Supplementary Information

List of variables

Variable	Signification
X_H	Stock of primary resource
X_L	Stock of waste
X_T	Total stock
X_S	Buffer stock
μ_H	Potential of primary resource
μ_L	Potential of waste
F_{HP}	Flow of primary resource
F_{LP}	Flow of waste
G	Final goods
G_S	Buffer flow
G_D	Demand of goods
$G_D^{\text{satisfied}}$	(or G_D^S) Satisfied demand
F_{HR}	Flow of recycled resource
F_{LR}	Flow of waste used in recycling
F_{RIn}	Flow of resource used to recycle
F_{NR}	Flow of natural recycling
R_P	Production viscosity
R_R	Recycling viscosity
J	Global Intensity
J_P	Production intensity
$J_P^{\max}$	Maximal intensity of production
τ	Time constant of intensity of production
J_R	Recycling intensity
$J_R^{\max}$	Maximal intensity of recycling
J_P	Production intensity
n_P	Production intensity modulation coefficient
n_R	Recycling intensity modulation coefficient

Table S1. Variables

Potentials, Entropy, and Exergy

In this subsection, we would like to clarify a few points concerning the thermodynamic measurement of resource quality. They are best illustrated by Carnot's yield, which stipulates that efficiency of a heat engine is necessarily lower than Carnot's factor $(1 - T_c/T_h)$, where T_c and T_h are the cold and hot tank temperatures respectively. The central issue, which alone sums up the singularity of thermodynamics, is that of the gauge. Indeed in most physical disciplines the choice of the gauge, *i.e.*, the zero of the scale, is arbitrary. This is the case in electrokinetics with the choice of the ground potential, or in mechanics with the choice of a zero of the altitudes at the sea level. Things are radically different in thermodynamics, where the gauge is rigidly defined in the temperature scale called thermodynamic temperature. If it were not so, it would be possible to arbitrarily define the following gauge $T_c = 0$ or any other value. We see then that with the choice $T_c = 0$, Carnot's efficiency would be equal to unity: thermodynamics would lose its singularity, and consequently, its very existence. The second principle and the notion of entropy thus boil down to the existence of an indisputable fundamental gauge. The concept of exergy then immediately follows from Carnot's factor: it is only a dual representation of the second principle, in which the fraction of energy really usable in the form of work—also called free energy—is the total available energy multiplied by Carnot's factor. However, and for practical reasons, the exergy approach often uses a chosen reference temperature, which amounts to defining an arbitrary gauge. As a consequence, the concept of exergy is only a weak version of the second principle, whereas the entropy concept is a strong version of the second law of thermodynamics.

If the question of the energy gauge is rather simple because it is based on the very definition of what the thermodynamic temperature is, it is not the same for the gauge which concerns material resources. As already said, the second principle also applies: like energy, matter is subject to dispersion. The measure of an element of matter in the sense of its environment is defined by its chemical potential. This does not, however, give access to an absolute gauge for matter, *i.e.*, an absolute zero of chemical potentials. An elementary reasoning supported by an equally elementary thermodynamic calculation shows that the energy cost for the concentration of a diluted resource diverges as a logarithm function with the dilution rate. The problem posed is therefore that of the existence of a gauge that would characterize the ultimate dilution, *i.e.*, the disappearance of matter. Because of the first law, no absolute gauge can therefore qualify matter in a strong version of the second law, such as the one that exists for temperature. Whereas there is indeed a scale of absolute temperatures, there is no absolute scale of chemical potentials. If, now, a weak version of the second law is accepted, an energy-efficient approach to material qualification can be considered. This approach has been widely developed in the last 20 years by Valero, and before him by Ayres, who first introduced the concept of exergy in the field of ecological economics. In order to realistically situate the problem at the scale of our Earth, which is a closed material system, the choice of gauge is usually defined by the ultimate concentrations that can be found in the Earth's crust, oceans, or atmosphere. This viewpoint allows, via a standard thermodynamic calculation, to evaluate the energy necessary to the concentration of a chemical element thus diluted, until reaching its initial ore concentration. Such a strategy may seem quite realistic in that it seems to be in perfect agreement with the laws of thermodynamics.

