Multi-source uncertainty propagation and sensitivity analysis of turbine blades with underplatform dampers

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Multi-source uncertainty propagation and sensitivity analysis of turbine blades with underplatform dampers

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Abstract Underplatform dampers (UPDs) can suppress blade vibration through interface friction dissipation. At present, most of the existing UPD analysis models ignore the uncertainties induced by multi-source factors such as manufacturing, excitation, and wear, resulting in insufficient accuracy of dynamic prediction and optimization of UPDs. Therefore, this paper will deeply consider the uncertain factors such as machining tolerances, assembly tolerances, working load environment, and contact surface wear encountered in the actual manufacturing and working process, and carry out the multi-source uncertainty dynamics research of the UPDs. To solve the "dimension disaster" caused by multi-source uncertainty analysis, an adaptive algorithm based on the non-embedded polynomial chaotic expansion (PCE) is proposed. The algorithm selects the integral accuracy of the best chaotic polynomial by generalizing error convergence criteria and then constructs a surrogate model. This algorithm is in agreement with the calculated results based on Monte Carlo simulation (MCS). Through the quantitative characterization of the above uncertain factors, the efficient prediction of the uncertainty zone of the UPDs dynamic response is realized, the influence law of multi-source uncertainty factors on the vibration attenuation characteristics of UPDs is obtained, and sensitivity analysis of all parameters is carried out, which provides data support for the robust optimization design of UPDs. The simulation results show that the adaptive algorithm based on the uncertainty of multi-source parameters can predict the dynamic response of UPDs structure with high precision and high efficiency, and the uncertainty factors have a significant influence on the nonlinear dynamic characteristics of UPDs. In this case, phenomena such as "resonance band" and "frequency shift" will appear, and the uncertainty of multiple source parameters can cause large fluctuations in the dynamic response of UPDs, especially at the formant of the amplitude response. In addition, compared with other nonlinear regions, the Sobol exponents of uncertain parameters near the formant change significantly.

Keywords: Nonlinear dynamics; Underplatform dampers; Dry friction; Quantification of uncertainty; Polynomial chaotic expansion; Adaptive algorithm;

1. Introduction

Aero-engines in operation mode. The resonance phenomenon of its blades is inevitable [1-3]. Effectively reducing these vibration levels is the main option for aero-engines to be
more durable and reliable, thus weakening high-cycle fatigue caused by resonance. The introduction of dry friction damping is one of the methods to reduce the amplitude of the blade, which provides high damping through the friction motion on the contact interface [4]. In dampers with dry friction, underplatform dampers are the most commonly used damping devices in turbine blades. They are placed in grooves under the two blade holes and are in contact with the blade boss as well as the damper. When the blade rotates, the contact surface between the blade and the UPDs displacements and generates friction, which provides damping and energy dissipation for the system [5,6], thus, the vibration amplitude of the blade is reduced. Due to dry friction, the dynamic response of the UPDs structure is nonlinear, and the vibration level and resonant frequency will directly affect the resonance amplitude. Therefore, it is very important to analyze the vibration reduction characteristics of UPDs for improving the working environment of blades.

In the design and use of UPD, it is necessary to accurately predict its nonlinear dynamic characteristics. At present, researchers have studied the vibration reduction characteristics of UPDs, but they mainly focus on the deterministic analysis model. By analyzing the normal load of the contact surface, Xu et al. [7] obtained the influence of the friction system on the vibration response when a micro-slip occurs on the contact surface. Wu et al. [8] proposed an improved design scheme for damper quality for different types of UPDs. Sun et al. [9] proposed a calculation method for aerodynamic excitation and used HBM to calculate the amplitude response of blades containing UPDs. Li et al. [10] calculated and analyzed the vibration reduction effect of typical aero-engine UPDs under broadband multi-order excitation based on the single degree of freedom model. Yang et al. [11] established a simplified concentrated mass model and gave the influence rule of design parameters on the vibration reduction characteristics of UPDs, which made it closer to the actual situation. Pesaresi [12] modeled the nonlinear behavior of the damper test stand on a turbine platform and correctly predicted the nonlinear characteristics of the blade constrained by the UPDs.
device. Based on the correlation of UPDs design parameters, Li et al. [13] analyzed the influence of linkage changes of multiple parameters on the damping effect and optimized the associated parameters.

The uncertainty exists objectively in the actual UPDs, to make the calculation result more accurate and reasonable, the influence of uncertain factors should be considered when analyzing the vibration characteristics of UPDs. For UPDs, first of all, real turbine UPDs have machining tolerances and assembly tolerances in the actual manufacturing process, and the tolerances vary randomly, resulting in the difference between the actual contact state and the ideal state of the UPDs, which ultimately affects the vibration reduction effect. Secondly, aero-engine operating conditions are changeable, turbine blades have multiple excitation orders, wide coverage frequency, and working speed changes violently and unpredictably, which makes it difficult for UPDs to achieve good vibration damping effect within the working range, and brings great difficulties to the design of UPDs. Finally, aero-engine turbine blades working at high temperatures for a long time will cause wear on the contact surface between blades and UPDs. First of all, the mass and positive pressure distribution of UPDs are changed, and secondly, the contact state of the contact surface is changed, which eventually leads to the change in contact stiffness, and also affects the vibration reduction effect. These uncertainties will lead to the original deterministic equation of motion into a combination of equations with uncertain terms, which cannot be solved by general methods. Therefore, several contact models are established to simulate the dynamic characteristics of the blade boss in contact with the damper. The simplest analytical model is the linear mass-spring model, which is a nonlinear macroscopic unit composed of a spring and slider [14-16]. However, these models often do not describe the behavior of contact surfaces well. Since the parameters involved are highly uncertain [17-19], they cannot be ignored. Petrov [20] studied the influence of the uncertainty of friction coefficient and structural parameters on the nonlinear behavior of the blade disk and revealed the influence of friction coefficient...
on the dynamic characteristics of the blade disk. Yuan et al. [21] quantified the influence of
the uncertainty of contact parameters on the dynamic characteristics of turbine blades and
obtained the uncertainty band of the nonlinear forcing response. These studies show that the
influence of the uncertainty of dry friction interface parameters on the dynamic characteristics
of the system response cannot be ignored. However, in the above work, the design parameters
of the flange plate damper mainly focus on the parameters of the contact surface, but it does
not take into account the changes of actual machining and other parameters in the work, such
as machining tolerance, assembly, working load environment, contact surface wear and other
uncertain factors encountered in actual machining.

In terms of uncertainty analysis methods, Jia et al. [22] proposed a new uncertainty
propagation analysis method to improve the fitting accuracy of system PDF based on the
extended Gaussian integral and maximum entropy principle. Xiong et al. [23] proposed a
statistical moment estimation and reliability analysis method, which was also based on sparse
mesh numerical integration. In addition, it is proved that the sparse grid scheme can solve the
strong interaction problem in high dimensional system response, which makes the method
widely applied in uncertainty analysis and has been successfully applied in aerospace,
electronics, and other fields [24,25]. To ensure the accuracy of response prediction, the
uncertainty of parameters must be quantified in nonlinear response simulation. Some methods
have been developed, such as Monte Carlo simulation (MCS) [26-28], Taylor expansion
[29,30], reliability method [31,32], polynomial chaotic expansion (PCE) [33-35], etc., which
are helpful for uncertainty analysis to some extent. A generally accepted Simulation scheme is
MCS. When there are enough sampling points, the calculation results are reliable and high
precision. However, its disadvantages are also obvious in engineering applications. The
deterministic analysis model requires complete execution of each sampling point, and too
large a number of samples makes calculation difficult, resulting in the low efficiency of such
algorithms. For large complex structures with multiple degrees of freedom, excessive
repetitive computation costs are too high, and some computation tasks are even difficult to realize under existing computation conditions, such as the Curse of Dimensionality. This fundamentally limits the application of such methods in large engineering calculations. Therefore, in theoretical research, a large number of statistical uncertainty analysis methods are often used as tools to verify other analytical methods, in order to highlight the efficiency and computational accuracy of new methods. When the analysis model is complex enough, the surrogate model (also known as metamodel) becomes an effective alternative to the original model. Therefore, this study will explore the application of the chaotic polynomial expansion (PCE) surrogate model in dynamic response analysis, aiming at analyzing the influence of potential uncertainties on structural dynamic response.

