# Forecasting Forex Rate Volatility with Cutting-Edge Fuzzy Time Series Models

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# Forecasting Forex Rate Volatility with Cutting-Edge Fuzzy Time Series Models

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## Abstract

The performance of fuzzy time series (FTS) prediction algorithms is impacted negatively in the presence of outlier(s), heterogeneity, or contamination in the data. As a result of these issues, standard forecasting algorithms will fail to produce reasonable forecast error rates for defuzzified outputs in understudy data. In this article, we present a robust technique for FTS by assessing how the prediction performance of the techniques is influenced by the outlier, not only to tackle this problem but also to increase forecasting accuracy. We proposed two novel robust fuzzy time series models, i.e. Trimmed Fuzzy Time Series (TFTS) and Winsorized Fuzzy Time Series (WFTS), and implemented to annual exchange rates (AERs) between the Pakistani rupee and the US dollar for comparison to other competitive models. The proposed models consider sub-partitioning in the universe of discourse, optimization of parameters method(s), and interval forecasting, which makes the forecast accuracy more precise forecasting than previously studied methods. Such forecasting techniques will assist all stakeholders, whether directly or indirectly involved, in making sensible data-driven business decisions across the country.

**Keywords:** Robust Methods, Robust Fuzzy Time Series (RFTS), Forex Rate, ARIMA moving average (MA), Average Forecast Error Rate (AFER).

## 1 Introduction

The exchange rate is an important macroeconomic indicator for every country's economy, commerce, and business, among other things. People associated with these indicators always keep a keen consideration of on-trend forecasts for forex which pays off them in the good decision-making process. So, improved forecasting methods for currency exchange rates (ERs) are essential in reducing businessmen's uncertainty risk. The world is a global village, forex plays significant in the world's economy. Gorbatiuk, Hryhoruk [1] proposed some enhancements to Stevenson and Porter's scheme [2] in FTS forecasting versus time series forecasting using AFER for predicting Ukrainian enterprise net income level. According to [3] Pakistan, a developing country in South Asia has a struggling economy, and the PKR-US $ ER swings regularly; ER stability is critical for economic progress If the PKR-US $ ER rises, the value of debt payable rises as well, disrupting the balance of payments; if the government chooses to raise the money supply, inflation causes a rise in the price of services and commodities in the local market, causing the general public to suffer in the end. Policymakers must keep a check on the ER; otherwise, an unfavorable departure may have an impact on other macroeconomic factors [4]. Currency depreciation is the official reduction in the value of domestic money concerning gold or foreign currency. Typically, governments use it to close the deficit external balance gap because policymakers believe that home currency depreciation is advantageous to the economy. Because when a lower currency boosts exports, which leads to a rise in employment and is good for economic growth. The IMF and the Central Bank have repeatedly advised the present government to enhance its external competitiveness by increasing exports and decreasing imports to improve trade and current account balances and repay loans. Due to strong demand for import payments, the Pakistani rupee has recently devalued to a record low of Rs. 176.50 per US $. The depreciation of the rupee results in high import expenses as well as local inflation, lowering the general public's purchasing power. Understandably, domestic currency depreciation undermines the country's economy in contrast to other nations, and it is not a long-term sustainable answer for increasing the country's economy [5].

Pakistan is dealing with several socioeconomic difficulties; consequently, monetary policymakers should consider using a forecasting tool to make informed data-driven decisions. Accurate forecasting allows policymakers to amend policies as needed [6]. The PKR-US $ ER is generally on the rise. Before General Pervez Musharraf's leadership, the US $ was trading in the interbank market at Rs. 45. However, throughout his eight-year reign, the ER surged by more than 30%. The currency rate rose rapidly during the duration of Asif Zardari's government, reaching up to Rs. 95, followed by Nawaz Sharif's tenure, when the highest ER reached more than Rs. 120. According to Imran Khan's government, the current ER of the Pakistani rupee versus the US $ is around Rs. 179.18, and it is predicted to rise higher soon. We noticed a slightly decreasing trend in the ER within the first Covid-19 days, but it was minimal. Forecasting is any claim about the future and is regarded as an important technique in research. For predicting purposes, the researcher(s) have been using the PKR-US $ ER as a variable. The major goal of this study is to produce an overall forecasting model for the assessment of the PKR-US $ AER. Most forecasting models limit the random sequence of variables by the markovian property, monotonous, scalarity, stationarity, and so on, resulting in changes in the character of economic indicators. Zadeh's seminal article on fuzzy set theory has paved the way for many other fields of research, including time-series data dealing with uncertainty. Fuzzy approaches increase the predictability of real-world processes.

This study is divided into five main components, as follows: 1) Introduction: This is a quick overview of the topic. 2) Literature review: This section includes an overview of the current and past literature. 3) Research Methodology: It includes the theoretical development of robust fuzzy time series (RFTS) i.e. Trimmed fuzzy times series (TFTS) and Winsorized fuzzy time series (WFTS). 4) Case Study: It comprises dataset-based outcomes that were drawn using the methodologies mentioned. 5) Discussion and Conclusion: This section includes the sequence and primary focused outcomes from the study plan. Finally, recommendations for future research work are derived from the findings. A list of abbreviations for this study is provided in the appendix.

The key contributions of this study to literature are as under.

- Considerable changing in UoD through Trimmed FTS and Winsorized FTS.
- Addition of the ARIMA model along with Two Years and Three Years Moving Averages, as a basic time series model for a better comparison to proposed methods.
- Simultaneous equation system for estimation of parameters of the proposed method.
- Interval estimation in support of future forecasting.
- Relative Efficiency for the overall performance of the proposed method.

## 2 Literature Review

In the modern scientific world, forecasting has a wide role in every field of business, run by either individuals or by a firm/ company. Data scientists help them in taking well-informed data-driven decisions based on relatively small forecast error rates. Uncertainty in the data and day-to-day changing scenarios of the country's economic policies either internal or external are the key factors that urge businessmen to adopt such a forecasting method to avoid any big loss in their business. It is obvious that the results of the forecast don't have a 100% ideal prediction but give us a future guess that any object in the future will look like that but not fully guaranteed. Therefore, choosing a good forecasting method is also a big question.

Fuzzy set theory has been used successfully to anticipate approaches in a variety of scientific domains. Yolcu and Lam [7] studied the performance basic model versus the proposed robust FTS models. Several adaptations are shown to demonstrate how the suggested model may produce more accurate and robust predicting results. For forecasting market prices, Qu, Zhang [8] suggested an RFTS forecasting method based on a multi-partition methodology and outlier identification. They concluded that the suggested strategy enhances robustness and AFER. Güler Dincer and Akkuş [9] investigated a novel FTS framework presented for monitoring air pollution based on the Fuzzy K-Medoid (FKM) clustering algorithm and concluded that the proposed technique delivers good forecasting results, particularly in time series with outliers. Gao and Duru [10] suggested a unique method for ensuring the parsimony principle of FTS models while maintaining a specific level of out-of-sample accuracy. They used MASE and RMSE to compare simple naïve forecasting to the proposed model for validation.

