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Original Paper

Fatigue life analysis of a composite materials structure using allowable strain criteria

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Abstract: In the current research, a method for estimation of the durability of a structure based on an integrated calculation of static and fatigue strength according to the criterion of allowable strain is presented. The proposed technique is demonstrated for the upper stringer panel of the wing design. In the course of calculations, several combinations of design service goals, levels of allowable strains, and levels of cyclic stresses were considered. As a result, the relative values of stiffness degradation and the required safety ratios of static strength were obtained.
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1. Introduction

The design of modern highly loaded structures with strict requirements for limiting weight is associated with overcoming of a large number of technical difficulties and requires to use latest achievements in science and technology. Thus, the use of polymer composite materials (PCM) is one of the factors determining the weight perfection of modern aircraft structures, which leads to the manufacture of aircrafts with a large proportion of the use of PCM. As for any critical structure, it is necessary to confirm strength of aircraft components made of PCM.

The criterion of allowable strain is one of the most common strength criteria used in design and calculation of the static strength of structures made of composite materials (CM) [1,2]. The strains that occur in the CM structure under the design load should not exceed the allowable values. The allowable strains level of a laminate can take into account the impact of the operational and fabrication-induced damage presence, as well as influence of humidity and temperature detrimental effect on material strength. In some cases, limiting strains can be obtained by having the values of failure stresses and elastic characteristics of the layer. Note that to assess the strength of a laminate consisting of mutually perpendicular orientations of layers, for example, 0/90 and +45/-45, it is possible to use the maximum strains along the fiber as allowable [3-5]. The use of this form of the allowable strain criterion leads to the most efficient use of the material in terms of the structural weight reduction, since the failure longitudinal strains (along the fiber) are larger than the transverse ones for a layer of typical carbon fiber based on an epoxy matrix [6].

It is necessary to provide a given durability for critical structures that require long-term operation i.e., to satisfy the requirements of fatigue strength.

For traditional aviation materials, such as metal alloys, the fatigue strength of a structure is estimated by using the $S-N$ fatigue curves, where the given value of cyclically applied stresses S corresponds to the certain value of cycles to failure N. The $S-N$ dependence can be used as an envelope of the residual strength of the element structure [7,8]. In the case of metals, intermediate values of residual life are not used, the accumulated damage is taken equal to the ratio $\frac{n}{N}$, where n is the number of applied cycles and N is the number of cycles to failure at a given cyclic load. The accumulation of fatigue damage to structural elements made of polymer composite materials (PCM) manifests itself not only in the form of strength, but also in the form of stiffness, what is showed in number of experiments [9, 10]. This is where the differences in the definitions in the methods for calculating the fatigue life of PCM structures: in some calculations, according to residual strength [11,12], in others, according to residual stiffness [12,13].

In modern industrial practice, for example, in aircraft structures, the design uses the “no growth approach” principle, following which it is required to confirm that the damage present in the structure does not grow during the operational life of the aircraft or the period between detailed periodic inspections depending on the severity of damage according to the accepted classification [14]. In tests, this is confirmed by the fact that after working out a certain number of cycles, with repeated static loading, the resulting strains do not exceed the allowable values, and the structure endures the operational and then the design load.

The behavior of the material under cyclic operational loads will depend on the chosen level of allowable strains under the design load. In particular, with conservatively accepted allowable strains for PCM structural elements, stresses and strains arising during operation are not sufficient for the manifestation of fatigue phenomena in the material, such as damage accumulation [15, 16]. It is possible to achieve a more efficient use of a composite material in a load-bearing structure by adapting the so-called the principle of controlled damage growth (slow growth approach). A step towards the implementation of this principle may be the calculation of the required safety ratios, taking into account the growth of damage under the influence of repeated loads in operation, that is, the accumulation of fatigue damage in the CM. The necessary basis for such a calculation will be the dependence of the damage measure on the number of cycles and the load level, as well as the rule for summing up the accumulated damage from cycles with different load parameters, that is, complex loading that occurs during the operation of the aircraft.

Bearing in mind the property of CM to lose stiffness during long-term, repetitive loading, it seems reasonable to measure the accumulated damage in the form of residual stiffness relative to the initial one [17, 18]. In the scientific literature, such approaches are called phenomenological due to the fact that the damage measure only indirectly reflects its physical content: fiber rupture, various forms of matrix cracking inside the layer, delamination, loss of fiber stability and other types of destruction at the material level. Due to the coupling of the emerging fracture modes during fatigue loading, it is rather difficult to track them in isolation during the experiment, and hence to formulate the equations for the rates and durations of damage growth [17]. At the same time, the experimental data for phenomenological theories are standard tests on samples for cyclic loading with registration of forces, displacements and strains, from which the values of mechanical characteristics corresponding to various cycles are subsequently obtained. At the same time, it was found [9] that the values of mechanical characteristics obtained during intermediate (between load cycles) quasi-static tests in the non-damaging range of strains are better suited for use in the stiffness degradation model.

A special requirement for the methodology of durability calculation according to the stiffness degradation model is to take into account the redistribution of forces during damage accumulation. This directly follows from the fact that the growth of damage in the model is reflected in the progressive change in the mechanical characteristics at each point, and therefore the values of the stresses acting at the point cannot be constant due to the need to satisfy the equilibrium equations in the mechanical system [19]. If the system is rather complicated, as is the case of real multicomponent structures, then it is efficient to perform stress redistribution using the finite element method [20, 21]. Thus, to clarify the stress-strain state at certain stages of cyclic loading, a recalculation of the reference static case is required. The highest accuracy of calculations can be achieved if the physical meaning of the fracture degradation parameters used in static and fatigue calculations is consistent, but at the same time, in order to speed up computational procedures, it is necessary to find a balance between the required accuracy, damage mechanics representation and the number of intermediate recalculations of stress-strain state [21-23].

