

# Supplementary Information for

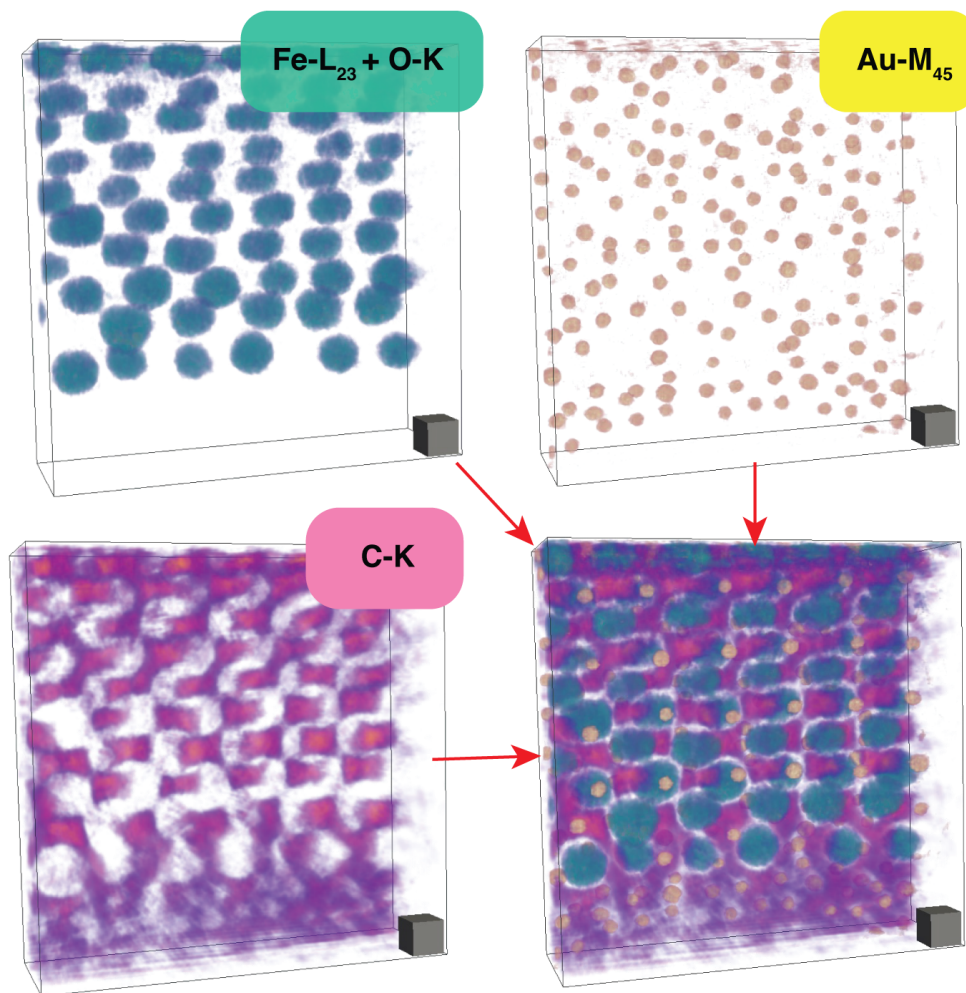
## Imaging 3D Chemistry at 1 nm Resolution with Fused Multi-Modal Electron Tomography

Jonathan Schwartz, Zichao Wendy Di, Yi Jiang, Jason Manassa, Jacob Pietryga, Yiwen Qian, Min Gee Cho, Jonathan L. Rowell, Huihuo Zheng, Richard D. Robinson, Junsi Gu, Alexey Kirilin, Steve Rozeveld, Peter Ercius, Jeffrey A. Fessler, Ting Xu, Mary Scott, and Robert Hovden

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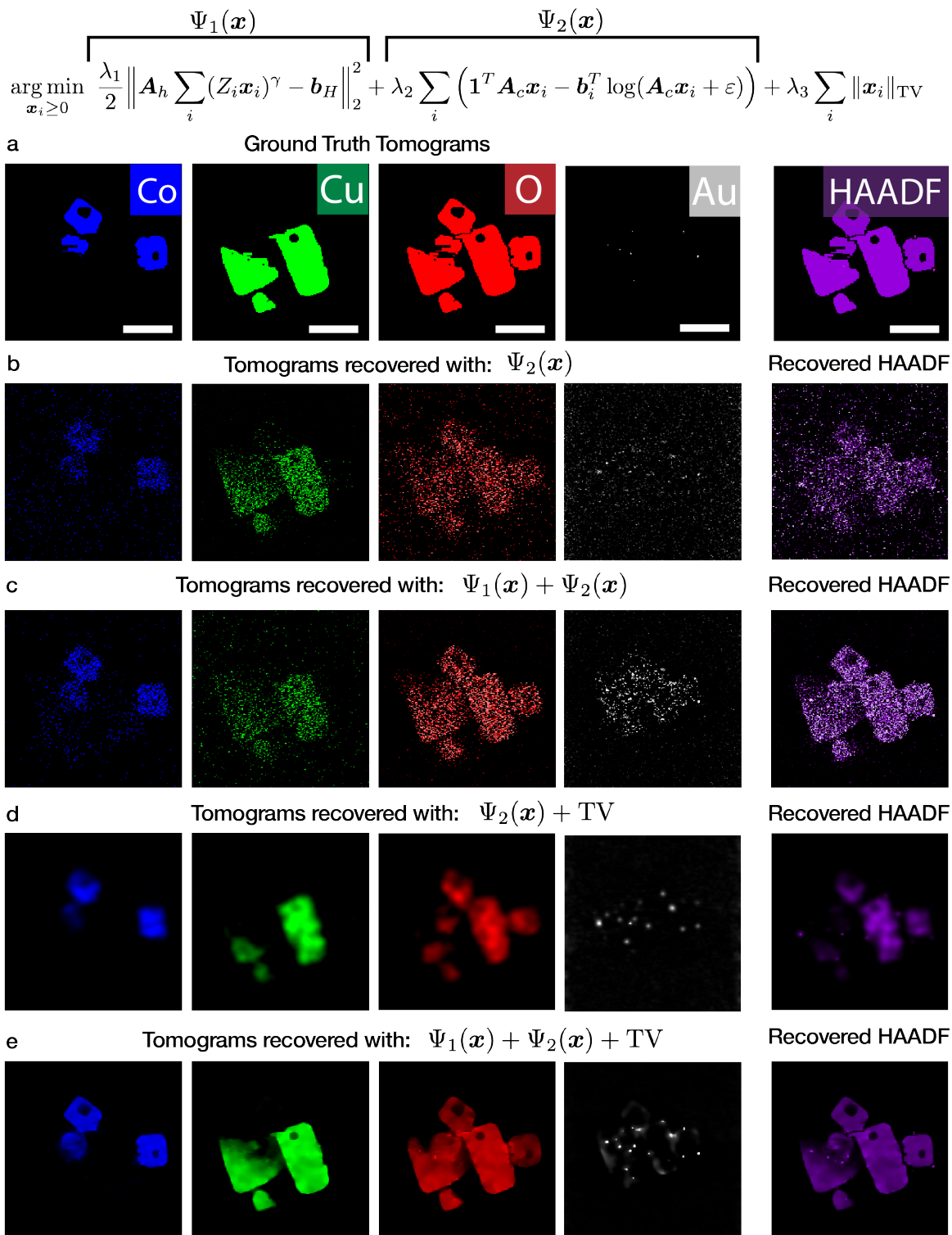
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## S1 Multi-Modal Reconstruction of Au-Fe<sub>3</sub>O<sub>4</sub> Nanoparticles Inside a Carbon Support



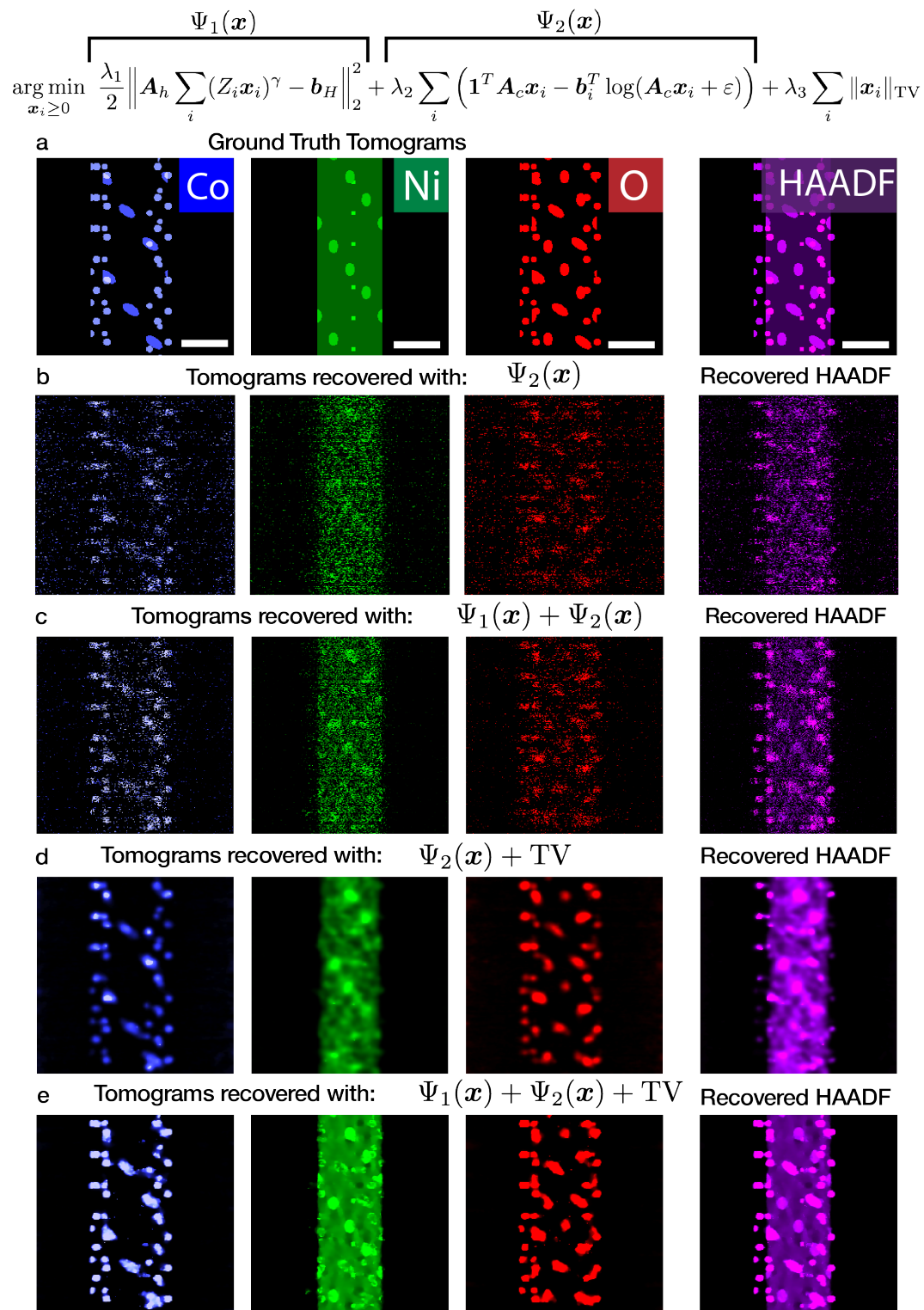
**Fig.S1** | The complete reconstruction of the Au-Fe<sub>3</sub>O<sub>4</sub> nanoparticle superlattice inside the carbon matrix (highlighted in purple). Scale cube, 10 nm<sup>3</sup>.

