

Supplementary Information for

Probing optical anapoles with fast electron beams

Carlos Maciel-Escudero^{1,2,†}, Andrew B. Yankovich^{3,†}, Battulga Munkhbat^{3,4}, Denis G. Baranov^{3,5}, Rainer Hillenbrand^{2,6}, Eva Olsson^{3,*}, Javier Aizpurua^{1,7,*}, and Timur O. Shegai^{3,*}

¹Materials Physics Center, CSIC-UPV/EHU, Paseo de Manuel Lardizabal,
Donostia-San Sebastián, 20018, Spain

²Department of Electricity and Electronics, CIC NanoGUNE BRTA, Tolosa
Hiribidea, Donostia-San Sebastián, 20018, Spain

³Department of Physics, Chalmers University of Technology, Gothenburg, 412 96,
Sweden

⁴Department of Photonics Engineering, Technical University of Denmark, Kgs.
Lyngby, Copenhagen, 2800, Denmark

⁵Center for Photonics and 2D Materials, Moscow Institute of Physics and
Technology, Dolgoprudny, Moscow, 141700, Russia

⁶IKERBASQUE, Basque Foundation for Science, Bilbao, 48011, Spain

⁷Donostia International Physics Center, Paseo de Manuel Lardizabal, Donostia-San
Sebastián, 20018, Spain

*Corresponding author(s). E-mail(s): eva.olsson@chalmers.se, aizpurua@ehu.eus,
timurs@chalmers.se

[†]These authors contributed equally to this work.

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Supplementary Note 1. COMSOL Multiphysics simulations

S1.1 Numerical simulations of EEL probability

To calculate the electron energy loss (EEL) probability $\Gamma(\omega)$ shown in Figs. 1, 3 and 4 of the main text, we used the classical dielectric theory [1, 2]. We assume that the induced electric field, coming from a response of the nanodisk to the incident electromagnetic field of the electron beam, acts back on the beam, thus causing an energy loss which reveals the energy and strength of the excitation induced in the nanodisk. We assumed that the electron beam is traveling in the z -direction and we modelled it as a vertical line (nonrecoil approximation) carrying a current $I = -I_0 e^{i\omega z/v}$, where $I_0 = 1$ A, ω is the angular frequency, $v = 0.7c$ (200 keV) is the velocity of the electron and c is the speed of light in vacuum. Then, the EEL probability $\Gamma(\omega)$ of the electron beam to lose an energy $\hbar\omega$ is given by

$$\Gamma(\omega) = \frac{e^2}{\pi \hbar \omega I_0} \int_{-L_{\text{PML}}/2}^{L_{\text{PML}}/2} dz \operatorname{Re}[E_z^{\text{ind}}(x_e, y_e, z) e^{-i\omega z/v}], \quad (1)$$

where e is the elementary charge, $\operatorname{Re}[x]$ represents the real part of the complex number x , $E_z^{\text{ind}}(\mathbf{r})$ is the z -component of the induced electromagnetic field in the nanodisk and $E_z^{\text{ind}}(\mathbf{r})$ is evaluated along the trajectory of the electron beam $\mathbf{r}_e(t) = (x_e = b, y_e = 0, z = vt)$. To calculate $E_z^{\text{ind}}(\mathbf{r})$ of the WS₂ disk, we performed the simulation for each frequency twice: one simulation considering the nanodisk with the permittivity function of WS₂ (see Supplementary Note 1.2, Supplementary Eqs. 3-4) and another simulation considering the nanodisk with permittivity $\varepsilon = 1$, preserving the same mesh in all domains. Their difference yields $E_z^{\text{ind}}(\mathbf{r})$, which can be integrated along the electron beam trajectory to find $\Gamma(\omega)$ (Supplementary Eq. 1).

To obtain the electron energy loss probability of the WS₂ flake (green lines in Figs. 3(e),(f) and 4(e),(j) of the main text), we calculate the non-retarded bulk loss probability $\Gamma^{\text{bulk}}(\omega)$ experienced by a fast electron beam traveling a distance $d = 70$ nm in WS₂. The calculation was performed using the the following expression [3]

$$\Gamma^{\text{bulk}}(\omega) = \frac{e^2 d}{(2\pi)^2 v^2 \hbar \varepsilon_0} \operatorname{Im} \left[-\frac{1}{\varepsilon_{xx}} \ln \left(\frac{\varepsilon_{zz} \omega^2 / v^2 + \varepsilon_{xx} q_c^2}{\varepsilon_{zz} \omega^2 / v^2} \right) \right], \quad (2)$$

where $\operatorname{Im}[x]$ represents the imaginary part of the complex number x , $q_c = 1 \text{ \AA}^{-1}$ is the maximum perpendicular wavevector of the electrons selected by the collection aperture of the EELS spectrometer and ε_{xx} , ε_{zz} are the in- and out-of-plane permittivities of WS₂, respectively, discussed below.

S1.2 Permittivities used in the simulations

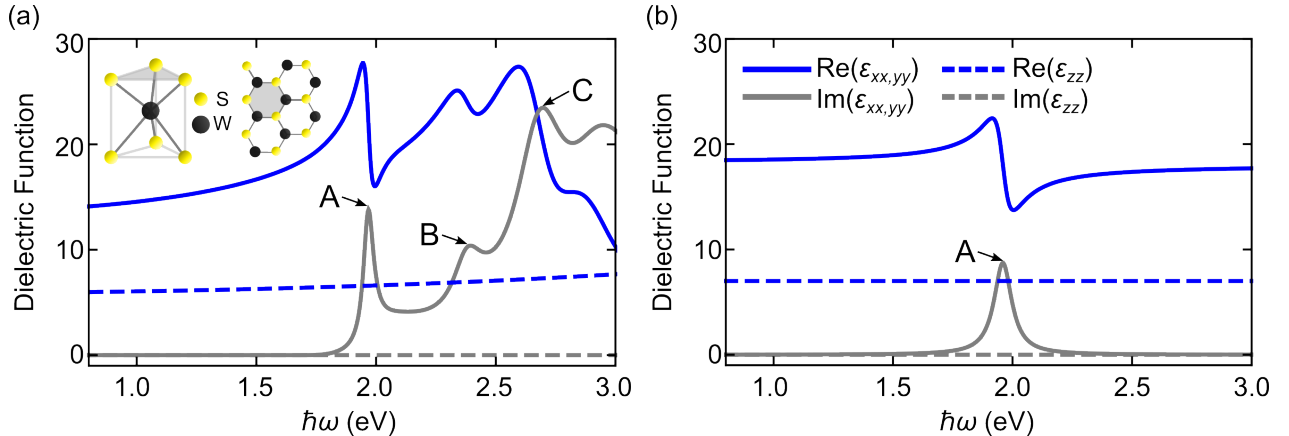
To obtain the numerical calculations shown in Fig. 1, we use a constant value of $\varepsilon = 18$ for the permittivity of the high-index dielectric nanodisk. For simplicity, these simulations were performed without considering any substrate whereas numerical calculations shown in Figs. 3 and 4 were performed with the nanodisk being on top of a 50 nm thick substrate layer with permittivity $\varepsilon_{\text{SiN}} = 4.1853$.

For the simulations shown in Figs. 3 and 4, we used the following in- and out-of-plane permittivities of WS₂ [4], $\varepsilon_{xx} = \varepsilon_{yy}$ and ε_{zz} , respectively

$$\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_{\infty} + f_0 \frac{\omega_0^2}{\omega_0^2 - \omega^2 - i\gamma\omega}, \quad (3)$$

$$\varepsilon_{zz} = 7, \quad (4)$$

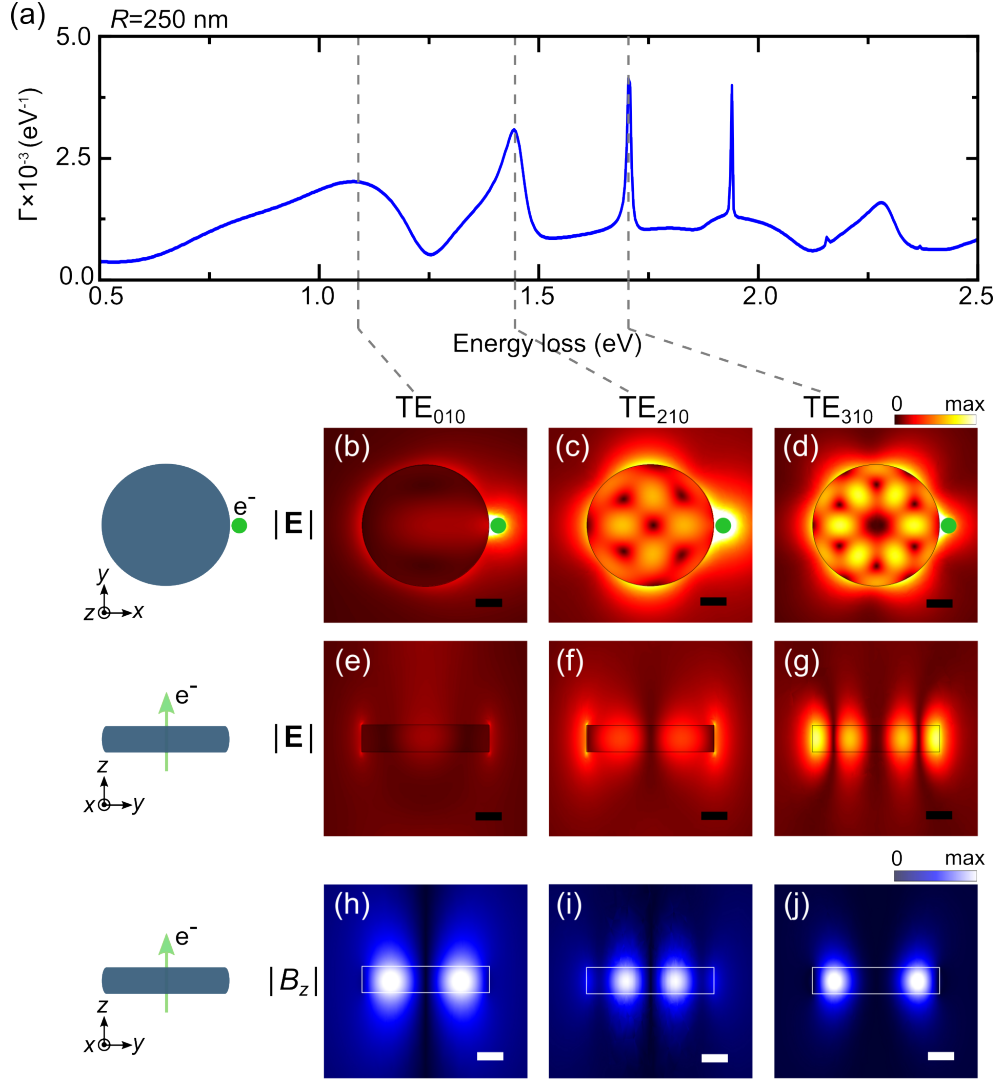
where $\varepsilon_{\infty} = 18$ is the high-frequency permittivity, $f_0 = 0.4$ is the oscillator strength, $\hbar\omega_0 = 1.96$ eV is the exciton frequency and $\hbar\gamma = 90$ meV is the exciton damping constant. The in-plane component ε_{xx} (Supplementary Eq. 3) considers only the A-exciton absorption band of the WS₂.



