\sigma} E(i,\sigma) c^+_{i,\sigma} c_{i,\sigma} + \frac{1}{2}\Sigma_{i,j,k}\Sigma_{\sigma,\sigma'}\langle i,j|V|k,l\rangle c^+_{i,\sigma} c^+_{j,\sigma'} c_{k,\sigma} c_{l,\sigma'} \qquad \text{(Eq. S1)}$$

Here, $\langle i,j|V|k,l\rangle$ are the Coulomb matrix elements computed in the basis of the two-dimensional harmonic oscillator states[2]. We will express all interaction energies in our system in terms of the exchange energy $E_{ex} = \sum_{i=1}^{\infty}\langle 00,0i|V|00,0i\rangle$, which gives the total exchange interaction of an electron in the orbital (0,0) with all spin-polarized electrons in the lowest LL. To make the system electrically neutral, we account for the positive background by distributing n fixed positive charges on orbitals similar to the electronic ones. We neglect the Zeeman energy in this model.

We assume that we are adding a spin-up electron to the central orbital (N, m) = (1, 1) of the second Landau level, while the remaining n electrons occupy the lowest N=0 Landau level with spin up. Our goal is to compute the spectral function of the added electron as

$$A(E) = \sum_I P_I \sum_F \left|\langle F|c^+_{1,1,\uparrow}|I\rangle\right|^2 \delta(E_F - E_I - E). \qquad \text{(Eq. S2)}$$

Here, the index I enumerates the initial states, i.e., the correlated states of interacting n electrons in the lowest LL. P_I denotes the occupation probability of the state I of the initial system, and E_I is the energy of that state. $P_I = \frac{1}{Z}e^{-E_I/k_BT}$ $Z = \sum_I e^{-E_I/k_BT}$ The index F enumerates the final states, i.e., the correlated states of interacting n+1 electrons, of which exactly one is in the second LL.

We start by computing the low-energy initial states of the n-electron system using the exact diagonalization technique in a limited basis, with variable number of electrons n. We carry out two calculations. In the first part, at early times in spin injection, we consider the addition of the spin-up electron to the second LL in the presence of the $\nu = 1$ electronic droplet. In the second

part, at later times in injection pulse, a fraction of the electrons relax from the N=1 spin-up Landau level to the spin-down N=0 Landau level due to spin-flip scattering. We then compute the spectral function for addition of a spin-up electron to the N=1 level in the presence of spin-up electrons in the lowest LL, along with a low density of spin down electrons.

The initial states of the ν = 1 system are considered by taking n = 8 electrons and distributing them on eight spin-up orbitals in the lowest LL. We construct the spin polarized ground state $|\nu=1\rangle=\prod_{i=1}^{n-1} c_{i,\uparrow}^{+}|0\rangle$, and spin-wave excitations from the ground state of the form $|i,j\rangle=c_{i,\downarrow}^{+}c_{j,\uparrow}|\nu=1\rangle$ [3,4]. We write our Hamiltonian as a matrix in the basis of all these configurations and diagonalize it numerically to obtain the energies and eigenstates of the initial system. As expected, we find the ground state of the ν = 1 system to be the maximally polarized, ferromagnetic state.

The final states are considered in a similar way, but with an extra electron (the n+1st electron) in the second LL, that is, $c_{1,m,\uparrow}^{+}|\nu=1\rangle$. Even though the extra electron can be in principle added anywhere in the second LL, we choose to focus on m=1 as a representative subspace. The diagonalization of the Hamiltonian in this basis gives us the correlated final states. As the last step, we use the initial and final states to compute the spectral function $A(E)$ as described above. The resulting probability of injection of the spin-up electron into the second LL as a function of the energy E is shown in Fig. 4b in the main text (see the red line). For a noninteracting system, we would expect the addition peak at the cyclotron energy $E = \hbar\omega_c$, which is taken here to be equal to E_{ex}. The inclusion of interactions leads to a shift of the addition peak towards the lower energy. In simplest terms, that energy corresponds to the self-energy of the spin-up electron added to the second LL in the presence of fully occupied and spin-polarized lowest LL. We find further that the extra electron in the second LL leads to relatively little mixing between different configurations of the ν = 1 system. Indeed, in our more extended calculations (not shown), in spite of taking a large number of configurations in the initial (several hundred) and final (several thousand) states, the addition signature appears consistently at roughly the same energy of order of $0.5E_{ex}$.