## S2 Consequence of Reconstructing CuO/CoO Simulation with Individual Components in Cost Function



**Fig.S2** | **a** Synthetic CuO/CoO nanocubes ground truth EELS maps and simultaneous HAADF reconstruction. **b** Slice of the reconstructed nanocubes dataset with only  $\Psi_2$  (data consistency),  $\lambda_1, \lambda_3=0$ . The recovered EELS maps and HAADF reconstruction are noisy. **c** Slice of the reconstructed multi-modal nanocubes dataset with  $\Psi_1$  and  $\Psi_2$  (model + data-consistency),  $\lambda_3=0$ . The EELS maps are still noisy but show improvement with the extra  $\Psi_1$  term. **d** Slice of the reconstructed Multi-Modal nanocubes dataset with  $\Psi_2 + \text{TV}$ ,  $\lambda_1=0$ . The expression  $\Psi_2 + \text{TV}$  is equivalent to a denoising problem; thus the resulting reconstructions produce common staircase artifacts associated with TV [1]. **e** Slice of the reconstructed nanocubes dataset with fused Multi-Modal Electron Tomography. Scale bar, 75 nm.

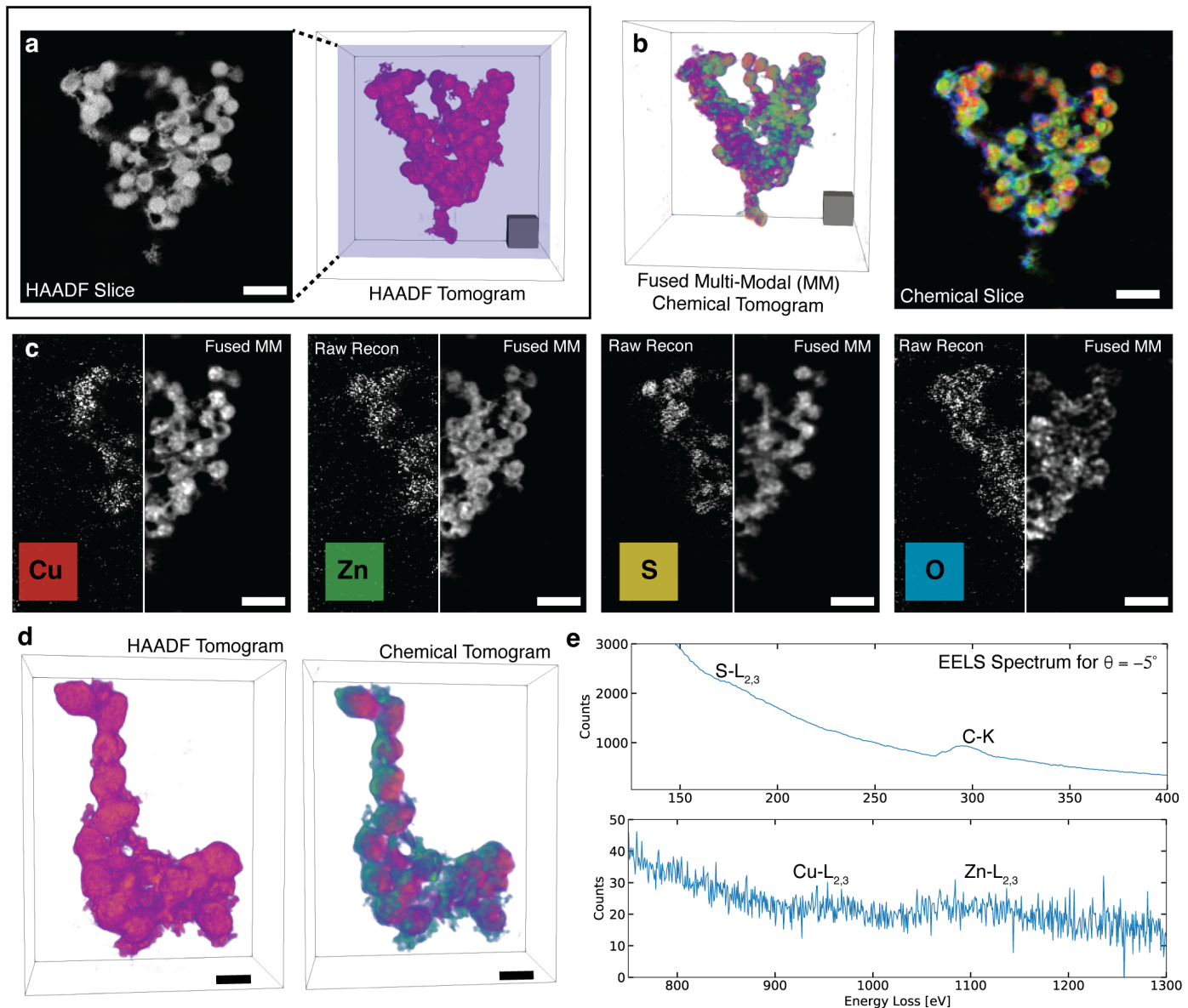
### S3 Consequence of Reconstructing CoNiO Simulation with Individual Components in Cost Function



**Fig.S3** | **a** Synthetic CoNiO nanotube ground truth EELS maps and simultaneous HAADF reconstruction. **b** Slice of the reconstructed nanotube dataset with only  $\Psi_2$  (data consistency),  $\lambda_1, \lambda_3=0$ . The recovered EELS maps and HAADF reconstruction are noisy. **c** Slice of the reconstructed multi-modal nanotube dataset with  $\Psi_1$  and  $\Psi_2$  (model + data-consistency),  $\lambda_3=0$ . The EELS maps are still noisy but show improvement with the extra  $\Psi_1$  term. **d** Slice of the reconstructed multi-modal nanotube dataset with  $\Psi_2 + TV$ ,  $\lambda_1=0$ . The expression  $\Psi_2 + TV$  is equivalent to a denoising problem; thus the resulting reconstructions produce common staircase artifacts associated with TV [1]. **e** Slice of the reconstructed nanotube dataset with fused Multi-Modal Electron Tomography. Scale bar, 50 nm.



## S4 Multi-Modal EELS Tomography of $\text{ZnS} - \text{Cu}_{0.64}\text{S}_{0.36}$ Heterostructured Nanoparticles

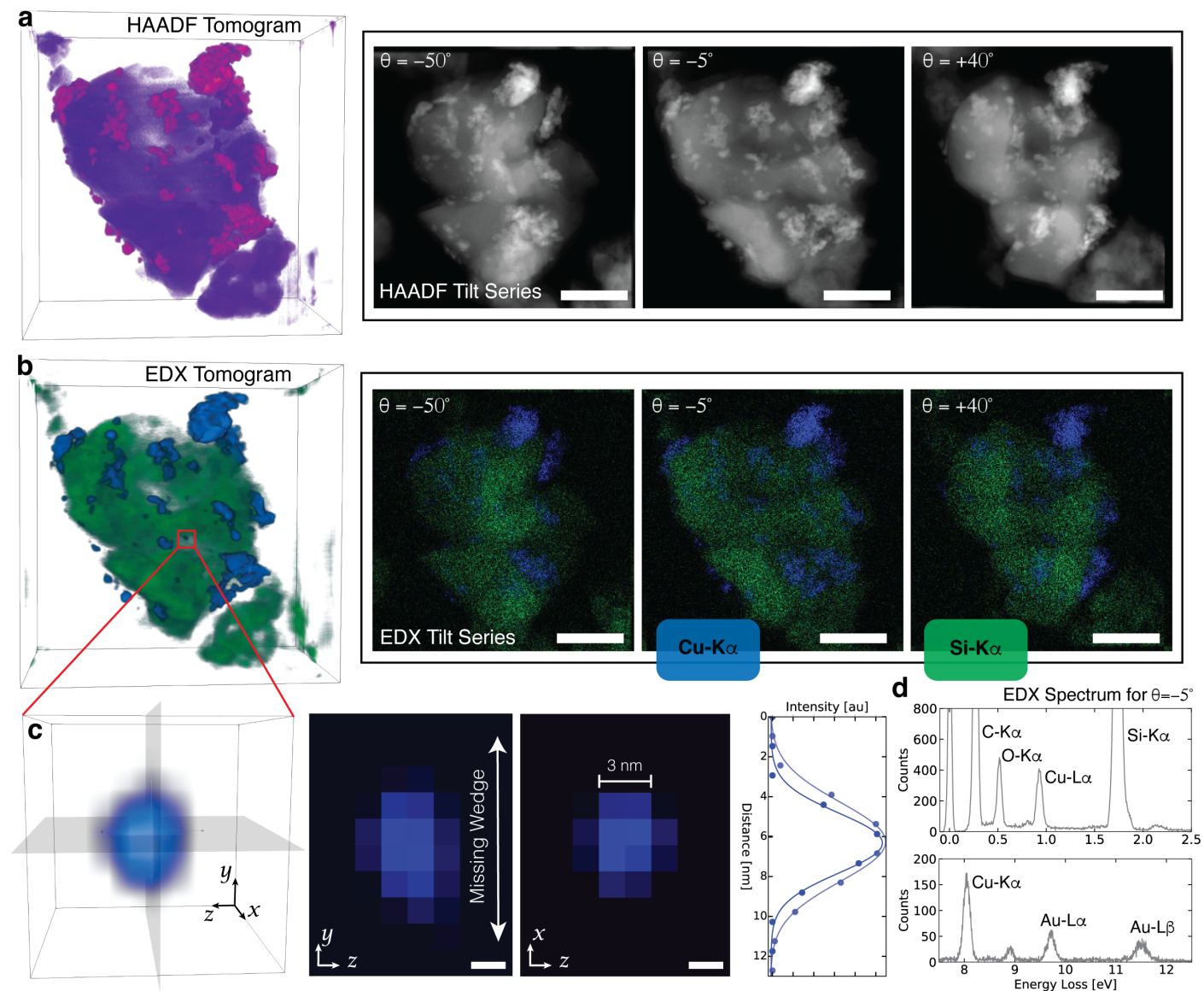


**Fig.S4 |** **a** HAADF tomography on heterostructured nanocrystals with applications in photovoltaic devices and battery electrodes [2]. The copper sulfide properties are sensitive to stoichiometry and crystal structure at the interface. The HAADF reconstruction and 2D slice are shown on the left. **b** The fused multi-modal reconstruction illustrating CuS or ZnS-rich nanoparticles and oxidized shells. **c** 2D slices of the chemical reconstructions with the noisy traditional reconstructions highlighted on the left of each image. Scale bar, 50 nm. **d** The HAADF and fused multi-modal chemical tomogram for a smaller field of view. **e** Representative EELS spectra for the S, Cu, and Zn core loss edges.