However, three criticisms can be formulated. The first one, already developed, concerns the weak form of the second law that represents the exergetic approach. In more practical terms this means that as soon as the gauge becomes an adjustable parameter, the exact relation to thermodynamics weakens strongly to the point that to assert that the energetic approach to the treatment of matter is more realistic than another becomes questionable. The second criticism comes from the numerical estimates that emerge from these calculations and their economic validity. Indeed, the energy cost of recycling a diluted resource becomes dependent on the choice of gauge, which reduces the relevance of the estimate not better than to an order of magnitude. This criticism can be modulated by the fact that, since the dependence is logarithmic, the sensitivity to the choice of gauge is limited for a wide range of concentrations, except for concentrations close to the minimal value set by the gauge. It should be added that while the energy cost of concentrating a resource can be estimated, nothing is said about the quality of the energy required for this operation. Indeed, in certain thermodynamic favorable situations, simple fusion-aggregation may suffice, which amounts to using only heat. In other cases, the input of work will be indispensable, as in the case of osmotic separation processes. To mention only the ecological field, it can be noted that the sustainability of living organisms results from the supply of very high quality energy in the form of solar photons, and that a simple supply of heat in equal quantities would not lead to the closing of the cycle of living organisms via the photosynthetic process. As everywhere in thermodynamics, it is through a high quality energy input that the cycling processes, which are counter-entropic by definition, are carried out. Adding this quality dimension to the discussion, we can see that the strictly thermodynamic exergetic approach does not allow an economic estimate that is as well founded as a first reading would suggest. This is why we have not chosen a strictly thermodynamic formulation of resource potentials, so that the analytic expressions of the potentials are given in normalized form in equations. (**??**) and (**??**). Note that, although these definitions do not correspond analytically to that of a chemical potential, they nevertheless define two potentials, and, in doing so, make it possible to consider at the same time (*i*) an assessment of the available resource, (*ii*) an assessment of the unavailable resource, (*iii*) a measure of the criticality of supply of the resource, and (*iv*) a criterion for pinching production due to the accumulation of the unavailable resource, and therefore the need for recycling.

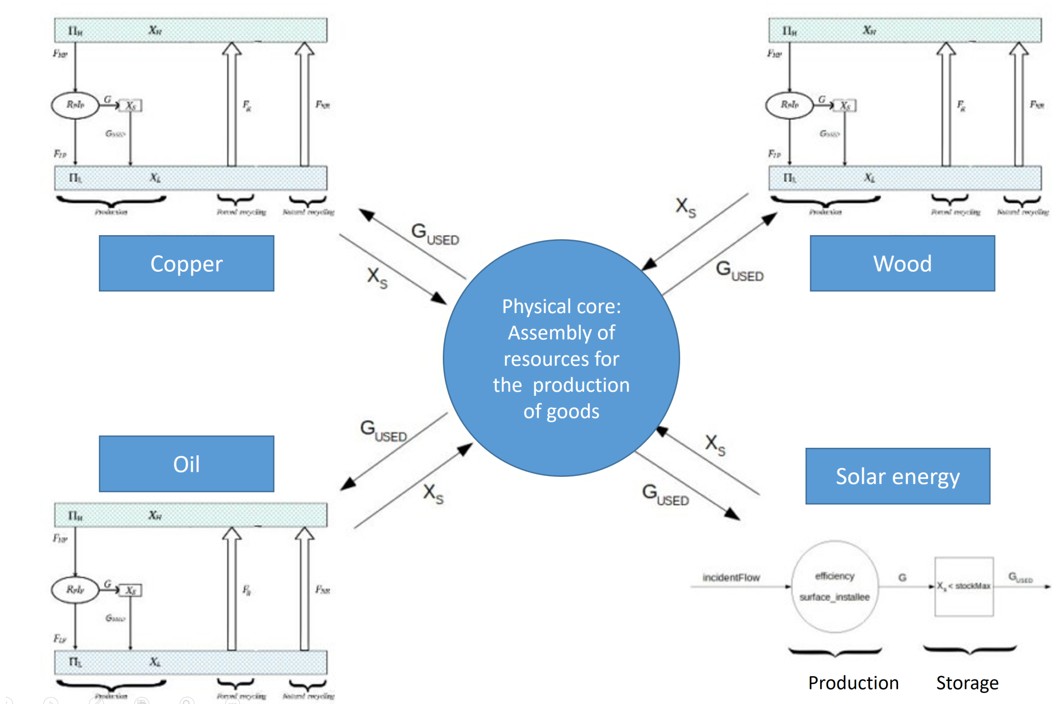

Figure S1. Schematic representation of the physical sphere composed of four sheets—in this example Oil, Copper, Wood are stock resources and Solar Energy is flux resource—connected to the economical sphere thanks to production of goods demand.

Multi-resources case study

We present here an illustration of a multi-resources case study. All the material necessary to run the following example is available online[?] as a free software, and accompanied by a comprehensive manual on the use and modification of the software. We will refer to this software as " the model ECODYCO ", or shorter ECODYCO in the following.