This study proposes a method for estimation of the durability of a structure based on a integrated numerical calculation of static and fatigue strength according to the allowable strains criterion. The main assumptions and calculation dependences of the method are given. The approach is illustrated by the example of calculating the structure of a stringer panel loaded with cyclic compression for several combinations of design service objectives, levels of allowable strains and cyclic stresses.

2. Methods

To implement the “slow growth approach” and calculate the required safety ratios with taking into account the development of damage under the influence of repeated loads, the following methodology is proposed, which, for example, is applied to the design of a stringer panel (Fig. 1).

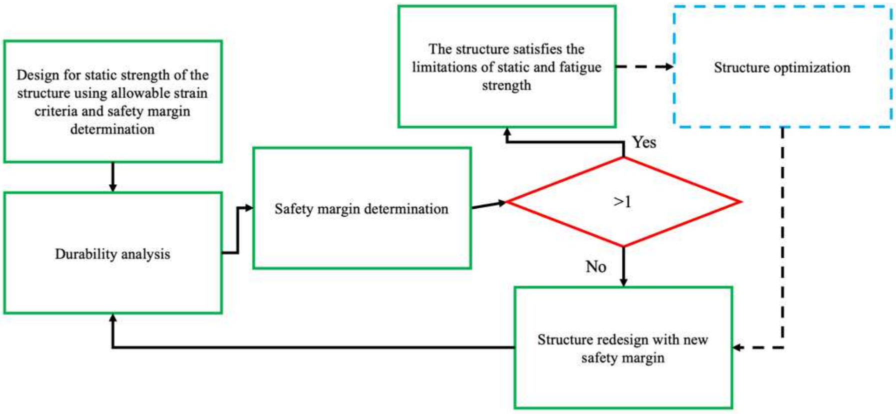

Fig. 1 The proposed method for determining the required safety ratio to satisfy static and fatigue strength

At the first stage, the design of the structure is carried out according to the restrictions on static strength and the safety ratio is determined. Design is conducted according to known relations [24] in the following sequence:

- The thickness of the layers of the laminate is set in the first approximation;
- The width of the skin between the stringer walls is found from the condition

$$\frac{N_z}{N_z^{cr}} + \left(\frac{N_{zx}}{N_{zx}^{\text{cr}}}\right)^2 \leq 1,$$

$$N_z^{\text{cr}} = 2.67\frac{\pi^2\sqrt{D_{11}D_{22}}}{l^2}\left(1.73 + \frac{D_{33}}{\sqrt{D_{11}D_{22}}}\right),$$

$$N_{zx}^{\text{cr}} = \frac{k\pi^2}{l^2}\sqrt{D_{33}D_{22}}\sqrt{4 + \frac{3\sqrt{D_{11}D_{22}}}{D_{33}} + \frac{D_{33}}{\sqrt{D_{11}D_{22}}}},$$

where N_z – applied compressive forces,

N_{zx} – applied shear forces,

N_z^{cr} – critical compressive forces,

N_{zx}^{cr} – critical shear forces,

D_{11}, D_{22}, D_{33} – flexural stiffness of the skin,

l – skin width between stringers.

- Next, the flexural stiffnesses of the reinforced panel are determined:

$$D_{11} = \frac{d_1 + d_2 + d_3 + d_4}{l}, d_1 = B_{11}^{(1)}\left[H\left(\frac{H}{2} - \delta_0\right)^2 + \frac{H^3}{12}\right],$$

$$d_2 = B_{11}^{(2)}[d(H - \delta_0)^2], d_3 = B_{11}^{(3)}\beta{\delta_0}^2, d_4 = B_{11}^{(4)}(b - \beta){\delta_0}^2$$

$$D_{22} = \frac{B_{22}^{(2)}dH + D_{22}^{\text{обш}}(l - d)}{l},$$

where H – stringer height,

d - stringer cap width,

δ_0 – distance from the skin to the center of stiffness of the stringer section,

β – width between stringer webs.

- Then the critical compressive load of the global buckling is found

$$N_z^{\mathrm{cr}} = \frac{\pi^2\sqrt{D_{11}D_{22}}}{b^2}\left[\sqrt{\frac{D_{11}}{D_{22}}}\left(\frac{mb}{a}\right)^2 + \frac{2D_{33}}{\sqrt{D_{11}D_{22}}} + \sqrt{\frac{D_{22}}{D_{11}}}\left(\frac{a}{mb}\right)^2\right],$$

where a – panel length,

b – panel width,

m is determine from condition

$$m(m-1) < \frac{a^2}{b^2}\sqrt{\frac{D_{22}}{D_{11}}} < m(m+1).$$

- Lamina strains are determined

$$\varepsilon_z^0 = \frac{N_z}{B_{11}},$$

where B_{11} – panel membrane stiffness.

- Safety ratio is determined

$$\eta_{st}^0 = \frac{[\varepsilon_z]}{\varepsilon_z^0},$$

where $[\varepsilon_z]$ –allowable strains,

ε_z^0 – applied strains.

The superscript 0 in the designations of the safety ratio and strain means the corresponding value before the application of a given number of load cycles.