Supplementary Figure 1: Permittivity of WS₂. (a) Complex in- and out-of-plane permittivities of WS₂, $\varepsilon_{xx} = \varepsilon_{yy}$ (solid lines) and ε_{zz} (dashed lines), respectively. Blue lines represent real parts and gray lines imaginary parts. The dielectric functions were obtained from Ref. [5]. The labels A, B and C indicate the A-, B-, and C-exciton absorption bands of WS₂. (b) Same as (a) but the components of the dielectric function are given by Supplementary Eqs. 3 and 4. Inset in panel (a) illustrates the atomic structure of WS₂.

For comparison, we show in Supplementary Fig. 1 the dielectric function of WS₂ obtained from experimental data [5] (see Supplementary Fig. 1(a)) and the dielectric function of WS₂ used in the simulations (see Supplementary Fig. 1(b)). The labels A, B and C in Supplementary Fig. 1 indicate the resonance peaks associated to the A-, B- and C-exciton absorption bands of WS₂.

Supplementary Note 2. Modes of the cylindrical nanoresonator



Supplementary Figure 2: Disk modes excited in the nanodisk by the fast electron. (a) Simulated EEL probability $\Gamma(\omega)$ for a nanodisk with $R = 250$ nm and $d = 55$ nm. Gray dashed lines mark the first three peak positions at $\hbar\omega_{\text{TE}_{010}} = 1.09$ eV, $\hbar\omega_{\text{TE}_{210}} = 1.44$ eV and $\hbar\omega_{\text{TE}_{310}} = 1.70$ eV. The field plots in panels (b)-(d) show the amplitude of the total electric field $|\mathbf{E}(\omega)|$ in the plane $z = 0$ for the energies marked by the gray dashed lines. Green dots indicate the electron beam position. (e)-(g) Same as (b)-(d) but for the plane $x = 0$. The maximum value of $|\mathbf{E}(\omega)|$ in each case is: (b) 2×10^9 V/m, (c) 1.2×10^9 V/m, (d) 3×10^9 V/m and (e)-(g) 2×10^9 V/m. (h)-(j) Same as (e)-(g) but showing the amplitude of the z -component of the total magnetic field $|B_z(\omega)|$ in the plane $x = 0$. The maximum value of $|B_z(\omega)|$ in each case is: (e) 1.7 T, (f) 1.6 T and (g) 20 T. Insets at the left of panels (b) and (e), (h) illustrate top and side views of the nanodisk being probed by the electron beam. The scale bar in panels (b)-(j) is 100 nm.

As we pointed out in the main text, one can notice that the EEL probability $\Gamma(\omega)$ spectra exhibit different peaks (see spectrum in Fig. 1(c) of the main text). They are attributed to the different modes excited in the nanodisk by the fast electron beam as we discuss in the following paragraph.

In Supplementary Fig. 2(a) we show the $\Gamma(\omega)$ spectrum of a high-index dielectric ($\varepsilon = 18$) nanodisk of radius $R = 250$ nm and thickness $d = 55$ nm (same as in Fig. 1(c) of the main text). In the simulations, the electron beam travels with velocity $v = 0.7c$ (200 keV, c being the speed of light in vacuum) along the z -axis (symmetry axis of the nanodisk) with impact parameter $b = 1.1R$, as depicted in the inset at left of Supplementary Fig. 2(b). For simulation details, we refer the reader to the Methods section and Supporting Note 1. We observe in Supplementary Fig. 2(a) clear peaks at around 1.09 eV, 1.44 eV, 1.70 eV, 1.8 eV and 2.25 eV. To elucidate the nature of these peaks, we show the amplitude of the total electric field $|\mathbf{E}(\omega)|$ inside the nanodisk in the plane $z = 0$ (half of the disk height) at the three energies indicated by the gray dashed lines: 1.09 eV (panel (b)), 1.44 eV (panel (c)) and 1.70 eV (panel (d)). In panel (b), we observe a mode that is localized at the center and edge of the nanodisk, whereas the field plots in panels (c) and (d) reveal modes with 4 and 6 nodes along the azimuthal direction with zero nodes along the z -direction (panels (e)-(g)). Additionally, the amplitude of the z -component of the total magnetic field $|B_z(\omega)|$ is different from zero inside the disk at the energies corresponding to the peaks (see panels (h)-(j)), and thus we classified the modes as transverse electric (TE). Furthermore, to label these modes we use the standard nomenclature of cylindrical resonator modes [6, 7]: $\text{TE}_{n,k,p}$ where n, k, p indices denote azimuthal, radial and axial wavenumbers, respectively.

Supplementary Note 3. Multipole decomposition

The electric spherical and Cartesian multipole moments of the nanodisks were calculated using the standard expressions reported in Refs [8, 9, 10, 11]. In this section, we present the close expressions for the spherical electric dipole $\mathbf{P}_{\text{sph}}(\omega)$ and the spherical electric quadrupole $\hat{\mathbf{Q}}_{\text{sph}}(\omega)$ moments. We also show that in the long-wavelength approximation, electric spherical dipole and quadrupole moments are a combination of Cartesian electric and toroidal multipole moments.

S3.1 Electric dipole

We calculate the electric spherical dipole moment $\mathbf{P}_{\text{sph}}(\omega)$ in the nanodisk using the following expression [9]

$$\mathbf{P}_{\text{sph}}(\omega) = -\frac{1}{i\omega} \int_V d^3r \left\{ \mathbf{J}_{\text{ind}} j_0(k_0 r) + \frac{k_0^2}{2} [3(\mathbf{r} \cdot \mathbf{J}_{\text{ind}})\mathbf{r} - r^2 \mathbf{J}_{\text{ind}}] \frac{j_2(k_0 r)}{(k_0 r)^2} \right\}. \quad (5)$$

Here the integral extends over the whole volume V of the nanodisk, $k_0 = \omega/c$ is the magnitude of the wavevector in vacuum, $j_\ell(x)$ is the spherical Bessel function of the first kind, r is the magnitude of the position vector $\mathbf{r} = (x, y, z)$ and $\mathbf{J}_{\text{ind}} = \mathbf{J}_{\text{ind}}(\mathbf{r})$ is the induced current density distribution in the nanodisk.

In the long-wavelength approximation ($k_0 r \ll 1$) the spatial size of $\mathbf{J}_{\text{ind}}(\mathbf{r})$ is much smaller than the wavelength of the electromagnetic field, and thus one can expand the spherical Bessel functions $j_0(k_0 r)$, $j_2(k_0 r)$ using the small argument limit [12]

$$j_0(k_0 r) \approx 1 - \frac{(k_0 r)^2}{6}, \quad (6)$$

$$j_2(k_0 r) \approx \frac{(k_0 r)^2}{15}. \quad (7)$$

By substituting Supplementary Eqs. 6 and 7 into Supplementary Eq. 5, we can obtain the following expression for the electric spherical dipole moment in the long-wavelength approximation

$$\begin{aligned} \mathbf{P}_{\text{sph}}^{k_0 r \ll 1}(\omega) &\approx -\frac{1}{i\omega} \int_V d^3r \left\{ \mathbf{J}_{\text{ind}} \left[1 - \frac{(k_0 r)^2}{6} \right] + \frac{k_0^2}{2} [3(\mathbf{r} \cdot \mathbf{J}_{\text{ind}})\mathbf{r} - r^2 \mathbf{J}_{\text{ind}}] \frac{(k_0 r)^2}{15(k_0 r)^2} \right\} \quad (8) \\ &\approx \underbrace{-\frac{1}{i\omega} \int_V d^3r \mathbf{J}_{\text{ind}}}_{\mathbf{P}_{\text{car}}(\omega)} + i k_0 \underbrace{\left\{ \frac{1}{10c} \int_V d^3r [(\mathbf{r} \cdot \mathbf{J}_{\text{ind}})\mathbf{r} - 2r^2 \mathbf{J}_{\text{ind}}] \right\}}_{\mathbf{T}_{\text{car}}(\omega)}, \end{aligned}$$

where one recognizes from the last equality the conventional expressions of Cartesian electric and toroidal dipole moments [10]. From this analysis, we conclude that in the long-wavelength approximation electric spherical dipole moment $\mathbf{P}_{\text{sph}}(\omega)$ is determined by the sum of Cartesian electric $\mathbf{P}_{\text{car}}(\omega)$ and toroidal $\mathbf{T}_{\text{car}}(\omega)$ dipole moments as:

$$\mathbf{P}_{\text{sph}}(\omega) \approx \mathbf{P}_{\text{car}}(\omega) + ik_0 \mathbf{T}_{\text{car}}(\omega). \quad (9)$$

S3.2 Electric quadrupole

To calculate the electric spherical quadrupole moment $\hat{\mathbb{Q}}_{\text{sph}}(\omega)$ induced in the nanodisk, we use the following expression [9]:

$$\begin{aligned} \hat{\mathbb{Q}}_{\text{sph}}(\omega) = & -\frac{3}{i\omega} \int_V d^3r \left\{ \left[3(\mathbf{r}\mathbf{J}_{\text{ind}} + \mathbf{J}_{\text{ind}}\mathbf{r}) - 2(\mathbf{r} \cdot \mathbf{J}_{\text{ind}})\hat{\mathbb{I}} \right] \frac{j_1(k_0r)}{k_0r} \right. \\ & \left. + 2k_0^2 \left[5\mathbf{r}\mathbf{r}(\mathbf{r} \cdot \mathbf{J}_{\text{ind}}) - (\mathbf{r}\mathbf{J}_{\text{ind}} + \mathbf{J}_{\text{ind}}\mathbf{r})r^2 - r^2(\mathbf{r} \cdot \mathbf{J}_{\text{ind}})\hat{\mathbb{I}} \right] \frac{j_3(k_0r)}{(k_0r)^3} \right\}, \end{aligned} \quad (10)$$

with $\hat{\mathbb{I}}$ being the identity tensor. In the long-wavelength approximation, one can expand the spherical Bessel functions $j_1(k_0r)$, $j_3(k_0r)$ as [12]

$$j_1(k_0r) \approx \frac{k_0r}{3} - \frac{(k_0r)^3}{30}, \quad (11)$$

$$j_3(k_0r) \approx \frac{(k_0r)^3}{105}, \quad (12)$$

and substituting Supplementary Eqs. 11 and 12 into Supplementary Eq. 10 one finds that

