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Shared rules between planetary orbits existing in multi-planet systems and their functions

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Abstract: Nearly 300 systems containing three or more confirmed planets have been discovered, which have the potential to reflect more orbital features in addition to mean motion resonances (MMRs) since they are all controlled by the same central gravitation. By using the method of orbital ratio parameter analysis, we calculated the observation data of the planetary orbits of these systems. The calculations show the orbital relationships between all planets of each system in a clearer quantification way, including the planets of MMRs. By comparisons and statistics, we found some features shared by different systems. We summarized these features into the shared rules between planetary orbits, from which we derived related equations, principles and traits. We will offer explanations. The shared rules, more likely appearing along with system formation, can help learn about the true planetary configuration of a system and its change. Because of varying factors, many systems show large deviation rates, which affect the clear display of the shared rules. Even so, the calculation with corresponding equations and judgment according to corresponding principles still reflect that the shared rules exist in 88 of the 101 extrasolar systems containing four or more planets. We further analyzed these systems, provided the suggestions of migratory and absent exoplanets, and calculated related orbits. Meanwhile, we summarized five main reasons for large deviation rates, which can objectively explain why some systems do not clearly show the shared rules. The shared rules provide a new approach for studying the changes in planetary configurations, the completeness of multi-planet systems, and their evolution.

Introduction

Limited by observation ability and the number of discovered systems, there is not an approach of studying the orbit relations between all planets in any one system. However, scientists have found several approaches in studying the completeness of some systems and the stable motion of some celestial bodies, such as orbital formulas represented by the Titius-Bode law (temporarily called T-B-related formulas) and mean motion resonances (MMRs). They play important roles in system study. Although T-B-related formulas provide some help in the observations of solar planets and exoplanets (Dermott [1968](#); Hayes & Tremaine [1998](#)), they remain disputed because they are obtained in the way that is not widely accepted. Because MMRs were discovered through the analysis of observations, they have been deeply researched, well explained and widely accepted (Dermott [1968](#); Borucki et al. [2011](#); Petrovich et al. [2013](#); Fabrycky et al. [2014](#); Mills et al. [2016](#); Morrison et al [2020](#)). They are also supported by computer simulations (Migaszewski [2016](#); Cimerman et al. [2018](#)). By analyzing exoplanets showing the features of MMRs, especially exoplanets showing the chains of MMRs, astronomers explain how exoplanets maintain stable motion and influence each other and how multi-planet systems form and evolve (Batygin [2015](#); Hardy & Gong [2017](#); Morrison et al. [2020](#); MacDonald et al [2022](#)). However, because only some exoplanets show the orbital relations of near MMRs and the number of systems showing the chains of near MMRs is small, MMRs can not help analyze all planets in any one system. Astronomers also noticed the result. Fabrycky et al (2014) researched resonant exoplanets and found that exoplanet pairs showed little statistical preference to be near MMRs. Although over 5,000 confirmed exoplanets have been discovered now (NASA Exoplanet Archive [2022](#)), the

proportion of exoplanets showing MMRs does not increase obviously, and the number of systems showing the chains of near MMRs is still small. For example, of the 36 systems with five or more exoplanets (NASA Exoplanet Archive [2022](#)), only 11 systems show the chains of near MMRs (see the systems of TRAPPIST-1, HD 219134, K2-138, Kepler-80, TOI-178, HD 158259, HD 23472, Kepler-102, Kepler-292, Kpler-444 and TOI-561 in Supplementary Tables S21-S56). Thus, MMRs show relatively large limitations in the study of the changes of planetary orbits. Since all planets of a system, being a whole, are controlled by the same central gravitation, some specific orbital relations should exist between their orbits. Meanwhile, all systems are controlled by the same gravitation. Therefore, the relations may be shared by different systems. If one finds the relations, one may obtain a more effective approach to analyze the orbital changes of all planets of all or most systems. Then, the research of the changes in the planetary configurations may be more founded, the study of planetary stable motion may extend to more planets, the analysis of the completeness of multi-planet systems may be more objective, the calculation of the orbits of migratory and absent planets may be more accurate, and the discussion of the evolution of corresponding systems may be more convincing.

Therefore, we tried different ways to calculate, compare and analyze the planetary observation data of multi-planet systems (NASA Exoplanet Archive [2022](#); Extrasolar Planets Encyclopaedia [2022](#)), including planetary orbits, planetary masses, stellar masses, stellar ages, stellar temperatures etc, and possible connections among them. Finally, we found that the calculations with the application of the concept of the orbital ratio parameter can reflect the orbital relationships between all planets of a system in a clearer quantification way, including the planets of MMRs. Based on further quantified orbital relationships, the planetary configuration and its change can be better reflected. After comparison, we summarized the calculations into the method of orbital ratio parameter analysis. By calculating multi-planet systems with the method, one can find more orbital features. Based on the features, one can also derive related equations and principles. According to the equations and principles, one can more objectively research the planetary configurations and their changes. Compared with the T-B-relation formulas and MMRs feature, the equations and principles derived by the method of orbital ratio parameter analysis provide a different approach in studying the completeness of multi-planet systems and their evolution.

The paper contains eight sections. Following the introduction, the methods section explains how the orbital ratio parameters are calculated. Then, we present results, which show the shared rules between planetary orbits, the related equations and principles, and the statistics of corresponding calculations of multi-planet systems. In Sect. 4, we present the main reasons for large deviation rates, which are illustrated in five parts. The 5th section shows the analyses of all multi-planetary systems discovered. Sect. 6 explains the appearance time of the shared rules. Finally, the discussion and conclusion are presented. To make the analysis of extrasolar systems relatively fair, the observation data of each extrasolar system in the paper come from the Confirmed Planets of NASA Exoplanet Archive ([2022](#)) if not specifically annotated.

2. Methods

At present, research on the changes in planetary configurations and evolution of extrasolar systems is mainly performed through exoplanets showing the features of MMRs, especially the chains of near MMRs. Based on such exoplanets, astronomers have performed extensive studies

and well explained how multi-planet systems evolve (Batygin 2015; Mills et al. 2016; Pichierri et al. 2019; Hara et al. 2020). However, only a small fraction of exoplanets show MMRs. If one intends to study the non-MMR exoplanets of a system, it seems that he has to seek other ways. Furthermore, even if the planets of a system are in a chain of MMRs, one cannot see the degree to which the orbital ratios of the corresponding planets are equal. The understanding of the degree of equality may promote further studies of multi-planet systems. For example, the planets of HD 158259 show continuous near 3:2 resonances of period ratios, presenting a long chain of near 2:3 MMRs. However, one cannot see the degree to which the period ratio of each planet pair is equal to that of others (Table 1). If a system contains both MMRs and non-MMRs planets, the analysis with the concept of MMRs can neither reflect the degree to which the orbital ratios of MMRs planets are equal nor the orbital ratios of all planets are different, such as the six planets of K2-138 (Table 1). Moreover, some systems only show the resonances of neighbouring orbits with large ratios or do not show any resonance, such as the Kepler-207 system, which shows two near 19:10 period ratios, and Kepler-1468, which shows period ratios of 2.3240 and 2.3775 (Supplementary Table S1). For such systems, the features of MMRs may not play an important role in the analysis of orbital changes. If one knows the degrees to which the orbital ratios of all planets of a system are equal or different, one may learn about more orbital relationships between the planets of the system and then analyze the orbital changes of more planets and the evolution of more systems. After varying attempts, we found that the application of the concept of the orbital parameter ratio can serve this purpose. It is temporarily called the method of orbital ratio parameter analysis. The method mainly shows and explains the calculations of the orbital ratio parameters of all planets in a system containing three or more planets. If orbital ratio parameters are regarded as planetary physical parameters, the comparison of the parameters can reflect the changes in planetary configurations and completeness of multi-planet systems.