Parametrization

We propose to illustrate the functioning of ECODYCO by a case study based on the following resource configuration:

- Four resources. Some are stock resources, some are flow resources.
  - Two are material; copper and wood, which are both stock resources
  - Two are energies; oil and solar energy, which are respectively stock and flux energies
- Three goods can be produced, the quantity produced depends on demand and production capacity. For each good there is a specific " recipe " that can contain energy and/or material
  - *Good 0*; can be obtained with 1 unit of copper and 2 units of energy,
  - *Good 1*; can be obtained with 1 unit of copper and 2 units of wood and 3 units of energy
  - *Good 3*; (copper recycling) can be obtained with 1 unit of energy

In addition a target energy mix is proposed including 90% of solar energy and 10% of oil. The simulation will try to full-filled this condition. If the condition is not full-filled then the code automatically switch to the most appropriate mix.

We reproduce below the contains of a parametrization file for a given sheet of a stock resource, here the copper's one. All the definitions of the parameters can be found in the online tutorial.

```
type : stock
name : copper
total stock : 500000
Xh_init : 500000
Xl_init : 0
Rp0 : 0.003
K0 : 1
recyclingEnergyFlux : 1
isEnergy : False
r : 0
to : 9
xc : 2.8e-5
x0 : 5.8e-3
```

In addition, we also report the contain of the flux sheet of the solar power production used in this modelling

```
type : flow
name : solar energy
incidentFlow : 1e10
eff_init : 0.15
installed_surface : 1e-9
isEnergy : True
stockMax_init : 50
```

The physical world is defined by a specific sheet named **world.txt**. The recipe matrix, which ensures the distribution of the different resources for each production of goods, are defined within this file.

The economic engine is set in **main.py** file. We provide as an example two economic engines, based respectively on the descriptions of Solow and Goodwin production function. In this example, the production function is of Solow-type. Parameterization is available in a dedicated files that can be modified by the user. Alternatively, another economic engine corresponding to the production function the user wishes to implement can be proposed. The production function is the only interface between the physical and economic spheres. Knowing that, all the physical parameters of each sheet are accessible at each time step of the simulation, the user can use them to decide the economic choices he wants to implement according to the state of the resources. As an illustration in the proposed example, a target energy mix is initially defined (composed of 90% solar and 10% oil) which may or may not be achieved depending on the resources.

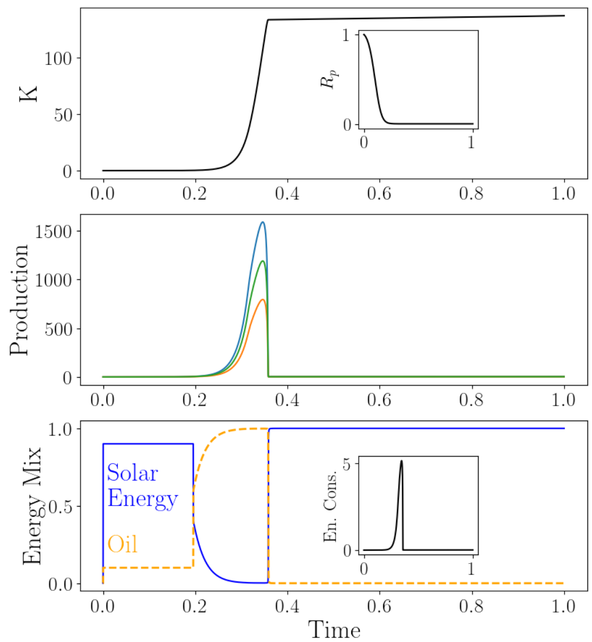

Figure S2. Output selections from the simulation conducted with four resources—composed of either energy and matter—and Solow economics production function, see the text for details. All quantities are shown in arbitrary units, in function of a normalized time. From top to bottom: *i*-Capital variation. Inset shows R_p dissipation parameter of the wood resource. $R_p = 1$ corresponds to 100% of its initial value. *ii*-Production of goods 0, 1 and 3 as defined in main text. *iii*-Energy mix, composed of oil (orange dashed thick line) and solar power (blue solid line). The target energy mix is 10% oil. In inset we show the corresponding total energy consumption.

Results

Running the simulation via the module **main.py** leads to the results shown in figure S2 for production capital and energy. All the information related to each sheet being recorded, it is then possible to follow the coupled evolution of their parameters, for example see in figure S3 $\Delta\Pi$ for stock sheets (*Copper*, *oil* and *Wood*) and the waste fraction (not shown).