$$\begin{aligned} \hat{\mathbb{Q}}_{\text{sph}}^{k_0r \ll 1}(\omega) \approx & -\frac{3}{i\omega} \int_V d^3r \left\{ \left[3(\mathbf{r}\mathbf{J}_{\text{ind}} + \mathbf{J}_{\text{ind}}\mathbf{r}) - 2(\mathbf{r} \cdot \mathbf{J}_{\text{ind}})\hat{\mathbb{I}} \right] \frac{1}{k_0r} \left[\frac{k_0r}{3} - \frac{(k_0r)^3}{30} \right] \right. \\ & \left. + 2k_0^2 \left[5\mathbf{r}\mathbf{r}(\mathbf{r} \cdot \mathbf{J}_{\text{ind}}) - (\mathbf{r}\mathbf{J}_{\text{ind}} + \mathbf{J}_{\text{ind}}\mathbf{r})r^2 - r^2(\mathbf{r} \cdot \mathbf{J}_{\text{ind}})\hat{\mathbb{I}} \right] \frac{(k_0r)^3}{105(k_0r)^3} \right\} \\ \approx & \underbrace{-\frac{1}{i\omega} \int_V d^3r \left[3(\mathbf{r}\mathbf{J}_{\text{ind}} + \mathbf{J}_{\text{ind}}\mathbf{r}) - 2(\mathbf{r} \cdot \mathbf{J}_{\text{ind}})\hat{\mathbb{I}} \right]}_{\hat{\mathbb{Q}}_{\text{car}}^{(\text{e})}(\omega)} \\ & + i3k_0 \underbrace{\left\{ \frac{1}{42c} \int_V d^3r \left[4\mathbf{r}\mathbf{r}(\mathbf{r} \cdot \mathbf{J}_{\text{ind}}) - 5r^2(\mathbf{r}\mathbf{J}_{\text{ind}} + \mathbf{J}_{\text{ind}}\mathbf{r}) + 2r^2(\mathbf{r} \cdot \mathbf{J}_{\text{ind}})\hat{\mathbb{I}} \right] \right\}}_{\hat{\mathbb{Q}}_{\text{car}}^{(\text{T})}(\omega)}, \end{aligned} \quad (13)$$

where $\hat{Q}^{(e)}(\omega)$ and $\hat{Q}^{(T)}(\omega)$ are the conventional electric Cartesian and toroidal quadrupole moments [10], respectively. Thus, in the long-wavelength approximation electric spherical quadrupole moment $\hat{Q}_{\text{sph}}(\omega)$ is determined by the sum of Cartesian electric $\hat{Q}_{\text{car}}^{(e)}(\omega)$ and toroidal $\hat{Q}_{\text{car}}^{(T)}(\omega)$ quadrupole moments as:

$$\hat{Q}_{\text{sph}}(\omega) \approx \hat{Q}_{\text{car}}^{(e)}(\omega) + i3k_0\hat{Q}_{\text{car}}^{(T)}(\omega). \quad (14)$$

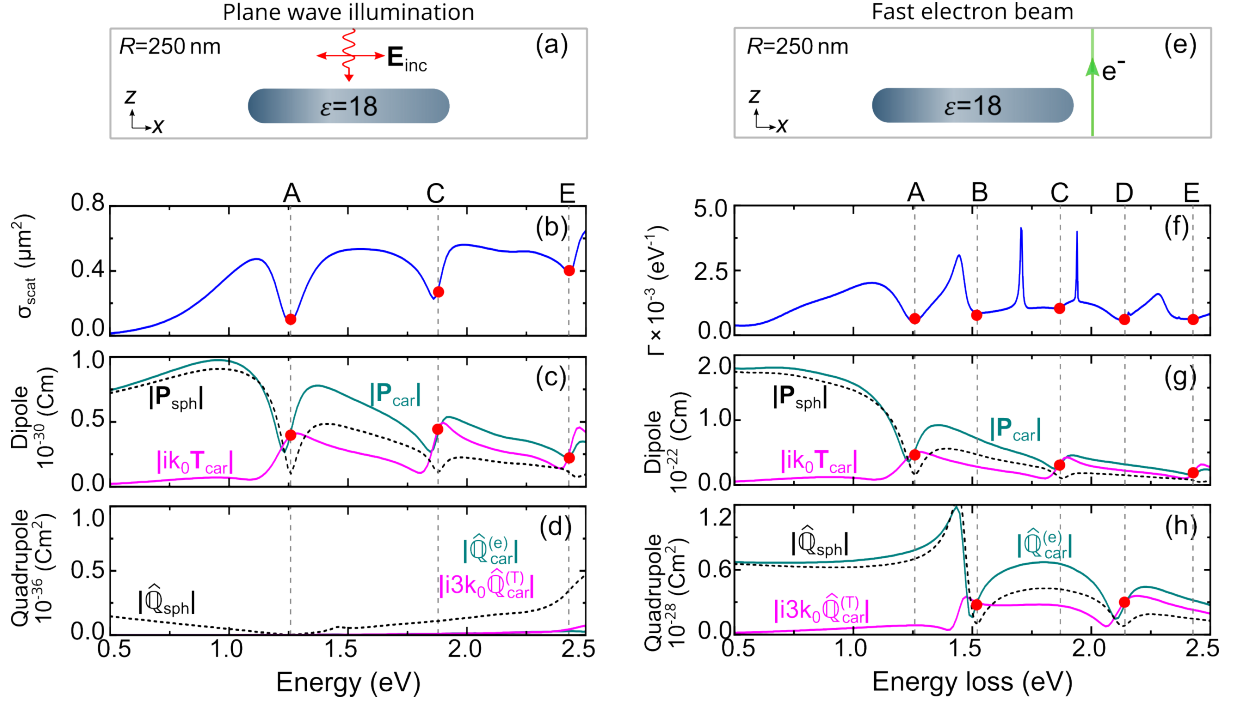
In our simulations, we calculate the multipole moments $\mathbf{P}_{\text{sph}}(\omega)$, $\mathbf{P}_{\text{car}}(\omega)$, $\mathbf{T}_{\text{car}}(\omega)$, $\hat{Q}_{\text{sph}}(\omega)$, $\hat{Q}_{\text{car}}^{(e)}(\omega)$ and $\hat{Q}_{\text{car}}^{(T)}(\omega)$ applying the following procedure:

- (i) We calculate the total electric field $\mathbf{E}(\mathbf{r})$ inside the nanodisk. The total electric field is the one produced by the probing electron plus the induced electric field in the nanodisk.
- (ii) We next calculate the induced current density as $\mathbf{J}_{\text{ind}}(\mathbf{r}) = -i\omega\varepsilon_0(\varepsilon - 1)\mathbf{E}(\mathbf{r})$ and performed the volume integrals (Supplementary Eqs. 5, 8, 10 and 13) with functions predefined in COMSOL Multiphysics software.

Supplementary Note 4. Comparison of far-field and EEL spectroscopies

In this section we show that electric quadrupole anapole states are not excited in the nanodisk when it is illuminated with a plane wave at normal incidence. To that end, we calculate the scattering cross section $\sigma_{\text{scat}}(\omega)$ of the model high-index dielectric disk ($\epsilon = 18$) with $R = 250$ nm and $d = 55$ nm. The nanodisk is illuminated with a plane wave linearly polarized along the x -axis as sketched in Supplementary Fig. 3(a). The scattering cross section of the disk was calculated as $\sigma_{\text{scat}} = \frac{1}{S_0} \oint_A d^2r \mathbf{S}_{\text{scat}}(\mathbf{r}) \cdot \hat{\mathbf{n}}$, where $S_0 = |\mathbf{E}_{\text{inc}}|^2/(2Z_0)$, $|\mathbf{E}_{\text{inc}}| = 1$ V/m is the amplitude of the incident plane wave and Z_0 is the impedance of vacuum. $\mathbf{S}_{\text{scat}}(\mathbf{r})$ is the time-averaged Poynting vector, d^2r is the surface element with outward normal unit vector $\hat{\mathbf{n}}$ and the integration is performed along the complete surface A of a sphere that encloses the disk.

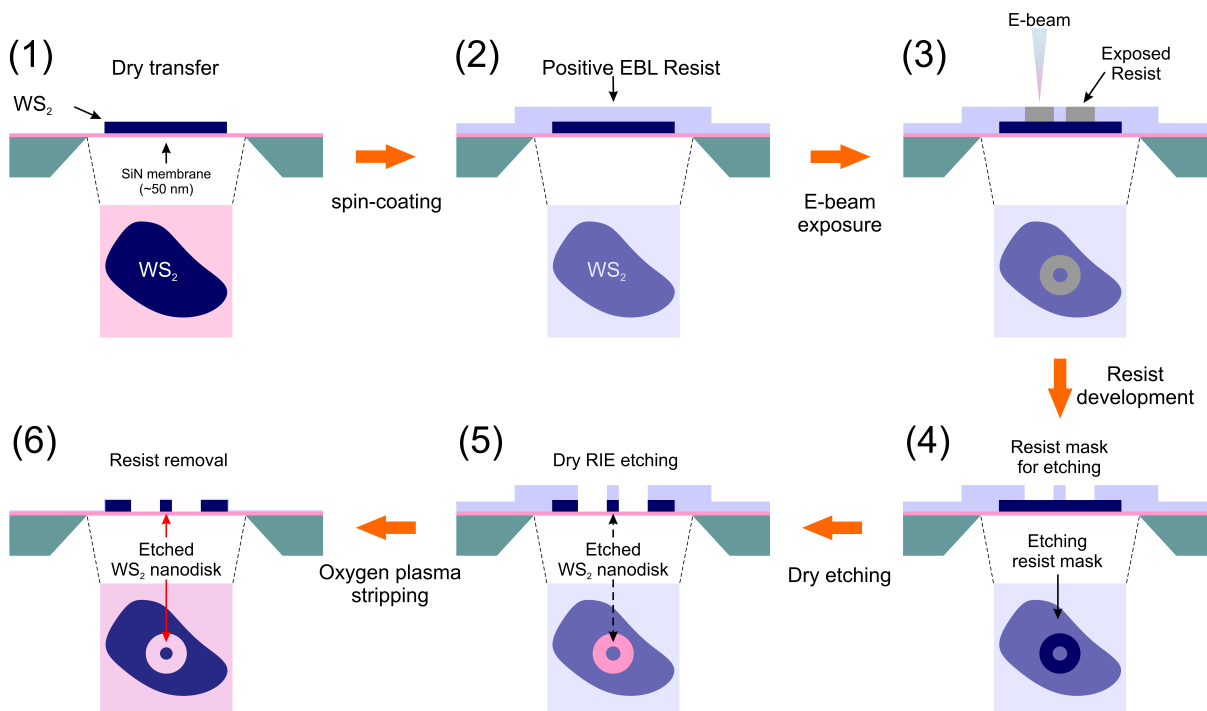
In Supplementary Fig. 3(b) we show the scattering cross section $\sigma_{\text{scat}}(\omega)$ spectrum of the $R = 250$ nm disk. We can recognize three attenuated dips at around 1.26 eV (labelled as A), 1.86 eV (labelled as C) and 2.4 eV (labelled as E), which can be associated with the excitation of the first, second and third electric dipole anapole states in the disk, respectively. We verify this by calculating the amplitude of the electric spherical $|\mathbf{P}_{\text{sph}}(\omega)|$ (black dashed line in Supplementary Fig. 3(c)) dipole moment, as well as the Cartesian electric $|\mathbf{P}_{\text{car}}(\omega)|$ (green line in Supplementary Fig. 3(c)) and scaled toroidal $|ik_0 \mathbf{T}_{\text{car}}(\omega)|$ (magenta line in Supplementary Fig. 3(c)) dipole moments. By comparing the green and magenta spectra we can see that the Cartesian and toroidal dipole moments sustain the same amplitude but opposite phase at around 1.26 eV, 1.86 eV and 2.4 eV, which confirms that the dips in the $\sigma_{\text{scat}}(\omega)$ spectrum corresponds to the excitation of ED anapole states. Most importantly, the plane wave is not able to efficiently excite quadrupolar disk modes and thus induces negligible Cartesian electric $\hat{\mathbf{Q}}_{\text{car}}^{(\text{e})}(\omega)$ and toroidal $\hat{\mathbf{Q}}_{\text{car}}^{(\text{T})}(\omega)$ quadrupole moments in the disk, as we show in Supplementary Fig. 3(d). As a consequence, the disk EQ anapole states are not excited by the incident plane wave in contrast to the case of the nanodisk being illuminated by the fast electron beam. For comparison, we show in Supplementary Figs. 3(e)-(h) the EEL spectrum, together with the amplitude of the induced electric dipole and quadrupole moments of the $R = 250$ nm nanodisk.