With a large safety ratio or a safety ratio < 1, the number of layers of the package and / or the dimensions of the elements of the stringer panel are changed and the calculation is carried out again.

After determining the parameters of the panel and lay-up, a calculation is carried out using the FEM to confirm the resulting safety ratio and calculate the reference case for the fatigue analysis.

Next, the durability calculation is carried out. In this work it was carried out using the Simcenter 3D software package from Siemens, which has a specialized submodule for calculating PCM for durability Van Paepegem Analysis [25]. This method belongs to the group of methods based on the property of a composite material to lose stiffness during cyclic loading.

The basic analysis equation for the case of uniaxial loading can be represented as:

$$\frac{dD}{dN} = c_1 \cdot \Sigma \cdot exp\left(-c_2\frac{D}{\sqrt{\Sigma}}\right) + c_3 \cdot D \cdot \Sigma^2 \cdot \left[1 + exp\left(c_5(\Sigma - c_4)\right)\right],$$

if $\sigma \geq 0$,

$$\frac{dD}{dN} = \left[c_1 \cdot \Sigma \cdot exp\left(-c_2 \frac{D}{\sqrt{\Sigma}}\right)\right]^3 + c_3 \cdot D \cdot \Sigma^2 \cdot \left[1 + exp\left(\frac{c_5}{3}(\Sigma - c_4)\right)\right],$$

if $\sigma < 0$.

where $D = 1 - {}^{E}/_{E_0}$ – stiffness degradation parameter ($D = [0 \dots 1]$),

E – current material elastic modulus,

E_0 – elastic modulus of intact material.

When $D = 0$ – there is no damage, $D = 1$ means that the material has completely lost its stiffness, which is referred to as destruction. The definition of the stiffness degradation parameter is implemented as follows: first, this parameter is calculated in the reference static analysis as the initial damage, after which it is evaluated during the accumulation of fatigue and recalculation of the static analysis, which takes into account the current decrease in the stiffness of the material that occurred during fatigue loading and emerged new stress-strain state (SSS), within which a redistribution of stress is obtained due to the loss of stiffness.

$$\Sigma(\sigma, D) = \frac{\tilde{\sigma}}{X} = \frac{\sigma}{\frac{1-D}{X}} = \frac{E_0 \varepsilon}{X} \begin{cases} X = X_T, if\ \sigma \geq 0 \\ X = X_c, if\ \sigma < 0' \end{cases}$$

where N – current load cycle,

Σ – material failure index,

σ – applied stress,

X_T, X_c – tensile and compressive strength of the material, respectively,

c_1, c_2 – material constants characterizing the initial stiffness loss (stage 1, Fig. 2)

c_3, c_4, c_5 – material constants characterizing the progressive stiffness loss (stages 2 and 3 Fig. 2).

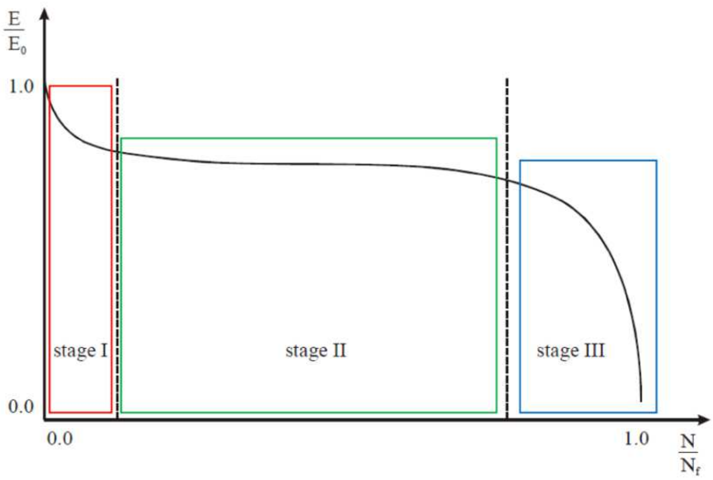

Fig. 2 Stages of progressive composite material stiffness loss

N_f – number of load cycles to failure.

The main analysis equation in the original work of the author of the method [13] was successfully used in calculating the durability of a woven prepreg with [0]n and [±45]n lay-up. For the case of an orthotropic layer of a unidirectional material, the main calculation equation was adopted in [20]:

if $\sigma_{11} \geq 0$:

$$\frac{dD_{11}}{dN} = c_{1,11} \cdot \Sigma_{11} \cdot exp\left(-c_{2,11}\frac{D_{11}}{\sqrt{\Sigma_{11}}}\right) + c_{3,11} \cdot D_{11} \cdot {\Sigma_{11}}^{2} \cdot \left[1 + exp\left(c_{5,11}(\Sigma_{11} - c_{4,11})\right)\right]$$

if $\sigma_{11} < 0$

$$\frac{dD_{11}}{dN} = \left[c_{1,11} \cdot \Sigma_{11} \cdot exp\left(-c_{2,11}\frac{D_{11}}{\sqrt{\Sigma_{11}}}\right)\right]^{3} + c_{3,11} \cdot D_{11} \cdot {\Sigma_{11}}^{2} \cdot \left[1 + exp\left(\frac{c_{5,11}}{3}(\Sigma_{11} - c_{4,11})\right)\right]$$

if $\sigma_{22} \geq 0$:

$$\frac{dD_{22}}{dN} = c_{1,22} \cdot (1 + D_{12}^{2}) \cdot \Sigma_{22} \cdot exp\left(-c_{2,22}\frac{D_{22}}{(1 + D_{12}^{2})\sqrt{\Sigma_{22}}}\right) + c_{3,22} \cdot D_{22} \cdot {\Sigma_{22}}^{2} \cdot \left[1 + exp\left(c_{5,22}(\Sigma_{22} - c_{4,22})\right)\right]$$

if $\sigma_{22} < 0$:

$$\frac{dD_{22}}{dN} = \left[c_{1,22} \cdot (1 + D_{12}^{2}) \cdot \Sigma_{22} \cdot exp\left(-c_{2,22}\frac{D_{22}}{(1 + D_{12}^{2})\sqrt{\Sigma_{22}}}\right)\right]^{3} + c_{3,22} \cdot D_{22} \cdot {\Sigma_{22}}^{2} \cdot \left[1 + exp\left(\frac{c_{5,22}}{3}(\Sigma_{22} - c_{4,22})\right)\right]$$

$$\frac{dD_{12}}{dN} = c_{1,12} \cdot (1 + D_{12}^{2}) \cdot \Sigma_{12} \cdot exp\left(-c_{2,12}\frac{D_{12}}{(1 + D_{12}^{2})\sqrt{\Sigma_{12}}}\right) + c_{3,12} \cdot D_{12} \cdot {\Sigma_{12}}^{2} \cdot \left[1 + exp\left(c_{5,12}(\Sigma_{12} - c_{4,12})\right)\right]$$

In equation [20]:

D_{11}, D_{22}, D_{12} – degradation parameters of longitudinal, transverse and shear stiffness, respectively;

$c_{i,jk}$ – material constants for longitudinal direction ($c_{1,11}$), transverse direction ($c_{1,22}$) and in the plane of the layer ($c_{1,12}$), $i = 1 \ldots 5, jk = 11, 22, 12$;

Σ_{jk} – failure indices in the respective directions:

$$\Sigma_{11} = \frac{\sigma_{11}}{\frac{1-D}{X}} \begin{cases} X = X_T, if\ \sigma_{11} \geq 0 \\ X = X_c, if\ \ \sigma_{11} < 0, \end{cases}$$

$$\Sigma_{22} = \frac{\sigma_{22}}{\frac{1-D}{Y}} \begin{cases} Y = Y_T, if\ \sigma_{22} \geq 0 \\ Y = Y_c, if\ \ \sigma_{22} < 0, \end{cases}$$

$$\Sigma_{12} = \frac{\tau_{12}}{\frac{1-D}{S_{12}}},$$

where $\sigma_{11}, \sigma_{22},\ \tau_{12}$ – applied longitudinal, transverse and shear stress in the plane of the layer, respectively;

X_T, X_C – longitudinal tensile and compressive strength of the material, respectively;

Y_T, Y_C – transverse tensile and compressive strength of the material, respectively;

S_{12} – shear strength of the material in the plane of the layer.

After carrying out the calculation for durability, the value of the safety ratio is calculated at the design load after the cycles of applied loads have expired:

$$\eta_n = \frac{[\varepsilon_z]}{\varepsilon_z^n}$$

where ε_z^n − strain value after application of prescribed number of load cycles.

If $\eta_n > 1$, then the panel satisfies the static and fatigue strength constraints. In this case $\eta_n \gg 1$, it is possible to carry out optimization in order to obtain the smallest weight of the structure. If $\eta_n < 1$, then it is necessary to redesign the panel taking into account the new limitation on static strength or reduce the load acting on the panel. In the general case, to calculate the required safety ratio for statics η_0, it is possible to carry out an iterative procedure for changing η_0 depending on the value of η_n, which is determined numerically in each successive calculation loop. A simpler one-step procedure, appropriate in individual cases, is described in the example below.

The reduction of safety factor after the end of fatigue cycles can be expressed as:

$$\eta_0 - \eta_n = \frac{[\varepsilon_z]}{\varepsilon_z^0} - \frac{[\varepsilon_z]}{\varepsilon_z^n} = \eta_0\left(\frac{\varepsilon_z^n - \varepsilon_z^0}{\varepsilon_z^n}\right) = \eta_0\chi,$$

thus, the relative loss of stiffness can be expressed by the coefficient χ.

3. A calculation example

3.1 Design cases

In order to test the methodology and the “slow growth approach”, several combinations of design service goals (DSO), levels of allowable strains and the level of cyclically acting load were considered for the case of a stringer panel loaded with zero compression cycles:

Var. №1. DSO 500,000 cycles, applied strain 25% of the allowable, allowable strain$[\varepsilon_z] = 0.4\%$ (limit approximately corresponds to the ultimate strain in the presence of BVID (Barely Visible Impact Damage);
Var. №2. DSO 500000 cycles, applied strain 60% of the allowable, allowable strain$[\varepsilon_z] = 0.4\%$;
Var. №3. DSO 2000000 cycles, applied strain 25% of the allowable, allowable strain$[\varepsilon_z] = 0.4\%$;
Var. №4. DSO 2000000 cycles, applied strain 60% of the allowable, allowable strain $[\varepsilon_z] = 0.4\%$;
Var. №5. DSO 500000 cycles, applied strain 25% of the allowable, allowable strain$[\varepsilon_z] \approx 0.8\%$ (the limit corresponds to the maximum strain of the panel when the criterion of static strength is met);
Var. №6. DSO 500000 cycles, applied strain 60 of the allowable, allowable strain $[\varepsilon_z] \approx 0.8\%$;
Var. №7. DSO 2000000 cycles, applied strain 25% of the allowable, allowable strain$[\varepsilon_z] \approx 0.8\%$;
Var. №8. DSO 2000000 cycles, applied strain 60% of the allowable, allowable strain $[\varepsilon_z] \approx 0.8\%$.