Guiffrida and Nagi [11] introduced a simulation-based model for forecasting the demand-supply of electric power having load components of day timings in form of linguistic terms. He has done fuzzy forecasting by dividing his basic logic into three categories as Delphi method for qualitative forecasting, time series by quantitative method, and regression analysis. Articles that have mostly been published so far used the University of Alabama's enrollment data as a test sequence for 20-year-long periods, AFER and MSE were used to compare models to select the best technology. Jilani, Burney [12] developed a first-order and time-variant model for enhanced fuzzy forecasting by employing frequency distribution partitioning of the enrollment historical data. Stevenson and Porter

[2] proposed a modified method for university enrollment data by using consecutive years' change in percentage as the UoD. Boiroju, Venugopala Rao [13] used FTS and ARIMA based on MAE, MSE, and MAPE with training and testing concepts to conduct a comparative analysis for US $ and Indian Rupee ER. Abdullah [14] proposed a distance-based FTS model for ER data of the New Taiwan Dollar (NTD) vs the US $. For the NTD/US $ ER, his approach excelled in the ANN and random walk models. In his work, he projected the Malaysian Ringgit (MYR) ER against US $ and examined its performance using a distance-based fuzzy time series model US $ ER. The experiment findings demonstrate that the predicted MYR/US $ ER performed better under the distance-based FTS model. Forecasting, according to Yusoff [15] is a challenging problem with significant nonlinearity and data irregularity. Furthermore, predicting ERs is reliant on inaccurate and inconsistent data. Forecasting model analysis that correlates to the ER has always been sensitive to swings. As a result, modeling ERs has become a tricky issue in business. Several studies have demonstrated that stand-alone forecasting models like time series, FTS, and Markov chain have flaws and are ineffective in prediction. As a result, he suggested a MCFTS hybrid model to anticipate the future ER. MCFTS is a time series data analysis paradigm that combines the standard FTS model with the Markov chain model. The proposed strategy was tested for robustness using the MAPE performance indicator. Finally, a MAPE assessment between the developed framework and the FTS model was performed. As a consequence, the proposed model's prediction ability was demonstrated to be superior to that of the FTS model. Maneesilp, Kruatrachue [16] investigated how the appropriateness of a forex system's parameters influences its efficiency using a performance indicator. The use of parameter forecasting with fuzzy time series is a straightforward and efficient solution. Its performance, however, is determined by the similarity of the time series pattern to learning and forecast, which is represented as a fuzzy connection. They created a technique called a time-varying fuzzy relationship. The system works better in all periods as a result of the forgetting component. The implications of time series length on forecasting system performance, as well as advantages derived from different approaches on predicting investment choices in actual markets, were explored. Fuzzy inference systems have been frequently used in the literature for time series forecasting, Bas, Yolcu [17]. Adaptive network fuzzy inference systems, FTS methodologies, and fuzzy regression functions approaches stand out among fuzzy inference systems. In recent years, intuitionistic fuzzy sets have been used in fuzzy modelling, and innovative fuzzy inference methods based on them have been published. They employed an intuitionistic fuzzy time series functions

method in their study, which is based on a new intuitionistic fuzzy regression functions technique for predicting based on intuitionistic fuzzy sets. They proposed a novel intuitionistic fuzzy inference system in their study and the obtained results from the intuitionistic FTS functions approach were compared to some other methods using RMSE and MAPE criteria, and the intuitionistic FTS functions approach outperformed all other methods in terms of forecasting performance. Bas, Egrioglu [18] developed PFTS, a FTS extension with a definition based on picture fuzzy sets. As a consequence, employing picture fuzzy sets rather than classical fuzzy sets contributes more information to the modeling procedure. He created a high-order multivariate PFTS forecasting model as well as an algorithm to go with it and observed that the proposed technique produces the best performance.

Asadullah, Bashir [19] did similar research to this work in the context of comparing Pak-US$ ERs predicted the ER using a blend of several Poon and Granger [20] models. They include three univariate time series models and one multivariate model Non-Linear Auto Regressive Distributive Lag (NARDL). They attempted to use several forecasting approaches to merge univariate models with NARDL to forecast the PKR-US $. Following that, the models were evaluated using the equal weightage and var-cor approaches. In terms of forecasting the ER, NARDL surpasses all individual time series models. Similarly, the combination of NARDL and Naive models outperformed all individual and combined models using MAPE, indicating that the Pakistani Rupee ER versus the US $ is determined by macroeconomic fundamentals and recent time-series observations. Muhammad, Ahmad [4] investigated predicting future prices of the Pakistani Rupee vs US $. They utilized the ARIMA model to anticipate future ERs on a given dataset. ARIMA (1,1,9) was shown to be the best model for forecasting the ER. Forecasting error was found to be less than 1%, indicating that ARIMA is a robust model that can assist government bodies, monetary policymakers, economists, and other stakeholders in identifying the future direction of the ER and making data-driven intelligent judgments. According to Cheng and Chen [21], FTS has been utilized to foresee future difficulties based on data. The suggested new method for advancing the weighted FTS model for forecasting issues is based on Apriori's large datasets. To verify the proposed model, gold price data sets and ERs are employed as experimental data sets. Their study compared the proposed model to other methods for forecasting accuracy, and the results revealed that the proposed technique outperformed other methods in the RMSE and MAPE criteria. Tsaur [22] conducted a comparison between the New Taiwan Dollar (NTD) and the United States Dollar (US $). He developed a FTS-Markov chain strategy for linguistic analysis, i.e. small sample time series data is recommended to improve prediction accuracy even more. He converted FTS data to a fuzzy logic group and then calculated a Markov chain transition matrix. He gained minimal forecasting error with adjusted parameters of several FTS approaches. Finally, an illustrative case of ER forecasting is utilized to validate the proposed model's performance with a very minimal MAPE. Leu, Lee [23] used a FTS model established to forecast stock prices and foreign currency rates, taken from the Central Bank of Taiwan. To forecast the ER, they suggested a novel FTS model called distance-based FTS (DBFTS). Unlike previous FTS models that demand a perfect match of the fuzzy logic relationships (FLRs), the distance-based FTS model determines prediction rules based on the distance between two FLRs. A two-factor distance-based FTS model was developed to forecast the ER. The first element in the model is the ER itself, and the second factor consists of various possible factors that impact the variation of ERs. According to the results, the distance-based FTS excelled over the random walk model and the artificial neural network model by MSE. Cheng, Chen [24] suggested a new FTS forecasting method and similarity initiatives between the subscript of the fuzzified historical testing datum's fuzzy set on the previous trading day and the subscripts of the fuzzy sets making an appearance in the current forms of the fuzzy logical relationships in the chosen fuzzy logical relationship group. PSO algorithms were utilized to optimize interval partitioning in the UoD. In terms of predicting accuracy, the experimental findings suggest that the proposed fuzzy forecasting approach beats existing methods. Chen and Phuong [25] suggested a new FTS forecasting approach based on optimum UoD interval partitions and optimal TSFTLRG weighting vectors. In the suggested method, PSO methods were employed to achieve optimal interval partitions as well as optimal weighting vectors at the same time. In terms of forecasting accuracy, the proposed FTS forecasting method gives an efficient for forecasting the TAIEX and the NTD/US $ ERs. Long memory time series, according to  Sadaei, Enayatifar [26] are stationary processes with statistical long range dependency between the present value and values at other periods in the series, with delayed decay of the autocorrelation function. For long-term memory time series forecasting, they developed a hybrid technique that combines ARFIMA models with FTS. The suggested approaches are divided into two stages:  Autoregressive Model and Simple Moving Average Methods. The combined ARFIMA and FTS model was introduced based on methology. For parameter estimation, the PSO approach was applied. The suggested ARFIMA–FTS technique was applied to two stock index databases, TAIEX and DJIA, as well as ER data of nine major currencies versus the US $, concluding that the proposed hybrid method outperformed traditional ARFIMA models.