Supplementary Figure 3: Comparison of the far-field and the EEL spectra obtained for the disk with 250 nm radius. (a) Sketch of the system. The blue pill shape represents the high-index dielectric nanodisk with dielectric function $\varepsilon = 18$, radius $R = 250$ nm and thickness $d = 55$ nm. The red arrow represents the incident plane wave, polarized along the x -axis. (b) Simulated scattering cross section $\sigma_{\text{scat}}(\omega)$ of the nanodisk. The red dots mark the spectral dip positions. (c) Amplitude of the induced dipole moment in the nanodisk. (d) Amplitude of the induced quadrupole moment in the nanodisk. (e) Same as (a) but the dielectric disk is excited by an electron beam (green arrow) traveling in aloof trajectory close to the disk. (f) Simulated EEL spectrum of the $R = 250$ nm disk. The red dots mark the spectral dip positions. (g) and (h) are the same as (c) and (d), respectively. The simulated spectra in panels (f)-(h) are the same as the ones shown in Fig. 1 of the main text.

Supplementary Note 5. Nanodisk fabrication

In this section we show a detailed schematics (see Supplementary Fig. 4) of the step-by-step procedure we applied to fabricate the WS_2 nanodisks on a thin SiN membrane. The details of each step are described in the Methods section of the main text.



Supplementary Figure 4: Fabrication of the WS_2 disks. Fabrication scheme for the WS_2 nanodisks on a thin SiN TEM membrane. It starts with (1) a dry-transfer of a mechanically-exfoliated WS_2 flake onto a ~ 50 nm thick SiN membrane, and followed by (2) a spin-coating of positive an e-beam resist. In order to pattern WS_2 into various sizes of nanodisks, (3) the deposited e-beam resist was exposed using e-beam lithography with donut-shape patterns. (4) Subsequently, the exposed positive resist was developed for creating dry-etching resist mask. As a final step, we performed (5) dry-etching of the WS_2 flake through a donut-shaped resist mask, and (6) removed the left-over resist with an oxygen-plasma stripping. This process of e-beam exposure of a positive e-beam resist combined with a dry etching, enables the fabrication of donut-shaped etched patterns with isolated nanodisks at their center that have controllable diameters.

Supplementary Note 6. Nanodisk thickness determination

We determine the thickness of the WS₂ nanodisks using the following three methods: (i) electron energy loss spectroscopy, (ii) tilted-view STEM imaging and (iii) the combination of normal-incidence optical reflectivity measurements with a proper transfer-matrix fitting using a permittivity of WS₂ obtained from Ref. [5]. As we discuss in the following, the thickness d extracted from each of these methods is consistent and close to the value of 70 nm.

S6.1 Electron energy loss spectroscopy

In EELS experiments, electrons that pass through the sample are separated into a spectrum based on how much energy they lose while interacting with the sample due to inelastic scattering events. The probability that electrons undergo inelastic scattering while passing through the sample increases as the thickness of the sample is increased. As a consequence, one obtains a stronger EEL signal (associated to inelastic scattering events) when the thickness of the sample is increased. This relationship between inelastic scattering events and sample thickness allows us to measure the nanodisk thickness d by comparing the number of electrons that do not undergo inelastic scattering to the number of electrons that do. The electrons that do not undergo inelastic scattering are contained within the zero-loss peak, which is typically the largest peak in an EEL spectra located at 0 eV. The relative magnitudes of these two components are determined by the inelastic mean-free-path of the electrons in the material and thus it is straightforward to obtain the thickness of the sample in the unit of the number of inelastic mean free paths. From this value, the sample thickness can be estimated by utilizing known experimental and sample parameters with an accuracy of $\pm 10\%$ to 20% , as described in Ref. [13].

In practice, we estimate the thickness of the WS₂ nanodisks using this EELS method that is implemented in the Gatan Microscopy Suite 3 software (Gatan, Inc.). For our experiments, determining the nanodisk thickness using EELS is further complicated due to the SiN substrate below and residual resist material above the nanodisk. When acquiring an EEL spectrum from a nanodisk, the electron beam passes through all three layers (resist material, WS₂ and SiN) and thus the measured EEL signal is a combination of the three contributions. Disentangling the signal coming from each of these three layers is complicated without knowing the precise thickness and chemical composition of each layer. Therefore, to estimate the nanodisk thickness we first assumed that the EEL signal acquired from the nanodisk only comes from WS₂, and the SiN and the residual resist contributions

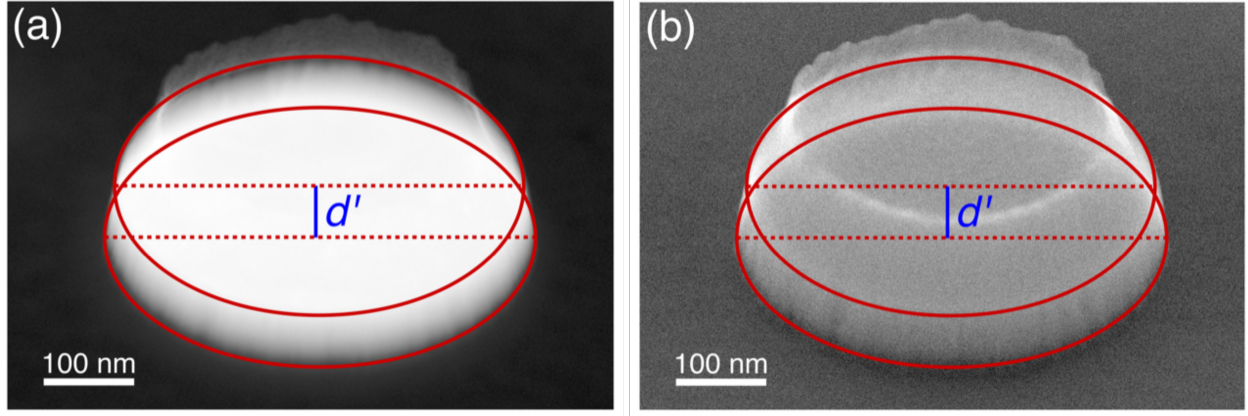
are disregarded in the thickness calculation. The WS₂ nanodisk is the thickest and most strongly inelastically scattering component of the sample, and therefore should be the dominant contributor to the inelastic EEL signal. However, this assumption increases the inaccuracy of the method by an amount that is difficult to quantify. This overestimates the actual nanodisk thickness d because of the added inelastic scattering from the SiN and resist material in the EELS signal. To attempt to correct for part of this overestimation, we measure the thickness of the SiN membrane on an area of the sample where there is no WS₂ or resist material using the same EELS method, and subtract this value from the measured thickness of the nanodisk to get our final estimated nanodisk thickness. This method yields a nanodisk thickness d of 60 nm – 65 nm. Because of the relatively large and unpredictable uncertainty associated with this methodology, we also use two other independent and more straightforward methods.

S6.2 Tilted-view STEM imaging

As a complementary method to the one discussed above, we use STEM images of tilted nanodisks to determine their thicknesses. Specifically, by acquiring a STEM image of a nanodisk that is tilted at an angle θ from the [0001] crystallographic direction of WS₂, one can measure the projected disk thickness d' defined in Supplementary Fig. 4. Once we know d' , we obtain the nanodisk thickness d using the following expression:

$$d = \frac{d'}{\sin \theta}. \quad (15)$$

We acquire simultaneous SE and HAADF images of various nanodisks at a tilt angle of $\theta = 53^\circ$ and we identify the top and bottom nanodisk edges in the images, as shown by the red ellipses in Supplementary Figs. 5(a),(b). The projection of the tilted circular nanodisk edges creates ellipses in the images, which can be used to determine the projected disk thickness d' (distance between the center of the top and bottom ellipse) marked by the blue line in Supplementary Figs. 5(a),(b). From different tilted images, we find a projected disk thickness of $d' = 57$ nm consistently from multiple nanodisk diameters and using Supplementary Eq. 15, we obtain a disk thickness of $d = 71$ nm. The uncertainty in this approach is limited by the accuracy of identifying the ellipse-shaped edges of the nanodisk in the images and then measuring the distance between them. We estimate the error in measuring d' to be about 5 nm, which translates to an uncertainty in d of around $\pm 10\%$.



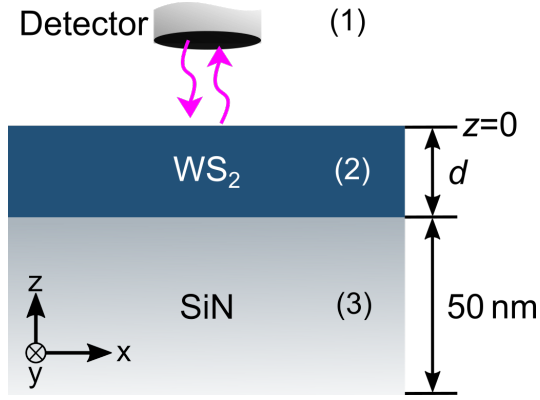
Supplementary Figure 5: Tilted view of a WS₂ disk. Simultaneously acquired (a) HAADF and (b) SE STEM images of a nanodisk with overlays to portray how the projected disk thickness d' is determined. The red ellipses indicate the estimated edges of the top and bottom circular surfaces. The red dashed lines indicate the center line of each ellipse. The blue line indicates the distance between the center of the top and bottom ellipse, that is, d' .

S6.3 Optical reflection spectroscopy

To corroborate the nanodisk thickness obtained with EELS and STEM imaging, we additionally performed a normal-incidence optical reflectivity measurements of the WS₂ flake. Then we employed transfer-matrix simulations to find the value of the thickness d required to reproduce the position of the experimental dips in the reflectivity spectra.

Reflection spectra at normal-incidence from the WS₂ flake were collected using a 20 $\times$ objective (Nikon, NA = 0.45), directed to a fiber-coupled spectrometer and normalized with reflection from a standard dielectric-coated silver mirror. The obtained reflectivity spectra were reproduced and fitted by employing the transfer-matrix simulations using a previously obtained permittivity of WS₂ obtained from Ref. [5]. Then we extracted the thickness d of the WS₂ flake from the fitting. The transfer-matrix formalism allows to obtain the total reflection $r_{\text{total}}^{s(p)}$ and transmission $t_{\text{total}}^{s(p)}$ coefficients of a $s(p)$ -polarized planewave, for any layered system, in terms of the thickness d_i and the dielectric function ε_i of each layer i . For our case, the nanodisk is located on top of a 50 nm SiN membrane (see Supplementary Note 5) and thus the multilayer system is composed of a WS₂ layer of thickness d on top of a 50 nm thick layer made of SiN, as depicted in Supplementary Fig. 6.