This paper will consider the multi-source uncertainty factors such as machining tolerance, assembly tolerance, working load environment, and contact surface wear in the actual manufacturing and working process, quantitatively characterize these multi-source uncertainty factors and deeply analyze various uncertainties. Influence of deterministic factors on the damping characteristics of the UPDs. In order to solve the problem of the "dimensional curse" when there are too many uncertain parameters, this paper proposes an adaptive algorithm based on non-embedded polynomial chaos expansion, which combines sparse grid technology with extended Gaussian integral [22] to reduce Calculate the cost, and select the best PCE integral precision k according to the generalization error convergence criterion, and then construct the PCE surrogate model to realize the efficient prediction of the UPDs dynamic response uncertainty zone. The calculation results obtained by Carlo sampling have a high degree of agreement. In addition, the adaptive algorithm overcomes the strong interaction encountered in the system response, effectively solves the huge error problem caused by the univariate dimensionality reduction method [36], and provides important application potential for uncertainty analysis. In addition, an important advantage of PCE is that the mean and variance of the responses can be easily obtained from the surrogate model, and the sensitivity
analysis based on variance can also be performed: Sobol analysis [35,37]. Therefore, sensitivity analysis will be used in this study.

The purpose of this paper is to apply the uncertainty analysis method to nonlinear UPDs, quantitatively evaluate the influence of multi-source uncertainty parameters on turbine blade dynamics characteristics, obtain the uncertainty band of nonlinear forcing response, and compare the calculated results with the calculated solutions obtained by sampling $10^5$ MCS of the original model. In addition, the uncertain parameters have different degrees of influence on the nonlinear vibration response. The sensitivity analysis based on variance was used to obtain the Sobol index of each parameter, to order the sensitivity of each uncertain parameter, and lay a foundation for the dynamic robust design of UPDs. The rest of the work is arranged as follows: In the first section, a simplified two-blade-wedge-shaped flange plate damping model is established, and the uncertainty analysis of key parameters affecting the damping effect of the flange plate in this model will be carried out later. In Section 2, an adaptive scheme is proposed based on non-embedded polynomial chaos expansion theory, and its accuracy and efficiency are verified. In the third section, the uncertainty analysis of key parameters is carried out to obtain the uncertainty analysis results of the damping effect of key parameters on the flange plate damping structure. Based on the multi-source uncertainty parameters, the dynamic response boundary of the lower edge plate damping structure with different levels of uncertainty is obtained. Finally, the sensitivity analysis of each parameter is carried out. The fourth section summarizes the work.

<table>
<thead>
<tr>
<th>Nomenclature</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$, $C$, $K$</td>
<td>Mass matrix, Damping matrix, Stiffness matrix</td>
</tr>
<tr>
<td>$F$</td>
<td>Linear contact force</td>
</tr>
<tr>
<td>$F_{nl}$</td>
<td>Nonlinear contact force</td>
</tr>
<tr>
<td>$m_d$</td>
<td>Mass of damper</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Damper rolling degree of freedom</td>
</tr>
<tr>
<td>$k_{ld}$, $k_{rd}$</td>
<td>The stiffness of the damper in three degrees of freedom directions</td>
</tr>
<tr>
<td>$F_C$</td>
<td>Centrifugal force</td>
</tr>
<tr>
<td>$\omega_L$, $\omega_R$</td>
<td>Displacement of friction on left and right contact surfaces</td>
</tr>
<tr>
<td>$F_{sl}$, $F_{sr}$</td>
<td>An exciting force acting at the blade root</td>
</tr>
<tr>
<td>$F_{xL}, F_{xR}, F_{yL}, F_{yR}$</td>
<td>Nonlinear forces on the contact surface</td>
</tr>
<tr>
<td>$N_L$, $N_R$</td>
<td>Normal contact pressure</td>
</tr>
<tr>
<td>$f_x, f_r$</td>
<td>Tangential friction force</td>
</tr>
</tbody>
</table>
2. Dynamic modeling of UPDs

2.1. Modeling of the blade-UPDs

The actual aero-engine UPDs structure has complex geometric characteristics. In view of the actual blade-UPDs model and considering the nonlinear phenomenon of dry friction contact, high requirements are put forward for the analysis and calculation ability. A widely accepted research scheme is to extract the core dynamic characteristics from the actual blade-UPD structure, simulate the actual shape of the blade, and establish the FEM of the blade with UPDs. The frictional mechanics model was used to simulate the boundary conditions of the blades, and the geometric characteristics of the contact surface between the double-blade boss and the UPD were mainly preserved. The establishment of the system model is shown in Fig.1:

![Simplified blade-UPDs model](image)
The equation of motion of the UPDs structure can be written as:

$$M\ddot{U}(t) + C\dot{U}(t) + KU(t) = F(t) - F_{nl}(\dot{U}(t), U(t))$$  \hspace{1cm} (1)$$

Where $F$ is linear excitation, and $F_{nl}$ is the nonlinear force on the contact surface. The damper is located between the two blades and is in contact with the left and right boss of the blade, and the normal displacement and Angle of the blade root are fully constrained. The mass of the damper is $m_d$. Considering that the axial displacement of the blade is negligible compared with the normal displacement and the torsion of the blade, the nodes on the blade have two degrees of freedom, and each node of the flange plate damper has three degrees of freedom, including a rolling degree of freedom $\theta$, whose equivalent constraint stiffness is $k_{\theta d}$, $k_{yd}$, $k_{\theta d}$, and the tangential stiffness and normal phase stiffness of the two contact points are $k_i$ and $k_n$ respectively. $d_L$ and $d_R$ are the relative displacements between the damper and the left and right contact surfaces of the blade, $N_L$ and $N_R$ are the positive pressure on the contact surface, and $F_C$ is the centrifugal force on the damper. Here, the deformation and contact pressure of the contact surface are assumed to be uniformly distributed, and the contact pairs are in the same state, which can more accurately describe the dry friction behavior of the blade-UPDs structure.