Maciel and Ballini [27] introduced a fuzzy rule-based modeling technique (iFRB) for forecasting interval-valued data. iFRB is a fuzzy rule-based model with affine implications that offers a nonlinear way of processing interval-valued data. They used a real dataset to estimate the one-step-ahead forecast of EUR/US $ and BRL/US $ ERs with interval values. For both low and high ER prices, iFRB forecasts outscored classic econometric time series approaches and interval models. The results reveal that iFRB excels in the random walk and other competing techniques in out-of-sample interval-valued ER forecasting, implying that the proposed method is a potential option for financial ITS forecasting. Mukminin, Irawanto [28] provided a novel approach for forecasting ER data by combining certain models, (1) they utilized the average-based interval to generate ideal interval numbers, and (2) they used frequency density-based partitioning for optimal partitioning. Later, they subdivided the three highest frequency intervals into 4, 3, and 2 sub-intervals, eliminating intervals if no data was disseminated. (3) The FLRG was built using K-means clustering, which separated the FLR into 16 initial clusters. MSE and AFER assessed the model. Daily ER data (US $ to IDR) from January to May, with its erratic volatility induced by Pandemic Covid-19, was utilized as a case study to compare the proposed model's performance to rivals. Their research intends to develop a forecasting model using ER (US $ to IDR) data as a means of preparing for and evaluating future scenarios. Reyes, Aké [29] conducted a comparison of the volatility forecasting of traditional time series models, such as ARIMA, EGARCH, and PARCH, with two new proposed models based on fuzzy theory i.e. FTS-Fuzzy ARIMA Tseng's and FTS-Fuzzy ARIMA Tanaka's, and applied them to the Mexican Peso - US $ ER. The outcome of their study is that models based on fuzzy theory produce a more accurate estimate of volatility. In both in and out-of-sample testing, the fuzzy models exhibit the least forecast error than the typical time series. As a result, the fuzzy models were more efficient and better-mirrored market information. Permana and Fitri [30] used Bank Indonesia data to develop a model for dynamic currency rate data between Riyals and Rupiah. The study attempted to forecast the currency rate between Riyal and Rupiah in the future using the MCFTS approach using AFER and MEA criteria and managed forecasting for the next ten days.

Efendi, Deris [31] attempted to include FTS into non-stationary time series data forecasting, such as electrical load demand, ERs, university enrolment, and so on. The proposed data predictions were created by combining the weightage and linguistic out-sample approaches. The findings indicate that the FTS may be used to improve the accuracy and efficiency of these non-stationary data forecasting prospects. Forex forecasting has been the topic of multiple thorough analyses because of its relevance in appraising the rewards and hazards of international business contexts, according to Efendi, Ismail [32]. Many strategies have been investigated with the ultimate objective of improving the forecasting method's reliability and efficiency. However, because data is dynamic and complicated, developing an effective forecasting approach remains a challenging task. They developed a new weight for the FTS model to forecast the ER market on daily basis. To get regularly repeating FLR in the FLG, weights were applied to the fuzzy relationships using a probabilistic technique. The US $ to Malaysian ringgit (MYR) ERs dataset were analyzed and the suggested method's efficiency is compared to other conventional methods, revealing that the proposed method has improved the forecast accuracy. Tinh [33] explored how the partition UoD, the formation of FRGs, and the defuzzification of forecasting output values impact the predicting outcomes of these models. For the aforementioned concerns, he proposed a hybrid FTS model that integrated PSO and FCM. FCM clustering was used to partition the historical data into initial intervals of varying sizes. The data was then fuzzified into fuzzy sets to construct FLGs in chronological sequence. FLGs data assisted in forecasting value using a novel defuzzification approach. The PSO method assisted in obtaining optimal interval lengths in the UoD to improve forecast accuracy. In comparison to competing models, the proposed model exhibits greater forecast accuracy against both first-order and high-order FLR. Singh [34] reviewed prior research in the FTS field and recognized domain-specific difficulties and research trends, and attempted to classify them with implications for future study. Wulandari, Surarso [35] provided a new advancement in establishing the UoD, historical data variation, and partitioning stage. They tried to define the UoD for computing the base value to determine how many intervals should be employed. Second, the primary intervals are divided into several sub-intervals. According to the empirical data, the sub-interval led the fuzzy number to move closer to a crisp and better-forecasted value. For simulation, annual petroleum production data from Indonesia were utilized. The updated technique excelled in earlier approaches in forecasting with lower MSE and AFER. Xian and Cheng [36] investigated the gaps in the combination of n-PFTS and time series, and then proposed n-PFTS and its forecasting method n-IMWPFCM to employ n-PFCM to overcome the subjectivity of directly assigning membership and non-membership values, thereby improving the accuracy of partitioning the UoD. As a consequence, the suggested technique surpasses existing models in order to forecast accuracy. All of the examples presented in this section demonstrate the utility of FTS strategies for predicting in various research problems and decision making.

## 3 Research Methodology

In this study, we introduced robust modifications to the model [1] to enhance the forecasting accuracy of the modified model, as evident by AFER. The model [1] is based on the Stevenson and Porter Model [2], where they directly applied their proposed model to Ukrainian's enterprise net income level without checking the nature of the data. The presence of outliers and heterogeneity in the data restricts the model performance. Therefore, before applying any specific model, data visualization should be given utmost preference to see the actual behavior of the data. Therefore, data cleaning is an important statistical feature that guarantees the accuracy of the model. Trimming and winsorization are two statistical techniques that help in removing the contamination from the data and eventually the model with homogeneity gains more accurate results.

Consider a time series of an economic indicator, denoted by $y_i$, $i = 2, 3, ..., n$. For modeling this time series, the considered method uses the following indicator as chain growth rate:

$$T_i = \left( \frac{y_i}{y_{i-1}} - 1 \right) \times 100\ \% \qquad (1)$$

### 3.1 Trimmed Technique

For trimming (1), calculate the lower quartile $Q_1$ and upper quartile $Q_3$ against (1) and eliminate all values that occur before $Q_1$ and preceding to $Q_3$. So, we are left with 50% of the total data in between these cutpoints as the growth rate becomes $K_i = [Q_1\ , Q_3]$ and our modified model contains the following steps:

**Step 1.** Determine the universe of discourse UoD as the Set $U$, $U = [\min K_i, \max K_i]$ where, $i = 2, 3, ..., n$ and divide into equal "m" intervals by using the frequency distribution class boundaries method.

$$m = 1 + 3.3logN \qquad (2)$$

Where "m" is the class interval and "N" is the total number of observations in "U".