The simultaneously acquired HAADF and EELS tilt series for the specimen were collected on a Talos F200X G2 (Thermo Fisher) operated at 200 keV with a probe semi-angle of roughly 10.5 mrad and inner collection semi-angle of 50 mrad. For the larger cluster (**a-c**), the HAADF projections were collected from  $-74^\circ$  to  $+70^\circ$  with a  $3^\circ$  angular increment using a Model 2021 Fischione Analytical Tomography Holder. At each tilt angle, a STEM image with a  $24 \mu\text{s}$  dwell time at each pixel of a lateral dimension of 1.26 nm. Simultaneously acquired HAADF and EELS spectra were acquired at acquired with a  $15^\circ$  angular increment with a dwell time of 2 ms receiving a total electron dose of  $8.25 \times 10^4 \text{ e}/\text{\AA}^2$  ( $3.87 \times 10^3$ ,  $7.86 \times 10^4$  for the HAADF and EELS modality, respectively).

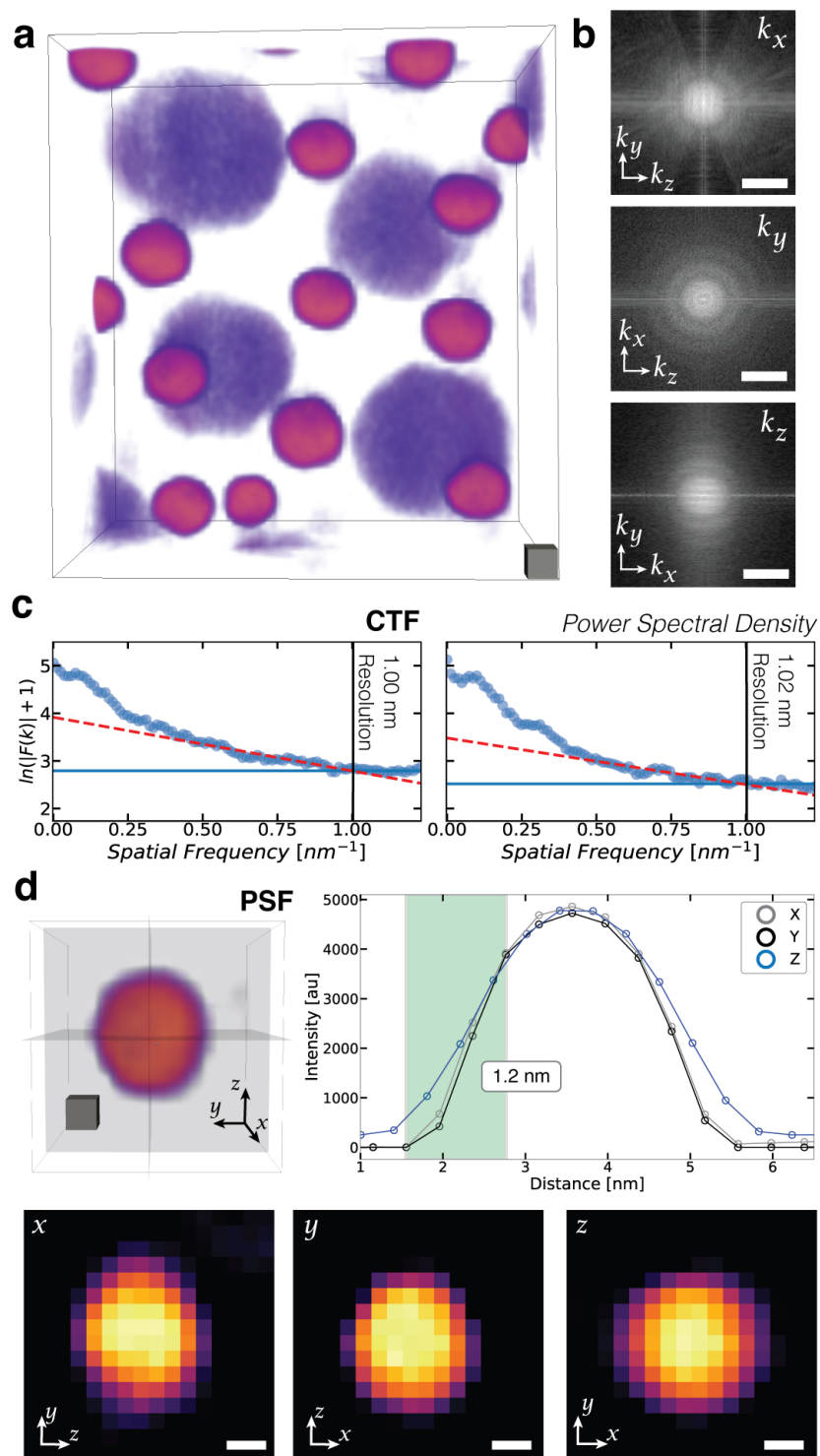
For the smaller cluster (**d**), the HAADF projections were collected from  $-74^\circ$  to  $+67^\circ$  with a  $3^\circ$  angular increment using a Model 2021 Fischione Analytical Tomography Holder. At each tilt angle, a STEM image with a  $30 \mu\text{s}$  dwell time at each pixel of a lateral dimension of 1.12 nm. Simultaneously acquired HAADF and EELS spectra were acquired at acquired with a  $15^\circ$  angular increment with a dwell time of 3.5 ms receiving a total electron dose of  $2.07 \times 10^5 \text{ e}/\text{\AA}^2$  ( $6.87 \times 10^3$ ,  $2.01 \times 10^5$  for the HAADF and EELS modality, respectively).

## S5 Multi-Modal EDX Tomography of Cu-SiC Nanoparticles



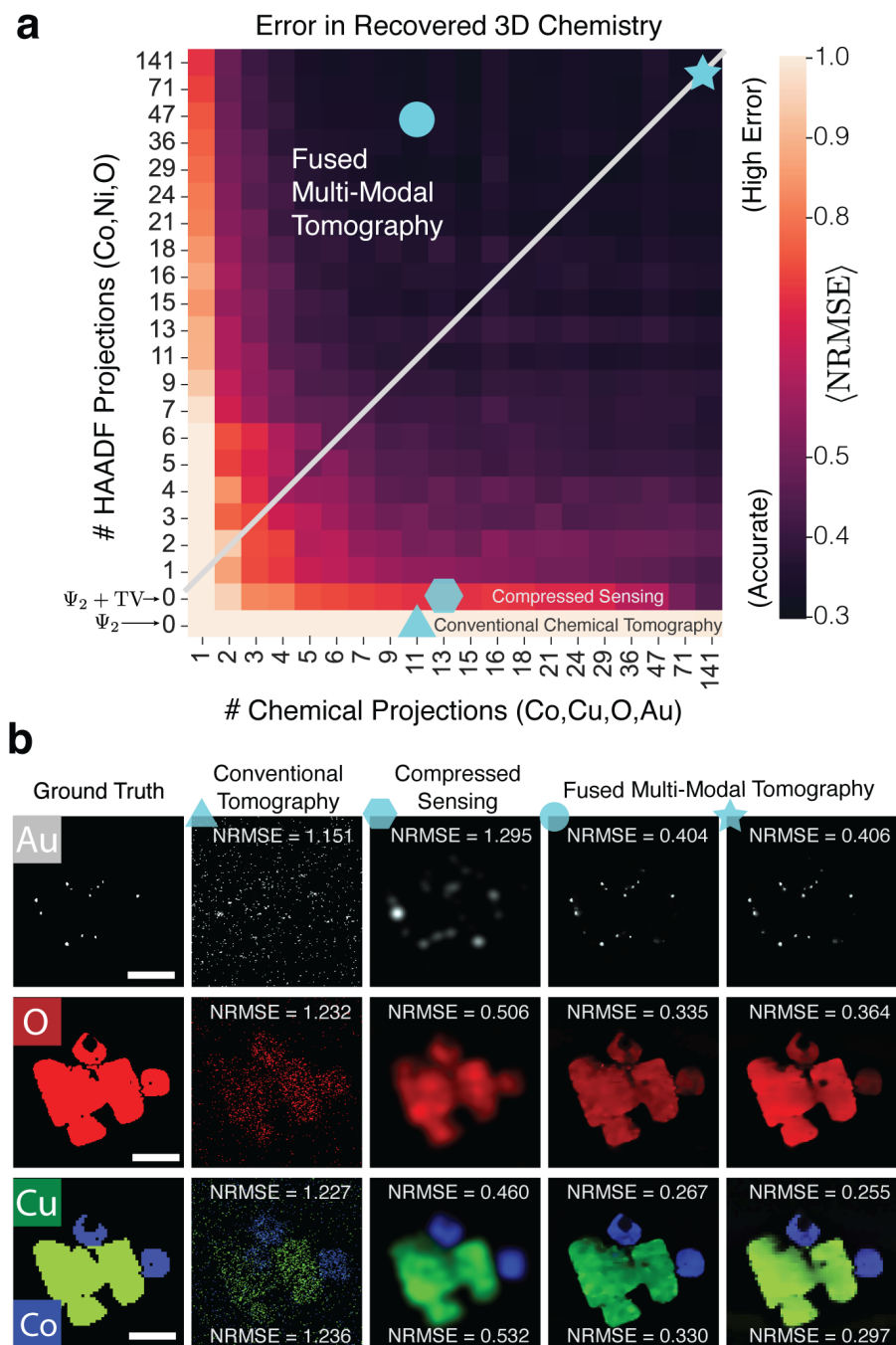
**Fig.S5 | a** Cu-supported silicon carbon (SiC) catalysts designed for the production of fuels and chemicals from biomass. These silica-supported copper catalysts efficiently convert ethanol into acetaldehyde because of their high selectivity and stability [3, 4]. The HAADF reconstruction with a few tilt micrographs is shown on right. **b** The fused multi-modal reconstruction highlighting Cu nanoparticles embedded inside the SiC support and raw EDX maps are shown on the right. Scale bar, 50 nm. **c** The PSF of an individual 3 nm nanoparticle inside the SiC. A few 2D slices of the reconstruction are shown on the right where we see the structure is sharp along the orthogonal axis perpendicular to the missing wedge and approximately a 20% reduction in resolution along the missing wedge direction. Scale bar, 2 nm. **d** EDX spectra for a single tilt.

## S6 Measuring the Resolution of Au-Fe<sub>3</sub>O<sub>4</sub> Nanoparticles in the HAADF



**Fig.S6 | HAADF Resolution Analysis of Au-Fe<sub>3</sub>O<sub>4</sub> superlattice nanoparticles.** **a** Fused EELS tomograms of Au-Fe<sub>3</sub>O<sub>4</sub> nanoparticles. Power spectral density of the HAADF reconstruction along the principal axial directions shown on the right. Scale cube, 2 nm<sup>3</sup>. Scale bar, 0.5 nm<sup>-1</sup>. **b** Power spectral density profiles for YZ and XY planes. **c** Line scan profiles of a 2.5 nm Au nanoparticle give a resolution of 1.00, 1.03, and 1.01 nm along the x, y, and z directions.