It is worth mentioning that it is impractical to calculate directly with a finite element model. Firstly, the nonlinear calculation process takes a long time. Secondly, the subsequent uncertainty analysis also requires a certain amount of samples to simulate the original model, which requires a large amount of calculation. Therefore, to reduce the computational load, the author adopts the fixed mode synthesis method to reduce the model. Only the friction force and the external excitation degree of freedom on the blade are retained, and other relevant degrees of freedom are ignored. The calculated results are compared with the non-reduction model. Steady-state response calculation results show that blade tip responses before and after shrinking are consistent, as shown in Fig. 2:
2.2. Dynamic equation of blade -UPDs

In this paper, a hysteretic Coulomb friction model with contact stiffness is used to describe the mechanical characteristics of UPDs. There are three contact states in the two-dimensional contact model of plane variable pressure: slip, viscosity, and separation. The relation between normal pressure $N$ and normal relative displacement $v$ is:

$$N = \max(k_n v, 0)$$  \hspace{1cm} (2)

The relation between the friction force $T$ in the direction of the contact surface and the relative displacement $u$ is:

$$T = \begin{cases} 
  k_s (u - u_0) & \text{stick} \\
  \text{sgn}(\dot{u}) \mu N & \text{slip} \\
  0 & \text{separation} 
\end{cases}$$  \hspace{1cm} (3)

Among them, $\text{sgn}$ represents the sign function, and $u_0$ represents the tangential relative displacement value at the initial moment of viscosity, and the dynamic equation of the blade-UPDs is constructed according to Eq. (1) as follows:

![Fig. 2 Comparison of steady-state response between the reduction model and the finite element model.](image-url)
\[ M_L \ddot{u}_{sl} + C_L \dot{q}_{sl} + K_L q_{sl} = F_{xL} - F_{xL} \]
\[ I_{thL} \ddot{\theta}_{thL} + C_{thL} \dot{\theta}_{thL} + K_{thL} \theta_{thL} = F_{\thetaL} H \]
\[ m_d \ddot{q}_{sd} + c_d \dot{q}_{sd} + k_d q_{sd} = F_{xL} + F_{xL} \]
\[ m_c \ddot{q}_{cd} + c_d \dot{q}_{cd} + k_d q_{cd} = F_{xL} + F_{xL} + CF \]
\[ I_{thL} \ddot{\theta}_{thL} + c_{thL} \dot{\theta}_{thL} + k_{thL} \theta_{thL} = F_{xL} + F_{xL} + CF \]
\[ M_R \ddot{u}_{sr} + C_R \dot{q}_{sr} + K_R q_{sr} = F_{xR} - F_{xR} \]
\[ I_{thR} \ddot{\theta}_{thR} + C_{thR} \dot{\theta}_{thR} + K_{thR} \theta_{thR} = F_{\thetaR} H \]

In Eq. (4), \( I_d \) represents the moment of inertia of the damper, \( H \) is the length of the excitation force point from the frictional node on the blade, \( F_{xL} \) and \( F_{xR} \) are the same phase excitation force acting on the left and right blades respectively, \( F_{xL}, F_{xR} \) is the nonlinear force along the left and right contact points, and \( F_{yL}, F_{yR} \) is the nonlinear force along the left and right contact points. Its expression is Eq. (5):

\[
\begin{align*}
F_{xL} &= f_x \cos \alpha_x + N_x \sin \alpha_x \\
F_{xR} &= f_x \sin \alpha_x - N_x \cos \alpha_x \\
F_{yL} &= f_y \cos \alpha_y - N_y \sin \alpha_y \\
F_{yR} &= -f_y \sin \alpha_y - N_y \cos \alpha_y
\end{align*}
\]

Where \( N_L, N_R \) is the normal contact pressure; \( f_L, f_R \) represents tangential friction force, and \( a_L, b_L, a_R, b_R \) are the projected distance from the center of mass of the damper to the left and right contact points in the \( x \) and \( y \) directions respectively; the position of the contact point is the center of mass to the vertical points on both sides. It can be obtained from Eq. (4) and Eq. (5). The relation between tangential displacement \( u_L, u_R \) and normal relative displacement \( v_L, v_R \) and relative displacement in the cartesian coordinate system is as follows:

\[
\begin{align*}
u_L &= (q_x - q_d \cos \alpha_x + q_d \sin \alpha_x) - \sqrt{a_x^2 + b_x^2} \theta_x \\
v_L &= (-q_d + q_d \sin \alpha_x) - (q_x \sin \alpha_x + q_d \cos \alpha_x) \\
u_R &= (q_x - q_d \cos \alpha_x + q_d \sin \alpha_x) - \sqrt{a_x^2 + b_x^2} \theta_x \\
v_R &= (q_d + q_d \sin \alpha_x) - (q_x \sin \alpha_x - q_d \cos \alpha_x)
\end{align*}
\]

Where \( q_x, q_d, q_y, q_a, q_b \) and represents the normal direction and angular displacement of the node.
2.3. **Nonlinear steady-state response of UPDs**

The contact parameters and geometric parameters of the blade-UPDS model are shown in Table 1. Based on these deterministic parameters, the response of UPDs in in-phase mode (IP) and out-of-phase mode (OOP) can be obtained by using the time-domain numerical calculation method, as shown in Fig.3:

<table>
<thead>
<tr>
<th>Physical values of geometric and structural parameters of the UPDs model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>Length of blade L/m</td>
</tr>
<tr>
<td>Width of blade B/m</td>
</tr>
<tr>
<td>The thickness of the blade H/m</td>
</tr>
<tr>
<td>The radius of wheel R/m</td>
</tr>
<tr>
<td>The density of material kg/m³</td>
</tr>
<tr>
<td>Modulus of elasticity E/Gpa</td>
</tr>
<tr>
<td>Shear correction factor k</td>
</tr>
</tbody>
</table>

**Fig. 3** Deterministic steady-state response of UPDs.

As shown in Fig. 3, the vibration response of the UPDs model under different exciting forces (3N, 6N, 9N, 12N, 15N) and centrifugal loads. As shown in Fig. 1, an exciting force in the direction of x is applied to the blade root. The dynamic characteristics of UPD structures were quantified by drawing the response curve at the blade tip. At the first natural frequency, there is softening behavior of stiffness and a certain frequency shift. At the same time, the
damping performance of different vibrations is better than that of in-phase vibration because of the rolling phenomenon and contact loss in in-phase vibration. This motion reduces the contact area and slightly degrades the performance of the damper. In addition, for the in-phase mode, Fig. 3(c) shows the comparison of linear and nonlinear amplitude responses at the UPDs interface under resonant frequencies at different excitation levels when the external excitation force $F_a$ increases from 0.5N to 8N. Under the maximum excitation level, the amplitude decreases by 35.5%. For the OOP mode, Fig. 3(d) shows the comparison of linear and nonlinear amplitude responses at the damping interface of the lower edge plate of resonant frequency at different excitation levels when the external excitation force $F_a$ increases from 0.5N to 8N. Under the maximum excitation level, the amplitude decreases by 48.9%.

3. Uncertainty propagation

3.1. Sparse Grid Numerical Integration (SGNI)

3.1.1. Polynomial chaotic expansion

To carry out the relevant content of UPDs uncertainty analysis, polynomial chaos expansion theory is used to construct the surrogate model. Consider the finite variance calculation model as mapping $Y = G(X)$, the polynomial is truncated for the convenience of calculation. Where the highest order of the polynomial $\Psi(\zeta)$ does not exceed the $p$-order, The truncated form of the equation is as follows:

$$Y = G(X) = \sum_{i=0}^{P} \hat{b}_i \Psi_i(\zeta)$$

(7)

Where the coefficient $\hat{b}_i$ representing the polynomial expansion, $\Psi_i(\zeta)$ represents the product of a random variable $\zeta_1, ..., \zeta_d$ corresponding to a 1-dimensional orthogonal polynomial basis function, so it is called the mixed polynomial, namely:
\[ \Psi_i(\zeta) = \prod_{k=1}^{d} \psi_k^i(\zeta_k) \]  

(8)

Where, \( \Psi_i^k(\zeta_k) \) represents the 1-dimensional orthogonal polynomial basis function corresponding to the \( k \)-dimensional random variable \( \zeta_k \).