**Step 2.** After frequency distribution, the large classes are subdivided into smaller partitions for accuracy. In our trimmed dataset, we have divided the large classes into 4-2-3 respectively for improving accuracy.

**Step 3.** Fuzzy sets $X_k$, $k = 1, 2, 3, ..., m$ on each partition interval as the triangular fuzzy number, which carriers are defined by three values: lower limit, midpoint, and upper limit. For the trimmed dataset, it is to determine which fuzzy set will describe each value. So that fuzzifies the dataset into the initial series.

**Step 4.** Defuzzification of the fuzzy set to crisp values is done by the following formula:

$$t_j = \begin{cases} \frac{1+0.5}{\frac{1}{a_1}+\frac{0.5}{a_2}} & j = 1 \\ \frac{0.5+1+0.5}{\frac{0.5}{a_{j-1}}+\frac{1}{a_j}+\frac{0.5}{a_{j+1}}} & 2 \leq j \leq m-1 \\ \frac{0.5+1}{\frac{0.5}{a_{j-1}}+\frac{1}{a_j}} & j = m \end{cases} \tag{3}$$

Where $a_{j-1}$, $a_j$, and $a_{j+1}$ are the midpoints of the interval of fuzzy sets $X_{k-1}$, $X_k$, and $X_{k+1}$ carriers respectively, and initially, the value 0.5 was assigned to each parameter as a specified value.

$$\hat{y}_i = y_{i-1}\left(1+\frac{t_j}{100}\right) \tag{4}$$

Where $i = 2,3,4,...,n \,\&\, j = 1,2,3,...,m$

### 3.2 Winsorized Technique

For winsorizing (1), calculate the lower quartile $Q_1$ and upper quartile $Q_3$ against (1) and replace all values with $Q_1$ that occurs before $Q_1$ and and with $Q_3$ that falls above $Q_3$ value. So, the data becomes homogeneous as the growth rate becomes $W_i = [Q_1, Q_3]$ and 15-2-2-10 sub-partitioned the frequency distribution's larger classes for better accuracy, our modified model follows the same steps as discussed in 3.1.

## 4 Case Study

Forex plays an important role in a country's economy. Pakistan's international trade is linked to the US $ so, any fluctuation in the exchange rate has a severe effect on Pakistan's economy as under developing country. An available open-source full dataset of annual PKR-US $ from 1960 to 2021 has been taken from World Bank's website and presented graphically in figure 1 as follows:

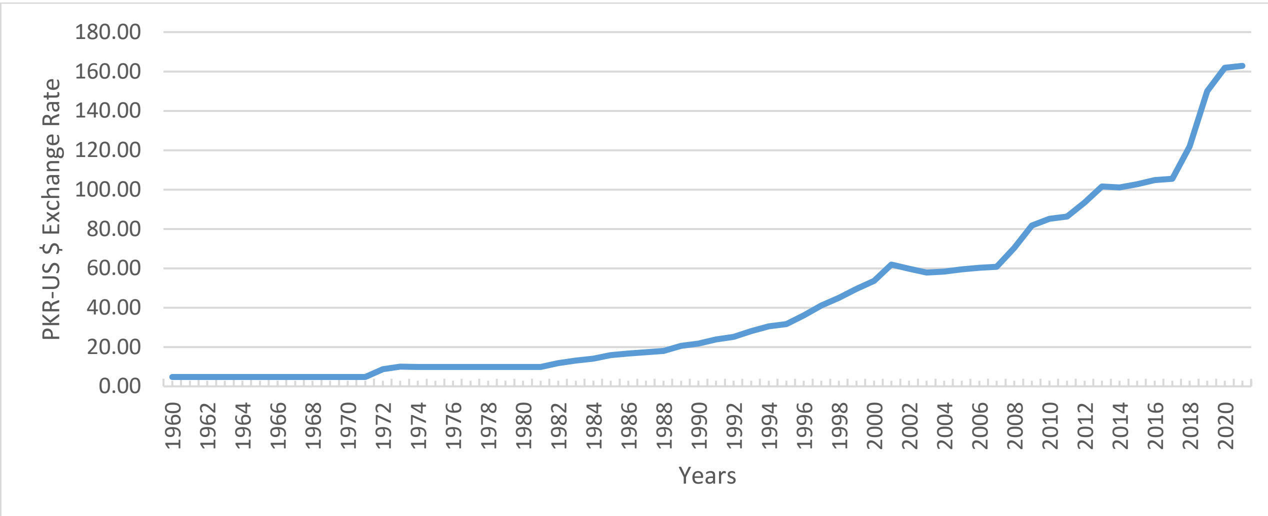


Fig. 1. PKR performance against US $ from 1960 to 2021

## 4.1 Trimmed FTS Calculations

The growth rates as (1) were calculated, trimmed, and presented in table 1 as follows:

**Table 1.** Year to year Trimmed PKR US $ exchange rate and corresponding Growth Rate (T)

| Years | PKR-USD (y) | T | Years | PKR-USD (y) | T |
|---|---|---|---|---|---|
| 1974 | 9.9 | 0 | 1990 | 21.707375 | 3.474329647 |
| 1975 | 9.9 | 0 | 1991 | 23.80076667 | 3.520483011 |
| 1976 | 9.9 | 0 | 1992 | 25.08279167 | 4.259945509 |
| 1977 | 9.9 | 0 | 1993 | 28.10718333 | 4.512937623 |
| 1978 | 9.9 | 0 | 1994 | 30.56659167 | 4.514684733 |
| 1979 | 9.9 | 0 | 1995 | 31.64268333 | 5.38648615 |
| 1980 | 9.9 | 0 | 1996 | 36.07868333 | 5.67574815 |
| 1981 | 9.9 | 0.62583467 | 1997 | 41.111525 | 7.085157468 |
| 1982 | 11.84746667 | 0.654816108 | 1998 | 45.04666667 | 7.866249352 |
| 1983 | 13.116975 | 0.775129393 | 1999 | 49.50069158 | 8.167173461 |
| 1984 | 14.04633333 | 0.875929311 | 2000 | 53.6481865 | 8.378660548 |
| 1985 | 15.92839167 | 1.271724232 | 2001 | 61.92716167 | 8.750105993 |
| 1986 | 16.64750833 | 1.349354985 | 2002 | 59.72378167 | 8.815980081 |
| 1987 | 17.3988 | 1.651020496 | 2003 | 57.75199667 | 9.571869851 |

| | | | | | |
|---|---|---|---|---|---|
| 1988 | 18.00329167 | 1.945956605 | 2004 | 58.25786333 | 9.643688685 |
| 1989 | 20.54149167 | 2.15698207 | | | |

In Table 1, we observed that the growth rate varies from 0 to 9.64 as the minimum and maximum limits. The universe of discourse as a set can be given as $U = [0, 9.64\,]$, and its frequency distribution using class boundaries limits system by using (2), huge contributing classes i.e 1st, 4th, and 7th were further divided into 4-3-2 sub-intervals, fuzzy sets $X_i,\ i = 1,2,3,\dots,13$ were applied in the fuzzification process. By using (3), we can fit our proposed model (4) to defuzzified data as in table 2.

