Supplementary material - Generalized Linear Mixed Modeling

By means of multiple generalized linear mixed model (GLMM), we tested whether the factor space (short, equal, long) affected Kappa effect for each modality and place. GLMM accounts for the effect of the experimental variables and for the variability between participants by means of fixed- and random- effect parameters, respectively[1]. Models were of the form:

$$\Phi^{-1}[P(Y=1)] = \beta_0 + u0 + \beta_1 \cdot x + u1 \cdot x + \beta_2(S) + \beta_3(L) \quad (1)$$

where Φ^{-1} is the probit link function, $P(Y=1)$ is the probability of perceiving the first stimulus as longer than the second, β_* and u_* are the fixed- and random- effect parameters, respectively. S and L are two dummy variables correspondent to the levels of the factor Space short(S) and long(L).

Tables 1 and 2 report the coefficients of the unimodal and multimodal models, respectively. In agreement with the previous data analysis, for all modalities and places the coefficient β_0, representing the PSE of control condition, reports values close to zero, therefore always not significant. The coefficient β_1, significant in all modalities and places, is related to the inverse of standard deviation of the psychometric function (i.e., JND = $0.75/\beta_1$). The coefficients β_2 and β_3 are related to the short and long condition, respectively: the PSE of the corresponding psychometric functions can be calculated as $-(\beta_0+\beta_2)/\beta_1$ and $-(\beta_0+\beta_3)/\beta_1$. The value of such coefficients, usually specular to zero, changes with modalities in agreement with Fig. **??**. For instance, β_2 in the visual modality is twice the tactile modality. Regarding the factor Place, instead, for all the modalities the coefficients are within the range of the SE. In general,the random effect in the models showed a limited variance ($0.04-0.08$) on the intercept, whereas it showed a greater variance on the slope ($0.3-0.8$), meaning that the discrimination thresholds were different among the participants. We investigated also if the subject-specific effects were correlated with any of the static predictors. In particular, age and arm length values were standardized, and linear models were fitted between such predictors and both random intercepts and slopes. Whereas the coefficients were always not significant, the AIC parameter increased in all the configurations tested, meaning that such parameters were not improving the model fit.

Figure 1a and 1b report the GLMM of the unimodal visual and tactile modalities applied on the experimental data for each participants, respectively. Although the GLMM only provides 3 significant parameters for each factor combination of Modality and Place, it nevertheless allows to represent the Magnitude of the Kappa effect (i.e., the translation of the PSE on the $x-$axis) and the JND of most of the participants. However, compared to the psychometric functions individually computed on the experimental data, the GLMM is unable to fit the data of specific participants which report an unbalanced Kappa effect (e.g., Fig.1a sub1 long, sub9 short, sub 11 short).

Table 1. Coefficients of fixed effects the unimodal models (with SE) Significance levels: *=0.01, **=0.001, ***=0.0001

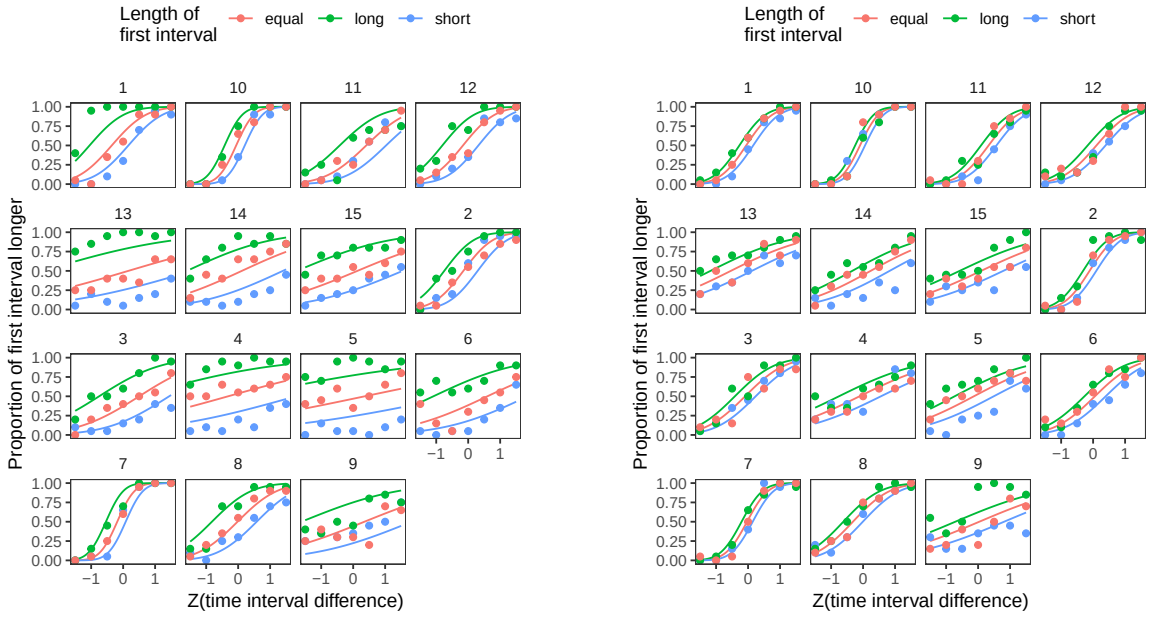
Modality	**Visual**		**Tactile**	
Place	Palm	Forearm	Palm	Forearm
β_0	0.15 (0.07)	0.01 (0.07)	0.06 (0.06)	0.03 (0.06)
β_1	1.05 (0.18)***	0.92 (0.17)***	1.21 (0.21)***	1.08 (0.14)***
β_2	-0.59 (0.05)***	-0.61 (0.05)***	-0.28 (0.05)***	-0.34 (0.05)***
β_3	0.76 (0.05)***	0.82 (0.05)***	0.40 (0.05)***	0.33 (0.05)***

Table 2. Coefficients of fixed effects the models (with SE) Significance levels: *=0.01, **=0.001, ***=0.0001

Modality	**BC**		**BIV**		**BIT**	
Place	Palm	Forearm	Palm	Forearm	Palm	Forearm
β_0	0.08 (0.08)	0.00 (0.07)	0.07 (0.08)	-0.01 (0.06)	0.07 (0.08)	-0.01 (0.07)
β_1	1.27 (0.22)***	1.16 (0.20)***	1.24 (0.22)***	1.24 (0.23)***	1.22 (0.23)***	1.16 (0.21)***
β_2	-0.66 (0.05)***	-0.55 (0.05)***	-0.60 (0.05)***	-0.51 (0.05)***	-0.04 (0.05)	-0.12 (0.05)*
β_3	0.74 (0.05)***	0.83 (0.05)***	0.64 (0.05)***	0.80 (0.05)***	0.13 (0.05)**	0.11 (0.05)*

References

1. Moscatelli, A., Mezzetti, M. & Lacquaniti, F. Modeling psychophysical data at the population-level: the generalized linear mixed model. *J. vision* **12**, 26–26 (2012).



(a) Visual modality

(b) Tactile modality

Figure 1. Individual results and psychometric functions calculated with GLMM of duration discrimination task in the visual (left) and tactile (right) modality on the forearm data