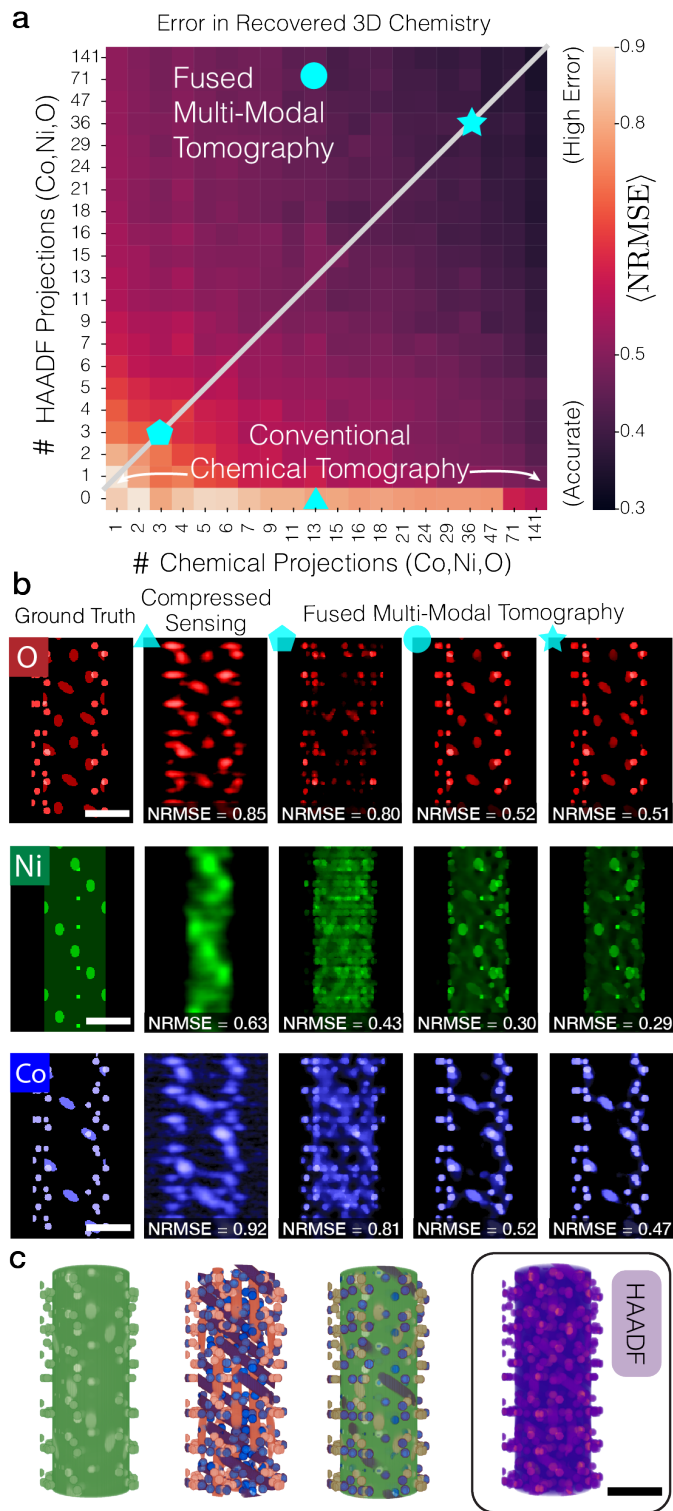
## S7 Sampling Requirements of Multi-Modal Tomography on CoO/CuO Nanocubes



**Fig.S7** **a** An NRMSE map representing reconstruction error as a function of the number of HAADF and chemical tilts. The bottom two rows provide results from non-multi-modal algorithms where the HAADF is not included in the reconstruction process. The last row uses the Poisson Maximum Likelihood term to reconstruct the individual chemical distributions. The second to last row adapts the compressed sensing framework into the reconstruction process by including total variation minimization – thus providing a 2-3 fold reduction in average error. **b** Visualization of four points in the phase diagram corresponding to conventional chemical tomography, regularized tomography (compressed sensing), and low or high-dose fused multi-modal electron tomography. The 3D models were then rendered and colored in Tomviz. Scale bar, 75 nm.



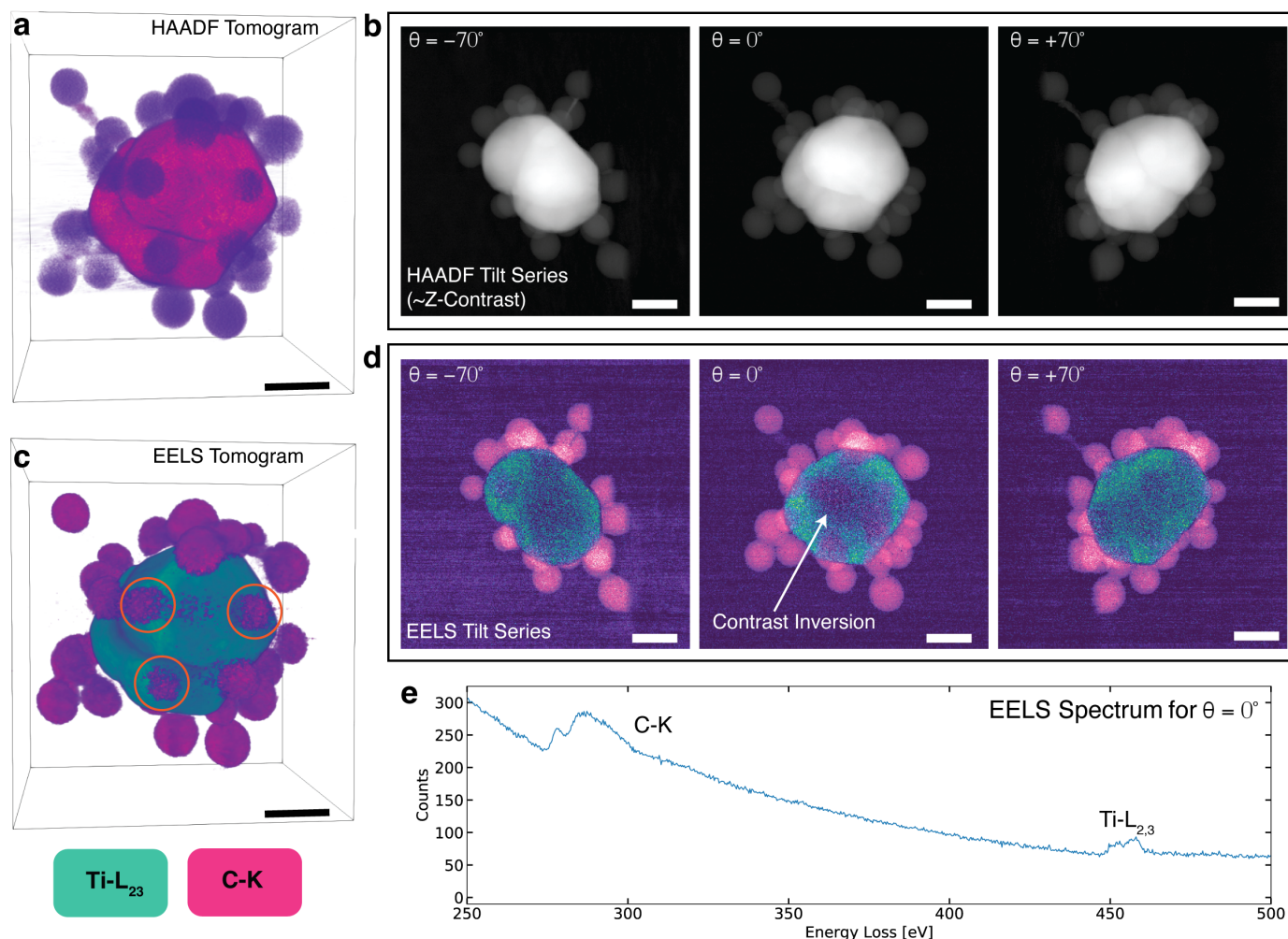
## S8 Estimating the Sampling Requirements for a CoNiO Composite



**Fig.S8** **a** A normalized root-mean-square-error (NRMSE) heatmap of a Fused Multi-Modal CoNiO nanotube reconstruction compared to the known ground truth structure as a function of the number of HAADF and chemical tomographic projections. Brighter pixels denote higher levels of error in the reconstruction when compared to the ground truth. The SNR for the Co, Ni, O, and HAADF modalities were 2.66, 6.46, 3.17, and 2156.16 respectively. **b** Visualization of four points in the phase diagram corresponding to conventional chemical tomography, regularized tomography (compressed sensing), and low or high-dose fused multi-modal electron tomography. **c** The synthetic CoNiO nanotube ground truth 3D models generating synthetic chemical and ADF projections. The 3D models were then rendered and colored in Tomviz. Scale bar, 50 nm.



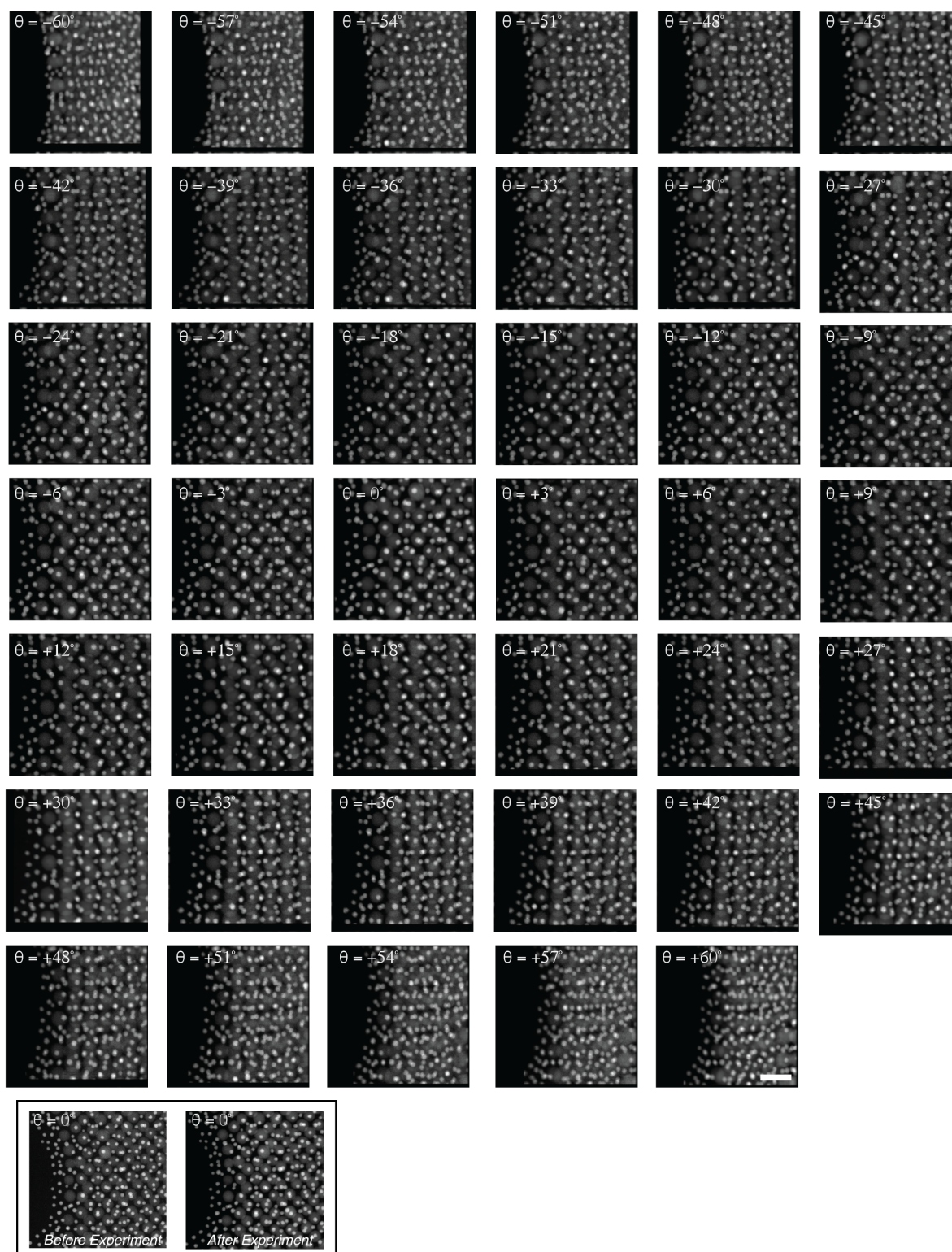
## S9 Consequence of Reconstructing Thick 280 nm Nanoparticles with EELS Tomography