In Figure S2, it is shown, as expected for a Solow-type production function, that capital and production of the three goods follow exponential laws up to approximately 30% of the duration of the simulation. The goods are produced in unequal quantities, corresponding to recipes and demand. Over this period, the investment allowed by the growth of capital decreases the resistance R_p of each of the sheets, as shown for *Wood* as an example. The decrease of R_p allows for the growth of resources production in agreement with the exponential growth production of goods. This eventually depletes the stocks until they are exhausted and the potential difference collapses simultaneously, as can be seen in figure S3, for *Wood*, *Copper* and *Oil*. The physical sphere is then no longer able to provide the resource for the economic sphere and the production of goods is almost interrupted, leaving capital to grow linearly.

The target energy mix, Figure S2 with 90% solar energy, is respected during the initial high growth period. As the growth of production is faster than the capacity to install new capture surfaces, see figure S3, *Oil* is then the only energy source able to provide the energy needed to produce the goods demanded by the economy thanks to its available stock. This constant *Oil* stock allows for the exponential growth phase. After the collapse of the production of goods, thanks to the surfaces already installed and because of the depletion of the *Oil* resource, the energy mix becomes composed almost entirely of solar energy. It is interesting to note, as shown in inset of the figure, that the total energy consumption collapsed with the production of goods.

Figure S3 shows the comparative evolution of *Copper*, *Wood* and *Oil* sheets. Buffer stocks (X_S, orange lines) are generated for all resources as soon as the simulation begins. It can be seen that the exponential growth in demand generates an almost synchronised collapse of $\Delta\Pi$ (blue lines). It is *Oil* stock (X_H not shown) that, in this example, is depleted first. Once the stock is consumed, the sheet is no longer able to supply energy and no longer shows any flow. This resource is no longer used in the simulation. In the case of *Wood* and *Copper* a large buffer stock (X_S) is formed very quickly at the time of the production collapse, the depletion of their stocks being slightly later than for *Oil*. In the continuation of the simulation we note that the recycling, natural in the case of *Wood* and forced in the case of *Copper*, makes it possible to go up $\Delta\Pi$ for both resources and thus to maintain—a very reduced but non zero—production until the end of the simulation. This production allows for the linear growth of capital observed in figure S2.

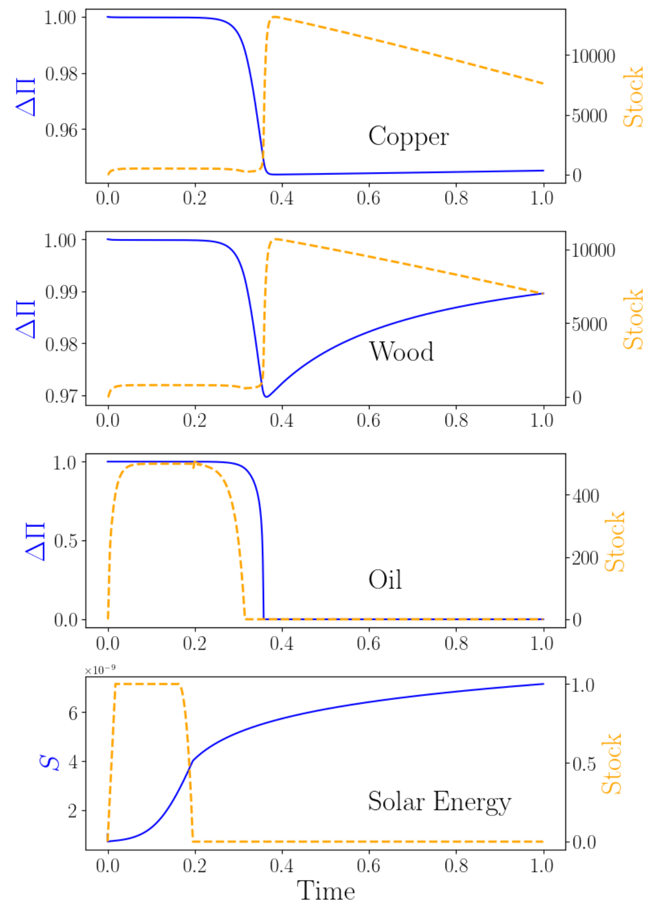

Figure S3. Coupled time evolution of the four resources potentials and buffer stock (X_S), see figure **??**b), involved in the simulation. See the main text for details of the simulation. We show from top to bottom: *Copper*, *Wood* and *Oil* which are stock resources and *Solar Energy* which is flux resource. For stock resources (resp. flux resource), the thin solid blue line is the potential difference $\Delta\Pi$ (resp. the installed surface) and the thick dashed orange line is buffer stock. Time is normalized. Units are arbitrary.