**Table 2. Forecasting for Trimmed PKR-USD Exchange Rate**

| **Years** | $\mathbf{y_i}$ | **T** | $\mathbf{X_k}$ | $\mathbf{t_j}$ | $\mathbf{\hat{y}_i}$ | **AEFR** | **Years** | $\mathbf{y_i}$ | **T** | $\mathbf{X_k}$ | $\mathbf{t_j}$ | $\mathbf{\hat{y}_i}$ | **AEFR** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1974 | 9.9 | 0 | X1 | 0.221412 | | | 1991 | 23.80077 | 3.520483 | X6 | 3.10573 | 22.38155 | 0.001988 |
| 1975 | 9.9 | 0 | X1 | 0.221412 | 9.92192 | 7.38E-05 | 1992 | 25.08279 | 4.259946 | X7 | 4.29833 | 24.8238 | 0.000344 |
| 1976 | 9.9 | 0 | X1 | 0.221412 | 9.92192 | 7.38E-05 | 1993 | 28.10718 | 4.512938 | X7 | 4.29833 | 26.16093 | 0.002308 |
| 1977 | 9.9 | 0 | X1 | 0.221412 | 9.92192 | 7.38E-05 | 1994 | 30.56659 | 4.514685 | X7 | 4.29833 | 29.31532 | 0.001365 |
| 1978 | 9.9 | 0 | X1 | 0.221412 | 9.92192 | 7.38E-05 | 1995 | 31.64268 | 5.386486 | X8 | 5.182874 | 32.15082 | 0.000535 |
| 1979 | 9.9 | 0 | X1 | 0.221412 | 9.92192 | 7.38E-05 | 1996 | 36.07868 | 5.675748 | X9 | 6.171463 | 33.5955 | 0.002294 |
| 1980 | 9.9 | 0 | X1 | 0.221412 | 9.92192 | 7.38E-05 | 1997 | 41.11153 | 7.085157 | X10 | 7.367015 | 38.73661 | 0.001926 |
| 1981 | 9.9 | 0.625835 | X2 | 0.369019 | 9.936533 | 0.000123 | 1998 | 45.04667 | 7.866249 | X10 | 7.367015 | 44.14022 | 0.000671 |
| 1982 | 11.84747 | 0.654816 | X2 | 0.369019 | 9.936533 | 0.005376 | 1999 | 49.50069 | 8.167173 | X10 | 7.367015 | 48.36526 | 0.000765 |
| 1983 | 13.11697 | 0.775129 | X3 | 0.786171 | 11.94061 | 0.002989 | 2000 | 53.64819 | 8.378661 | X11 | 8.349658 | 53.63383 | 8.92E-06 |
| 1984 | 14.04633 | 0.875929 | X3 | 0.786171 | 13.2201 | 0.001961 | 2001 | 61.92716 | 8.750106 | X12 | 8.943064 | 58.44598 | 0.001874 |
| 1985 | 15.92839 | 1.271724 | X4 | 1.210506 | 14.21637 | 0.003583 | 2002 | 59.72378 | 8.81598 | X12 | 8.943064 | 67.46535 | 0.004321 |
| 1986 | 16.64751 | 1.349355 | X4 | 1.210506 | 16.12121 | 0.001054 | 2003 | 57.752 | 9.57187 | X13 | 9.255858 | 65.25173 | 0.004329 |
| 1987 | 17.3988 | 1.65102 | X5 | 1.915966 | 16.96647 | 0.000828 | 2004 | 58.25786 | 9.643689 | X13 | 9.255858 | 63.09744 | 0.002769 |
| 1988 | 18.00329 | 1.945957 | X5 | 1.915966 | 17.73216 | 0.000502 | Total | | | | | | 0.046725 |
| 1989 | 20.54149 | 2.156982 | X5 | 1.915966 | 18.34823 | 0.003559 | **%AFER** | | | | | | **0.155751** |
| 1990 | 21.70738 | 3.47433 | X6 | 3.10573 | 21.17945 | 0.000811 | | | | | | | |

## 4.2 Winsorized FTS Calculations

By following section 3.2 for Winsorized scheme calculation, we proceeded with the steps as in the trimmed scheme. We observed that the growth rate varies from 0 to 9.89 as the minimum and maximum limits. The universe of discourse $U$ is given as $U = [0, 9.89]$ and its frequency distribution using class boundaries limits system by using (2), huge contributing classes i.e. 1st, 4th, 6th, and 7th were further divided into 15-2-2-10 sub-intervals, fuzzy sets $X_i,\ i = 1,2,3,\dots,32$ were applied in the fuzzification process. By using (3), we can fit our proposed model (4) to defuzzified data and represent its final form as in table 3.