3.1.2. Smolyak polynomials

Firstly, a multidimensional node \( U_n^k \) with \( n \)-dimensional random variables and integral precision, \( k \) is defined. Let \( \hat{g}^{d,k} \) represent the \( d \)-dimensional Smolyak polynomial function (interpolation), Where the interpolation of the coefficient grid is a linear combination of tensor product operators \( p^{j|l} \):

\[
\hat{g}^{d,k}(x_1, \ldots, x_d, b) = \sum_{\max(d,k+1) \leq |j| \leq d+k} (-1)^{d+k-|j|} \binom{d-1}{d+k-|j|} \sum_{|i|=i_1+i_2+\ldots+i_n} p^{j_1 \ldots j_n}(x_1, \ldots, x_d) 
\]

(9)

\(|i| (|i|=i_1+i_2+\ldots+i_n) \) denotes the sum of multiple exponents, \( i_j (j = 1, 2, \ldots, n) \) is the univariate exponent in the orthogonal scheme, and \((-1)^{k+n-|i|} \binom{n-1}{k+n-|i|}\) is the calculation coefficient. Among them:

\[
p^{j_1 \ldots j_n}(x_1, \ldots, x_d) = \sum_{i_1=1}^{m(i_1)} \ldots \sum_{i_d=1}^{m(i_d)} b_{i_1 \ldots i_d} \psi_{i_1}(x_1) \ldots \psi_{i_d}(x_d) 
\]

(10)

Where \( m(i_j) \) is the number of polynomial basis functions in dimension \( j \), and the number of nodes increases polynomially. In addition, SGNI provides multidimensional node \( U_n^k \) solutions:

\[
U_n^k = \bigcup_{k+1 \leq |i| \leq d+n} U_1^i \otimes U_1^i \otimes \ldots \otimes U_1^i 
\]

(11)

where \( \otimes \) denotes the tensor product notation, and by numerical integration using sparse grids, nodes based on Eq. (8) can be obtained.

According to \( m(i_j)(j=1, \ldots, d) \), the number of integral nodes on various dimensional random variables, it is pointed out in the literature that the relationship between the two is
Of course, other forms of functional relationship can also be specified, such as

\[ m(ij) = 2^{j-1} + 1, ij \geq 2 \text{ and } m(1) \equiv 1. \]

For each node, the expression for the weight is as follows:

\[
w_i = (-1)^{k+n-j} \left( \frac{n-1}{k+n-|i|} \right) w_{ij}^i, w_{ij}^j, \ldots, w_{ij}^n
\]

(12)

Where, \( w_{ij}^l \) represents the weight of the node \( \zeta_{ij} \), \( \zeta_{ij} \) is the \( j \) element of the node \( U_{ij} \), \( w_i \) represents the weight of the \( l \)-th dimensional node \( \zeta_{ij} = [\zeta_{ij}^1, \zeta_{ij}^2, \ldots, \zeta_{ij}^n] \in U_{ij}^k \).

3.1.3. Coefficient of solution

The spectral mapping method uses the inner product to map the response values of each basis function while using the orthogonal property of the polynomial family to calculate the coefficients to be solved. Using the Galerkin projection method, both sides of Eq. (12) are projected to each orthogonal polynomial successively at the same time, thus:

\[
\langle g\psi_j(\xi) \rangle = \left\{ \sum_{i=0}^P b_i \psi_i(\xi) \psi_j(\xi) \right\}, (j = 0, 1, \ldots, P)
\]

(13)

\( P \) is the number of truncated terms. From the definition of inner product and the orthogonality of orthogonal polynomials, we can get:

\[
\langle \psi_i(\xi) \psi_j(\xi) \rangle = \langle \psi_i^2(\xi) \rangle \delta_{ij}
\]

(14)

Eq. (14) is arranged as:

\[
b_i = \frac{E\left[g\psi_i(\xi)\right]}{E\left[\psi_i(\xi)\psi_i(\xi)\right]}, (i = 0, 1, \ldots, P)
\]

(15)

In the above equation, the denominator is the square of the norm of the multivariate orthogonal polynomial, which can be calculated by calculating the product of the square of the norm of one variable:

\[
E\left[\psi_i(\xi)\psi_i(\xi)\right] = \prod_{k=1}^d E\left\{\left(\phi_k^i(\xi_k)\right)^2\right\}
\]

(16)
\( \Phi_i(\zeta) \) is the product of orthogonal polynomials in each dimension. The computation of Eq. (16) comes from the calculation of molecules \( E[g\Phi_i(\zeta)] \). Mathematically, \( E[g\Phi_i(\zeta)] \) can be expressed as:

\[
E\left[g\psi_i(\xi)\right] = \int g\psi_i(\xi)F(\xi)d\xi
\]

(17)

Where \( F(\zeta) \) is the joint probability density function of \( \zeta \). For the calculation \( g\psi_i(\tilde{\zeta}) \), the specific steps are as follows:

1. Calculate the integral points and weights of 1-dimensional Gauss;
2. Calculate the dimension integral node and the corresponding weight;
3. Calculate the value of the response function on the integral node of each dimension;
4. The expected value is obtained according to the Gauss integral principle.

3.1.4. Post-processing of PC extensions

Once the value of the coefficient is obtained, an approximation of the following polynomials can be obtained:

\[
\tilde{G}_p(X) = \sum_{i=0}^{p} b_i\psi_i(X)
\]

(18)

The latter can be used as a metamodel (that is, an alternative model of the model's response to \( G(X) \)) so that quantities of interest can be obtained at a very cheap computational cost. In particular, the mean and the variance of the response output can be approximated as:

\[
\hat{\mu}_{y,p} = \hat{b}_0
\]

\[
\hat{\sigma}_{y,p}^2 = \sum_{i=0}^{p} b_i^2
\]

(19)

The sensitivity index of the uncertainty parameter is also easy to obtain. Sensitivity analysis is an analytical method to study the influence of the output of a model on the variation of input variables. It can effectively identify important input variables. Sensitivity analysis is an important prerequisite for optimization analysis in many fields. Sobol
decomposition is defined as:

\[ g(x_1, \ldots, x_M) = g_0 + \sum_{i=1}^{M} g_i(x_i) + \sum_{1 \leq i < j \leq M} g_{ij}(x_i, x_j) + \cdots + g_{1,2,\ldots,M}(x_1, \ldots, x_M) \]  

(20)

The following conditions are true:

1. \( g_0 \) is constant, the expected value of \( g(X) \);
2. The integrals of decimals are ignored concerning their variable \( g_{i_1,\ldots,i_s} \).

\[ \int_0^1 g_{i_1,\ldots,i_s}(x_{i_1}, \ldots, x_{i_s}) \, dx_{i_k} = 0 \quad \text{if} \quad 1 \leq k \leq s \]  

(21)

This extension was shown to exist and be unique in Sobol’ for the tunable function and the independent input variable \( X \) in the total variance \( D_X \). It also shows that all sums of the extension can be computed recursively by integration, as follows.

\[ g_0 = \int_{D_x} g(x) \, dx \]

\[ g_i(x_i) = \int_0^1 \cdots \int_0^1 g(x) \, dx_{-i} - g_0 \]  

(22)

\[ g_{ij}(x_i, x_j) = \int_0^1 \cdots \int_0^1 g(x) \, dx_{-ij} - g_0 - g_i(x_i) - g_j(x_j) \]

Where \( x = \{x_1, \ldots, x_d\} \), the symbol \( \sim \) indicates that the variable is excluded, e.g.:

\[ x_{-i} = (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_d) \]  

(23)