3.2 Material properties

The design and further static and fatigue calculations of the stringer panel were carried out with the properties of carbon fiber reinforced plastic (CFRP) presented in Table 1.

Table 1 CFRP properties

Property	Value
Elastic modulus under longitudinal tension, $\boldsymbol{E_{1t}}$, GPa	137
Elastic modulus under longitudinal compression, $\boldsymbol{E_{1c}}$, GPa	122
Elastic modulus under transverse tension, $\boldsymbol{E_{2t}}$, GPa	8.28
Elastic modulus under transverse compression, $\boldsymbol{E_{2c}}$, GPa	8.4
Poisson's ratio, $\boldsymbol{\nu_{12}}$	0.335
Shear modulus, $\boldsymbol{G_{12}}$, GPa	4.15

Ultimate longitudinal tension strength, $\boldsymbol{F_{1t}}$, MPa	2147
Ultimate longitudinal compression strength, $\boldsymbol{F_{1c}}$, MPa	979
Ultimate transverse tensile strength, $\boldsymbol{F_{2t}}$, MPa	50.3
Ultimate transverse compression strength, $\boldsymbol{F_{2c}}$, MPa	237
Shear strength, $\boldsymbol{F_{12}}$, MPa	81.9
Monolayer thickness, $\boldsymbol{t}$, mm	0.13

At the same time, in elements with multidirectional lay-up the influence of neighboring layers on the values of the transverse and shear strength (in-situ strength) of the layer was taken into account according to the formulas [5]:

$$F_{2t}^{is} = 1.12\sqrt{\frac{2t_t}{t_e}}F_{2t}$$

$$F_6^{is} = \sqrt{\frac{2t_t}{t_e}}F_6$$

where F_{2t}^{is} – local transverse tensile strength;

F_6^{is} – local shear strength;

$t_t = 4a_0$ – transition thickness,

$a_0 = 0.2$ – representative defect size for CFRP;

$t_e = \min(t_t, t_k)$,

t_k – monolayer thickness.

The material constants that determine the form of the monolayer stiffness degradation curve according to the Van Paepegem model were chosen according to the values recommended for CFRP due to the lack of special experimental data for the CFRP chosen in this study.

3.3 Description of the FE model

A general view of the geometry of the stringer panel is shown in the Fig. 1. The geometric parameters of the stringer model are presented in the Table 2.

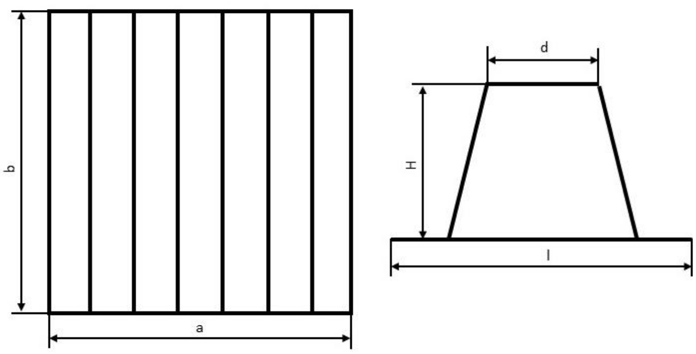

Fig. 3 Stringer panel geometry

Table 2 Geometric parameters of the stringer panel

Parameter	Value
Length, $\boldsymbol{a}$, мм	180
Width, $\boldsymbol{b}$, мм	180
Stringer's height, $\boldsymbol{H}$, мм	25
Cap's width, $\boldsymbol{d}$, мм	20
Stringer's pitch, $\boldsymbol{l}$, мм	60

The panel was designed in such a way that the material strength criterion was satisfied before the overall and local buckling of the panel, in accordance with the methodology presented in section 2 using the maximum strain limit.

The general view of the stringer panel FEM is shown in Figure 1. This model contains 3781 Mindlin Shell [T29] elements and 3906 nodes. The element size is 5 mm.

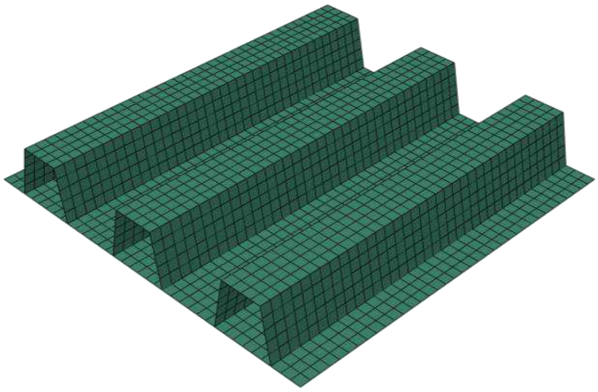

Fig. 4 Stringer panel FEM

Mindlin Shell elements [T29] were used in the simulation because Mindlin plate theory takes into account transverse shear strain. This is important for the laminated panel because it has low transverse shear stiffness due to the interlaminar interface. Thus, displacements and strains are defined as [26,27]:

$$u(x,y,z) = u^0(x,y) - z\theta_y, \varepsilon_x = \frac{\partial u^0}{\partial x} - z\frac{\partial \theta_y}{\partial x},$$