**Table 3. Forecasting for Winsorized PKR-USD Exchange Rate**

| Years | $y_i$ | T | $X_k$ | $t_j$ | $\hat{y}_i$ | AFER | Years | $y_i$ | T | $X_k$ | $t_j$ | $\hat{y}_i$ | AFER |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1960 | 4.7619 | | | | | | 1992 | 25.08279 | 3.47433 | X17 | 3.185054 | 24.55883 | 0.000342 |
| 1961 | 4.7619 | 0 | X1 | 0.060551 | 4.764783 | 9.93E-06 | 1993 | 28.10718 | 3.520483 | X17 | 3.185054 | 25.88169 | 0.001298 |
| 1962 | 4.7619 | 0 | X1 | 0.060551 | 4.764783 | 9.93E-06 | 1994 | 30.56659 | 4.259946 | X18 | 4.408114 | 29.34618 | 0.000655 |
| 1963 | 4.7619 | 0 | X1 | 0.060551 | 4.764783 | 9.93E-06 | 1995 | 31.64268 | 4.512938 | X18 | 4.408114 | 31.914 | 0.000141 |
| 1964 | 4.7619 | 0 | X1 | 0.060551 | 4.764783 | 9.93E-06 | 1996 | 36.07868 | 4.514685 | X18 | 4.408114 | 33.03753 | 0.001382 |
| 1965 | 4.7619 | 0 | X1 | 0.060551 | 4.764783 | 9.93E-06 | 1997 | 41.11153 | 5.386486 | X19 | 5.314751 | 37.99618 | 0.001242 |
| 1966 | 4.7619 | 0 | X1 | 0.060551 | 4.764783 | 9.93E-06 | 1998 | 45.04667 | 5.675748 | X20 | 6.266156 | 43.68764 | 0.000495 |
| 1967 | 4.7619 | 0 | X1 | 0.060551 | 4.764783 | 9.93E-06 | 1999 | 49.50069 | 7.085157 | X21 | 7.268722 | 48.32098 | 0.000391 |
| 1968 | 4.7619 | 0 | X1 | 0.060551 | 4.764783 | 9.93E-06 | 2000 | 53.64819 | 7.866249 | X22 | 8.027961 | 53.47459 | 5.3E-05 |
| 1969 | 4.7619 | 0 | X1 | 0.060551 | 4.764783 | 9.93E-06 | 2001 | 61.92716 | 8.167173 | X22 | 8.027961 | 57.95504 | 0.001052 |
| 1970 | 4.7619 | 0 | X1 | 0.060551 | 4.764783 | 9.93E-06 | 2002 | 59.72378 | 8.378661 | X22 | 8.027961 | 66.89865 | 0.001969 |
| 1971 | 4.7619 | 0 | X1 | 0.060551 | 4.764783 | 9.93E-06 | 2003 | 57.752 | 8.750106 | X24 | 8.683636 | 64.90998 | 0.002032 |
| 1972 | 8.681383 | 0 | X1 | 0.060551 | 4.764783 | 0.007396 | 2004 | 58.25786 | 8.81598 | X25 | 8.82494 | 62.84858 | 0.001292 |
| 1973 | 9.99425 | 0 | X1 | 0.060551 | 8.686639 | 0.002145 | 2005 | 59.51448 | 9.57187 | X30 | 9.531453 | 63.81068 | 0.001183 |
| 1974 | 9.9 | 0 | X1 | 0.060551 | 10.0003 | 0.000166 | 2006 | 60.27134 | 9.643689 | X31 | 9.672754 | 65.27116 | 0.00136 |
| 1975 | 9.9 | 0 | X1 | 0.060551 | 9.905995 | 9.93E-06 | 2007 | 60.73852 | 9.89 | X32 | 9.76752 | 66.15835 | 0.001463 |
| 1976 | 9.9 | 0 | X1 | 0.060551 | 9.905995 | 9.93E-06 | 2008 | 70.40803 | 9.89 | X32 | 9.76752 | 66.67116 | 0.00087 |
| 1977 | 9.9 | 0 | X1 | 0.060551 | 9.905995 | 9.93E-06 | 2009 | 81.71289 | 9.89 | X32 | 9.76752 | 77.28515 | 0.000888 |
| 1978 | 9.9 | 0 | X1 | 0.060551 | 9.905995 | 9.93E-06 | 2010 | 85.19382 | 9.89 | X32 | 9.76752 | 89.69421 | 0.000866 |
| 1979 | 9.9 | 0 | X1 | 0.060551 | 9.905995 | 9.93E-06 | 2011 | 86.34338 | 9.89 | X32 | 9.76752 | 93.51514 | 0.001362 |
| 1980 | 9.9 | 0 | X1 | 0.060551 | 9.905995 | 9.93E-06 | 2012 | 93.3952 | 9.89 | X32 | 9.76752 | 94.77699 | 0.000243 |
| 1981 | 9.9 | 0 | X1 | 0.060551 | 9.905995 | 9.93E-06 | 2013 | 101.6289 | 9.89 | X32 | 9.76752 | 102.5176 | 0.000143 |
| 1982 | 11.84747 | 0 | X1 | 0.060551 | 9.905995 | 0.002686 | 2014 | 101.1001 | 9.89 | X32 | 9.76752 | 111.5555 | 0.001695 |
| 1983 | 13.11697 | 0.625835 | X7 | 0.604903 | 11.91913 | 0.001497 | 2015 | 102.7693 | 9.89 | X32 | 9.76752 | 110.9751 | 0.001309 |
| 1984 | 14.04633 | 0.654816 | X7 | 0.604903 | 13.19632 | 0.000992 | 2016 | 104.7691 | 9.89 | X32 | 9.76752 | 112.8073 | 0.001258 |
| 1985 | 15.92839 | 0.775129 | X9 | 0.795036 | 14.15801 | 0.001822 | 2017 | 105.4552 | 9.89 | X32 | 9.76752 | 115.0025 | 0.001484 |
| 1986 | 16.64751 | 0.875929 | X10 | 0.88982 | 16.07013 | 0.000569 | 2018 | 121.8241 | 9.89 | X32 | 9.76752 | 115.7555 | 0.000817 |
| 1987 | 17.3988 | 1.271724 | X14 | 1.268067 | 16.85861 | 0.000509 | 2019 | 150.0363 | 9.89 | X32 | 9.76752 | 133.7233 | 0.001782 |
| 1988 | 18.00329 | 1.349355 | X15 | 1.46914 | 17.65441 | 0.000318 | 2020 | 161.8385 | 9.89 | X32 | 9.76752 | 164.6911 | 0.000289 |
| 1989 | 20.54149 | 1.65102 | X16 | 2.041833 | 18.37089 | 0.001732 | 2021 | 162.8513 | 9.89 | X32 | 9.76752 | 177.6461 | 0.001489 |
| 1990 | 21.70738 | 1.945957 | X16 | 2.041833 | 20.96091 | 0.000564 | Total | | | | | | 0.052555 |
| 1991 | 23.80077 | 2.156982 | X16 | 2.041833 | 22.1506 | 0.001137 | **%AFER** | | | | | | **0.086156** |

The trimmed FTS forecast for annual PAK US \$ forex 2022 value by (3) using the last fuzzy set definition $X_{13}$ gives us 177.9246 ER as the estimated value with (130.8342, 225.0151) as interval estimation against the full data set while the winsorized FTS forecasted value for annual PAK US \$ forex 2022 by (3) using the last fuzzy set definition $X_{32}$ is 178.7579 with corresponding interval estimation as (134.4681, 223.0476). This interval estimation for the annual 2022 will have huge consequences on Pakistan's economy either positive or negative.

The forecasting accuracy of (3) may be further enhanced by adjusting (2) using the optimized parameters rather than specified values as 0.5 for all parameters. The parameters $\gamma_1$, $\gamma_2$, $\delta_1$ and $\delta_2$ are to be optimized for said purpose. The adjusted defuzzified $t_j^{adj}$ is given as under:

$$t_j^{adj} = \begin{cases} \frac{1+\gamma_1}{\frac{1}{a_1}+\frac{\gamma_1}{a_2}} & j = 1 \\ \frac{\delta_1+1+\delta_2}{\frac{\delta_1}{a_{j-1}}+\frac{1}{a_j}+\frac{\delta_2}{a_{j+1}}} & 2 \le j \le m-1 \\ \frac{\gamma_2+1}{\frac{\gamma_2}{a_{j-1}}+\frac{1}{a_j}} & j = m \end{cases} \quad (5)$$

For optimization of these parameters, a simultaneous equation system through Maple-17 was used and found the optimized values for the trimmed modified model as $\gamma_1 = 0.500137$, $\gamma_2 = 0.500148$, $\delta_1$ = -0.386331 and $\delta_2$= -0.656105. AFER under (4) through (3) was found at 0.8147% which disturbed the forecast accuracy due to high spikes in the years 1995 and 1996 adjusted parameter values and we intentionally left this natural ambiguity to open the research forum for more learning. The optimized parameter values for winsorized modified model were estimated as $\gamma_1 =$ 0.371835, $\gamma_2 = 0.529928$, $\delta_1 = 0.087826$ and $\delta_2$= 0.01588. AFER under (4) through (3) was found at 0.08605%, improving than specified parameter values by 0.12850%. The forecasted value under (4) for the annual 2022 value using the last fuzzy set definition $X_{32}$ gives us 178.7548 as estimated PKR US $ forex with (134.4662, 223.0435) as interval estimation. The impact of this interval estimation will be huge for Pakistan's floating economy in 2022. The overall forecast performance under (4) is illustrated in figure 2 as under:

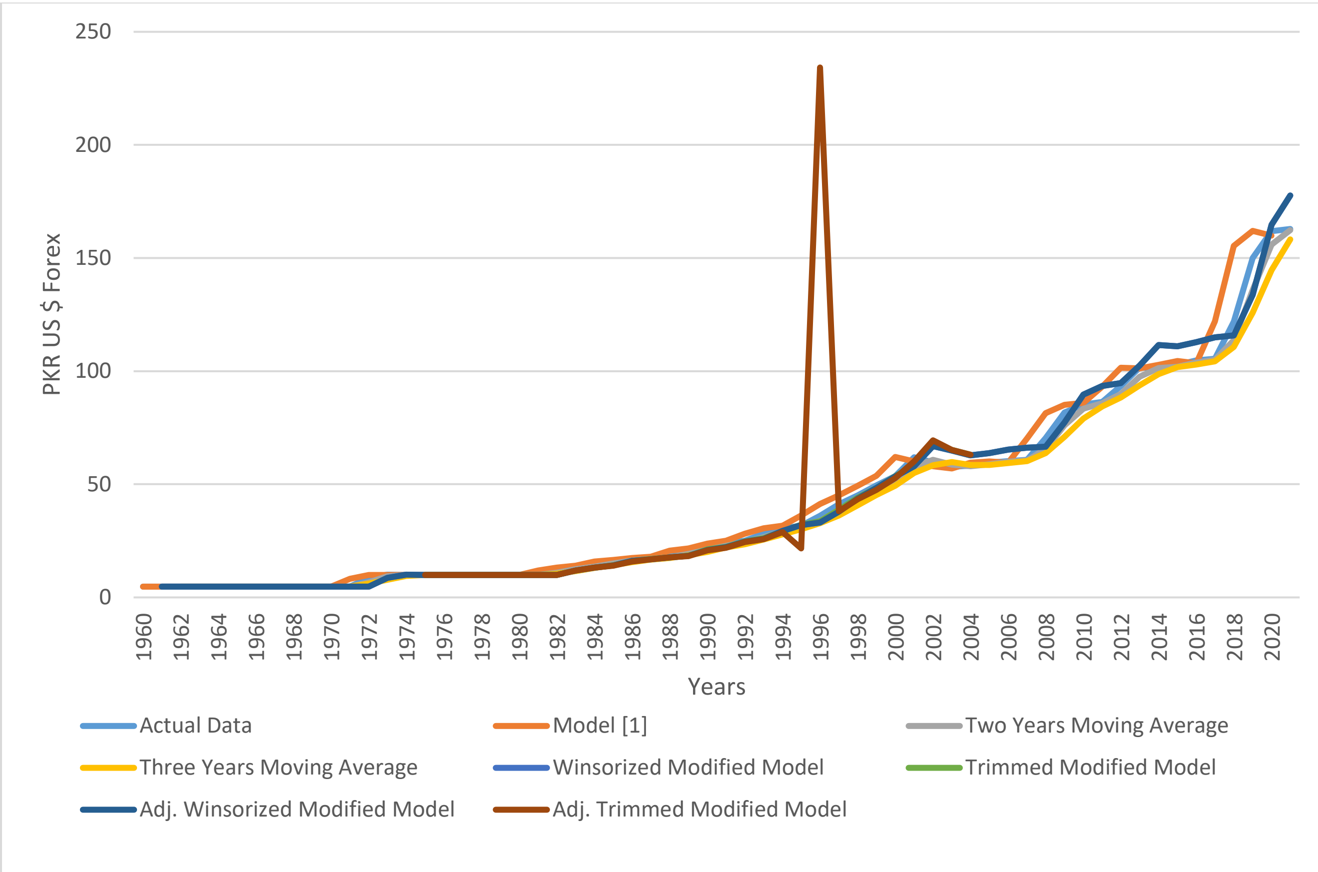


Fig. 2 The overall performance summary of PKR-US $ Actual Data, Model [1], Two years and Three years moving average time series methods, Trimmed FTS Model, Winsorized FTS Model, Adj. Trimmed FTS Model and Adj. Winsorized FTS Model.

The comparative analysis of overall performance as shown in figure 2 reveals that Winsorized FTS Model with adjusted parameters has much closer contact with actual data points and has the least AFER than all studied methods. The forecasting tendency of this model confirms the most trustful forecast.

**4.3 ARIMA MODEL** By using Gretl, we got ARIMA (0,2,2) through AIC and BIC model selection criteria for the case study with highly significant parameters, the results are as, given in Table 4(a), Table 4(b), Table 5, and figure 3.

Table 4(a). ARIMA (0,2,2) Estimation

| Model | Coefficient | Std. Error | z | P-Value |
|---|---|---|---|---|
| Constant | 0.132755 | 0.0413627 | 3.210 | 0.0013 *** |
| $\phi_1$ | −0.261625 | 0.0892098 | −2.933 | 0.0034 *** |
| $\phi_2$ | −0.738375 | 0.0828830 | −8.909 | 5.17e-019 *** |

Table 4(b). ARIMA (0,2,2) Performance

| | | | |
|---|---|---|---|
| Mean Dependent Var | 0.016881 | S.D. Dependent Var | 4.577102 |
| Mean Of Innovations | −0.171839 | S.D. Of Innovations | 3.340195 |
| R-Squared | 0.993860 | Adjusted R-Squared | 0.793754 |
| Log-Likelihood | −159.4014 | Akaike Criterion | 326.8028 |
| Schwarz Criterion | 335.1802 | AFER | 0.830797 |

ARIMA (0,2,2) model forecast for the next 5 years is as under:

Table 5. ARIMA (0,2,2) Forecasting using 95% Confidence Intervals

| **Years** | **Prediction** | **Std. Error** | **95% C.I** |
|---|---|---|---|
| 2022 | 167.911 | 3.34020 | (161.365, 174.458) |
| 2023 | 174.806 | 6.69869 | (161.677, 187.935) |
| 2024 | 181.834 | 8.86499 | (164.459, 199.209) |
| 2025 | 188.994 | 10.5973 | (168.224, 209.765) |
| 2026 | 196.287 | 12.0838 | (172.603, 219.971) |