To calculate the total reflection r_{total} and reflectivity $\mathcal{R}$, we follow the transfer-matrix procedure described in Ref. [14] which we briefly discuss next. For the multilayer system shown in Supplementary Fig. 6, considering that each layer has dielectric tensor $\hat{\varepsilon}_i = \text{diag}(\varepsilon_{i,x}, \varepsilon_{i,y}, \varepsilon_{i,z})$, and for normal incidence illumination, the Fresnel coefficients $r_{i,i+1}$ and $t_{i,i+1}$ of each interface between



Supplementary Figure 6: Schematics of the multilayer system. Schematic representation of the multilayer system: WS₂ layer of thickness d on top of a 50 nm SiN layer. Labels 1, 2 and 3 refer to Air, WS₂ and SiN, respectively. The half cylinder on top of the layer system represents the detector used to collect the reflected light.

layers i and $i + 1$ are given by

$$r_{i,i+1} = \frac{\sqrt{\varepsilon_{i,x}} - \sqrt{\varepsilon_{i+1,x}}}{\sqrt{\varepsilon_{i,x}} + \sqrt{\varepsilon_{i+1,x}}}, \quad (16)$$

$$t_{i,i+1} = \frac{2\sqrt{\varepsilon_{i,x}}}{\sqrt{\varepsilon_{i,x}} + \sqrt{\varepsilon_{i+1,x}}}. \quad (17)$$

We remove s and p indices from Fresnel coefficients, $r_{i,i+1}$ and $t_{i,i+1}$, since the angle of the incident illumination is zero (normal incidence). The total transfer matrix $\hat{\mathbb{T}}$ can then be obtained as

$$\hat{\mathbb{T}} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} = \hat{\mathbb{M}}_{1,2} \hat{\mathbb{P}}_2 \hat{\mathbb{M}}_{2,3} \hat{\mathbb{P}}_3 \hat{\mathbb{M}}_{3,1}, \quad (18)$$

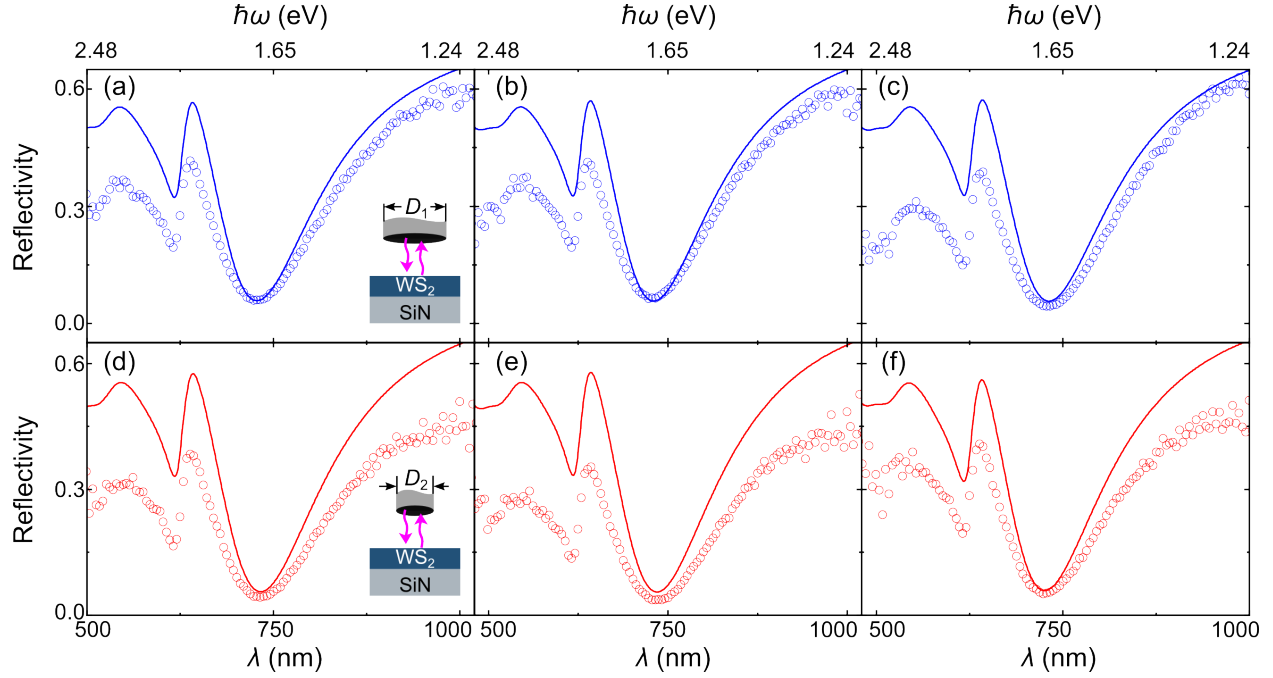
where each matrix $\hat{\mathbb{M}}_{i,i+1}$ and $\hat{\mathbb{P}}_i$ are determined by the following expressions

$$\hat{\mathbb{M}}_{i,i+1} = \frac{1}{t_{i,i+1}} \begin{pmatrix} 1 & r_{i,i+1} \\ r_{i,i+1} & 1 \end{pmatrix}, \quad (19)$$

$$\hat{\mathbb{P}}_i = \begin{pmatrix} e^{-ik_0 d_i \sqrt{\varepsilon_{i,x}}} & 0 \\ 0 & e^{ik_0 d_i \sqrt{\varepsilon_{i,x}}} \end{pmatrix}. \quad (20)$$

Once we obtain total matrix $\hat{\mathbb{T}}$, we use the matrix elements to calculate the reflection coefficient r_{total} and reflectivity $\mathcal{R}$ of the multilayer system as $r_{\text{total}} = \frac{T_{21}}{T_{11}}$ and $\mathcal{R} = |r_{\text{total}}|^2$.

In Supplementary Fig. 7 we show experimental reflectivity spectra (dots) together with the fits



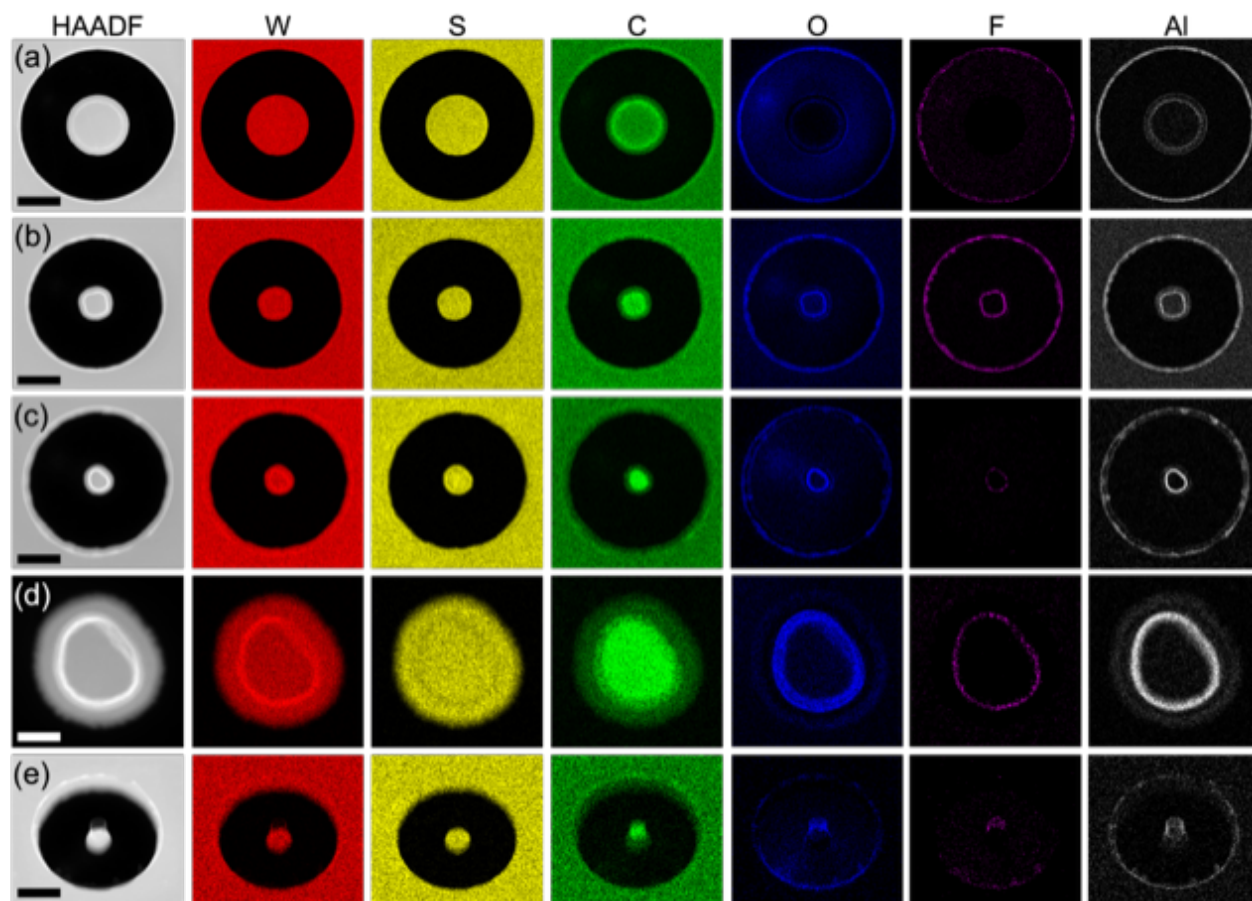
Supplementary Figure 7: Reflectivity spectra obtained from a WS₂ flake on top of the SiN membrane of 50 nm thickness. (a)-(c) Reflectivity spectra (blue dots) collected at three different positions in the WS₂ flake. The diameter of the detection spots is $\sim 4 \mu\text{m}$. (d)-(f) Reflectivity spectra (red dots) collected at three different positions in the WS₂ flake. The diameter of the detection spots is $\sim 2 \mu\text{m}$. The reflectivity spectra are normalized to that obtained on a Ag reference layer of 200 nm. In all panels, the dots are the experimental values of the reflectivity $\mathcal{R}$ and the solid lines are the simulations obtained with transfer-matrix formalism (Supplementary Eqs. 16-20). Insets in panels (a) and (d) illustrate the multilayer system and the detectors with different spot diameters $D_1 > D_2$. Fit values obtained for the WS₂ layer thickness, d , are: (a) 74.5 nm, (b) 74.1 nm, (c) 74.7 nm, (d) 75.0 nm, (e) 75.3 nm and (f) 73.7 nm.

(solid lines) obtained from the transfer-matrix formalism (Supplementary Eqs. 16-20). From the 9 fittings we obtain an average value of 74.5 nm for the thickness of the WS₂ which is in good agreement to the thicknesses obtained with EELS and STEM imaging.