$$v(x,y,z) = v^0(x,y) - z\theta_x, \varepsilon_y = \frac{\partial v^0}{\partial y} - z\frac{\partial \theta_x}{\partial y}$$

$$\gamma_{xy} = \frac{1}{2}\left(\frac{\partial u^0}{\partial y} + \frac{\partial v^0}{\partial x}\right) - \frac{z}{2}\left(\frac{\partial \theta_y}{\partial y} + \frac{\partial \theta_x}{\partial x}\right), \gamma_{yz} = \frac{1}{2}\left(\frac{\partial w^0}{\partial y} - \theta_x\right), \gamma_{zx} = \frac{1}{2}\left(\frac{\partial w^0}{\partial x} + \theta_y\right)$$

x, y – axis in layer plane;

z – axis perpendicular to the plane of the layer;

u^0, v^0 - in-plane displacements of the mid-surface;

w^0 - displacement of the mid-surface along z axis

θ_x, θ_y – angles of rotation around the axes x and y;

u, v, w – displacement along the axes x, y, z;

$\varepsilon_x, \varepsilon_y, \gamma_{xy}, \gamma_{yz}, \gamma_{zx}$ – strains.

Displacement inside FE [28]:

$$\{\boldsymbol{f}\} = \begin{Bmatrix} u(x,y,z) \\ v(x,y,z) \\ w(x,y) \end{Bmatrix} = [\boldsymbol{N}]\{\boldsymbol{\delta}\}^e = [N_i, \quad N_j, \quad N_m, \quad \ldots] \begin{Bmatrix} \delta_i \\ \delta_j \\ \delta_m \\ \ldots \end{Bmatrix}$$

$[N_i]$ – shape functions;

$\{\delta_i\}^e$ – node displacements;

$i, j, m, \ldots$ - node number.

For each element of the panel, the laminate stacking sequence is specified, presented in the Table 3.

Table 3 The laminate stacking of stringer panel elements

Stringer panel element	Laminate stacking
Skin	$[45,0,0,-45,0,\overline{90}]_s$
Stringer web	$[45,0,0,-45,0,\overline{90}]_{s2}$
Stringer cap	$[45,0,0,-45,0,\overline{90}]_{s2}$

4 Results

During the static analysis, the values of the criterion of maximum stresses and strainjs of the panel in the longitudinal direction were obtained for two design cases (Fig. 5). For the design case with limited allowable strains $[\varepsilon_z] = 0.4\%$ design load P_{d}, acting on the panel was assumed to be 300 kN.

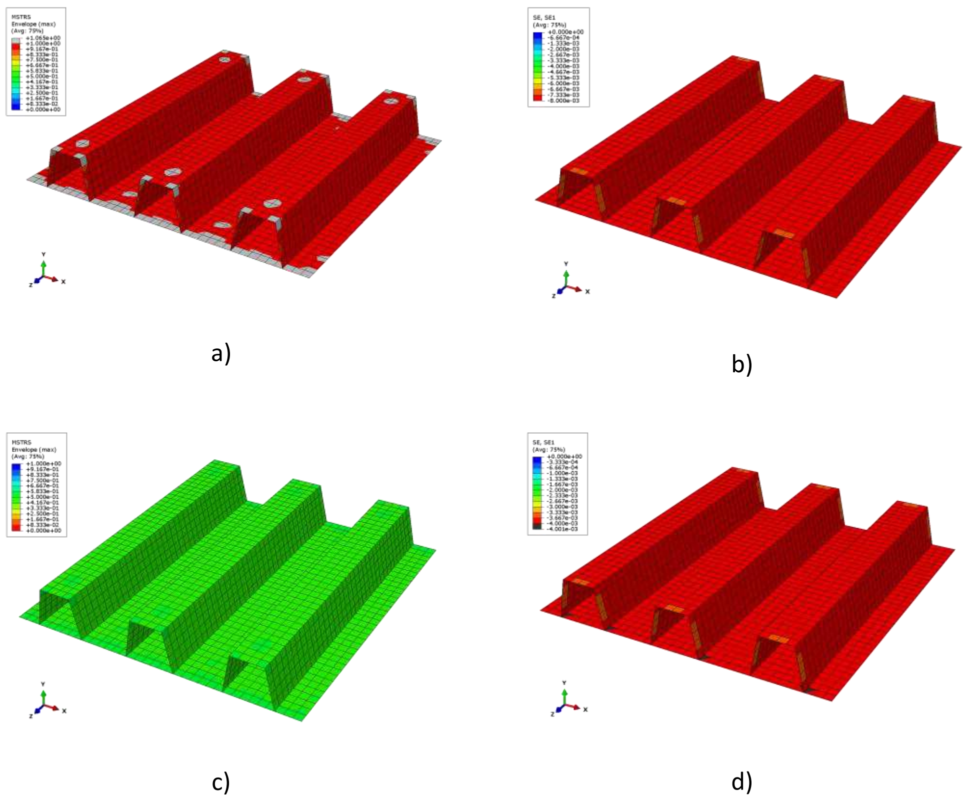

Fig. 5 Static analyses results. a) Maximum stress criterion for design case $P_d = 600\ kN$; b) Panel longitudinal strain for design case $P_d = 600\ kN$; c) Maximum stress criterion for design case $P_d = 300\ kN$; d) Panel longitdinal strain for design case $P_d = 300\ kN$;

Thus, the designed panel has a safety ratio of η_{st} when the maximum strains are limited under the design load $P_{\mathrm{d}} = 600\ \mathrm{kN}$ and when the allowable strains are $[\varepsilon_z] = 0.4\%$ under the design load $P_{\mathrm{d}} = 300\ \mathrm{kN}$.