Table 1. Calculation of orbital ratio parameters and deviation rates of adjacent planets in multi-planet systems

Kepler-229 ($M_{\text{star}}=0.8M_{\odot}$, Age= $3.09^{+6.09}_{-2.97}$ Gyr)							Kepler-229 System ($R_{\text{star}}=0.728 R_{\text{Sun}}$)						
Pla	P	Ratio	ω_p^*	θ_p^*	Mass	Ratio	Pla	a	Ratio	ω_a^*	θ_a^*	Radius	Ratio
net	(day)	($P_n:P_{n-1}$)		(%)	(M_{Jup})	($M_n:M_{n-1}$)	net	(AU)	($a_n:a_{n-1}$)		(%)	(R_{Jup})	($R_n:R_{n-1}$)
b	6.25297				5.48		b	0.062				0.196	
c	16.0686	2.5698	1.0012	0.12	21.5	3.923	c	0.117	1.8871	1.0018	0.18	0.439	2.240
d	41.1949	2.5637	0.9988	-0.12	14.2	0.661	d	0.220	1.8803	0.9982	-0.18	0.343	0.781
average ratio _(p) =2.56675							average ratio _(a) =1.8837						
arp=99.28%(arp _a) ^{3/2}													
HD 158259 ($M_{\text{star}}=1.08M_{\odot}$)							Kepler-296 A ($M_{\text{star}}=0.498M_{\odot}$, Age= $4.37^{+5.60}_{-2.49}$ Gyr)						
Pla	P	Ratio	ω_p	θ_p	Mass	Ratio	Pla	a	Ratio	ω_a	θ_a	Mass	Ratio
net	(day)	($P_n:P_{n-1}$)		(%)	(M_{Jup})	($M_n:M_{n-1}$)	net	(AU)	($a_n:a_{n-1}$)		(%)	(M_{Jup})	($M_n:M_{n-1}$)
b	2.1780				0.0070		c	0.0521				0.0101	
c	3.4320	1.5758	1.0353	3.53	0.0176	2.514	b	0.079	1.5163	1.0100	1.0	0.0147	1.455
d	5.19808	1.5146	0.9951	-0.49	0.0170	0.966	d	0.118	1.4937	0.9949	-0.51	0.0158	1.075

e	7.9510	1.5296	1.0049	0.49	0.0191	1.124	e	0.169	1.4322	0.9540	-4.6	0.0093	0.589
f	12.028	1.5128	0.9939	-0.61	0.0193	1.011	f	0.255	1.5089	1.0051	0.51	0.0123	1.322
benchmark ratio _(p) =1.5221							benchmark ratio _(a) =1.5013						
K2-138 (M _{star} =0.935M _☉ , Age=2.8 ^{+3.8} _{-1.7} Gyr)							Kepler-20 (M _{star} =0.948M _☉ , Age=7.60±3.70Gyr)						
Pla	P	Ratio	ω _p	θ _p	Mass	Ratio	Pla	a	Ratio	ω _a	θ _a	Radius	Ratio
net	(day)	(P _n :P _{n-1})		(%)	(M _{Jup})	(M _n :M _{n-1})	net	(AU)	(a _n :a _{n-1})		(%)	(R _{Jup})	(R _n :R _{n-1})
b	2.35309				0.0098		b	0.0463				0.1667	
c	3.56004	1.5129	0.9886	-1.14	0.0199	2.031	c	0.0639	1.3801	0.9350	-6.50	0.0772	0.4631
d	5.40479	1.5182	0.9921	-0.79	0.0249	1.251	c	0.0949	1.4851	1.0061	0.61	0.2718	3.5207
e	8.26146	1.5285	0.9988	-0.12	0.0408	1.639	f	0.1396	1.4710	0.9966	-0.34	0.0895	0.3292
f	12.7576	1.5442	1.0091	0.91	0.0051	0.125	g	0.2055	1.4721	0.9973	-0.27	0.42	4.693
g	41.9680	3.2897	2.1497	115	0.0136	2.667	d	0.3506	1.7061	1.1558	15.6	0.2448	0.583
benchmark ratio _(p) =1.5303							benchmark ratio _(a) =1.4761						

Note. (A) Symbol ω_p represents period ratio parameter, θ_p is deviation rate of ω_p , ω_a is semimajor-axis ratio parameter, and θ_a is deviation rate of ω_a . They are calculated with equations (1-4) and by the benchmark ratio. If ω_p or ω_a is marked with “*”, it is calculated by average ratio. (B) Comparison displays a phenomenon that the orbital ratio parameters of most exoplanets are almost equal to 1 and their deviation rates are very close to 0. (C) Comparison also shows although the six systems have different stellar masses or ages, the differences do not affect the display of the phenomenon. Moreover, the mass ratios of adjacent planets also do not affect the appearance of the phenomenon. The calculations of more systems shows the same result (Supplementary Tables S1, S2 and S21-S56). Therefore, we will not discuss the influences temporarily and will mainly discuss the orbital relations between planets in the paper though stellar and planetary masses affect planet formation and motion.

The method is not complex, but the results calculated with it can reflect planetary orbital relationships in a clearer quantification way and display more orbital features objectively, including some previously noted features. The method contains first calculating the orbital ratios of all adjacent planets in a system, then dividing each ratio by a benchmark ratio to further quantify the relations between all orbital ratios, subsequently comparing the quantified values to look for possible common results reflected by all or most planets, and finally analyzing the results displayed by different systems to find the features possibly shared by these systems. The quantified value is called the orbital ratio parameter of adjacent planets, temporarily denoted by ω and split into the period ratio parameter (ω_p) and semimajor axis ratio parameter (ω_a). Because the calculations of all orbital ratios of a system are based on the continuous calculations of adjacent orbits in the same standard, they may reflect the orbital changes of relevant planets. The orbital ratio parameters can be calculated by the following equations.

$$\omega_{P[n:(n-1)]} = (P_n : P_{n-1}) \div br_p, \quad (1)$$

$$\omega_{a[n:(n-1)]} = (a_n : a_{n-1}) \div br_a, \quad (2)$$

Equation (1) is named the equation of the period ratio parameter, where $\omega_{P[n:(n-1)]}$ is the period ratio parameter of any two adjacent planets, P is the period, n is Planet n, n-1 is the inner adjacent planet of n, and br_P is the benchmark ratio for all period ratios. Equation (2) is named the equation of the semimajor-axis ratio parameter, where $\omega_{a[n:(n-1)]}$ is the semimajor-axis ratio parameter of any two adjacent planets. Only the semimajor axis replaces period, and other abbreviations do not change.