**Fig.S9** | **a** The HAADF reconstruction with a few HAADF projections from the tilt series shown on right. These projections were square rooted to visually see the TiO and C nanoparticles. **b** The fused multi-modal reconstruction illustrating TiO nanoparticle decorated by C support with raw EELS maps shown on the right. Scale bar, 100 nm.

The simultaneously acquired HAADF and EELS tilt series for the specimen were collected on a Talos F200X G2 (Thermo Fisher) operated at 200 keV with a probe semi-angle of roughly 10.5 mrad and inner collection semi-angle of 50 mrad. The HAADF projections were collected from  $-70^\circ$  to  $+70^\circ$  with a  $+2^\circ$  angular increment using a Model 2021 Fischione Analytical Tomography Holder. At each tilt angle, a STEM image with a  $32 \mu\text{s}$  dwell time at each pixel of a lateral dimension of 2.015 nm. Simultaneously acquired HAADF and EELS spectrums were acquired at acquired with a  $10^\circ$  angular increment with a dwell time of 2 ms receiving a total electron dose of  $4.96 \times 10^4 \text{ e}/\text{\AA}^2$  ( $3.48 \times 10^3$ ,  $4.61 \times 10^4$  for the HAADF and EELS modality, respectively)

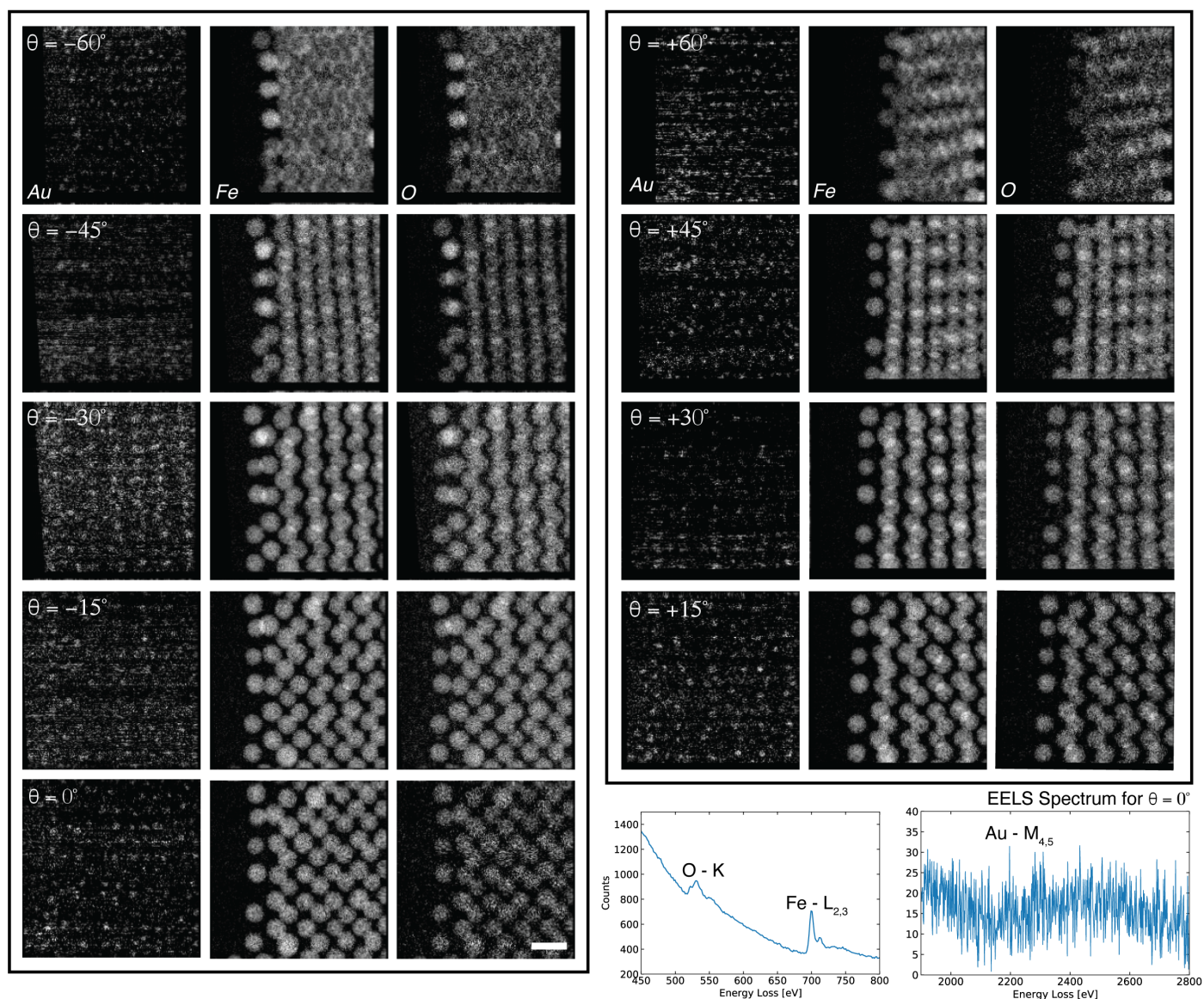
# S10 Raw HAADF Tilt Series for the Au-Fe<sub>3</sub>O<sub>4</sub> Nanoparticles



**Fig.S10** | The 45 projection images with a tilt range from  $-60^\circ$  to  $+60^\circ$  (shown at the top left of each panel) were measured using ADF-STEM. The total electron dose of the tilt series is  $1.72 \times 10^4 \text{ e}/\text{\AA}^2$ . Scale bar shown at the bottom right, 25 nm.

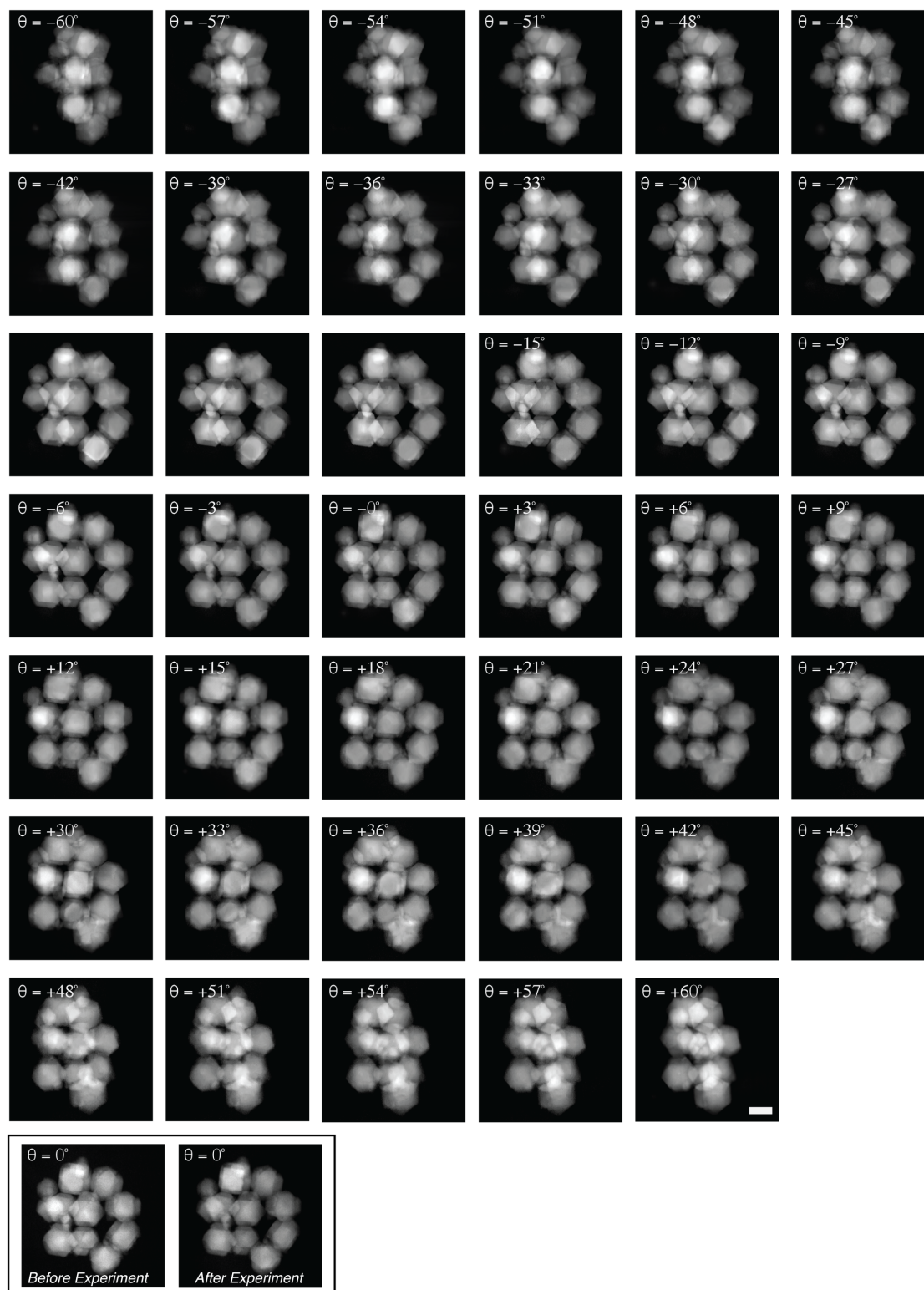


## S11 Raw EELS Tilt Series for the Au-Fe<sub>3</sub>O<sub>4</sub> Nanoparticles



**Fig.S11** | The 9 EELS maps with a tilt range from  $-60^\circ$  to  $+60^\circ$  (shown at the top left of each map) were measured using EELS spectroscopy. The total electron dose of the tilt series is  $4.73 \times 10^5 \text{ e}/\text{\AA}^2$ . Scale bar is shown at the bottom right, 50 nm.