Higher-order subterms can be constructed similarly. However, Eq. (20) is used only to calculate variance decomposition, which will be described next:

\[ D = \int_{D_x} g^2(x) \, dx - g_0^2 \]  

(24)

and the partial variances are computed as:

\[ D_{i_1,\ldots,i_s} = \int_0^1 \cdots \int_0^1 g_{i_1,\ldots,i_s}^2(x_{i_1}, \ldots, x_{i_s}) \, dx_{i_1} \cdots dx_{i_s} \quad 1 \leq i_1 < \ldots < i_s \leq d; s = 1, \ldots, d \]  

(25)

The sensitivity index is defined as:

\[ S_{i_1,\ldots,i_s} = \frac{D_{i_1,\ldots,i_s}}{D} \]  

(26)
Both sides of Eq. (20) are squared first and then integrated globally. According to the orthogonal properties of each subterm, we can know that:

\[
D = \sum_{i=1}^{n} D_i + \sum_{1 \leq i < j \leq n} D_{ij} + \cdots + D_{12\ldots n} \tag{27}
\]

The properties of all sensitivity indices and values of 1 can then be obtained:

\[
\sum_{i=1}^{n} S_i + \sum_{i=1}^{n} \sum_{j=i+1}^{n} S_{ij} + \cdots + S_{12\ldots n} = 1 \tag{28}
\]

The sensitivity index \( S_i \) represents the first-order influence of parameter \( x_i \) without considering the influence of cross-action. To evaluate the overall influence of parameter \( x_i \), the total sensitivity index \( S_i^T \) of parameter, \( x_i \) is defined as:

\[
S_i^T = 1 - S_{-i} \tag{29}
\]

### 3.2. Self-adaptive scheme

The convergence rate of the algorithm is related to the applicability of the surrogate model. A scheme based on polynomial basis function adaptation has been developed [38]. This scheme uses the regression method to solve the polynomial coefficients. However, using the regression method to find the coefficients involves matrix inversion and other operations, which is generally suitable for medium-scale problems, that is, random input variables. The number can not be too many, and when the number of samples is large, there will be numerical pathological problems, which has become the main disadvantage of this method. Whereas projection-based PCE does not require a truncation scheme for orthogonal-based coefficient computation since all coefficients up to a specified polynomial degree are computed simultaneously. Therefore, taking advantage of this advantage, this paper proposes an adaptive scheme for integral accuracy, which uses the projection method to calculate the coefficients of polynomials, combined with the Smolyak polynomials in Section 2.2.2, to alleviate the problem of the 'curse of dimensionality', while testing A certain optimization has

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been made on the sampling of the set, and by defining the standard of error convergence, while ensuring the improvement of calculation accuracy, the unnecessary calculation amount is reduced.

2.3.1 Estimation of approximate error

For different calculation models and different working conditions of the same model, to evaluate the appropriate value of integral accuracy, approximate error of model response should be introduced to compare the calculated results of real and surrogate model responses. After the surrogate model is constructed, its accuracy and predictive power can be quantified by estimating relative generalization errors. It is defined as:

\[
\mathcal{E}_{\text{gen}} = \frac{E\left[\left(G(X) - G^{\text{rc}}(X)\right)^2\right]}{\text{Var}(G(X))}
\]  

(30)

Prepare a separate validation set of inputs and outputs \([X_{\text{Val}}, Y_{\text{Val}} = M(X_{\text{Val}})]\), then the verification error can be calculated as:

\[
\mathcal{E}_{\text{Val}} = \frac{N - 1}{N} \sum_{i=1}^{N} \left( G\left(X^{(i)}_{\text{Val}}\right) - G^{\text{rc}}\left(X^{(i)}_{\text{Val}}\right) \right)^2 \\
\sum_{i=1}^{N} \left( G\left(X^{(i)}_{\text{Val}}\right) - \mu_{\text{Val}} \right)^2
\]

(31)

Where, \(\mu_{\text{Val}} = \frac{1}{N} \sum_{i=1}^{N} G\left(X^{(i)}_{\text{Val}}\right)\) is the sample mean of the verification set. For evaluating the performance of different proxy models on the same sample verification set, it is useful to define the relative generalization error at this time.

2.3.2 Self-adaptive scheme(\(k\)-SAA)

According to the sparse scheme in Section 2.2.2, for the polynomial of dimension \(d\) and integral accuracy \(k\), the training set generated is related to both of them. However, dimension \(d\) is generally directly related to the model and the number of parameters to be analyzed, and is generally set as a fixed value, which cannot be used to improve accuracy or reduce computation. Whereas the integral precision \(k\) does exactly that, in practice it is usually not
clear which integral precision will yield the best PCE approximation. According to the sparse scheme in Section 2.2.2, for the polynomial of dimension $d$ and integral accuracy $k$, the training set generated is related to both of them. However, dimension $d$ is generally directly related to the model and the number of parameters to be analyzed, and is generally set as a fixed value, which cannot be used to improve accuracy or reduce computation. Whereas the integral precision $k$ does exactly that, in practice it is usually not clear which integral precision will yield the best PCE approximation. With the increase of polynomial integral precision $k$, the training set will increase, and then the number of terms in the polynomial will increase, and the fitting surrogate model becomes more complex, which may lead to the over-fitting phenomenon, increasing generalization error. A key validation tool for any adaptive algorithm is the error estimation of the surrogate model (see Section 2.3.1) to estimate the accuracy of the model. To effectively evaluate the fitting accuracy of the proxy model, the following criteria are introduced: If the approximate error $\epsilon_{error}$ is increased twice in a row (such as $\epsilon_{error_i} < \epsilon_{error_{i+1}} < \epsilon_{error_{i+2}}$), the algorithm stops and the overfitting message is returned. The coefficient of correlation PCE is calculated by the projection method.

Finally, the best PCE is selected according to the generalization error, and the statistical distance of the response is directly output. It is worth reminding that:

(1) When the integration accuracy is iterated from $k$ to $k+1$, the idea of extended Gaussian integration is adopted [39-41], and the nodes of each layer contain the nodes of the previous layer in the SGNI schema, which effectively prevents the double calculation. This is different from the number of integral nodes on the random variables of each dimension in Section 2.2.2. In addition, some transformation is needed, that is, the non-normal distribution is transformed into the standard normal distribution. Limited by the space, see the literature [22] for details.

(2) The rich experimental design mentioned by the author here is by increasing the number of nodes per dimension in (1), which is different from the extended Gaussian
integral, which will directly increase the number of the training set. In addition, this experimental design is only used when the integration accuracy is iterated to $k_{\text{max}}$ and still does not meet the convergence conditions.

(3) After obtaining the surrogate model, the $10^5$ MCS simulation can be performed directly on the surrogate model, and statistical data such as the statistical distance of each order of the response, the response boundary, and the probability density function can be directly obtained. The $k$-SAA workflow is shown in Fig.4:
3.3. Verification of Adaptive Algorithms

3.3.1 Accuracy and efficiency of adaptive algorithms

To verify the accuracy and precision of the above adaptive algorithm in dealing with nonlinear UPDs. In this section, the fluctuation coefficient of the uncertain parameter is set at 5%. The results obtained through $10^5$ Monte Carlo simulations are used as the standard for verification. Select some frequency points around the natural frequency of the UPDs. For the verification part of the algorithm, the uncertain parameters are selected as the uncertainties on the contact surface, that is, namely quantifying the uncertainty of friction coefficient and contact stiffness. Curves of mean value and standard deviation as well as average calculation time were obtained, and MCS was used for comparison and verification, as shown in Fig.5:

![Comparison of mean values and standard deviations](image)

Fig.5 (a),(b): Comparison of mean values obtained by different algorithms; (c),(d): Compare the standard deviations obtained by different algorithms.