The benchmark ratio can be the orbital ratio of two adjacent planets that represents the initial orbital relationships of all planets in a system. However, due to the interference from the outside of planetary systems and the mutual perturbation between planets, all systems evolve, and all planets more or less change their orbits. Therefore, it is difficult to confirm such two adjacent planets in a system unless the system is in a specific or ideal state (explanations in later discussion). In this case, to relatively objectively reflect the orbital relationships of all planets of a system, the confirmation of the benchmark ratio can be based on comprehensive judgments of the results of all planetary behaviors. The confirmation can refer to the method of dive score calculation in the Olympic Games. That is, first remove the maximum and minimum orbital ratios in a system, then calculate the mean value with remaining orbital ratios, and finally set the mean value as the benchmark ratio of the orbital ratios of all planets in the system. Therefore, the possible influences of different factors on planetary stable motion can be avoided as much as possible and the benchmark ratio is more likely to represent the initial orbital relationships of all planets relatively objectively. Sometimes, the benchmark ratio is based on a set of orbital ratios, which possibly offset the mutual influence of the relative changes in planetary orbits. If it can be ensured that the orbital ratio of a planet pair reflects or temporarily reflects the initial orbital relationships of all or most planets in a system, the ratio can be directly taken as the benchmark ratio. It is called the specific benchmark ratio, denoted by sbr_P or sbr_a . At this moment, the orbital ratio parameter of the planet pair is equal to 1, which can also be regarded as the benchmark value or reference value of the orbital ratio parameters of other planet pairs. If the benchmark ratio is the mean value calculated by a set of orbital ratios, the reference value is not a fixed ratio; instead, it is a set of ratio parameters close to 1. At this moment, the mean value of the set of the ratio parameters is still equal to 1. A set of ratio parameters may sometimes be more objective and accurate. In addition, the benchmark ratio covers the meaning of the average value of all orbital ratios of a system and can reflect the orbital relationships of all planets more objectively, especially when there may be migratory or absent planets in the system. Therefore, the benchmark ratio is more suitable for system analysis. However, the benchmark ratio is only appropriate for the analysis of systems containing four or more planets. For a three-planet system, the average value, denoted by ar_P or ar_a , is usually used.

In Table 1, we show the orbital ratio parameters of several representative systems calculated with equations (1) and (2). For example, the HD 158259 system shows period ratio parameters of 1.0353, 0.9951, 1.0049 and 0.9939. Comparisons show that the ratio parameters of all period ratios are almost equal to 1. So is the Kepler-229 system. The calculations of the semimajor axis ratios to the Kepler-229 and Kepler-296 systems display the same result. Analyses show that the four systems simultaneously share a common result. We present twenty other systems that also show the result (Supplementary Tables S1 & S2). The common result implies some

interconnections between planetary orbits. K2-138 is another example. Based on the benchmark ratio calculated by $br_p = [(P_d:P_c) + (P_e:P_d) + (P_f:P_e)] \div 3 = 1.5303$, the calculation with equation (1) shows period ratio parameters of 0.9886, 0.9921, 0.9988, 1.0091 and 2.1497 (Table 1). The calculation shows that the period ratio parameter of Planet g to f greatly deviates from 1, showing $\omega_{P(g:f)} = (41.96797:12.75758) \div 1.5303 = 2.1497$. Even so, the system still shows most period ratio parameters almost equal to 1. According to planetary physical parameters, we may also take the period ratio of K2-138e to K2-138d as the specific benchmark ratio of the system. Now, the period ratio parameters are 0.9898, 0.9933, 1, 1.0103 and 2.1522 (Supplementary Table S5). They are almost identical to the parameters calculated with the benchmark ratio. Different benchmark ratios imply different meanings. We also calculated the semimajor axis ratios of the Kepler-20 system, which show the same result (Table 1). The calculations of more observed systems reflect that many systems show the result (Supplementary Tables S21-S56). The reason is that all or most adjacent planets in a system have similar orbital ratios and their ratios are almost equal to the benchmark ratio. The result that orbital ratio parameters are almost equal to 1 may suggest some specific orbital relationship between planets. The relationship may help analyze planetary orbital changes in a system.

To more directly and clearly reflect the degree to which all orbital ratios in a system are different, the orbital ratio parameter can also be further interpreted with the deviation rate and is expressed as percentage. Temporarily denoted by θ , the deviation rate can be calculated by following equations.

$$\theta_{P[n:(n-1)]}(\%) = \{(\omega_{P[n:(n-1)]}) - (m\omega_{rP})\} \times 100$$

or $\theta_{P[n:(n-1)]}(\%) = (\omega_{P[n:(n-1)]} - 1) \times 100$, (3)

$$\theta_{a[n:(n-1)]}(\%) = \{(\omega_{a[n:(n-1)]}) - (m\omega_{ra})\} \times 100$$

or $\theta_{a[n:(n-1)]}(\%) = (\omega_{a[n:(n-1)]} - 1) \times 100$, (4)

It is named the equation of the period ratio parameter expressed as the deviation rate and the equation of the semimajor-axis ratio parameter expressed as the deviation rate. In the equations, $m\omega_{rP}$ is the mean value of the period ratio parameters that are related to the period ratios calculating the benchmark ratio, while $m\omega_{ra}$ is that of the semimajor axis ratio parameters. They are calculated by $m\omega_{rP} = [(\omega_P)_{r1} + (\omega_P)_{r2} + \dots + (\omega_P)_{rm}] \div m = 1$ and by $m\omega_{ra} = [(\omega_a)_{r1} + (\omega_a)_{r2} + \dots + (\omega_a)_{rm}] \div m = 1$. Since both $m\omega_{rP}$ and $m\omega_{ra}$ are equal to 1, equations (3) and (4) can also be expressed in simplified ways. A deviation rate can be zero, negative or positive, which reflects different meanings. If a deviation rate is zero, it usually reflects that the corresponding orbital ratio is taken as the reference value for other orbital ratios in a system or the corresponding orbital ratio is equal to the reference value. If it is the latter, the calculation implies that there is an intrinsic link between the orbits of the calculated planets and the orbits of the reference planets. If the deviation rate is positive, it usually suggests that one of the two planets composing the corresponding orbital ratio has migrated relative to the orbit where it should be or relative to the orbits of the reference planets. If a deviation rate is very large, it usually suggests planet absence. If the deviation rate is negative, usually it usually suggests that one of the two planets composing the corresponding orbital ratio has migrated. However, the positive and

negative rates reflect that the migration directions are usually different if the analyzed planets are on the same side of the reference planets. We will show detailed analyses in later sections.

In Table 1, the calculations with equations (3) and (4) display the deviation rates in the six systems. It can be seen that the first four systems show all deviation rates very close to 0. HD 158259 is taken as an example. The deviation rates of the period ratio parameters of its five planets are 3.53%, -0.49%, 0.49% and -0.61% by $\theta_{P(n):(n-1)} = \{[(P(n):P_{(n-1)}) \div br_P] - (m\omega_{rP})\} \times 100$. For Planets f and e, the deviation rate of its period ratio parameter is $\theta_{P(f:e)} = \{[(12.028 \div 7.9510) \div 1.5221] - [(0.9951 + 1.0049) \div 2]\} \times 100 = -0.61\%$. Let us calculate the K2-138 system. The calculations with equation (3) clearly show the deviation rates of the period ratio parameters of all planets. Because the period ratios of the inner five planets are in a chain of near MMRs, they show four very small deviation rates. However, the outermost two planets show a very large deviation rate. The orbital relationships are displayed more directly. The 115% deviation rate displayed by the outermost two planets reflects the degree to which their period ratio is different from the reference ratios, implying that the orbital relationship of the outermost two planets is different from other planet pairs. Such a large deviation rate implies the evolution of the system or incompleteness of observations. If a system does not show the planets of MMRs, the calculations with equations (1)-(4) can also quantify their orbital relationships and show the degree to which all orbital ratios are different, such as the Kepler-207, Kepler-1468 and Kepler-20 systems (Supplementary Tables S1 and S4 [1] & [2]). For further contrast, we calculated the planetary orbits of K2-138 with equations (1) & (3) by using the benchmark ratio, specific benchmark ratio and average ratio (Supplementary Table S5 [1], [2] & [3]). We also calculated the Kepler-11 system with the three ratios (Supplementary Table S28 [3-1], [3-2] & [3-3]). The contrasts reflect that the benchmark ratio is the best choice in the analysis.