Supplementary Note 7. Chemical composition from energy dispersive X-ray spectroscopy (EDS)

We measure the spatial variations in chemical composition of the fabricated nanodisks using energy dispersive X-ray spectroscopy (EDS) STEM spectrum imaging. For these measurements, we raster scan a 1-2 Angstrom electron probe across the sample. For every electron probe position in the image (pixel), we collect an X-ray spectrum. This process creates a three dimensional data cube where the x and y dimensions of the data cube are the sample x and y coordinates, and the z dimension is the X-ray spectrum. We also collect the HAADF STEM image simultaneously to view the sample structure. From the EDS data cube, maps are generated which, for each pixel, integrate the EDS signal within a chosen characteristic X-ray peak. This enables the direct imaging of the nanodisk structure while simultaneously mapping the spatial variations of its constituent elements. We mapped the spatial variations of the W L, S K, C K, O K, F K, and Al L X-rays for three nanodisks with different diameters. All nanodisks were mapped at lower magnification to visualize the complete etched donut-shaped structure. The smallest nanodisk (with 88 nm radius) was also mapped both, at higher magnification to visualize smaller details around the nanodisk structure, and at a 30° tilt angle from the [0001] crystallographic direction of WS₂ to reveal information about the relative heights of the X-ray signals.

In Supplementary Fig. 8 we show the HAADF STEM images and extracted EDS maps of the $R = 295$ nm, $R = 110$ nm and $R = 88$ nm disks. We can observe that the W and the S signals come from the nanodisks at the center of the etched donut-shaped structure (see red and yellow panels). Additionally, the tungsten and sulfur X-ray signals obtained from the nanodisks are confined below the C, O, F, and Al X-ray signals (see panel (e)). The C, O, F, and Al X-ray signals come from the residual resist material (located above the nanodisk) that remains after the nanofabrication process (compare the first two columns in Supplementary Fig. 8(e) with the last four columns). From this analysis we can conclude that the residual resist material is not uniform in morphology or composition, and appears to have a tapered-edge structure and an outer cylindrical-shaped shell of different composition (higher W, O, F, and Al content) that is primarily filled with C.



Supplementary Figure 8: Chemical composition of the nanodisks. STEM-EDS composition data from (a) the $R = 295$ nm disk, (b) the $R = 110$ nm and (c)-(e) the $R = 88$ nm disk. (c) and (d) show a lower and higher magnification of the nanodisk, respectively. (e) Shows a 30° tilted view of the sample revealing information about the relative heights where the X-rays originate. The first column (gray) is the HAADF STEM image that is acquired simultaneously to the EDS data. The second (red), third (yellow), fourth (green), fifth (blue), sixth (magenta), and seventh (white) columns are the extracted EDS maps using the integrated signal from the W L, S K, C K, O K, F K, and Al K X-rays, respectively. The scale bars in panels (a)-(e) are $0.4\ \mu\text{m}$, $0.26\ \mu\text{m}$, $0.16\ \mu\text{m}$, $0.04\ \mu\text{m}$, and $0.2\ \mu\text{m}$, respectively, and are the same for all the EDS maps in the row.

Supplementary Note 8. Analysis of the anapole-exciton coupled system

In order to analyse the experimental and simulated EEL spectra of the WS_2 nanodisks (Figs. 3(a) and (d) in the main text), we reproduce the spectra using temporal coupled mode theory (TCMT). This theoretical formalism has been widely used to model the scattered and absorbed power of an object when it is illuminated with a monochromatic plane wave [15, 16, 17]. We thus develop here an extended TCMT assuming that the incident illumination is the one produced by a fast electron beam. We then apply this formalism to reproduce the EEL spectra of the WS_2 nanodisks and obtain the spectral dips that result from the coupling between the first ED anapole and the A-exciton resonance. We model the anapole-exciton system via three coupled modes excited in the disk by the probing electron. At the end of this section we show the results obtained with TCMT, together with the parameters extracted from it.

S8.1 Details of the incident electric field and the scattering channels

One of the key ingredients in TCMT is the expansion of the incident field over a set of appropriate scattering channels. In this subsection we thus show that the electric field, $\mathbf{E}_{\text{bg}}(\mathbf{r}; \omega)$, produced by a fast electron beam can be decomposed as a sum of incoming and outgoing spherical waves propagating toward and outward the electron beam, respectively. Each spherical wave defines a scattering channel as we discuss next.

We consider an electron traveling in vacuum along the z -axis, with impact parameter b and velocity v (see Fig. 1(a) of the main text). This fast electron produces a broadband electromagnetic field that can be expressed on the basis of vector spherical harmonics as follows [18]:

$$\mathbf{E}_{\text{bg}}(\mathbf{r}; \omega) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m}^{\text{TM}}(\omega) \mathbf{E}_{\ell m}^{\text{TM}}(\mathbf{r}; \omega) + a_{\ell m}^{\text{TE}}(\omega) \mathbf{E}_{\ell m}^{\text{TE}}(\mathbf{r}; \omega), \quad (21)$$

where

$$a_{\ell m}^{\text{TM}}(\omega) = -i \frac{ev}{\varepsilon_0} \frac{m}{\sqrt{\ell(\ell+1)}} \frac{\omega}{c^2} \phi_{\ell m}^{\text{TM}}(\omega), \quad (22)$$

$$a_{\ell m}^{\text{TE}}(\omega) = \frac{ev}{c\varepsilon_0} \frac{1}{\sqrt{\ell(\ell+1)}} \phi_{\ell m}^{\text{TE}}(\omega), \quad (23)$$

and the overlap integrals $\phi_{\ell m}^{\text{TM}}(\omega)$ and $\phi_{\ell m}^{\text{TE}}(\omega)$ are determined as:

$$\phi_{\ell m}^{\text{TM}}(\omega) = \frac{4\pi k_0}{v} i \int_{-\infty}^{\infty} dz e^{i\omega z/v} h_{\ell}^{(1)}(k_0 r_e) Y_{\ell m}^*(\theta_e, \phi_e), \quad (24)$$

$$\phi_{\ell m}^{\text{TE}}(\omega) = c_{\ell m} e^{-i\phi_e} \left(\frac{\partial}{\partial b} - \frac{i}{b} \frac{\partial}{\partial \phi_e} \right) \phi_{\ell m}^{\text{TM}}(\omega) - d_{\ell m} e^{i\phi_e} \left(\frac{\partial}{\partial b} + \frac{i}{b} \frac{\partial}{\partial \phi_e} \right) \phi_{\ell m}^{\text{TM}}(\omega). \quad (25)$$

Here $h_{\ell}^{(1)}(x)$ is the spherical Hankel function of the first kind, $Y_{\ell m}(\theta, \phi)$ is the scalar spherical harmonics, $c_{\ell m} = \sqrt{\ell(\ell+1) - m(m-1)}$, $d_{\ell m} = \sqrt{\ell(\ell+1) - m(m+1)}$ and the variables (r_e, ϕ_e, θ_e) are the spherical coordinates of the electron beam trajectory $\mathbf{r}_e(t) = (x_e = b, y_e = 0, z = vt)$. In Supplementary Eq. 21, the fields vectors $\mathbf{E}_{\ell m}^{\text{TE}}(\mathbf{r}; \omega)$ and $\mathbf{E}_{\ell m}^{\text{TM}}(\mathbf{r}; \omega)$ are defined according to the standard vector spherical harmonics relations:

$$\mathbf{E}_{\ell m}^{\text{TM}}(\mathbf{r}; \omega) = j_{\ell}(k_0 r) \mathbf{X}_{\ell m}(\theta, \phi), \quad (26)$$

$$\mathbf{E}_{\ell m}^{\text{TE}}(\mathbf{r}; \omega) = -\frac{i}{k_0} \nabla \times [j_{\ell}(k_0 r) \mathbf{X}_{\ell m}(\theta, \phi)], \quad (27)$$

$$\mathbf{X}_{\ell m}(\theta, \phi) = \frac{1}{\sqrt{\ell(\ell+1)}} \mathbf{L} [Y_{\ell m}(\theta, \phi)], \quad (28)$$

where $\mathbf{L} = -i(\mathbf{r} \times \nabla)$ is the angular momentum operator. By expressing the spherical Bessel function $j_{\ell}(k_0 r)$ as

$$j_{\ell}(k_0 r) = \frac{1}{2} [h_{\ell}^{(1)}(k_0 r) + h_{\ell}^{(2)}(k_0 r)], \quad (29)$$

with $h_{\ell}^{(2)}(x)$ being the spherical Hankel function of the second kind, one finds that the fields vectors $\mathbf{E}_{\ell m}^{\text{TE}}(\mathbf{r}; \omega)$ and $\mathbf{E}_{\ell m}^{\text{TM}}(\mathbf{r}; \omega)$ (Supplementary Eqs. 26 and 27) can be written as the following equally weighted superposition of incoming (+) and outgoing (-) spherical waves:

$$\mathbf{E}_{\ell m}^{\text{TM}}(\mathbf{r}; \omega) = \underbrace{\frac{h_{\ell}^{(1)}(k_0 r)}{2} \mathbf{X}_{\ell m}(\theta, \phi)}_{\mathbf{E}_{\text{TM}, \ell m}^+} + \underbrace{\frac{h_{\ell}^{(2)}(k_0 r)}{2} \mathbf{X}_{\ell m}(\theta, \phi)}_{\mathbf{E}_{\text{TM}, \ell m}^-}, \quad (30)$$

$$\mathbf{E}_{\ell m}^{\text{TE}}(\mathbf{r}; \omega) = \underbrace{-\frac{i}{2k_0} \nabla \times [h_{\ell}^{(1)}(k_0 r) \mathbf{X}_{\ell m}(\theta, \phi)]}_{\mathbf{E}_{\text{TE}, \ell m}^+} - \underbrace{\frac{i}{2k_0} \nabla \times [h_{\ell}^{(2)}(k_0 r) \mathbf{X}_{\ell m}(\theta, \phi)]}_{\mathbf{E}_{\text{TE}, \ell m}^-}. \quad (31)$$

Substituting Supplementary Eqs. 30 and 31 into Supplementary Eq. 21, one further finds that the electric field $\mathbf{E}_{\text{bg}}(\mathbf{r}; \omega)$ can be described as incoming and outgoing waves (propagating toward and

outward the electron beam) coupled to different scattering channels:

$$\begin{aligned} \mathbf{E}_{\text{bg}}(\mathbf{r}; \omega) = & \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m}^{\text{TM}}(\omega) [\mathbf{E}_{\text{TM}, \ell m}^{+}(\mathbf{r}; \omega) + \mathbf{E}_{\text{TM}, \ell m}^{-}(\mathbf{r}; \omega)] + \\ & a_{\ell m}^{\text{TE}}(\omega) [\mathbf{E}_{\text{TE}, \ell m}^{+}(\mathbf{r}; \omega) + \mathbf{E}_{\text{TE}, \ell m}^{-}(\mathbf{r}; \omega)]. \end{aligned} \quad (32)$$

Here, each TM or TE polarization together with the pair $\{\ell, m\}$ correspond to a distinct channel labeled as $\{q, \ell, m\}$ with q representing the polarization of the field. The power carried by the outgoing waves can be obtained by summing the radiated power along each scattering channel:

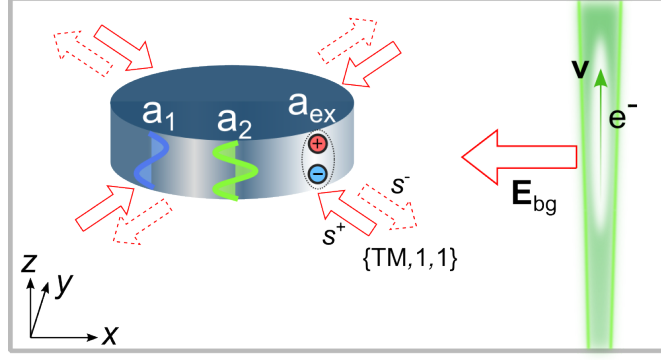
$$P_{\text{bg}}(\omega) = \frac{1}{8ck_0^2} \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \left(|a_{\ell m}^{\text{TM}}(\omega)|^2 + |a_{\ell m}^{\text{TE}}(\omega)|^2 \right). \quad (33)$$

The incident field decomposed in different scattering channels provides a convenient tool to calculate the scattered power of an object interacting with a fast electron beam. A model of the optical anapole (strong suppression of the scattered power) based on these scattering channels is discussed in the following section.

