Quantum loss sensing with two-mode squeezed vacuum state under noisy and lossy environment–Supplementary Information

Sang-il Park and Changsuk Noh
Kyungpook National University, Daegu 41566, Korea

Changhyoup Lee
Korea Research Institute of Standards and Science, Daejeon 34113, Korea

I. QUANTUM FISHER INFORMATION FOR GAUSSIAN STATES

The quantum Fisher information (QFI) can be calculated from the fidelity $\mathcal{F}(\rho_T, \rho_{T+dT})$ between the two states ρ_T and ρ_{T+dT} and is given by [1]

$$H(T) = \frac{8[1-\mathcal{F}(\rho_T, \rho_{T+dT})]}{dT^2}. \tag{1}$$

There is a well-known method of computing the fidelity of the Gaussian states [2]. For n bosonic modes described by quadrature operators $\mathbf{Q} = (x_1, \cdots, x_n, p_1, \cdots, p_n)^T$, the canonical commutation relations can be written as [3]

$$[\mathbf{Q}, \mathbf{Q}^T] = i\Omega_n, \ \Omega_n := \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \otimes I_n, \tag{2}$$

where I_n is the $n \times n$ identity matirx. The first- and second-order moments of Gaussian states can be written in terms of the mean ($\mathbf{u}$) and the covariance matrix (V) where

$$u_i = \langle Q_i \rangle, \tag{3}$$

$$V_{ij} = \frac{1}{2}\langle \{Q_i - u_i, Q_j - u_j\} \rangle. \tag{4}$$

Using a modified version of the covariance matrix $W := -2Vi\Omega_n$, the fidelity between two Gaussian states is given by

$$\mathcal{F}(\rho_1, \rho_2) = \mathcal{F}_0(V_1, V_2) \exp\left[-\frac{1}{4}\delta_u^T (V_1 + V_2)^{-1} \delta_u\right], \tag{5}$$

where $\delta_u := u_2 - u_1$ and $\mathcal{F}_0(V_1, V_2)$ is given by [2, 4, 5].

$$\mathcal{F}_0^2(V_1, V_2) = \frac{1}{\sqrt{\Delta + \Lambda} - \sqrt{\Lambda}}, \tag{6}$$

for single-mode Gaussian states and

$$\mathcal{F}_0^2(V_1, V_2) = \frac{1}{\sqrt{\Gamma} + \sqrt{\Lambda} - \sqrt{(\sqrt{\Gamma} + \sqrt{\Lambda})^2 - \Delta}}, \tag{7}$$

for two-mode Gaussian states. Here, the symplectic invariants are defined as $\Delta = \det(V_1 + V_2)$, $\Gamma = 2^{2n}\det(\Omega_n V_1 \Omega_n V_2 - I/4)$, and $\Lambda = 2^{2n}\det(V_1 + i\Omega/2)\det(V_2 + i\Omega/2)$.

II. QUANTUM ENHANCEMENTS FOR $n_{\rm th} = 0$

Figures S1(a) and (b) depict how Figs. 3 and 5 in the main text change, respectively, in the limit of vanishing thermal photon number, i.e., $n_{\rm th} \to 0$. For the original asymmetric setup (Fig. S1(a)), the 'No enhancement' region occupies a larger parameter space at the cost of reduced region for 'coin', while the region occupied by 'diff' stays more or less the same. The values of $\mathrm{R}_{\rm coin}$ have decreased significantly from those at $n_{\rm th} = 0.1$, while $\mathrm{R}_{\rm diff}$ exhibits mixed behavior. The latter has increased when T is large and γ is small (bottom right corner), but has decreased

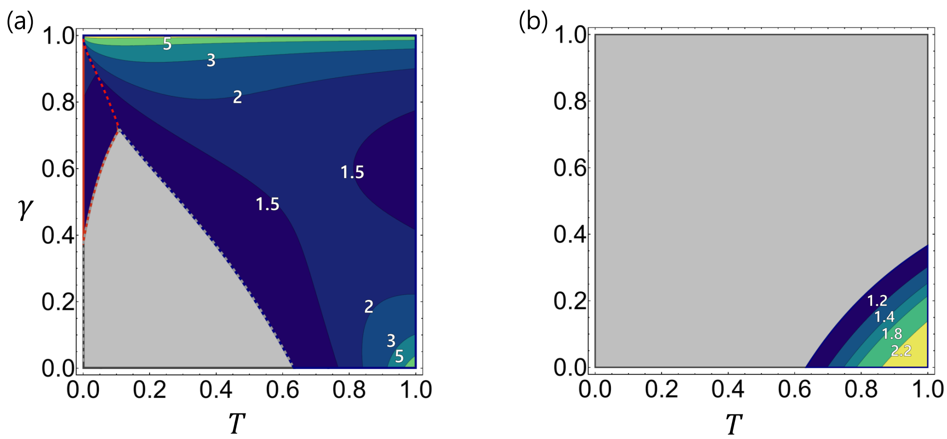

Fig. S1: Quantum enhancements achieved by the TMSV state as quantified by R_κ in (a) the original quantum setup and (b) the new quantum setup, for $n_{\mathrm{th}} = 0$.

when γ is large (for all T). The situation is similar for the alternative quantum setup, as shown in Fig. S1(b), except for the fact that the coincidence-counting scheme exhibits no quantum enhancement at all when $n_{\mathrm{th}} = 0$.

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