If one intends to emphasize the degree to which all orbital ratios of a system are equal, one can also use the equivalence rate to interpret the orbital ratio parameter as a percentage. Detailed explanations and examples are presented in Supplementary Table S3. For example, the equivalence rates of the period ratio parameters of the five planets of HD 158259 are 103.53%, 99.51%, 100.49% and 99.39%. The calculations reflect that the degree to which the period ratios of all planets are equal are very close to 100%, implying that all planets orbit the star in the same mode.

By further quantifying the orbital ratios of the exoplanets of all multi-planet systems observed (NASA Exoplanet Archive [2022](#); Extrasolar Planets Encyclopaedia [2022](#)), we found that most exoplanets show orbital ratio parameters very close to 1, although some exoplanets show orbital ratio parameters obviously deviating from 1. In other words, most exoplanets show small deviation rates, while some other exoplanets show very large deviation rates (statistics in Sect. 3.6). Further studies suggest that a large deviation rate in a system has its reason and usually implies that the system is disturbed greatly or needs more observations. In other words, a larger deviation rate suggests exoplanet absence or great migration. For example, the 115% deviation rate in the K2-138 system suggests two absent planets (discussed in Sect. 3.3). For Kepler-20, the 15.6% deviation rate suggests that Planet d migrated outward (discussed in Sect. 4.3). In fact, large deviation rates for different systems reflect the realistic meanings of the calculations with the method of orbital ratio parameter analysis. Conversely, if a system is not disturbed greatly, it will display small deviation rates, such as HD 158259. Suppose that a system had not been affected by any disturbance, it would show all orbital ratio parameters equal to 1 and all deviation rates equal

to 0. Such orbital relationships can be regarded as those of the ideal state, which should be the ultimate manifestation of the perfect operation of a natural system. The systems of Kepler-229, Kepler-207, Kepler-217, Kepler-289 and Kepler-249 are very close to such a system since they all show deviation rates of period ratios smaller than $\pm 0.25\%$ (Table 1 and Supplementary Table S1).

The calculations with the method of orbital ratio parameter analysis can show the orbital relationships between all planets of multi-planet systems in a clearer quantification way. The quantified orbital relationships can reflect some results shared by different systems. Based on the shared results, one may better research the orbital change of the planets of a system and its evolution.

3. Results

With the method of orbital ratio parameter analysis, we calculate the orbital ratio parameters and deviation rates of all exoplanets of observed multi-planet systems (NASA Exoplanet Archive [2022](#); Extrasolar Planets Encyclopaedia [2022](#)). Based on the calculations, we find more orbital features in addition to MMRs. Then, we summarize these features into the shared rules between planetary orbits. After comparing more calculations of observations and simulations, we derive the relation equations of planetary orbits from the shared rules and sum up the principles of judging the properties of planets and systems. According to simulation calculations, we also summarize the traits of reflecting planet properties. Meanwhile, we provide the statistics of the deviation rates of the systems containing four and more exoplanets and statistics of the systems with the shared rules. Although a three-planet system has only two orbital ratios, we still provided the statistics of the types of all three-planet systems. Because the corresponding rules, equations and principles are the outcomes reflected by the calculations of observation data, they show general practicability in the study of extrasolar systems. According to these equations, principles and traits, one can judge exoplanets that follow the shared rules, find exoplanets that migrate, disappear or are not detected, calculate the previous orbits of migratory exoplanets, infer the possible orbits of disappearing exoplanets, and predict the orbital ranges of undetected exoplanets. Because the shared rules are completely based on observations, they can help analyze more planets and more systems, show migratory and absent planets more objectively, and calculate their orbits more accurately. The shared rules provide the chance of researching planetary systems from a different angle.

3.1 Shared orbital features reflected by orbital ratio parameters and deviation rates

Based on exoplanetary observation data (NASA Exoplanet Archive [2022](#)), we calculated the orbital ratio parameters of all observed multi-planet systems with equations (1) and (2). By comparing the orbital ratio parameters of each system, we found a few phenomena. One of them is that many systems with three or more planets show that the period ratio parameters (ω_p) of all or most adjacent exoplanets tend to be equal to 1 within a certain precision, such as Kepler-229, HD 158259 and K2-138 (Table 1). The semimajor axis ratio parameters (ω_a) show the same result, such as Kepler-229, Kepler-296 A and Kepler-20 (Table 1). By comparison, the phenomenon is shared by more systems (Supplementary Tables S1, S2 and S21-S56). The phenomenon actually reflects two meanings. Most exoplanets maintain initial orbital relationships while others have migrated, disappeared or are not detected for some reason. The phenomenon can be regarded as an inherent feature reflected by the exoplanets that maintain initial orbital relationships, temporarily called **the feature that the orbital ratio parameters of adjacent planets tend to be equal to 1**

within a certain precision, for short, the feature of ratio parameter equal to 1. The feature also reflects a kind of alignment for all or most exoplanets in a system during their orbiting. Prof. D. C. Fabrycky and his colleagues deeply researched the architecture of multi-transiting systems. In the conclusion of their article, they affirm and strengthen the result that pairs of planets tend to be well aligned, to within a few degrees (Fabrycky et al. [2014](#)). The discovery of more exoplanets further confirms the alignment.

Because the planets of all systems are controlled mainly by the same central gravitation and orbit their host stars according to the same physical laws, the planets of every system should display some common features or social rules, similar to the common behaviour of individuals in different groups of the same sort in nature. The feature that the orbital ratio parameters are equal to 1 is likely to be one of them. However, because varying disturbances continue to affect the stable motion of every planet of a system, some planets may change their orbits and even disappear from the system, which is reflected as more or less deviations of their orbital ratio parameters from 1. Moreover, because of the influence of observation limitations, some planets of a system may not be detected simultaneously or never be detected. Undetected planets also make corresponding orbital ratio parameters obviously deviate from 1. Therefore, the feature cannot be perfectly reflected by all planets. As a result, most planets show orbital ratio parameters close to 1, while others show ratio parameters obviously or greatly deviating from 1. The result also makes the study of orbital relationships have realistic meaning. If the planets that follow the feature are taken as reference, the study of the planets that do not follow the feature will become more founded. To better reflect different influences, multi-planet systems can be classified into systems of different types according to the degree to which the orbital ratio parameters of all adjacent planets are equal to 1. Since the calculations with equations (3) and (4) show that the deviation rate can more clearly interpret the ratio parameter with percentage (Table 1), the types of multi-planet systems can also be judged according to deviation rates.

For easy analysis and application, we temporarily set a general standard for classification. If a system shows that the deviation rates of the period ratio parameters of all adjacent planets are equal to 0, it is regarded as an ideal system. An ideal system should be in the ideal state and should not be disturbed. If a system shows absolute values of all deviation rates smaller than 10%, it is temporarily regarded as a normal system. Usually, a normal system is not obviously disturbed during its formation and evolution, and its planets maintain the initial orbital relationships to a great extent. If a system shows absolute values of one or more deviation rates between 10-40%, it is temporarily regarded as a near-normal system. A near-normal system may experience relatively obvious disturbances, which make some planets migrate obviously. If a system shows one or two deviation rates larger than 40%, it is temporarily regarded as an abnormal system. Usually, an abnormal system is greatly disturbed. Disturbances may cause some planets to vanish or not be detected. Although the general standard can be adjusted, the adjustment usually does not affect the qualitative analysis of a system. In the natural universe, all systems can be regarded as natural systems since they are more or less influenced by different factors. An ideal system is the ultimate manifestation of the perfect operation of a natural system. Large deviation rates do not deny the existence of the feature. Instead, they reflect the evolution of multi-planet systems and limitations of observations.