<table>
<thead>
<tr>
<th>Table 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>The average calculation time for each method</td>
</tr>
<tr>
<td>Method of calculation</td>
</tr>
<tr>
<td>-----------------------</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

21
Note: MCS stands for the Monte Carlo method; $k$-SAA stands for the adaptive algorithm.

It can be seen from Fig. 6(a) and Fig. 6(b) that the adaptive algorithm is very close to the MCS results, which verifies the feasibility of their response analysis on the UPDs. In addition, this method uses fewer integral points to achieve a good approximation effect and does not need to modify the solution method of the original deterministic model, so it has the advantages of high precision and non-embedded.

The amplitudes of the PCE model and the real UPDs model at the natural frequency are compared on the response surface, sampling points of the surrogate model are evenly distributed on the response surface, but the sampling samples are still independent of the samples of the training set. It can be seen from Fig. 6 that the prediction of the response points of the surrogate model is very consistent with the real model. At the same time, the regression fitting of the response amplitude of the UPDs can be quickly obtained. As shown in Fig. 7, the regression coefficients are all close to 1, indicating the good characteristics of the surrogate model. The maximum error obtained by Eq. (31) is also within the acceptable range.

Fig. 6 Response comparison between the real model and the surrogate model.

Fig. 7 Regression fitting of surrogate model and original model.
Fig. 8 shows the upper and lower bounds of the uncertain dynamic response range calculated at the critical speed point of the UPDs and some surrounding frequency points. As we can be seen from Fig. 8, the $k$-SAA method is close to the Monte Carlo sampling results in $10^5$ times, which verifies their feasibility in the analysis of uncertain dynamic response, especially since the prediction of response boundary is very consistent with the real model.

![Fig. 8 Upper and lower bound of $k$-SAA prediction versus $10^5$MCS simulations](image)

### 3.3.2 Convergence analysis of output expectation and standard deviation

To compare the convergence of the adaptive algorithm, different sampling strategies are used for uncertainty propagation analysis. Of course, the change in sample number will directly affect the computational efficiency. In this case, the maximum relative error $\alpha_{Y,\mu}$ and maximum relative error $\alpha_{Y,\sigma}$ of the corresponding response amplitude at each frequency point can be expressed as:

$$\alpha_{Y,\mu} = \left| \frac{(\bar{Y})_{Y,\mu}^{cal}}{(\bar{Y})_{Y,\mu}^{ref}} - 1 \right| \times 100\%$$

$$\alpha_{Y,\sigma} = \left| \frac{(\sigma_{Y,\sigma}^{cal}}{(\sigma_{Y,\sigma}^{ref}} - 1 \right| \times 100\%$$

In the formula,

$(\bar{Y})_{Y,\mu}^{cal}$ -- The expectation obtained by numerical integration of sparse grid or non-reference solution MCS;

$(\bar{Y})_{Y,\sigma}^{cal}$ -- Standard deviation obtained by numerical integration of sparse grid or
non-reference solution MCS;

\[(\mathcal{Y})_{\text{ref}}^{\mu} - \text{The expectation obtained by } 10^5 \text{ sample MCS;}
\]

\[(\mathcal{Y})_{\text{ref}}^{\sigma} - \text{The standard deviation of the sample MCS of } 10^5.\]

Fig. 9 shows the convergence analysis of the MCS method with the mandatory limited sample number and sample number \(M=100,500,1000,2000,3000,4000\):

Here we do not pay attention to which surrogate model under the integral precision is the optimal surrogate model, it is only used to compare the convergence of adaptive schemes. However, it can be seen from Fig.9 that when the number of \(k\)-SAA samples is small in this case, the \(k\)-SAA method still shows a much faster convergence rate than MCS. It should also be emphasized that the accuracy level achieved by \(k\)-SAA method convergence is not its final accuracy level, which is limited by the accuracy level of the MCS reference solution with \(M=10^5\) samples.

\[\text{Fig.9 Comparison of mean and standard deviation convergence of } k\text{-SAA and MCS responses to UPDs.}\]

4 Uncertainty analysis of UPDs

In this section, the sources of uncertainty in existing research UPDs are described. Then, the mathematical characteristics of various uncertainty parameters are described, which is the premise of selecting an uncertainty analysis method suitable for UPDs.

The uncertainties in the dynamic simulation of damped blades come from a wide range of sources. During sliding, solid contact occurs between the two contact surfaces, resulting in
changes in surface roughness and stiffness. In addition, changes in damper geometry may lead to changes in blade coupling, resulting in changes in natural frequency. These factors will lead to random changes in input parameter values, which will lead to uncertainty in amplitude response and resonant frequency. Quantification of these uncertainties is very important for the interpretation of friction damper design [24]. In analyzing the characteristic law of uncertain parameters on damping performance, the influence of the mass of UPDs, geometric shape, contact interface parameters, excitation force, and other factors is generally considered. However, most of the existing damping analysis models only set the parameters as deterministic, which leads to insufficient accuracy of dynamic prediction and optimization of UPDs. In fact, in addition to the influence of these deterministic design parameters, the vibration reduction characteristics of UPDs are also not negligible due to various uncertain factors such as machining tolerance, assembly tolerance, working load environment and contact surface wear encountered in the actual manufacturing and working process, which is also proved by subsequent studies. All these factors lead to random changes in input parameter values, causing the amplitude response and resonant frequency to become uncertain. Potential uncertainty sources of UPDs can be summarized into the following categories:

(1) Uncertainty of machining tolerance and assembly tolerance:

Both real turbine blades and dampers inevitably have machining tolerances and assembly tolerances in the actual manufacturing process, and the tolerances are random. As a result, the contact state of the dry friction contact interface is inconsistent with the ideal state, which affects the vibration-damping effect of UPDs.

(2) Uncertainty in turbine blade operation:

The variable working state of aero-engine in service conditions makes turbine blades have numerous excitation orders, wide excitation coverage frequency, and drastic changes in working speed. As a result, it is difficult for UPDs to achieve good vibration-damping effects in a wide working range, which brings great difficulties to
(3) Uncertainty about turbine blades and dampers after working at high temperatures for a long time:

The long working time of aero-engine turbine blades at high temperatures will cause wear between the blade boss and damper contact surface. On the one hand, the mass and positive pressure distribution of the damper will be changed, and on the other hand, the state of the contact surface will change. Finally, the contact stiffness will change, which will affect the damping effect of the damper.

Through the quantitative characterization of these uncertain factors, the influence law of various uncertain factors on the vibration reduction effect is analyzed, which provides support for the optimization of UPDs.

In this study, according to the experimental data in Pesaresi [12], only the probability distribution form of the contact parameters is given, and the remaining parameters are assumed to obey the uniform distribution on the interval, see Table 3. A surrogate model is built using the adaptive scheme in Section 3.2, enabling the simulation of nonlinear dynamical system behavior at a very low computational cost.

4.1. UPDS response range at single parameter uncertainty level

After the verification of the algorithm, the dynamic response range of the UPDs under the level of single parameter uncertainty is considered here, to provide a reference for the subsequent sensitivity analysis. According to the calculation steps in Section 3.1, the distribution form of each uncertainty parameter is shown in Table 3, and the response of UPDs in two frequency ranges of IP and OOP modes is calculated.