S8.2 Anapole-exciton coupled system within TCMT

We next consider the case of the nanodisk being illuminated by the fast electron beam. The scattering of the disk, coming from the response to the electromagnetic field of the electron beam, couples to the infinite set of scattering channels $\{q, \ell, m\}$ determined by the incident field, as sketched in Supplementary Fig. 9. The optical anapole characterized by complete suppression of the scattered field by the disk is well described by TCMT and particularly, it can be modeled as a single-channel scattering problem as we discuss next.

The first ED anapole state originates from the destructive interference of the radiation produced by the Cartesian electric and toroidal dipoles, both of which radiate in the TM and $\ell = 1$ channel [9]. This constrains the first ED anapole to occur in three linearly-independent scattering channels: $\{\text{TM}, 1, -1\}$, $\{\text{TM}, 1, 0\}$ and $\{\text{TM}, 1, 1\}$. The Cartesian electric and toroidal dipoles having the fields of a linearly polarized dipole ($\mathbf{X}_{11} + i\mathbf{X}_{1-1}$) further constrains the first ED anapole to the $m = \pm 1$ channels which have identical scattering coefficients. To model the first ED anapole state excited by the fast electron beam, we thus assume that the scattered field of the nanodisk couples dominantly to the single scattering channel $\{\text{TM}, 1, 1\}$. We then approximate the scattered field of



Supplementary Figure 9: Schematics of the fast electron beam illuminating the nanodisk. Solid arrows represent incoming waves, s^+ , toward the disk whereas dashed arrows represent outgoing waves, s^- , outward the disk. In the schematics the two disk modes with amplitudes a_1 , a_2 interact with the exciton with amplitude a_3 . The nanodisk scatters along the channel $\{\text{TM}, 1, 1\}$ to produce the first ED anapole state.

the disk as follows

$$\mathbf{E}_{\text{scat}}(\mathbf{r}; \omega) \approx b_{11}^{\text{TM}}(\omega) \mathbf{E}_{\text{TM},11}^-(\mathbf{r}; \omega), \quad (34)$$

with $b_{11}^{\text{TM}}(\omega)$ being the amplitude of the outgoing wave in the $\{\text{TM}, 1, 1\}$ channel. We further assume that the first ED anapole is the result of the far-field coupling of two modes of the disk radiating in the channel $\{\text{TM}, 1, 1\}$. These modes have amplitudes $a_1(t)$, $a_2(t)$, resonant frequencies ω_1 , ω_2 and total decay rates γ_1 , γ_2 , respectively. Thus, the anapole-exciton coupled system can be modeled by considering the two disk modes coupled to a third nonradiating mode representing the exciton resonance with amplitude $a_{\text{ex}}(t)$, resonant frequency ω_{ex} and absorptive decay rate γ_{ex} .

Within TCMT, the dynamics of the anapole-exciton coupled system is determined by the following equation:

$$\frac{d\mathbf{A}_3(t)}{dt} = -i\hat{\mathbb{H}}_3 \mathbf{A}_3(t) + \mathbf{K}_3 s^+(t), \quad (35)$$

where $\mathbf{A}_3(t) = (a_1(t), a_2(t), a_{\text{ex}}(t))^{\top}$ is the vector with the amplitudes of the three resonant modes of the disk (symbol $\top$ denotes transpose of a matrix), $s^+(t)$ describes the incoming field (in our case the one produced by the electron beam) in the scattering channel $\{\text{TM}, 1, 1\}$, $\mathbf{K}_3 = (\kappa_1, \kappa_2, \kappa_{\text{ex}})^{\top}$ is the vector of radiative coupling coefficients to the incoming field, and the effective 3×3 Hamiltonian $\hat{\mathbb{H}}_3$ is given by

$$\hat{\mathbb{H}}_3 = \underbrace{\begin{pmatrix} \omega_1 & 0 & 0 \\ 0 & \omega_2 & 0 \\ 0 & 0 & \omega_{\text{ex}} \end{pmatrix}}_{\hat{\Omega}} - i \underbrace{\begin{pmatrix} \gamma_1^{\text{abs}}/2 & 0 & 0 \\ 0 & \gamma_2^{\text{abs}}/2 & 0 \\ 0 & 0 & \gamma_{\text{ex}}/2 \end{pmatrix}}_{\hat{\Gamma}_3^{\text{abs}}} - i \underbrace{\begin{pmatrix} \gamma_1^{\text{rad}}/2 & \gamma_{12} & 0 \\ \gamma_{12} & \gamma_2^{\text{rad}}/2 & 0 \\ 0 & 0 & 0 \end{pmatrix}}_{\hat{\Gamma}_3^{\text{rad}}} \quad (36)$$

$$+ \underbrace{\begin{pmatrix} 0 & 0 & g_{1\text{ex}} \\ 0 & 0 & g_{2\text{ex}} \\ g_{1\text{ex}} & g_{2\text{ex}} & 0 \end{pmatrix}}_{\hat{\mathbb{G}}_3}.$$

From Supplementary Eq. 36 one can see that the matrix $\hat{\mathbb{H}}_3$ has the information of the resonant frequencies $\omega_1, \omega_2, \omega_{\text{ex}}$, the absorptive decay rates $\gamma_1^{\text{abs}}, \gamma_2^{\text{abs}}, \gamma_{\text{ex}}$ and the radiative decay rates, $\gamma_1^{\text{rad}}, \gamma_2^{\text{rad}}$ of each mode. The total decay rate is the sum of the absorptive and radiative contributions ($\gamma = \gamma^{\text{abs}} + \gamma^{\text{rad}}$) and determines the linewidth (full width at half-maximum) of the resonance. Matrix elements $\gamma_{12} = \sqrt{\gamma_1^{\text{rad}}\gamma_2^{\text{rad}}/4}$ represent the far-field interference between the two nanodisk modes [19] and $g_{1\text{ex}}, g_{2\text{ex}}$ are the coupling strengths between the optical modes and the exciton.

The scattered field of the disk, together with the incident field, produce the following outgoing wave

$$s^-(t) = \mathbf{D}_3^\top \mathbf{A}_3(t) + C s^+(t), \quad (37)$$

where $\mathbf{D}_3 = \mathbf{K}_3 = (\sqrt{\gamma_1^{\text{rad}}}, \sqrt{\gamma_2^{\text{rad}}}, 0)^\top$ and C is a constant which includes all non-resonant scattering processes in the channel [17]. In our case we assumed $C = 1$. Through a time-to-frequency Fourier transform of Supplementary Eqs. 35 and 37, we find the following steady-state solution

$$\mathbf{A}_3(\omega) = \left[i(\hat{\mathbb{H}}_3 - \omega \hat{\mathbb{I}}) \right]^{-1} \mathbf{D}_3 s^+(\omega), \quad (38)$$

where the outgoing wave $s^-(\omega)$ can be written as

$$s^-(\omega) = \underbrace{\left[\mathbf{D}_3^\top \left[i(\hat{\mathbb{H}}_3 - \omega \hat{\mathbb{I}}) \right]^{-1} \mathbf{D}_3 + C \right]}_{\hat{\mathbb{S}}(\omega)} s^+(\omega), \quad (39)$$

with $\hat{\mathbb{A}}_3(\omega) = \mathcal{F} [\hat{\mathbb{A}}_3(t)]$ and $s^\pm(\omega) = \mathcal{F} [s^\pm(t)]$. Here, $\mathcal{F}$ is the time-to-frequency Fourier transform. Supplementary Eq. 39 relates the incoming wave $s^+(\omega)$ and the outgoing wave $s^-(\omega)$ via the scattering matrix $\hat{\mathbb{S}}(\omega)$, which, under our assumption of a single scattering channel, reduces to a single matrix element. Furthermore, the amplitude of the outgoing wave can be written as a sum of the contributions of the scattered and the incident fields, $s^-(\omega) = a_{11}^{\text{TM}}(\omega) + b_{11}^{\text{TM}}(\omega)$. Substituting this last relation into Supplementary Eq. 39, one finds that the amplitude of the scattered field is

$$b_{11}^{\text{TM}}(\omega) = a_{11}^{\text{TM}}(\omega) \left[\hat{\mathbb{S}}(\omega) - 1 \right], \quad (40)$$

where the incoming radiation is $s^+(\omega) = a_{11}^{\text{TM}}(\omega)$.

According to Supplementary Eq. 33, the radiated power by the nanodisk can be calculated as

$$P_{\text{scat}}(\omega) = |b_{11}^{\text{TM}}(\omega)|^2 = |\hat{\mathbb{S}}(\omega) - 1|^2 |s^+(\omega)|^2, \quad (41)$$

whereas the total absorbed power can be obtained as the sum of the absorbed power by each mode,

$$P_{\text{abs}}(\omega) = \gamma_1^{\text{abs}} |a_1(\omega)|^2 + \gamma_2^{\text{abs}} |a_2(\omega)|^2 + \gamma_{\text{ex}} |a_{\text{ex}}(\omega)|^2 = 2 \mathbf{A}_3^\dagger(\omega) \Gamma_3^{\text{abs}} \mathbf{A}_3(\omega), \quad (42)$$

where $\hat{\mathbb{M}}^\dagger$ denotes conjugate transpose of matrix $\hat{\mathbb{M}}$.

Finally, to calculate the electron energy losses within the TCMT formalism, we assume that the energy lost by the electron beam when interacting with the nanodisk is equal to the energy dissipated by the nanoresonator either via heating (ohmic losses) or radiation into the far field [20]. Thus, we assume that the electron energy loss probability $\Gamma(\omega)$ is proportional to the sum of the scattered and absorbed power (extinction) by the nanodisk excited by the probing electron

$$\Gamma(\omega) \propto P_{\text{scat}}(\omega) + P_{\text{abs}}(\omega) = |\hat{\mathbb{S}}(\omega) - 1|^2 |s^+(\omega)|^2 + 2 \mathbf{A}_3^\dagger(\omega) \Gamma_3^{\text{abs}} \mathbf{A}_3(\omega), \quad (43)$$

where the vector $\mathbf{A}_3(\omega)$ and the scattering matrix (a scalar in our case) $\hat{\mathbb{S}}(\omega)$ are determined by Supplementary Eqs. 38 and 39, respectively.