After carrying out fatigue calculations, the coefficient values χ were obtained (Table 4).

Table 4 Relative stiffness degradation

Design case	$\boldsymbol{\chi \cdot 100}$, %	
	500000 cycles	2000000 cycles
Restrictions on maximum strains, $\boldsymbol{P_o = 0.6P_d}$	1.07	4.97
Restrictions on maximum strains, $\boldsymbol{P_o = 0.25P_d}$	0.46	1.30
Restrictions on allowable strains $[\boldsymbol{\varepsilon_z}] = \boldsymbol{0.4\%}$, $\boldsymbol{P_o = 0.6P_d}$	0.54	1.57
Restrictions on allowable strains $[\boldsymbol{\varepsilon_z}] = \boldsymbol{0.4\%}$, $\boldsymbol{P_o = 0.25P_d}$	0.27	0.68

As can be seen from the Table 4, with a restriction on allowable strains $[\varepsilon_z] = 0.4\%$, there is practically no degradation of the panel stiffness in the longitudinal direction both for 500000 cycles and for 2000000 cycles, which is another confirmation of the fact that in a CM structure designed according to allowable strains of the BVID level, the principle of non-development of damage is performed automatically. When using the restriction on maximum strains, a slight drop in stiffness is also observed for 500000 cycles at $P_{\mathrm{o}} = 0.25P_{\mathrm{d}}$ and $P_{\mathrm{o}} = 0.6P_{\mathrm{d}}$, and also for 2000000 at $P_{\mathrm{o}} = 0.25P_{\mathrm{d}}$. Thus, a laminate designed according to the conditions of static and fatigue strength for allowable strains close to neat material failure strains, in a fairly wide range of durability and acting cyclic loads, shows resistance to damage development, but, at the same time, has half the thickness than the previously considered package with the accepted allowable strain $[\varepsilon_z] = 0.4\%$,. A distinct drop in stiffness is observed for the panel calculated using the maximum strain limit and high operating load level $P_{\mathrm{o}} = 0.6P_{\mathrm{d}}$ (5% drop in stiffness).

Figures 6-8 show the distribution of longitudinal strains of the panel for the design case using the maximum strain limit at $P_{\mathrm{o}} = 0.6P_{\mathrm{p}}$ for a different number of cycles.

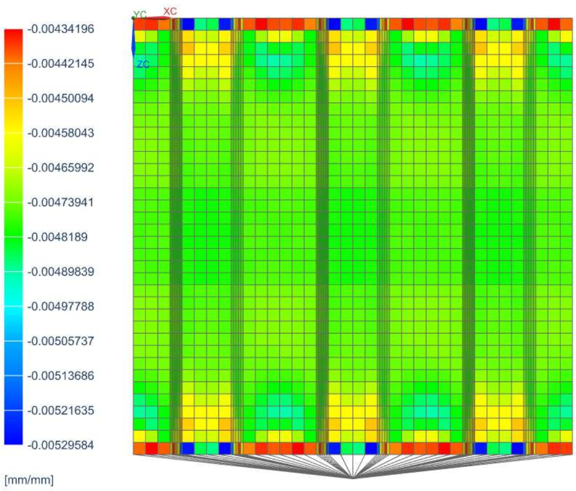

Fig. 6 Panel longitudinal strains after 500000 cycles

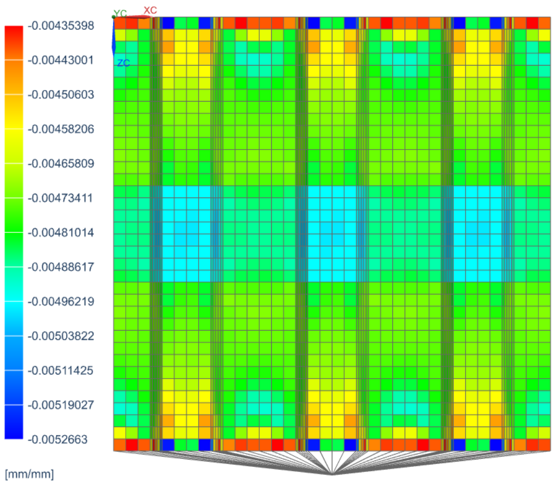

Fig. 7 Panel longitudinal strains after 2000000 cycles

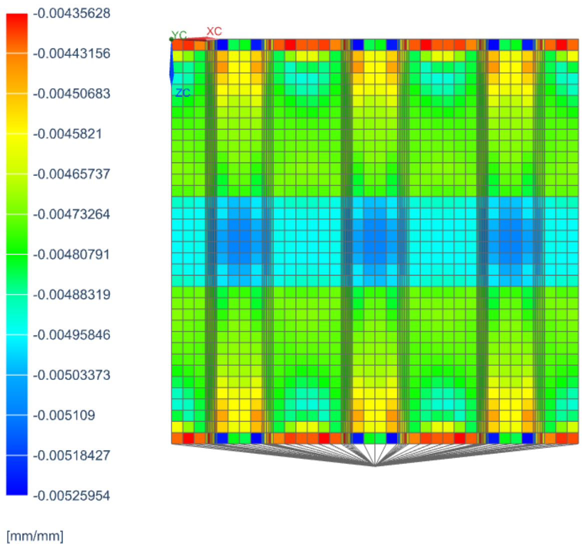

Fig. 8 Panel longitudinal strains after 3800000 cycles

Figure 1 shows a graph of stiffness degradation versus the number of cycles. It can be seen from the graph that in the range of application of load cycles $n = 10^6 - 2.5 * 10^6$ the degradation rate increases, and immediately then, after $n = 2.5 * 10^6$ it starts to slow down.