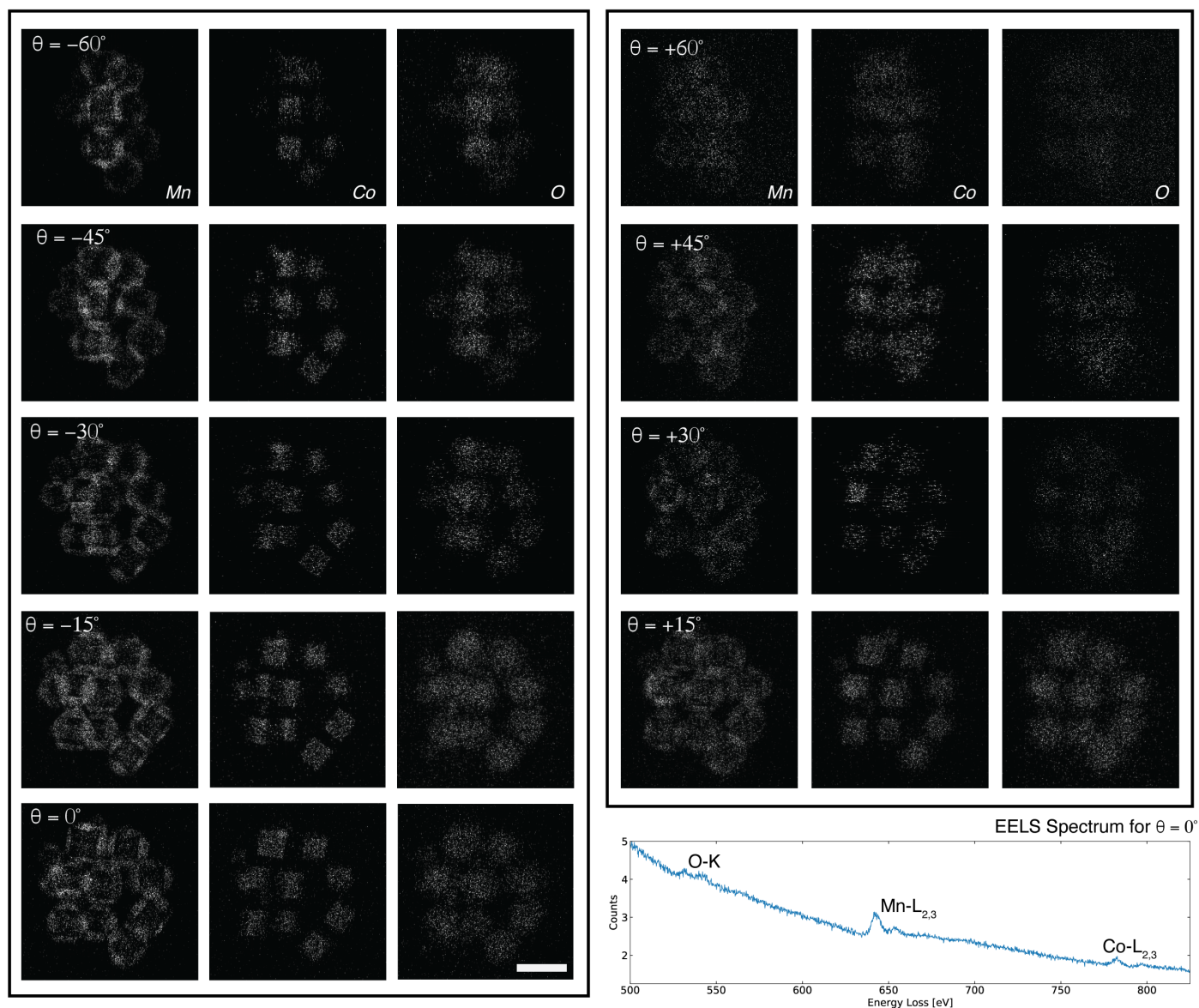
# S12 Raw HAADF Tilt Series for the $\text{Co}_3\text{O}_4$ - $\text{Mn}_3\text{O}_4$ Core-Shell Nanoparticles



**Fig.S12** | The 45 projection images with a tilt range from  $-60^\circ$  to  $+60^\circ$  (shown at the top left of each panel) were measured using ADF-STEM. The total electron dose of the tilt series is  $1.16 \times 10^4 \text{ e}/\text{\AA}^2$ . Scale bar is shown at the bottom right, 50 nm.



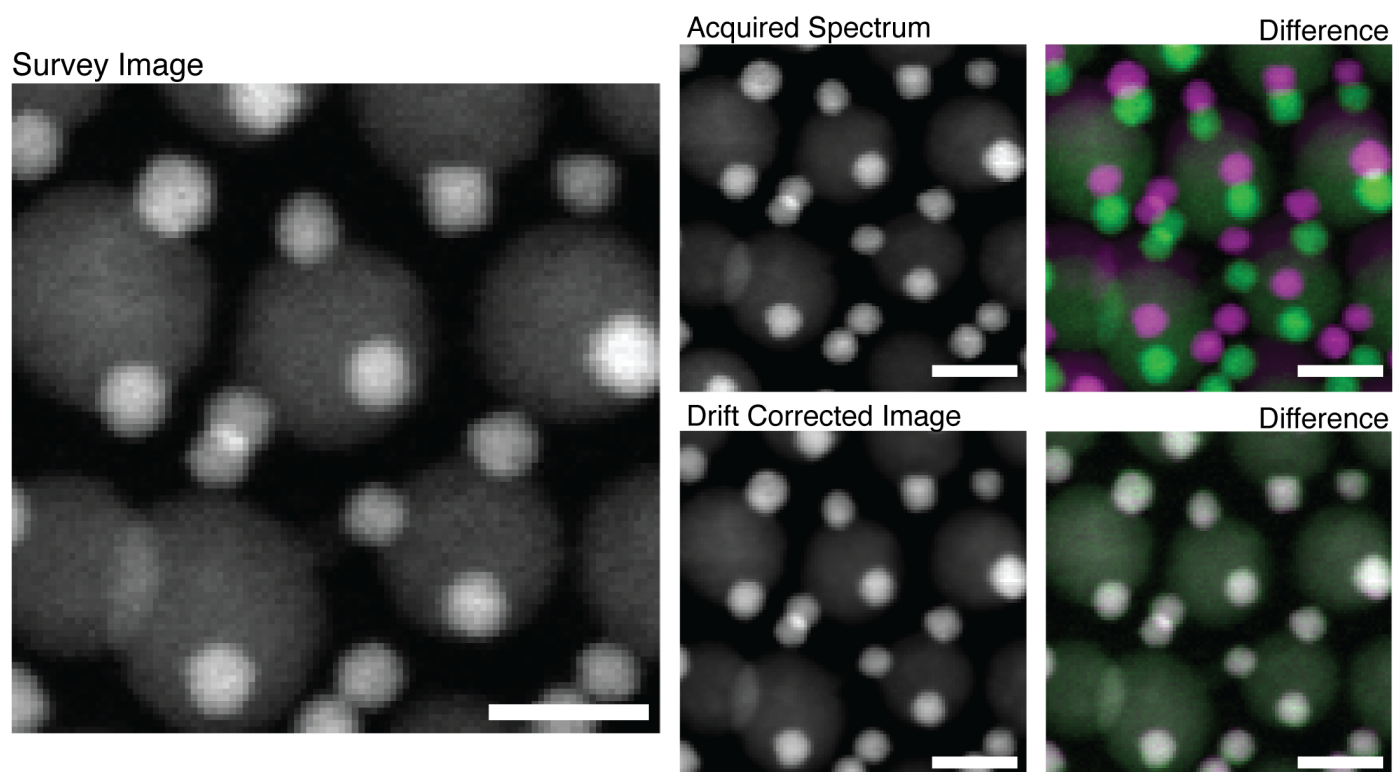
# S13 Raw EELS Tilt Series for the $\text{Co}_3\text{O}_4$ - $\text{Mn}_3\text{O}_4$ Core-Shell Nanoparticles



**Fig.S13** | The 9 EELS maps with a tilt range from  $-60^\circ$  to  $+60^\circ$  (shown at the top left of each map) were measured using EELS spectroscopy. The total electron dose of the tilt series is  $7.21 \times 10^4 \text{ e}/\text{\AA}^2$ . Scale bar is shown at the bottom right, 50 nm.