According to the feature of ratio parameters equal to 1, we further compared the calculations of different systems (NASA Exoplanet Archive [2022](#)). Comparisons show another phenomenon. In

an ideal system, the periods of all adjacent planets decrease with a shared divisor or increase with a shared multiple, such as A-HD (Table 2 [1]). In a normal system, the periods of adjacent planets tend to decrease with a shared divisor or increase with a shared multiple within a certain precision, such as HD 158259 (Table 1). Even in an abnormal system, the phenomenon still exists for some planets, such as K2-138 (Table 1). The calculations of the semimajor axes show the same phenomenon (Table 1). The phenomenon is temporarily called **the feature that the orbits of adjacent planets tend to vary with a shared divisor (or multiple)**, for short, the feature of a shared divisor (or multiple). The feature can be expressed by equations.

$$P_n = (P_{n+1}) \div (D_p) \text{ or } P_n = (P_{n-1})(M_p), \quad (5)$$

$$a_n = (a_{n+1}) \div (D_a) \text{ or } a_n = (a_{n-1})(M_a), \quad (6)$$

In equation (5), P is the period, n is any one planet, n+1 is the outer neighbour of Planet n, n-1 is the inner, D_p is the divisor of period change and M_p is the multiple. In equation (6), only the semimajor axis replaces the period. In an ideal system, D_p or M_p is usually constant, which is the period ratio of any two adjacent planets. So is D_a or M_a . In a normal system, D_p , M_p , D_a , and M_a are relatively constants. For the planets of an ideal system, the orbits calculated with equation (5) or (6) are equal to their observed orbits (Table 2 [1]). For the planets of a normal system, the orbits calculated with equation (5) or (6) are equal to their observed orbits within a certain precision (Table 1).

The feature of shared divisors actually reflects the essence of logarithmic spirals (Logarithmic spiral [2021](#)). For an ideal system, the semimajor axes of adjacent planets change at an identical ratio while for a logarithmic spiral, the radii of the spiral at the same coordinate axis also change at an identical ratio. For a normal system, the orbits of adjacent planets change in the mode of a logarithmic spiral to a great extent (Table 1). Comparing the semimajor axes of the planets of Kepler-130 and the radii of the spiral of a nautilus's section shows that the planetary orbits overlap the spiral of the nautilus at the same axis (Supplementary Fig. S1). Therefore, the phenomenon is temporarily called **the feature that the orbits of adjacent planets change in the form of a logarithmic spiral**, for short, the feature of spiral orbit. The feature has been noticed and is not lonely. In nature, the logarithmic spiral can often be seen, such as the shell of a nautilus (or snail), the approach of an insect to a light source, and the whirling cloud of a typhoon. In the universe, some protoplanetary disks show matter distributions presenting the pattern of logarithmic spirals, such as MW C758 and SAO 206462 (Benisty et al. [2015](#), Xie et al. [2021](#)) Some galaxies also show their arms presenting the shapes of logarithmic spirals, such as M74 (Newman et al. [2013](#); Ban & Su [2019](#)). All these phenomena should have connections and originate from some kind of the same effect.

3.2 Shared rules between planetary orbits summarized from shared orbital features

The calculation of observation data with the method of orbital ratio parameter analysis shows some common orbital features, which are shared by multi-planet systems. To better study the orbital relationships between all planets of natural systems, we temporarily summarized them into two rules.

The standard rule between planetary orbits: In an ideal system, the orbital ratios of adjacent planets are equal, the orbital ratio parameters are equal to 1, the deviation rates of orbital ratio parameters are equal to 0, the orbits of adjacent planets vary with a shared divisor (or multiple),

and the orbital changes of adjacent planets present the mode of logarithmic spiral.

The tendency rule between planetary orbits: In a normal system, the orbital ratios of adjacent planets tend to be equal within a certain precision, the orbital ratio parameters tend to be equal to 1 within a certain precision, the deviation rates of orbital ratio parameters tend to be equal to 0 within a certain precision, the orbits of adjacent planets vary with a shared divisor (or multiple) within a certain precision, and the orbital changes of adjacent planets present the mode of logarithmic spiral to a great extent.

Although a perfect ideal system has not yet been detected, some extrasolar systems still display nearly the same orbital relationships as perfect ideal systems, such as the systems of Kepler-229, Kepler-207, Kepler-217, Kepler-289, Kepler-249, Kepler-31, Kepler-398, Kepler-332, Kepler-130, HD 184010, Kepler-1468, K2-219, Kepler-1130, Kepler-445 and YZ Cet (Table 1 and Supplementary Table S1). If a system that shows the absolute values of all deviation rates smaller than 1.25% is regarded as an ideal system, the 15 systems can be considered. An ideal system is the ultimate manifestation of the perfect operation of a natural multi-planet system and can be used as a reference standard for analysing a natural system. As we know, all planets are controlled by central gravitation and other forces with the same properties. If they formed and evolved in ideal conditions and environments, they would display the same orbital relationship, embodied as the standard rule between planetary orbits. However, all planets are affected by unbalanced forces and external disturbances. Therefore, the standard rule is more or less impacted and behaves as a tendency or a vague trace, even vanishing in some systems. However, because central gravitation plays the dominant role in the formation and evolution of all systems, most planets still follow the standard rule to a great extent. The result agrees with observations. In addition, unlike individual behaviour, group behaviour is more complex as the latter reflects mutual influences. Therefore, a common behaviour shown by individuals is often reflected as a trend. In this case, the standard rule acts as a tendency rule. The tendency rule is the real representation of the standard rule in the natural universe and shared by natural systems. Hence, the two rules can be collectively called the shared rules between planetary orbits. To avoid verbose expression, we will call them the shared rules in later discussion.

The shared rules describe the common orbital relationships reflected by different systems. Based on the shared rules, one can learn about the types and properties of a natural system and analyze the change in the planetary configuration and completeness of the system. Although calculations reflect that some systems have large deviation rates, suggesting that they are greatly disturbed and are abnormal systems, they do not deny the existence of the shared rules. Conversely, they reflect the evolution of these systems and limitations of observations. If an abnormal system has three or more planets that follow the shared rules, one can restore the real planetary configuration of the system (discussed in Sect. 3.4). Abnormal systems make the shared rules have realistic significance.

Because the shared rules are the results reflected by calculating observation data with the method of orbital ratio parameter analysis, instead of that by hypotheses, simulations and computations, they are more natural and convincing. Moreover, computer simulations of system formation also reflect that the newly-formed planets show the orbital relationships of MMR chains (Mills et al. 2016; MacDonald et al. 2022). Since the planets in some normal systems also show the chains of near MMRs, such as Kepler-229 and HD 158259 (Table 1), the computer simulations should support the existence of the shared rules.

3.3 Relation equations of planetary orbits derived from the shared rules

According to calculation and comparison, we derived some relation equations of planetary orbits. The empirical equations are the natural reflections by the calculations of simulated and observed systems. Meanwhile they are supported by the process of exoplanet discovery. They do not contain any man-made factors or limitations. For an ideal system, the equations hold completely, and the values on both sides of equal signs are completely equal. For a natural system, they also hold true and the values are equal within a certain precision. Because the equations reflect the orbital relationships of group behavior in the natural state, the equality within a certain range of precision should be acceptable. As a qualitative analysis, the equation-calculated values have a certain reference significance in the observation and analysis of multi-planet systems. We will show the equations and their function.