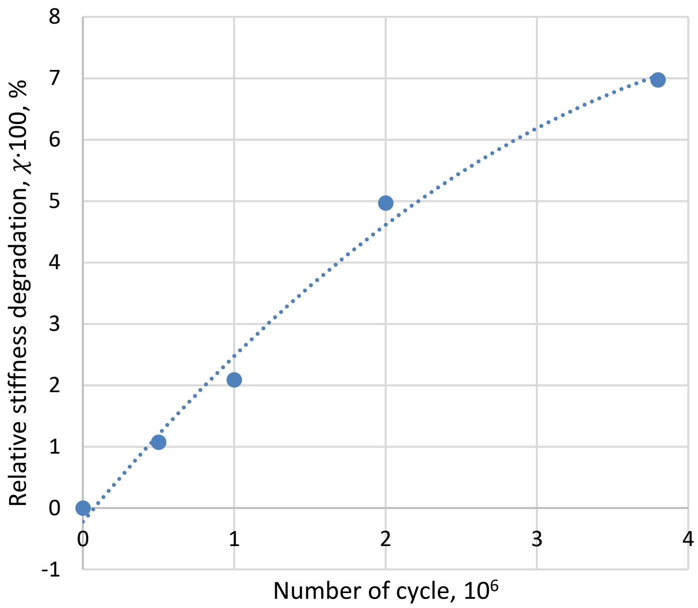

Рис. 1 Graph of stiffness degradation vs. number of cycles

Further, in accordance with the methodology, the total safety ratios for statics and durability were obtained for each design case (Table 5).

Table 5 Static strength margin after fatigue cycles, η_n

Design case	$\boldsymbol{\eta_n}$, 500000 cycles	$\boldsymbol{\eta_n}$, 2000000 cycles
Restrictions on maximum strains, $\boldsymbol{P_o = 0.6P_d}$	0,99	0,95
Restrictions on maximum strains, $\boldsymbol{P_o = 0.25P_d}$	1,00	0,99
Restrictions on allowable strains $[\boldsymbol{\varepsilon_z}] = \mathbf{0.4}\%$, $\boldsymbol{P_o = 0.6P_d}$	0,99	0,98
Restrictions on allowable strains $[\boldsymbol{\varepsilon_z}] = \mathbf{0.4}\%$, $\boldsymbol{P_o = 0.25P_d}$	1,00	0,99

As can be seen from Table 5, for the design case with a restriction on maximum strains at $P_{\mathrm{o}} = 0.6P_{\mathrm{d}}$, it is required to redesign the panel, while it is necessary to achieve the required safety ratios presented in Table 6. With small drops in stiffness $\leq 5\%$, this approach is quite legitimate and contains the following logic. The increase in strains $\Delta\varepsilon$ revealed at the end of the durability calculation is a measure of the accumulated damage of the material. The required static strength safety ratio, taking into account the accumulated damage, can be calculated by determining the new required level of applied strains ε_{req}^{new} relative to the initial ε_{req}^{old}, namely: $\varepsilon_{req}^{new} = \varepsilon_{req}^{old} - \Delta\varepsilon$. The new required safety ratio η_0^{new} will automatically be greater than the old η_0^{old}: $\eta_0^{new}(= \frac{[\varepsilon]}{\varepsilon_{req}^{new}}) > \eta_0^{old}(= \frac{[\varepsilon]}{\varepsilon_{req}^{old}})$. A lamina redesigned for a new safety ratio will guarantee to have a lower accumulated damage than $\Delta\varepsilon$.

Table 6 Required static strength safety ratio ${\eta_0}^{new}$ for several design cases

Design case	$\boldsymbol{{\eta_0}^{new}}$, 500000 cycles	$\boldsymbol{{\eta_0}^{new}}$, 2000000 cycles
Restrictions on maximum strains, $\boldsymbol{P_o = 0.6P_d}$	1,01	1,05
Restrictions on maximum strains, $\boldsymbol{P_o = 0.25P_d}$	1,00	1,01
Restrictions on allowable strains $[\boldsymbol{\varepsilon_z}] = \mathbf{0.4}\%$, $\boldsymbol{P_o = 0.6P_d}$	1,01	1,02
Restrictions on allowable strains $[\boldsymbol{\varepsilon_z}] = \mathbf{0.4}\%$, $\boldsymbol{P_o = 0.25P_d}$	1,00	1,01

5 Discussion and conclusion

This study proposes a method for estimation of the durability of a structure based on a complex numerical calculation of static and fatigue strength according to the criterion of allowable strains. The main assumptions and calculation dependences of the method are given. The approach is illustrated by the example of analysis of the structure of a stringer panel loaded by cyclic compression for several combinations of DSO, levels of allowable strains and cyclic stresses. The above procedure for increasing the required safety ratio will be relevant if there is a requirement for a CM structure to withstand the full design load throughout the entire DSO, taking into account the reliability factor. It should be noted that taking into account the degradation of the strength of the material and the different requirements for static strength for the structure will require modification of the proposed procedure for determining the required safety ratio.

Declarations

The authors declare no conflict of interest

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