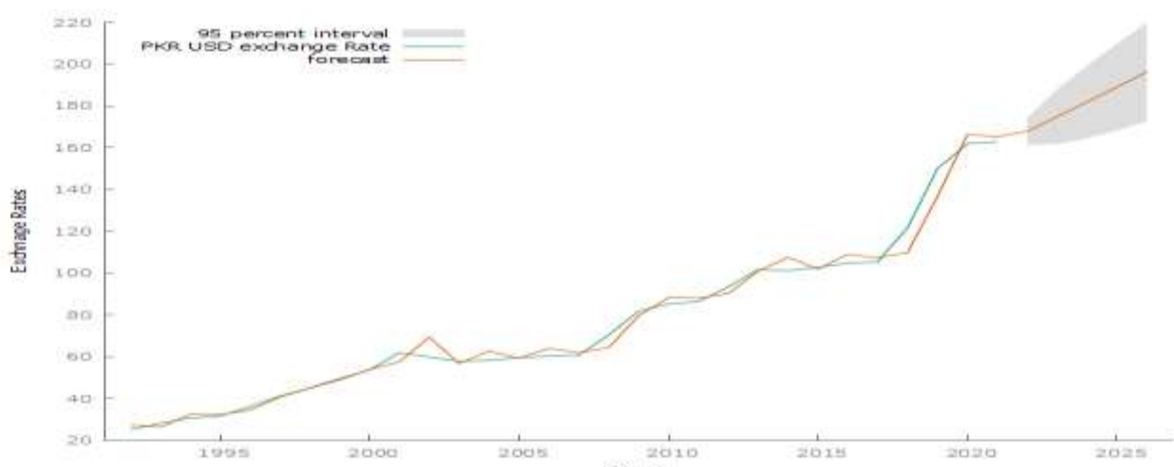

Fig. 3 Actual Data Versus ARIMA (0,2,2) Plotting and Forecasting

The comprehensive comparative analysis of all competitive models in this study is given in table 6, the fuzzy proposed model that outperformed all the competitive class models is specified as well as with optimized parameters by the least AFER (highlighted as bold). The relative efficiency for the said case is better than all methods, highlighted in bold. We witnessed in figure 2 that the proposed models have closer contact with actual data points and hence perform better than all studied methods.

**Table 6. Overall Comparison of All Models**

| Model | | | AFER (%) | Forecast for 2022 | 95% C.I | Relative Efficiency |
|---|---|---|---|---|---|---|
| **Specified Parameters** | SP | | 0.9412 | 158.6337 | 117.8123, 202.3021 | 100 |
| | Proposed | Trimmed | 0.1558 | 177.9246 | 130.8342, 225.0151 | 604.1078 |
| | | Winsorized | **0.0862** | 178.7579 | 134.4681, 223.0476 | **1091.879** |
| **Adjusted Parameters** | SP | | 0.8791 | 161.2916 | 119.1294, 201.7119 | 107.064 |
| | Proposed | Trimmed | 0.8147 | 168.13 | 122.4102, 216.6529 | 115.5272 |
| | | Winsorized | **0.0861** | 178.7548 | 134.4662, 223.0435 | **1093.148** |
| Two Years Moving Average | | | 4.7041 | 157.493 | 122.1825, 196.1081 | 20.00808 |
| Three Years Moving Average | | | 9.2367 | 156.2387 | 122.9123, 199.2912 | 10.18979 |
| ARIMA | | | 0.8308 | 167.911 | 161.3650, 174.4580 | 113.2884 |

# 5 Discussion and Conclusion

It was observed that robust methods like model [1] and conventional time series models can be applied to PKR-US $ data and gives a minimum AFER with the highest relative efficiency against all competitive methods. In RFTS methods, the proposed WFTS outperformed TFTS in forecasting i.e. minimum AFER and higher relative efficiency. WFTS can be enhanced for further research for more effective RFTS methods. Besides, the idea extension under model [1], interval estimation contains more information for the future and makes the idea more appealing to the goal of the study under WFTS. One step ahead of short-term prediction for the year 2022 with interval estimation will be useful for all businessmen across the country.

This RFTS study is an attempt to find out a more reliable forecasting method along with a point estimate for the future with minimum AFER among all competitive techniques. We found that the partitioning of UoD plays an important role in the forecasting accuracy of the concerned method and its partitioning is a subjective matter for different data sets rather than fixed to be called natural partitioning as 4-3-2 as used in the model [1], the natural sub-partitioning rule as 4-3-2 may not be accurate for all studies as like in our study where we proposed 4-2-3 and 15-2-2-10 sub-partitioning rule under TFTS and WFTS respectively and got the targeted results. So, the smaller the partitioning of the UoD, the larger the forecast accuracy can be obtained. The available literature is lacking in a multiple-step ahead forecast concerning RFTS. Therefore, RFTS methods open new dimensions of research studies towards modeling economic indicators for a country's betterment as better forecasting methods than conventional methods and will help make policies by data-driven wiser decision(s) towards the country's betterment.

## Appendix

| S# | Abbreviation | Full Form | S# | Abbreviation | Full Form |
|---|---|---|---|---|---|
| 1 | FTS | Fuzzy Time Series | 17 | PARCH | Power Autoregressive Conditional Heteroscedasticity |
| 2 | AER | Annual Exchange Rates | 18 | MCFTS | Markov Chain Fuzzy Time Series |
| 3 | AFER | Average Forecast Error Rate | 19 | FCM | Fuzzy C-Means Clustering |
| 4 | MA | Moving Average | 20 | *n*-PFTS | *n*-Pythagorean Fuzzy Sets |
| 5 | IMF | International Monetary Fund | 21 | *n*-IMWPFCM | n-Improved Markov Weighted Pythagorean Fuzzy C-Mean Clustering Method |
| 6 | ARIMA | Autoregressive Integrated Moving Average | 22 | *n*-PFCM | *n*-Pythagorean Fuzzy C-Means Clustering Method |
| 7 | MAE | Mean Absolute Error | 23 | UoD | Universe Of Discourse |
| 8 | MSE | Mean Squared Error | 24 | FOREX | Foreign Exchange Rate |
| 9 | MAPE | Mean Absolute Percent Error | 25 | EGARCH | Exponential General Autoregressive Conditional Heteroskedastic |
| 10 | ER | Exchange Rates | 26 | ARFIMA | Auto Regressive Fractional Integrated Moving Average |
| 11 | ANN | Artificial Neural Network | 27 | DJIA | Dow Jones Industrial Average |
| 12 | TAIEX | Taiwan Stock Exchange Capitalization Weighted Stock Index | 28 | iFRB | Rule-Based Modeling Approach |
| 13 | PFTS | Picture Fuzzy Time Series | 29 | FLRG | Fuzzy Logical Relationship Group |
| 14 | PSO | Particle Swarm Optimization | 30 | FLR | Fuzzy Logical Relationship |
| 15 | TSFTLRGS | Two-Factors Second-Order Fuzzy-Trend Logical Relationship Groups | 31 | GA | Genetic Algorithm |
| 16 | RFTS | Robust Fuzzy Time Series | 32 | MASE | Mean Absolute Scaled Error |
| 17 | TFTS | Trimmed Fuzzy Times Series | | | |
| 18 | WFTS | Winsorized Fuzzy Time Series | | | |

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### Declarations

**Conflicts of interest:** The authors have no conflicts of interest to declare that are relevant to the content of manuscript.

**Ethics approval and consent to participate:** This article does not contain any studies with human participants or animals performed by any of the authors.

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