Relation equations of orbital ratio parameter and benchmark ratio:

$$\omega_{P[n:(n-1)]} = (br_P)^N, \quad (7)$$

$$\omega_a[n:(n-1)] = (br_a)^N, \quad (8)$$

In equation (7), $\omega_{P[n:(n-1)]}$ represents the period ratio parameter of any two adjacent planets, P is the period, n is Planet n, n-1 is the inner neighbor of Planet n, br_P is the period benchmark ratio, and N is the power, which can be any natural number. In equation (8), only the semimajor axis replaces the period. Based on the equations, one can judge the number of absent planets. If N is 0, the equation suggests no planets between Planet n and Planet n-1. If N is 1, it suggests one planet between them. If N is 2...

Relation equations of planetary orbits:

$$P_x = (P_n) \div [(lr_P)^{(snn-snx)}], \quad (9)$$

$$a_x = (a_n) \div [(lr_a)^{(snn-snx)}], \quad (10)$$

In equation (9), P is the period, x is Planet x, n is any planet that serves as the reference planet, lr_P is the leading ratio of all period ratios in a system, snn is the sequence number of Planet n in the system and snx is that of Planet x. Usually, lr_P is the ratio that most likely reflects the initial period relations of all or most planets. It can be the benchmark ratio, shared divisor of all or part planets, or ratio of two representative adjacent planets. In equation (10), only the semimajor axis replaces the period. The two equations are developed from equations (5) and (6). Based on the equations, one can calculate the orbit of an undetected, vanishing, migratory or unconfirmed planet. If one intends to learn about the possible shared divisor (sd) of all or part planets of a natural system, the equations can also play a role, showing $sd_P = (P_n \div P_m)^{1/(snn-snm)}$ and $sd_a = (a_n \div a_m)^{1/(snn-snm)}$. In addition, the orbital relationship of any two planets can be expressed as $(P_n):(P_m) = (sd_P)^{(snn-snm)}$ and $(a_n):(a_m) = (sd_a)^{(snn-snm)}$, where m is Planet m, which is inside the orbit of Planet n.

Relation equation of period benchmark ratio and semimajor axis benchmark ratio:

$$br_P = (br_a)^{3/2}, \quad (11)$$

where br_P is the period benchmark ratio and br_a is the semimajor axis benchmark ratio. The degree to which the two ratios are equal reflects the degree of precision of the orbital observation data of all planets in a system as a whole.

Relation equation of period ratio parameter and semimajor axis ratio parameter:

$$\omega_{P[n:(n-1)]} = \{\omega_{a[n:(n-1)]}\}^{3/2}, \quad (12)$$

where $\omega_{P[n:(n-1)]}$ is the period ratio parameter of any two adjacent planets and $\omega_{a[n:(n-1)]}$ is the semimajor axis ratio parameter of the corresponding planets. Equation (12) can reflect the precision of the observation data of the period or semimajor axis of any one related planet. Further calculations also show that equation (12) can be expressed as $(P_n:P_{n-1})=(a_n:a_{n-1})^{3/2}$, which can help display the unconfirmed period or semimajor axis of a planet with high precision.

In addition, the precision of orbital observation data of a planet can also be checked by the equations of classical mechanics, such as the relation equation of the planetary orbit and stellar mass (Standard gravitational parameter 2021). It is expressed as $\mu=GM$, where μ is the standard gravitational parameter, G is the gravitational constant and M is the mass of a central celestial body. For a very smaller celestial body orbiting the central celestial body with an elliptical orbit, μ can also be expressed as $\mu=4\pi^2\alpha^3/P^2$, where α is the semimajor axis of the smaller body and P is its period. Within the range of a high accuracy, they can be expressed as $GM=4\pi^2\alpha^3/P^2$. For convenient comparison, the equation can be converted into an equation with a comparable value, showing:

$$\beta = GMP^2 / 4\pi^2\alpha^3 \quad (13)$$

where β is temporarily called the relation parameter of the mass and orbit. The equation is temporarily called the equation of the parameter reflecting the relation between the mass and orbit. Usually, β is equal to 1 within a very high accuracy for an orbiting celestial object that has accurate observation data. For example, the Earth shows $\beta=1.0002214$. Calculations show that equations (12) and (13) can come to the same conclusion.

For better understanding, we will explain the equations with simulated and actual systems and the discovery process of some planets. A-HD is an ideal system, showing all deviation rates equal to 0 (Table 2 [1]). Supposing Planet A-c is not detected, the system displays four planets (Table 2 [2]). If calculated with equation (1), the period ratio parameter of Planet A-d to A-b is $\omega_{P[(A-d):(A-b)]}=1.533$, and the period benchmark ratio is $br_p=1.533$. They reflect the content of equation (7) by $\omega_{P[(A-d):(A-b)]}=100\%(br_p)^1$. The power of br_p is 1, suggesting an undetected planet between A-d and A-b, temporarily called Planet A-x. Then, we tried to calculate its period. Taking Planet A-e as the reference planet, the period ratio of A-f to A-e as the divisor, and the difference between the sequence numbers of Planets A-e and A-x as the power, the calculation shows $P_x=(P_{A-e})\div[(P_{A-f}):(P_{A-e})]^{(snA-e)-(snA-x)}$, which shows the content of equation (9). The calculation provides 3.3386 days by $P_x=7.8461\div(1.533)^{(4-2)}$, which is equal to the period of Planet A-c before supposition. The equal values state that equations (7) and (9) exist and hold.

Table 2. Influence of planetary absence or migration of simulated systems on period ratio parameters (ω_P) and deviation rates (θ_P)

Analysis of the A-HD system [1]					Supposing A-c is not detected [2]					If a stray object made A-d migrate [3]				
Planet	P	Ratio	ω_P	θ_P	Planet	P	Ratio	ω_P	θ_P	Planet	P	Ratio	ω_P	θ_P
	(day)	of P		(%)		(day)	of P		(%)		(day)	of P		(%)
A-b	2.1780				A-b	2.1780				A-b	2.1780			

A-c	3.3386	1.533	1	0	Undetected planet	↓				A-c	3.3386	1.533	1	0
A-d	5.1181	1.533	1	0	A-d	5.1181	2.350	1.533	53.3	A-d*	5.8858	1.763	1.150	15.0
A-e	7.8461	1.533	1	0	A-e	7.8461	1.533	1	0	A-e	7.8461	1.333	0.870	-13.0
A-f	12.028	1.533	1	0	A-f	12.028	1.533	1	0	A-f	12.028	1.533	1	0
benchmark ratio _(P) =1.533					benchmark ratio _(P) =1.533					benchmark ratio _(P) =1.533				
Analysis of the HD 158259 system [4]					Supposing c is not detected [5]					If a stray object made d migrate [6]				
Planet	P	Ratio	ω_P	θ_P	Planet	P	Ratio	ω_P	θ_P	Planet	P	Ratio	ω_P	θ_P
	(day)	of P		(%)		(day)	of P		(%)		(day)	of P		(%)
b	2.1780				b	2.1780				b	2.1780			
c	3.4320	1.576	1.035	3.5	Undetected planet	↓				c	3.4320	1.576	1.020	2.0
d	5.1981	1.515	0.995	-0.5	d	5.1981	2.387	1.560	56.0	d*	5.9778	1.742	1.128	12.8
e	7.9510	1.530	1.005	0.50	e	7.9510	1.530	1	0	e	7.9510	1.330	0.861	-13.9
f	12.028	1.513	0.994	-0.6	f	12.028	1.513	0.989	-1.1	f	12.028	1.513	0.979	-2.1
benchmark ratio _(P) =1.5225					benchmark ratio _(P) =1.530					benchmark ratio _(P) =1.545				

Note. (A) ω_P and θ_P are calculated with equations (1) and (3). (B) The A-HD system is an ideal system obtained on the basis of the HD 158259 system after a tiny adjustment of the periods of Planets c, d and e (See [1] & [4]). The details are presented in Supplementary Table S8 [1]-[3] and Note 2. (C) The calculations of actual and simulated systems provide and explain corresponding equations, principles and traits.