Based on the above analysis, Table 5 is regarded as the uncertainty parameter respectively, and the corresponding dynamic response range of uncertainty is obtained when the uncertainty level is 5% and 10%, as shown in Fig.10(a~h) and Fig.11(a~h).
Table 3
The probability distribution of key parameters

<table>
<thead>
<tr>
<th>Number</th>
<th>Parameter Type</th>
<th>Distribution of obedience</th>
<th>Nominal value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>Left base corner of UPDs $\theta_{L}$</td>
<td>Uniform</td>
<td>$30^\circ$</td>
</tr>
<tr>
<td>A2</td>
<td>Right base Angle of UPDs $\theta_{R}$</td>
<td>Uniform</td>
<td>$30^\circ$</td>
</tr>
<tr>
<td>A3</td>
<td>The amplitude of exciting force $F_{A}/N$</td>
<td>Uniform</td>
<td>10</td>
</tr>
<tr>
<td>A4</td>
<td>Coefficient of friction $\mu$</td>
<td>Gaussian</td>
<td>0.34</td>
</tr>
<tr>
<td>A5</td>
<td>Normal contact stiffness $K_{n}(N/m)$</td>
<td>Gaussian</td>
<td>$5.5 \times 10^5$</td>
</tr>
<tr>
<td>A6</td>
<td>Tangential contact stiffness $K_{t}(N/m)$</td>
<td>Gaussian</td>
<td>$4.3 \times 10^6$</td>
</tr>
<tr>
<td>A7</td>
<td>Vane speed $\omega(rad/s)$</td>
<td>Uniform</td>
<td>200</td>
</tr>
<tr>
<td>A8</td>
<td>The quality of the UPDs $m_{d}(kg)$</td>
<td>Gaussian</td>
<td>0.002</td>
</tr>
</tbody>
</table>

(a) Parameter of uncertainty-$\mu$
(b) Parameter of uncertainty-$\theta_{L}$
(c) Parameter of uncertainty-$\theta_{R}$
(d) Parameter of uncertainty-$F_{A}$
(e) Parameter of uncertainty-$K_{n}$
(f) Parameter of uncertainty-$K_{t}$
The following conclusions can be drawn from Fig.10 and Fig.11:

Due to the existence of uncertain factors, the dynamic characteristics of UPDs change obviously. The original deterministic response becomes a dynamic response interval under the
influence of uncertainty. This is necessary because definite solutions can be considered a special case of dynamic solutions. Due to the existence of nonlinearity, the dynamic response boundaries obtained by different parameters have frequency shifts, and the upper and lower boundaries have shifted in opposite directions. As the uncertainty level (fluctuation coefficient) increases, the interval value of the dynamic response increases, especially near the formant, and the uncertainty changes the position of the format of the system to a certain extent. In addition, the influence of some uncertain parameters is smaller than frequency part of the system resonance, such as friction coefficient, damping block mass, link speed, and other parameters, and the influence of some uncertain parameters is greater than the frequency part of the system resonance, such as the external excitation amplitude.

Under the current working conditions, it can be seen that the uncertainty fluctuation range of the two parameters of friction coefficient and damping block mass is small, and the deterministic curve has no obvious deviation under the uncertainty level of less than 10%. This phenomenon indicates that the sensitivity of UPDs to the friction coefficient is not high, and the inherent properties of the system do not change significantly. The tangential contact stiffness is next, the fluctuation of the external excitation amplitude is the most significant, and the nonlinear fluctuation of the normal contact stiffness is more severe. This is because the normal contact stiffness directly affects the magnitude of the friction force. It was reflected in the subsequent sensitivity analysis.

4.2. UPDS response range at multi-source uncertainty level

In the actual UPDs work of turbine blades, it is not only affected by a single parameter, but also affected by multiple parameters at the same time, and there are correlations among parameters, which will be reflected in the subsequent sensitivity analysis. Based on times, all the above single parameters are regarded as uncertainty parameters at the same time, and the corresponding dynamic response range of uncertainty at the uncertainty level of 5% and 10%
is obtained, as shown in Fig.12:

![Graph of excitation vs. response](image)

**Fig.12 Dynamic response of UPDs with multiple source parameter uncertainty.**

Note: Y-Cer represents the deterministic response, and 5%Vol and 10%Vol represent the volatility coefficient.

It can be seen from Fig. 12 that blade failure often occurs at the upper and lower boundaries of response prediction, and multi-source uncertainty parameters will increase the prediction of response boundaries. Compared with Fig. 5(a) and Fig. 5(b), the uncertain interval becomes wider, the boundary appears unsmooth (jagged) with the increase of the wave coefficient, and the uncertain interval far from the formant changes little. This is also related to the nonlinearity of the model itself, which is a feature that distinguishes it from the linear model. At the same time, the main influence of uncertainty is less than the frequency component of the resonant frequency of the system when the vibration is in the same direction, and with the increase of the fluctuation coefficient, the resonant frequency of the response to the upper boundary decreases and the resonant frequency of the lower boundary increases, while the main influence of uncertainty is greater than the frequency component of the response to the resonance frequency of the upper boundary increases and the resonant frequency of the lower boundary decreases when the vibration is in the different direction.

Therefore, in the design and research of UPDs, to obtain more reliable and safe research results, it is necessary to consider the machining tolerance and parameter uncertainty generated during the working state, which is also extremely important for the subsequent optimization of UPDs.

To evaluate the effect of different excitations and excitation levels, two modes of the
The boxplot in Fig. 13 were studied at resonance. When the external excitation force increases, the overall change trend of the vibration in the same direction and the vibration in the opposite direction is consistent, and both show the phenomenon that the amplitude increases gradually and the resonance frequency decreases. For the change of excitation amplitude, as shown in Fig. 13(a) and Fig. 13(c), the change range of the amplitude of the vibration in the same direction and the opposite direction increases, which leads to an increase in the change range of the amplitude when the amplitude variation range increases with the increase of excitation force. At lower excitation levels (0.5N, 1N), the contact state is viscous, there is no dry friction at this time, and the amplitude fluctuation range is small; when the excitation force level increases (3N, 4N, 6N), the contact state behaves At this time, the dry friction force starts to act, and the increase of the amplitude is suppressed at this time, and the increase is no longer linear; when the excitation amplitude increases again (9N, 12N), the contact surface begins to separate, and the UPD's damping effect on the amplitude becomes weaker. For the phenomenon of frequency offset, the trend of vibration in the same direction is consistent with that in the opposite direction, as shown in Fig.13(b) and Fig.13(d). At lower excitation levels (0.5N, 1N), due to no dry friction, The frequency deviation is not obvious, and the frequency fluctuation range is small; when the excitation force level increases (3N, 4N, 6N), under the action of dry friction, the overall frequency begins to decrease, but the frequency fluctuation range becomes larger; When the excitation amplitude is increased again (9N, 12N), the contact surface will be separated to a certain extent, and the frequency shift will no longer be obvious when the excitation amplitude increases, and the frequency fluctuation range will change slightly.
4.3. Sobol index

After the uncertainty analysis of output parameters is completed, some sensitivity analysis calculations are carried out. Sensitivity analysis can quantify the influence of uncertain parameter fluctuation on the output parameters of the simulation model, and sort the input parameters according to the impact degree, so as to identify which parameters have an important impact on the output parameters and which have little impact on the output parameters. The results of sensitivity analysis can guide the design and optimization of UPDs, and focus on the parameters with large sensitivity coefficients in some safety measures. At the same time, it can also provide references for the selection of input parameters in future uncertainty analysis. This can simplify the complexity of the analysis model and reduce the amount of calculation. In addition, sensitivity analysis can also evaluate the behavior of the output parameters (smooth or non-smooth, single-peak or multi-peak), which is of great significance for uncertainty analysis and algorithm selection for model optimization. In Post-optimization analysis, sensitivity information is useful for determining whether the
output parameters are robust to small changes at the optimization design point.