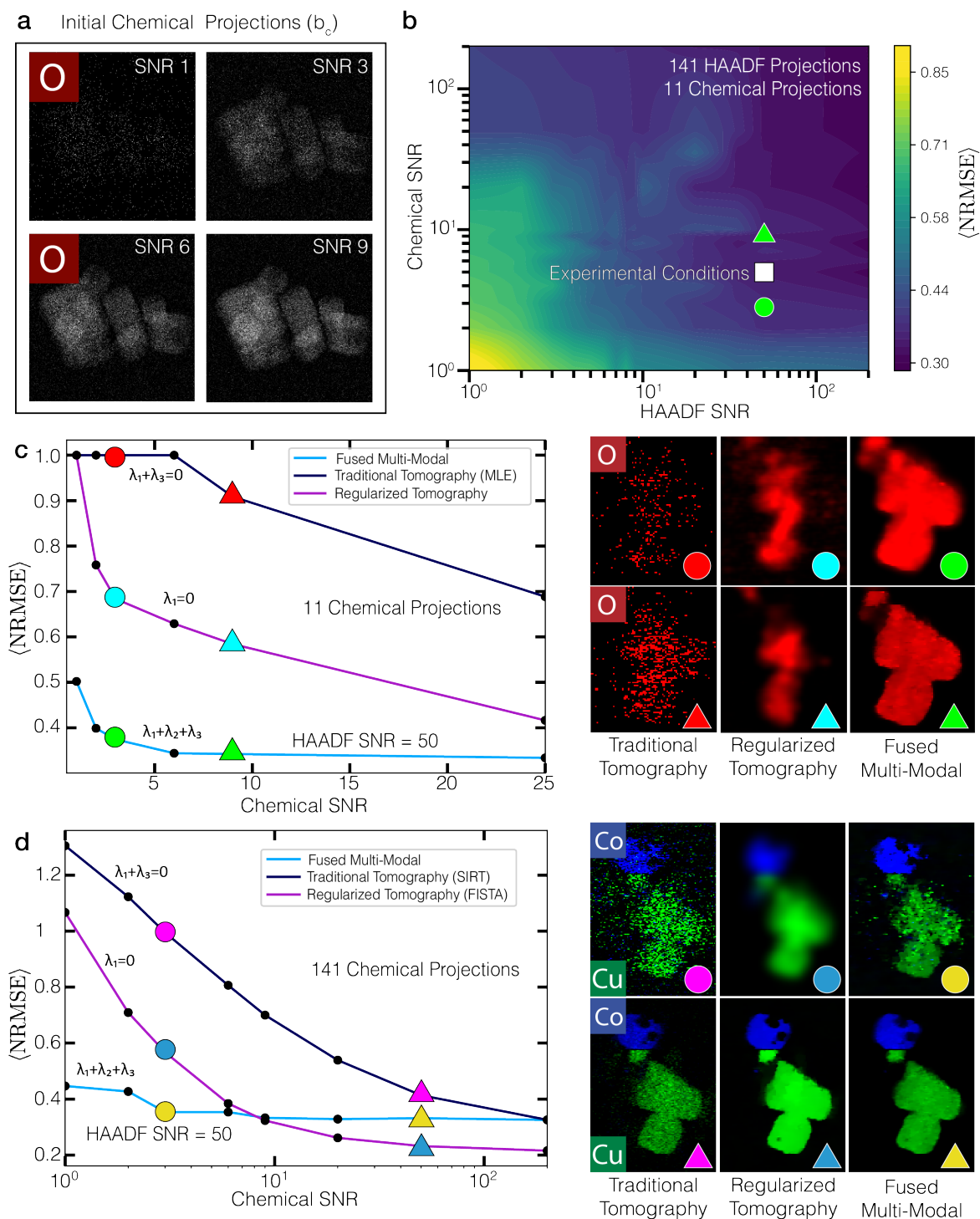


## S14 Drift Correction of Spectrum Maps



**Fig.S14** | Drift correction of the spectrum images: a survey image  $I(x,y)$ , recorded before the spectrum image acquisition, is used as a reference to calculate the affine transformation on the acquired spectrum map. These parameters are subsequently used to correct for drift in the acquired image and all associated spectroscopic signals.

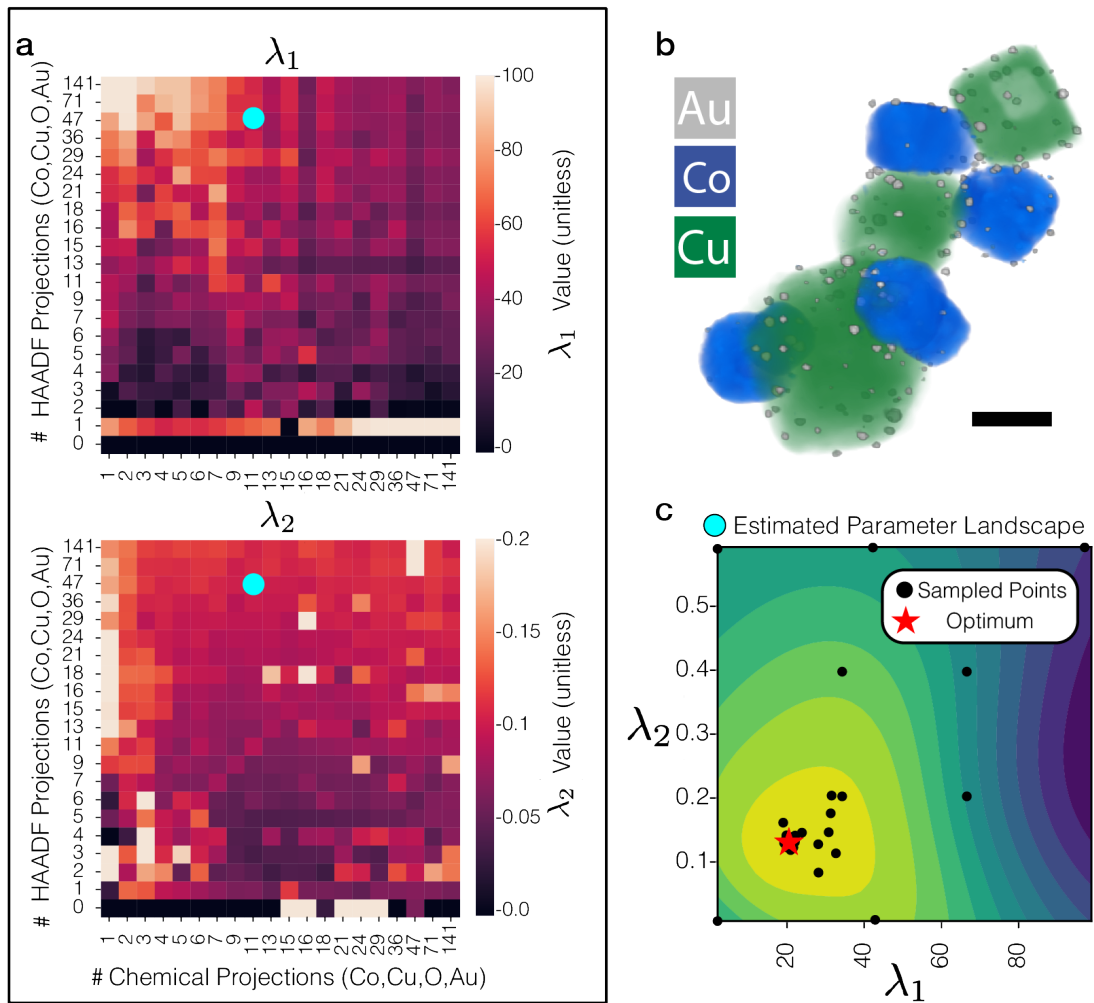
## S15 SNR Dependency for Successful Fused Multi-Modal Recovery



**Fig.S15** **a** The initial corrupted chemical distributions for oxygen in the CuO-CoO synthetic dataset with increasing SNR. We see as we exceed the Rose criterion (SNR  $\approx 5$ ) internal pores and fine features are visible. **b** A heat map expressing the relationship between average reconstruction error and SNR for either modality (HAADF or Chemistry) when 11 chemical maps ( $\Delta\theta = +12^\circ$ ) and 141 HAADF projections ( $\Delta\theta = +1^\circ$ ) are available. **c** SNR plot highlighting the average NRMSE as a function of chemical SNR for reconstructions without any regularization or fusion (traditional tomography), without data-fusion (regularized tomography) and within the multi-modal framework. We see substantial improvements in reconstruction quality as we incorporate more terms in the cost function. Most notably, data fusion converges quite rapidly when the SNR is above 6. Visualization of the oxygen elemental reconstruction of the 2D slices from the reconstructions from each of the three curves. **d** SNR plot highlighting the average NRMSE as a function of chemical SNR for traditional tomography with the SIRT reconstruction algorithm, without data-fusion with the FISTA reconstruction algorithm and within the multi-modal framework [5, 6]. When the sampling between the chemistry and HAADF is equivalent and chemical SNR is larger, regularized tomography outperforms multi-modal tomography. Visualization of the cobalt and copper elemental reconstruction of the 2D slices from the reconstructions from each of the three curves.

# S16 Hyperparameter Estimation with Bayesian Optimization for the CuO-CoO Nanocubes

$$\arg \min_{\mathbf{x}_i \geq 0} \frac{\lambda_1}{2} \left\| \mathbf{A}_h \sum_i (Z_i \mathbf{x}_i)^\gamma - \mathbf{b}_H \right\|_2^2 + \lambda_2 \sum_i \left( \mathbf{1}^T \mathbf{A}_c \mathbf{x}_i - \mathbf{b}_i^T \log(\mathbf{A}_c \mathbf{x}_i + \varepsilon) \right) + \lambda_3 \sum_i \|\mathbf{x}_i\|_{\text{TV}}$$

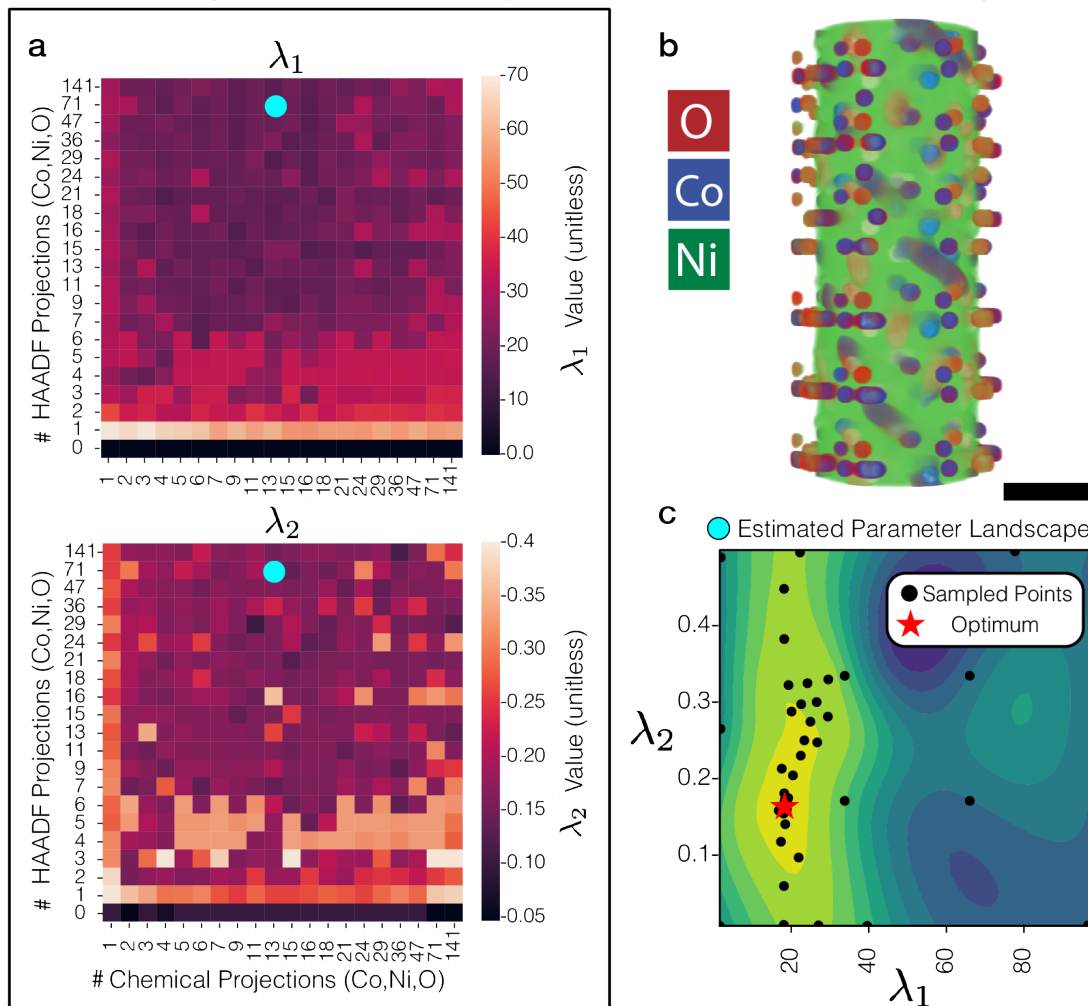


**Fig.S16 | a** Bayesian optimization optimizes the data fusion cost function (shown above) when provided a given number of chemical and HAADF tilts. We see the weights between HAADF and chemical modality ( $\lambda_1$  and  $\lambda_2$  respectively) can vary depending on our certainty in either modality. When the number of tilts is low, the corresponding  $\lambda$  value would decrease and vice-versa. Overall, these maps can guide future scientists to produce multi-modal reconstructions with reasonable hyperparameter selections. **b** 3D visualization of the ground truth Au decorated CuO/CoO nanocubes. Scale bar, 75 nm. **c** Bayesian optimization parameter selection landscape where each black dot represents one of the many attempts to find the minimum NRMSE.