HD 158259 is a normal system (Table 2 [4]). Supposing Planet c is not detected, it shows four planets (Table 2 [5]). The calculations with equation (1) show $\omega_{P(d,b)}=101.96\%(br_P)^1$. It reflects the content of equation (7) within an acceptable precision, suggesting that a planet is not detected, called Planet x*. If taking Planet e as the reference planet, the benchmark ratio as the divisor, and the difference in the sequence numbers of Planet e and Planet x* as the power, the calculation shows equation (9) by $P_{x^*}=(P_e)\div(br_P)^{(sne-snx^*)}$, showing 3.3966 days by $P=7.9510\div1.530^{(4-2)}$. It is 101.04% equal to the period of Planet c before supposition. The simulation calculations of the planetary semimajor axes with the normal Kepler-444 system display the contents of equations (8) and (10) (Supplementary Table S6 Note 3). Because some planets migrated slightly, the simulations of normal systems do not display 100% equality. However, the degree of equality should be acceptable for the qualitative analysis of group behavior.

We will analyze the planetary configuration of an actual system. K2-138 has six planets now. With equations (1-4), we quantified their orbital relationships (Table 1 [5] and Table 3 [1]). Both calculations of periods and semimajor axes show a large deviation rate. It is an abnormal system. Here, we will analyze the semimajor axes. According to $br_a=1.3280$ and $\omega_{a(g:f)}=1.6657$, $\omega_{a(g:f)}=94.45\%(br_a)^2$. Supposing equation (8) holds true, the 94.45% equality still implies that there should be two undetected planets between Planets g and f, temporarily called Planets x and y. To confirm the undetected planets, we first calculated the semimajor axis of all observed planets with equation (10). By taking the largest Planet d as the reference planet and the semimajor axis

ratio of Planet d to c as the leading ratio, the calculations by $\alpha_{n(cal)} = \alpha_d \div (\alpha_d : \alpha_c)^{(snd-snn)}$ are 0.03377, 0.04461, 0.05893, 0.07785, 0.10284 and 0.23706 AU. Comparisons show that the calculated semimajor axes of the six planets are 100.24, 100, 100, 100.45, 101.59 and 97.48% equal to their observed semimajor axes. The degrees of equality, especially for the inner five planets, can demonstrate that equation (10) exists in the system and holds true though it is an abnormal system. In this case, equation (10) should also be suitable to calculate the orbital ranges of the undetected planets. Still by $\alpha_{n(cal)} = \alpha_d \div (\alpha_d : \alpha_c)^{(snd-snn)}$, their orbits are 0.135845 AU by $\alpha_x = 0.05893 \div 1.3210^{(3-6)}$ and 0.17945 AU by $\alpha_y = 0.05893 \div 1.3210^{(3-7)}$. If considering Planets x and y, all deviation rates become very small (Table 3 [2]). In addition, the calculated semimajor axis of Planet g is 0.23706 AU, which is 97.48% equal to its observed semimajor axis by $[(0.23109 \div 0.23706)] \times 100\%$, or 2.52% larger than its observed semimajor axis. Therefore, Planet g should migrate inwards. The migration conversely explains why the above equation is 94.45% equal. We also calculated the periods. The calculations show the same results (Supplementary Table S5 [5] and [6]). The calculations not only attest equation (7-10) but also display the actual planetary configuration of K2-138. In later sections, we will analyze the discovery process of Kepler-20g, Kepler-90i and HD 108236f. The calculated orbits agree well with their observed orbits, which objectively supports equations (7-10).

Table 3. Calculation of the semimajor axis ratio parameters (ω_a) and deviation rates (θ_a) of adjacent planets in the K2-138 system, and influence of undetected planets on planetary configuration

Analysis of semimajor axes of observed planets [1]						Analysis of semimajor axes after considering undetected planets [2]						
No.	Planet	a	Ratio	ω_a	θ_a	Mass	Ratio	Planet	a	Ratio	ω_a	θ_a
		(AU)	($a_n:a_{n-1}$)		(%)	(M_{Earth})	($M_n:M_{n-1}$)		(AU)	($a_n:a_{n-1}$)		(%)
1	b	0.03385				3.10		b	0.03385			
2	c	0.04461	1.3179	0.9924	-0.76	6.31	2.0355	c	0.04461	1.3179	1.0002	-0.02
3	d	0.05893	1.3210	0.9947	-0.53	7.92	1.2552	d	0.05893	1.3210	1.0026	0.06
4	e	0.07820	1.3270	0.9993	-0.07	12.97	1.6376	e	0.07820	1.3270	1.0071	0.71
5	f	0.10447	1.3359	1.0060	0.60	1.63	0.1257	f	0.10447	1.3359	1.0139	1.39
6	g	0.23109	2.2120	1.6657	66.6	4.32	2.6503	x	0.1359	1.3009	0.9873	-1.27
7	benchmark ratio(a)=1.3280							y	0.1795	1.3210	1.0026	0.26
8								g	0.23109	1.2874	0.9771	-2.29
benchmark ratio(a)=1.3176												

Note. (A) ω_a and θ_a are calculated with equations (2) and (4). (B) According to equation (10) and principle (1), the inner five planets follow the shared rules, which reflect the existence of the shared rules in the system. (C) The calculations with equations (8) and (10) suggest two undetected planets, which cause the large deviation rate and provide an entry point to analyze system evolution. According to the physical parameters of observed planets, Planets x and y likely have disappeared because of mutual dragging forces. A pass-by stray planet may be the inducement.

Equation (11) can also be explained with the K2-138 system (Tables 1 and 3). The period benchmark ratio ($br_p=1.5303$) and semimajor-axis benchmark ratio ($br_a=1.3280$) display $br_p=99.995\%(br_a)^{3/2}$. The degree of equality suggests that equation (11) holds. With equation (11), the calculations of the systems with three and four exoplanets demonstrate that equation (11) holds (Supplementary Table S1 and S2). We calculated all systems with five and more exoplanets. Statistics reflect that over 92% of the systems show $br_p=100(\pm 2)\%(br_a)^{3/2}$ (Supplementary Table S20). They all support equation (11). However, the calculation of the Kepler-32 system shows $br_p=108.72\%(br_a)^{3/2}$ and Kepler-186 shows $br_p=94.18\%(br_a)^{3/2}$. The results suggest that the orbital data of some planets need to be checked (Supplementary Tables S6 and S45).