In this section, we will focus on the Sobol index of uncertain parameters, focusing on the Sobol index of amplitude as the main evaluation criteria. Use the results analyzed in Section 4.1 as a reference. Latin hypercube sampling method was adopted for sampling, sensitivity analysis of uncertain parameters of UPDs to response amplitude Y was carried out, and the Sobol index in IP mode was obtained only. At this time, global and first-order sensitivity data were shown in Table 4. The histogram of sensitivity of each design parameter to response amplitude Y was drawn based on Table 4, as shown in Fig.14.

Table 4
The Sobol index for each uncertain parameter

<table>
<thead>
<tr>
<th>Number</th>
<th>Parameter</th>
<th>IP</th>
<th>OOP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Total Sobol's</td>
<td>1-order Sobol's</td>
</tr>
<tr>
<td></td>
<td></td>
<td>indices</td>
<td>indices</td>
</tr>
<tr>
<td>A1</td>
<td>$\mu$</td>
<td>0.38202</td>
<td>0.30440</td>
</tr>
<tr>
<td>A2</td>
<td>$K_T$</td>
<td>0.21772</td>
<td>0.17151</td>
</tr>
<tr>
<td>A3</td>
<td>$K_w$</td>
<td>0.09888</td>
<td>0.07245</td>
</tr>
<tr>
<td>A4</td>
<td>$F_a$</td>
<td>0.06157</td>
<td>0.05584</td>
</tr>
<tr>
<td>A5</td>
<td>$\omega$</td>
<td>0.05115</td>
<td>0.04183</td>
</tr>
<tr>
<td>A6</td>
<td>$m_d$</td>
<td>0.01435</td>
<td>0.00462</td>
</tr>
<tr>
<td>A7</td>
<td>$\theta_L$</td>
<td>0.21284</td>
<td>0.18061</td>
</tr>
<tr>
<td>A8</td>
<td>$\theta_R$</td>
<td>0.09042</td>
<td>0.07410</td>
</tr>
</tbody>
</table>

As can be seen from Table 4 and Fig.14, within the range of parameter values in this paper, the parameter $\mu$ has the most significant effect on the amplitude response, and the global sensitivity is greater than the first-order sensitivity, indicating that the interaction with other parameters has a significant influence on Y, which is also an issue to be considered when optimizing the damper. The Sobol behavior of the two modes is different. Meanwhile, the contact state and dissipated energy are correlated with the Sobol index under different
excitation. Here, the percentage of contact state is the ratio of the number of contact states of hysteretic loops in a single period to the total number of contact states in the whole closed period in the steady-state response, and the dissipated energy is the area surrounded by hysteretic loops. Fig. 15(a–i) and Fig. 16(a–i):

Fig. 15 IP mode: Sobol index under different external excitation: (a) 1.5N; (b) 5N; (c) 10N.
Based on the comprehensive analysis of Fig. 15 and Fig. 16, the following conclusions can be reached:

In the case of 1.5N load, Fig.15(a) and Fig.16(a) show that the percentage of sliding contact points is almost zero in the fully viscous state, which is also reflected in the dissipated energy (Fig.15(d) and Fig.16(d)). This low dissipation energy is reflected in the Sobol index (see Fig. 15(g) and Fig. 16(g)), since the Sobol exponent for the friction coefficient $\mu$ is zero in most frequency ranges, while $K_t$ and $F_A$ cause most of the variation in amplitude. In the case of 5N load, in Fig. 15(h) and Fig. 16(h), regardless of the vibration in the same direction or the opposite direction, when the exciting force increases, in the weak nonlinear region, the external excitation amplitude $F_A$, the disk speed $\omega$ tangential contact stiffness $K_t$, the normal the contact stiffness $K_n$ has the most significant impact on the response $Y$. Once it enters the slip state, the effect of the damper is reflected. In the case of 10N load, in Fig. 15(i) and Fig. 16(i), when the vibration is in the same direction, the Sobol index of the friction coefficient $m$ changes to a certain extent, and the changing trend of the Sobol index of other parameters is almost unchanged, while the changing trend of the different vibration is drastic. In the strong nonlinear range, the friction coefficient $m$ and the normal contact stiffness change significantly, which can be expected. The influence of the damper mass began to decrease, and the Sobol index of the excitation amplitude $F_A$ and the disc speed $\omega$ dropped sharply when the formant was far away.

When we analyze the specific working conditions and study the output response, we focus on considering the parameters with large Sobol. When designing the damper, the
influence of some uncertain parameters can be ignored, thus simplifying the problem and reducing the failure probability effectively. The use of Sobol indices to better understand the dynamic behavior of UPDs systems with dry friction contacts. The Sobol index is used in nonlinear dynamics analysis and has good performance in capturing contact nonlinear behavior. Therefore, Sobol analysis can be well applied to the dynamic response of structures with dry friction to understand the complex physics of vibration.

5. Conclusions

This paper mainly quantifies the influence of uncertain parameters including contact surface, machining tolerance, and assembly error on turbine blade dynamic response. Based on the reduced bladder-UPDs structure of the model, the application of the sparse grid technique and the extension of Gaussian integral in the calculation of uncertain UPDs dynamic response and the calculation steps are introduced. An adaptive algorithm is proposed, which has a good prediction effect on the nonlinear model response. The feasibility and accuracy were verified by comparing them with Monte Carlo. Different uncertainty parameters were selected to analyze the dynamic response range of UPDs with typical uncertainty levels.

The purpose of quantifying the dynamic response of UPDs in this paper is to quantify the uncertainty band of the response and determine the influence of each uncertainty parameter on the output. The uncertainty band is calculated by numerical simulation, and the nonlinear solution method of UPDs is combined with the surrogate model constructed by polynomial chaos theory. Then, the polynomial basis of the polynomial chaotic expansion alternative model is defined according to the probability distribution of different parameters. The polynomial surrogate model can be used to generate numerical calculations with a wide range of uncertain parameter combinations at very low computational cost, to obtain the uncertainty band of UPD's dynamic response. The results show that the uncertainty of each parameter
cannot be ignored in the simulation, because it causes a change in the dynamic response of the system. The uncertainty of parameters has a great influence on the dynamic response, and it increases with the increase of the uncertainty fluctuation coefficient. The resonance region is affected by uncertainty obviously, while the other regions are relatively weak. The resonance region of the UPDs will be extended, and the position of the formant will also change to varying degrees due to the existence of nonlinear phenomena, in some cases there will be a "resonance band", "frequency shift" and other phenomena. In addition, it is easy to obtain the Sobol index of each parameter at different frequencies through sensitivity analysis and prioritize the design decisions. The results show that the multi-source uncertainty parameters, including machining tolerances, can not be ignored in the simulation because they can lead to great changes in the output response.

With regard to the optimization of damper design, the uncertainty quantization method suggested in this paper is useful because it results in the output response becoming an uncertainty band. With this data information, the design direction of the damper is provided. Finally, the surrogate model constructed by PCE can realize the process more effectively, allowing UPDs research to be carried out with less computational effort. To make the simulation results more convincing, this method will be experimentally verified in future work.

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Data availability

All data generated or analyzed during this study are included in this article. But the processed data required to reproduce these findings cannot be shared at this time due to legal or ethical reasons.

Declarations

Conflict of interests

The authors declare that there is no conflict of interest regarding the publication of this article.

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