In the second part of the calculation, we account for an extra spin down electron which relaxed from the N=1 to N=0 (lowest) LL, that is, a spin-down electron added to the ν = 1 spin-up state. For our calculations, we take n = 9 electrons distributed in the lowest LL on eight orbitals. We map out the lowest energy states of such a correlated system as a function of the total angular momentum and the total spin projection S_z. The results are presented in Fig. 4a of the main text. We find that the global ground state corresponds to a fully depolarized system (spin polarization 1/2, shown in black) with total angular momentum of 31. We identify this state with the correlated skyrmion $|s\rangle$ [5–7]. Similar spin depolarizations in droplets in the vicinity of the ν = 1 state have been observed in quantum dots confining many electrons [8,9]. The state with maximal spin polarization of 7/2 is found at a higher energy (shown in blue). In the simplest terms, this state can be approximated as $|\nu = 1+\rangle = c_{0,m,\downarrow}^{+}|\nu = 1\rangle$, i.e., an uncorrelated state obtained from ν = 1 by adding one spin-down electron to it.

We follow with the numerical study of the system of n = 10 electrons, of which exactly one occupies an orbital from the second LL. Owing to the depolarized and correlated nature of the states of ν = 1 plus one electron, we have to account for all possible spin polarization and all possible angular momenta, resulting in several tens of Hilbert spaces, each of the size of hundreds to thousands of configurations (depending chiefly on spin polarization). Systematic diagonalization of all these cases gives us all necessary final states entering the calculation of $A(E)$.

We compute the spectral function assuming the skyrmion |s> as well as the maximally polarized $|\nu = 1+>|$ as the initial state of the n=9 electron system. The results are presented in Fig. S2 with the black and blue lines, respectively. We see a significant difference between these two spectral functions and that calculated for ν = 1 (see Fig. 4b of the main text). Indeed, the spectral function of the skyrmion is very broad, reflecting spin depolarization processes in the ν = 1 initial state, with overall blueshift from $0.5E_{ex}$ for ν = 1 to $0.8E_{ex}$, for ν = 1+ phase. We find therefore that the addition of a minority spin-down electron to the ν = 1 spin-up electron state is reflected by the spectral function of N=1 spin-up electron blue-shifted from the spectral function of N=1 electron added to the ν = 1 state. Hence, one expects a transition in the spectral function as a function of pumping density, from the peak at low energy to the peak at higher energy as spin-down electrons begin to populate the lowest spin-up Landau level.

Further details for these calculations will be presented in a future publication.

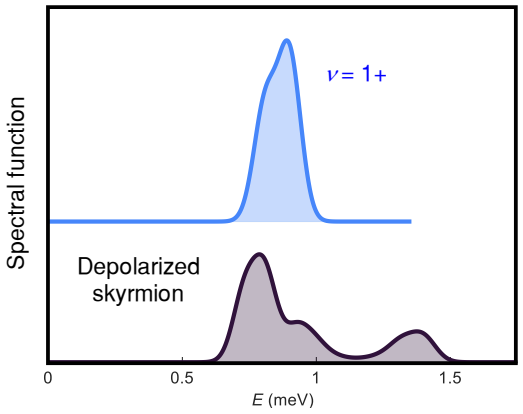

Fig. S2. Calculated spectral functions of ν = 1+ and skyrmion. The horizontal axis is the injection energy in the N = 1 LL in units of E_{ex}. The spectral function of a depolarized skyrmion state is broadened due to electronic correlations that give rise to nontrivial spin configurations (see Fig. 4b of the main text for the spectral functions of ν = 1).

References

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