## S17 Hyperparameter Estimation with Bayesian Optimization for CoNiO Composite

$$\arg \min_{\mathbf{x}_i \geq 0} \frac{\lambda_1}{2} \left\| \mathbf{A}_h \sum_i (Z_i \mathbf{x}_i)^\gamma - \mathbf{b}_H \right\|_2^2 + \lambda_2 \sum_i \left( \mathbf{1}^T \mathbf{A}_c \mathbf{x}_i - \mathbf{b}_i^T \log(\mathbf{A}_c \mathbf{x}_i + \varepsilon) \right) + \lambda_3 \sum_i \|\mathbf{x}_i\|_{TV}$$



**Fig.S17 | a** Bayesian optimization optimizes the data fusion cost function (shown above) when provided a given number of chemical and HAADF tilts. We see the weights between HAADF and chemical modality ( $\lambda_1$  and  $\lambda_2$  respectively) can vary depending on our certainty in either modality. When the number of tilts is low, the corresponding  $\lambda$  value would decrease and vice-versa. Overall, these maps can guide future scientists to produce multi-modal reconstructions with reasonable hyperparameter selections. **b** 3D visualization of the ground truth CoNiO composite structure. Scale bar, 75 nm. **c** Bayesian optimization parameter selection landscape where each black dot represents one of the many attempts to find the minimum NRMSE.

## S18 Fusion Multi-Modal Electron Tomography Pseudo Code

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### Algorithm 1 Fused Multi-Modal 3D Chemical Tomography

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1: Objective Function:  $\Psi(\mathbf{x}) = \frac{\lambda_1}{2} \|\mathbf{A}_h \sum_i (Z_i \mathbf{x}_i)^\gamma - \mathbf{b}_h\|_2^2 + \lambda_2 \sum_i (\mathbf{1}^T \mathbf{A}_c \mathbf{x}_i - \mathbf{b}_i^T \log(\mathbf{A}_c \mathbf{x}_i + \varepsilon)) + \lambda_{TV} \sum_i \|\mathbf{x}_i\|_{TV}$ 
2:
3: Input:  $\mathbf{b}_h \in \mathbb{R}^{n_y \cdot N_{\text{HAADF}}^{\text{proj}} \times n_x}$  ▷ HAADF Tilt Series
4: Input:  $\mathbf{b}_c \in \mathbb{R}^{n_i \cdot n_y \cdot N_{\text{chem}}^{\text{proj}} \times n_x}$  ▷ Chemical Tilt Series
5: Output:  $\mathbf{x} \in \mathbb{R}^{n_i \cdot n_y \cdot n_y \times n_x}$  ▷ Reconstructed Chemical Tomogram
6:
7:  $N_{\text{iter}} = 100, \varepsilon = 0.1$ 
8:  $\mathbf{x}_i^0 = \arg \min_{\mathbf{x}} \Psi_2(\mathbf{0})$  ▷ Initialize first iterate with reconstruction from raw chemical maps
9:
10: for  $k = 1, N_{\text{iter}}$  do ▷ Main Loop
11:   for  $s = 1, n_x$  do ▷ Apply the Gradient for Each Slice Along the Axis Parallel to Tilt Axis
12:      $\mathbf{x}_s^k = \mathbf{x}_s^{k-1} - (\lambda_1 \nabla_{\mathbf{x}} \Psi_1(\mathbf{x}_s^{k-1}) + \lambda_2 \nabla_{\mathbf{x}} \Psi_2(\mathbf{x}_s^{k-1}))$ 
13:   end for
14:   for  $i = 1, n_i$  do
15:      $\mathbf{x}_i^k = \text{tv\_fgp\_3D}(\mathbf{x}_i^k, \lambda_{TV})$  ▷ Channel-Wise TV Minimization
16:   end for
17: end for
18: return  $\mathbf{x}$ 
19:
20: Comments:
21:  $\nabla_{\mathbf{x}} \Psi_1(\mathbf{x}) = -\gamma \text{diag}(\mathbf{x}^{\gamma-1}) \Sigma^T \mathbf{A}_h^T (\mathbf{A}_h (\Sigma \mathbf{x}^\gamma) - \mathbf{b}_h)$ 
22:  $\Sigma \in \mathbb{R}^{n_y \cdot n_y \times n_y \cdot n_y \cdot n_i}$  expresses the summation operation as matrix vector multiplication.
23:  $\nabla_{\mathbf{x}_i} \Psi_2(\mathbf{x}_i) = \mathbf{A}_c^T ((\mathbf{A}_c \mathbf{x}_i - \mathbf{b}_i) \oslash (\mathbf{A}_c \mathbf{x}_i + \varepsilon))$ 
24:  $\oslash$  is element-wise division.
25:  $\mathbf{x}_i \in \mathbb{R}^{n_y \cdot n_y \times n_x}$  is the reconstruction for element  $i$ .
26:  $\mathbf{b}_i \in \mathbb{R}^{n_y \cdot N_{\text{chem}}^{\text{proj}} \times n_x}$  is the chemical tilt series for element  $i$ .
27:  $\mathbf{x}_s \in \mathbb{R}^{n_y \cdot n_y \cdot n_i}$ .

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1: Input:  $\mathbf{b} \in \mathbb{R}^{n_x \times n_y} \rightarrow$  3D Volume,  $\lambda \rightarrow$  Regularization Parameter,  $ng \rightarrow$  Number of Iterations
2: Output:  $\mathbf{x}^* \rightarrow$  Optimal Solution
3:
4: function TV_GP_3D( $\mathbf{x}, \lambda, ng$ )
5:    $\mathbf{p}_x^0 = \mathbf{0} \in \mathbb{R}^{(m-1) \times n \times k}$ ,  $\mathbf{p}_y^0 = \mathbf{0} \in \mathbb{R}^{m \times (n-1) \times k}$ ,  $\mathbf{p}_z^0 = \mathbf{0} \in \mathbb{R}^{m \times n \times (k-1)}$ 
6:   for  $k = 1, ng$  do ▷ Main Loop
7:      $(\mathbf{p}_x^k, \mathbf{p}_y^k, \mathbf{p}_z^k) = P_{\mathcal{P}} \left[ (\mathbf{p}_x^{k-1}, \mathbf{p}_y^{k-1}, \mathbf{p}_z^{k-1}) + \frac{1}{26\lambda} \mathcal{L}^T (P_C[\mathbf{b} - \lambda \mathcal{L}(\mathbf{p}_x^{k-1}, \mathbf{p}_y^{k-1}, \mathbf{p}_z^{k-1})]) \right]$ 
8:   end for
9:   return  $\mathbf{x}^* = P_C[\mathbf{b} - \lambda \mathcal{L}(\mathbf{p}_x^{ng}, \mathbf{p}_y^{ng}, \mathbf{p}_z^{ng})]$ 
10: end function
11:
12: function  $\mathcal{L}(p, q, r)$  ▷ Linear Operation
13:   for  $i = 1, \dots, n_x$  do
14:     for  $j = 1, \dots, n_y$  do
15:       for  $k = 1, \dots, n_k$  do
16:         if  $i == 0$  or  $i == n_x$  do  $p_{i,j,k} = 0$ 
17:         if  $j == 0$  or  $j == n_y$  do  $q_{i,j,k} = 0$ 
18:         if  $k == 0$  or  $k == n_k$  do  $r_{i,j,k} = 0$ 
19:          $\mathcal{L}(p, q, r)_{i,j,k} = p_{i,j,k} + q_{i,j,k} + r_{i,j,k} - p_{i-1,j,k} - q_{i,j-1,k} - r_{i,j,k-1}$ 
20:       end for
21:     end for
22:   end for
23:   return  $\mathcal{L}(p, q, r)$ 
24: end function
25:
26: function  $\mathcal{L}^T(x)$ 
27:   for  $i = 1, \dots, n_x - 1$  do
28:     for  $j = 1, \dots, n_y - 1$  do
29:       for  $k = 1, \dots, n_z - 1$  do
30:          $p_{i,j} = x_{i,j} - x_{i+1,j,k}$ 
31:          $q_{i,j} = x_{i,j} - x_{i,j+1,k}$ 
32:          $r_{i,j} = x_{i,j} - x_{i,j,k+1}$ 
33:       end for
34:     end for
35:   end for
36:   return  $(p, q, r)$ 
37: end function
38:

```

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```
1: function  $P_C(x)$  ▷ Orthogonal Projection Operator Onto Convex Set (Non-negativity)
2:   return  $\max\{0, x\}$ 
3: end function
4:
5: function  $P_P(p, q, r)$  ▷ Projection Operator for Isotropic TV Norm
6:   for  $i = 1, \dots, n_x - 1$  do
7:     for  $j = 1, \dots, n_y - 1$  do
8:       for  $k = 1, \dots, n_z - 1$  do
9:          $denom = \sqrt{p_{i,j,k}^2 + q_{i,j,k}^2 + r_{i,j,k}^2}$ 
10:         $p_{i,j,k} = \frac{p_{i,j,k}}{\max\{1, denom\}}$ 
11:         $q_{i,j,k} = \frac{q_{i,j,k}}{\max\{1, denom\}}$ 
12:         $r_{i,j,k} = \frac{r_{i,j,k}}{\max\{1, denom\}}$ 
13:      end for
14:    end for
15:  end for
16:  return  $(p, q, r)$ 
17: end function
18:
19: function  $P_{\mathcal{P}}(p, q, r)$  ▷ Projection Operator for Anisotropic TV Norm
20:   for  $i = 1, \dots, n_x - 1$  do
21:     for  $j = 1, \dots, n_y - 1$  do
22:       for  $k = 1, \dots, n_z - 1$  do
23:         $p_{i,j,k} = \frac{p_{i,j,k}}{\max\{1, |p_{i,j,k}|\}}$ 
24:         $q_{i,j,k} = \frac{q_{i,j,k}}{\max\{1, |q_{i,j,k}|\}}$ 
25:         $r_{i,j,k} = \frac{r_{i,j,k}}{\max\{1, |r_{i,j,k}|\}}$ 
26:      end for
27:    end for
28:  end for
29:  return  $(p, q, r)$ 
30: end function
```

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## References

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