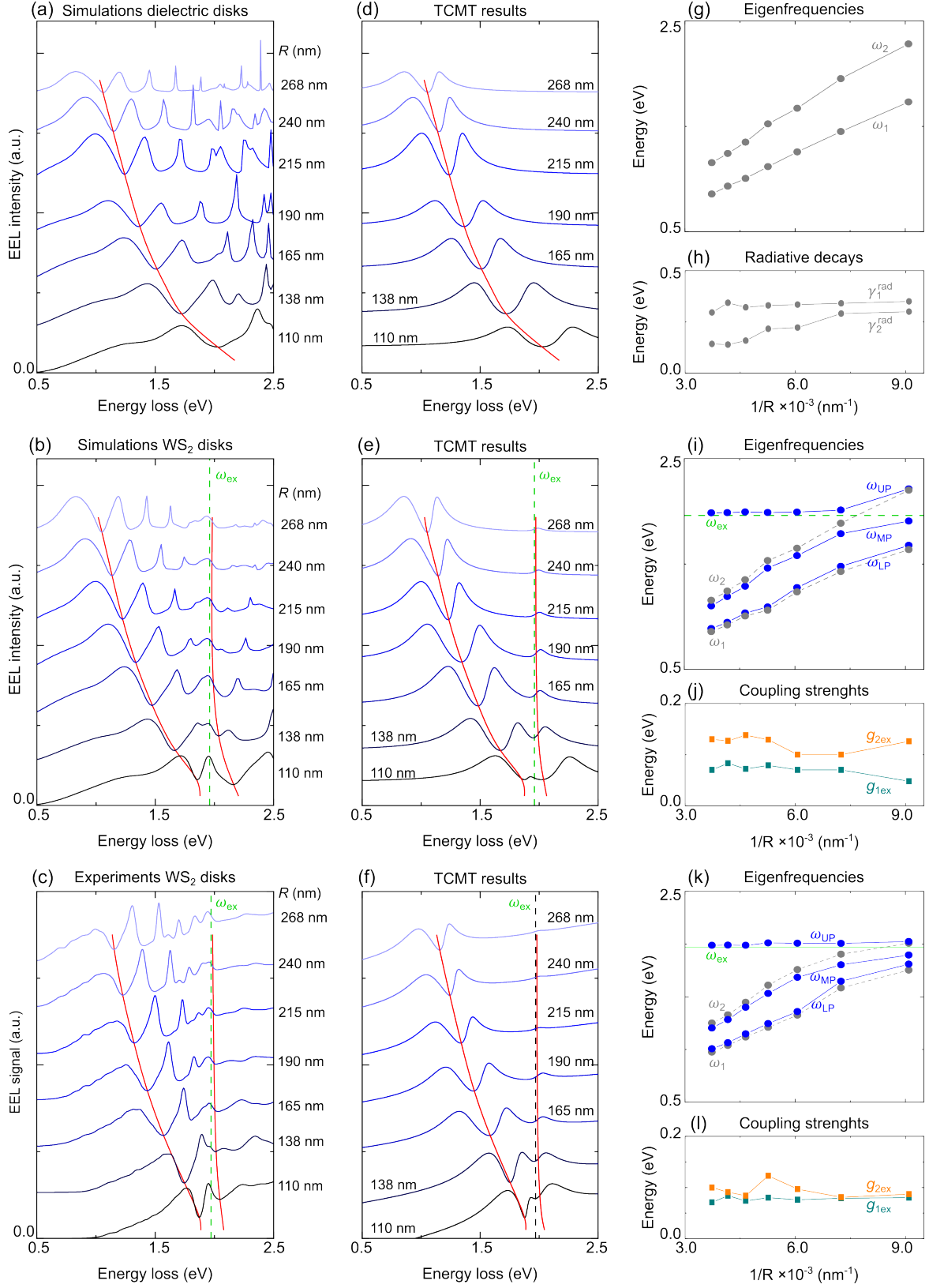
S8.3 Reproducing the EEL spectra with TCMT

To extract the parameters ω_1 , ω_2 , γ_1^{rad} , γ_2^{rad} , $g_{1\text{ex}}$ and $g_{2\text{ex}}$, we reproduce the EEL spectra shown in Figs. 3(a) and 3(d) of the main text. To that end, we apply the following procedure:

1. We normalize the simulated and experimental EEL spectra to the peak maximum in the energy range between 0.5 eV and 2.5 eV. Following the description discussed in the previous section, we assume that the energy losses of the disks are determined by Supplementary Eq. 43:

$$\Gamma(\omega) = \alpha \frac{P_{\text{scat}}(\omega) + P_{\text{abs}}(\omega)}{|s^+(\omega)|^2} + \beta, \quad (44)$$

where α is a proportionality constant and β is a parameters that accounts for the background in the EEL signal.



Supplementary Figure 10: Reproducing the EEL spectra with TCMT. Electron energy loss spectra obtained from (a) the COMSOL simulations of the high-index dielectric disks, (b) the COMSOL simulations of the WS₂ disks and (c) the experiments. Panels (d)-(f) show the TCMT results described in this section. Panels (g)-(l) show the parameters extracted from the TCMT results as a function of the inverse radius, $1/R$.

2. We reproduce using TCMT the first dip in the EEL spectra of the high-index dielectric ($\varepsilon = 18$) disks with radius $R = 268, 240, 215, 190, 165, 138, 110$ nm and thickness $d = 70$ nm (see Supplementary Figs. 10(a) and 10(d)). The red lines in Supplementary Figs. 10(a) and 10(d) are guides to the eye that indicate the position of the first dip in the EEL spectra. From the TCMT results (Supplementary Fig. 10(d)), we extract the eigenfrequencies ω_1, ω_2 and radiative decay rates $\gamma_1^{\text{rad}}, \gamma_2^{\text{rad}}$ for each disk. The extracted parameter as a function of the inverse radius $1/R$ are shown in Supplementary Figs. 10(g) and 10(h). For this step, we set $g_{1\text{ex}} = g_{2\text{ex}} = 0$ and we assume that β is constant. Note in Supplementary Figs. 10(g) and 10(h) the typical linear dispersion of the Mie-resonances sustained by high-index dielectric nanodisks [21].
3. We then reproduce with TCMT the simulated EEL spectra of the WS_2 disks (see Supplementary Figs. 10(b) and 10(e)) and we extract the coupling strengths $g_{1\text{ex}}$ and $g_{2\text{ex}}$. The red lines in Supplementary Figs. 10(b) and 10(e) are guides to the eye that indicate the position of the anapole-exciton-hybrids in the EEL spectra. In the TCMT results, the maximum value of the coupling strengths $g_{1\text{ex}}$ and $g_{2\text{ex}}$ were limited to that of the bulk polariton [22, 14] which in our case is around 0.14 eV. In this step, $\gamma_{\text{ex}} = 0.09$ eV and $\omega_{\text{ex}} = 1.96$ eV were fixed according to the WS_2 dielectric function (Supplementary Eq. 3) and we limit $\omega_1, \omega_2, \gamma_1^{\text{rad}}$ and γ_2^{rad} within a few meV from its initial value obtained in step 2. From the parameters, we also obtain the eigenfrequencies $\omega_{\text{LP}}, \omega_{\text{MP}}$ and ω_{UP} of the hybrid modes by calculating the eigenvalues of the effective 3×3 Hamiltonian $\hat{\mathbb{H}}_3$ (Supplementary Eq. 36). The parameter are shown in Supplementary Figs. 10(i) and 10(j).
4. We repeat step 3 but for the experimental EEL spectra (Supplementary Fig. 10(c)). In this step, $\gamma_{\text{ex}} = 0.11$ eV and $\omega_{\text{ex}} = 1.97$ eV were fixed according to the values obtained when reproducing with TCMT the experimental EEL spectra of an unpatterned WS_2 flake (green lines shown in Figs. 3(b) and 3(c) of the main text). We assume that the background of the EEL signal, β , is a linear function of energy. The TCMT results together with the extracted parameters are shown in Supplementary Figs. 10(f)-(l).

S8.4 Parameters obtained from the TCMT results

Supplementary Tables 1, 2 and 3 show all parameters obtained from the TCMT results shown in Supplementary Figs. 10(d), 10(e) and 10(f).

R (nm)	ω_1 (eV)	γ_1^{rad} (eV)	ω_2 (eV)	γ_2^{rad} (eV)
110	1.73	0.35	2.28	0.30
138	1.45	0.34	1.95	0.29
165	1.26	0.33	1.67	0.22
190	1.12	0.33	1.52	0.22
215	1.01	0.32	1.35	0.16
240	0.93	0.34	1.24	0.14
268	0.86	0.30	1.15	0.14

Supplementary Table 1: Parameters obtained by reproducing with TCMT the EEL spectra of the high-index dielectric disks shown in Supplementary Fig. 10(a).

R (nm)	ω_1 (eV)	γ_1^{rad} (eV)	$g_{1\text{ex}}$ (meV)	ω_2 (eV)	γ_2^{rad} (eV)	$g_{2\text{ex}}$ (meV)	ω_{LP} (eV)	ω_{MP} (eV)	ω_{UP} (eV)
110	1.64	0.36	48	2.20	0.30	126	1.68	1.91	2.21
138	1.43	0.34	70	1.89	0.29	100	1.48	1.79	2.01
165	1.24	0.33	70	1.65	0.22	100	1.27	1.58	1.99
190	1.06	0.33	79	1.53	0.22	129	1.09	1.46	1.99
215	1.01	0.32	72	1.35	0.16	138	1.03	1.29	1.99
240	0.92	0.34	83	1.24	0.14	127	0.94	1.19	1.99
268	0.86	0.30	70	1.15	0.14	130	0.89	1.10	1.98

Supplementary Table 2: Parameters obtained by reproducing with TCMT the simulated EEL spectra of the WS₂ disks shown in Fig. 3(d) of the main text and Supplementary Fig. 10(b).

R (nm)	ω_1 (eV)	γ_1^{rad} (eV)	$g_{1\text{ex}}$ (meV)	ω_2 (eV)	γ_2^{rad} (eV)	$g_{2\text{ex}}$ (meV)	ω_{LP} (eV)	ω_{MP} (eV)	ω_{UP} (eV)
110	1.75	0.36	80	2.01	0.30	87	1.81	1.90	2.02
138	1.58	0.34	79	1.90	0.29	81	1.65	1.80	2.01
165	1.33	0.33	76	1.76	0.22	97	1.36	1.68	2.01
190	1.21	0.33	80	1.61	0.22	123	1.25	1.53	2.01
215	1.12	0.32	74	1.45	0.16	84	1.15	1.40	1.99
240	1.04	0.34	84	1.33	0.14	91	1.06	1.28	1.99
268	0.97	0.30	71	1.25	0.14	100	1.01	1.20	1.99

Supplementary Table 3: Parameters obtained by reproducing with TCMT the experimental EEL spectra of the WS₂ disks shown in Fig. 3(a) of the main text and Supplementary Fig. 10(c).

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