For equation (12), we will show explanations with a few actual systems. K2-138 shows that $\omega_{P[n:(n-1)]}$ is equal to $[\omega_{a[n:(n-1)]}]^{3/2}$ within a high precision for each planet pair (Supplementary Table S5 [4]), such as $\omega_{P(e:d)}=99.985\%[\omega_{a(e:d)}]^{3/2}$. Kepler-62 is a five-planet system (Supplementary Table S8). It shows $\omega_{P(c:b)}=99.915\%[(\omega_{a(c:b)})^{3/2}]$, $\omega_{P(d:c)}=99.357\%[(\omega_{a(d:c)})^{3/2}]$, $\omega_{P(e:d)}=100.603\%[(\omega_{a(e:d)})^{3/2}]$ and $\omega_{P(f:e)}=100.085\%[(\omega_{a(f:e)})^{3/2}]$. If checked by equation (13), the five planets show $\beta_b=0.99915$, $\beta_c=0.99884$, $\beta_d=0.98779$, $\beta_e=0.99529$ and $\beta_f=0.99857$. The calculations with equation (12) are supported by equation (13), the equation of classical mechanics. Different calculations state that equation (12) exists and holds true. In addition, equation (12) can also reflect inaccurate observations. We simulated the calculation to Kepler-62d, which true orbit is $\alpha_d=0.12(\text{AU})$. Assuming its orbit was $\alpha_{d*}=0.14(\text{AU})$, the inaccurate orbit can be revealed by equation (12), showing $\omega_{P(d*:c)}=78.85\%[(\omega_{a(d*:c)})^{3/2}]$ and $\omega_{P(e:d*)}=126.4\%[(\omega_{a(e:d*)})^{3/2}]$ (Supplementary Table S8 [5]). Since both equations are related to Planet d*, its orbit should be doubtful. If Planet d* is calculated with equation (13), it shows $\beta_{d*}=0.62199$, also suggesting doubt about its orbit. In addition, because equation (11) suggests the doubt of the Kepler-32 system, we calculated the orbits of all planets with equation (12) (Supplementary Table S7). The calculations show $\omega_{P(c:b)}=56.48\%[(\omega_{a(c:b)})^{3/2}]$ and $\omega_{P(d:c)}=137.9\%[(\omega_{a(d:c)})^{3/2}]$. Since the two equations are related to Kepler-32c, its orbital data should be doubtful. If checked by equation (13), Kepler-32c shows $\beta_c=0.45694$, also suggesting a doubtful orbit.

3.4 Principles of judging the properties of planets and systems

Because the shared rules exist in ideal and normal systems, all planets in such a system can be considered to follow the shared rules and should orbit the star in some common manner. If the manner is discovered, it can be taken as the principle of judging a planet that follows the shared rules. The confirmation of the planets that follow the shared rules can also help judge the property of a system and make the judgment of migratory and absent planets more founded. By calculating simulated and observed systems, we found the manner and derived the following principles.

Principle of judging a planet that follows the shared rules, principle (1): If a planet shows the calculated orbit equal to its observed orbit, the planet can be judged to follow the shared rules, provided that the calculation is based on the relation equations of planetary orbits and based on the orbits of other planets as reference.

For an ideal system, the principle completely holds because each planet shows the calculated orbit 100% equal to its observed orbit. For a natural system, the principle holds true because the calculated orbit of the analyzed planet is equal to its observed orbits within a certain range of precision. The reason is that all planets exist in the natural universe and are disturbed by varying factors (discussed in Sect. 3). As qualitative judgment, the precision can be set within an

acceptable range. To facilitate judgment and application, we temporarily set a criterion that the calculated period of a planet is equal to its observed period. For the planets in ideal systems, the criterion is set at 100%, called criterion (I). For the planets in natural systems, the criterion is temporarily set at the range of 95-100% (or 100-105%), called the criterion (N-A). If more different influences are considered, the criterion can be relaxed slightly and set in the range of 90-100% (or 100-110%), called the criterion (N-B). The judgment according to semimajor axes can also follow the criteria.

Principle (1) is the result reflected by the calculations of simulated and observed systems and is supported by the process of planet discovery. Therefore, it can be applied in the judgment of the planet property of most systems. In fact, the judgment of the planet that follows the shared rules is based on the tenet that the analyzed planet has the same property as reference planets. Therefore, only planets that follow the shared rules can make an analyzed planet show a calculated orbit equal to its observed orbit. The principle also implies that the corresponding planets maintain the initial orbital relationships among them, reflecting the essence of the shared rules. According to equations (9) and (10), the orbital calculation of a planet should be based on the orbits of at least two other planets if the calculated planet itself is not taken as a reference planet. Therefore, if the calculation of a planet satisfies criterion (I), (N-A) or (N-B), both the calculated planet and two reference planets can be simultaneously regarded as the planets that follow the shared rules. Thus, only when the number of relevant planets in a system reaches three or more can they prove themselves as planets following the shared rules.

Principle of judging a system that shows the shared rules, principle (2): If a system shows three or more planets that follow the shared rules, the shared rules can be judged to exist in the system.

According to principle (1), only when the number of planets that follow the shared rules reaches three in a system can they prove their properties. Therefore, if the shared rules are considered to exist in a system, the system should have at least three planets that follow the shared rules even though it may have some planets that do not follow the shared rules. If a system shows that all planets observed follow the shared rules and the absolute values of all deviation rates are smaller than 10%, the system can be judged to be a normal (ideal) system. However, if a system shows that all observed planets follow the shared rules but shows the absolute value of a deviation rate larger than 10%, it can only be regarded as a near-normal or abnormal system with the shared rules. Therefore, the property of a system is sometimes different from its type.

Principle of judging a migratory planet, principle (3): If the calculated orbit of a planet is obviously different from its observed orbit, usually the planet does not follow the shared rules and is regarded as a migratory planet, provided that the calculation is performed with the relation equations of planetary orbits and based on the planets that follow the shared rules.

Principle of judging an absent planet, principle (4): If the calculated orbit of a planet does not belong to an observed planet, usually it belongs to an absent planet, provided that the calculation is performed with the relation equation of planetary orbits and based on the planets that follow the shared rules.

First, we will explain principle (1) with an ideal and a normal system. A-HD is an ideal system (Table 2 [1]). With equation (9), we calculated the period of Planet A-f. By taking Planet A-c as the reference planet and the period ratio of Planet A-c to A-b as the leading ratio, the calculation by $P_{A-f(cal)} = (P_{A-c}) \div \{(P_{A-c}) : (P_{A-b})\}^{[sn(A-c)-sn(A-f)]}$ shows 12.078 days. It is equal to its observed period.

Then, we calculated other planets with the same equation. All calculated periods are equal to their observed periods. If other planets are taken as the reference planet or as the leading ratio, the results are the same (Supplementary Table S9 Note 3). Comparisons show that all planets follow the same motion manner. That is, the calculated period of each planet is equal to its observed period when the calculation is performed with equation (9) and by referring to other planets. The equality is not by chance. This reflects that the calculated planet has the same property as the reference planets. The planets following the shared rules can prove each other. Therefore, the motion manner can be regarded as the principle of judging a planet that follows the shared rules. HD 158259 is a normal system (Table 2 [4]). The calculations with equation (9) also reflect the content of principle (1). Only the calculated period of each planet is equal to its observed period within a certain precision (Supplementary Table S10 [1-6]). For example, if Planet f is taken as the reference planet and the benchmark ratio is taken as the leading ratio, the calculated periods of Planets b, c, d, e and f are 97.12%, 100.62%, 100.12%, 100.62% and 100% equal to their observed periods, respectively. The degree of equality satisfies criterion (N-A), suggesting that they all follow the shared rules. As a qualitative judgment, the calculations reflect that principle (1) exists and is correct.

Now, we will explain the four principles. Based on the A-HD system (Table 2 [1]), we carried out several simulation calculations. The first simulation is the migration of a middle planet. Supposing Planet A-d migrated outward, equivalent to 15% of its initial period, the system shows $P_{A-b}=2.1780d$, $P_{A-c}=3.3386d$, $P_{A-d^*}=5.8858d$, $P_{A-e}=7.8461d$ and $P_{A-f}=12.028d$ (Table 2 [3]). According to equation (3), their deviation rates are 0, 15%, -13.04% and 0. It is now a near-normal system. To confirm which planet migrates or still follows the shared rules, we first calculated the period of Planet A-f with equation (9) by taking Planet A-c as the reference planet and the period ratio of Planet A-c to A-b as the leading ratio. The calculated period is 12.028 days by $P_{A-f(cal)}=(P_{A-c})\div\{(P_{A-c}):(P_{A-b})\}^{[sn(A-c)-sn(A-f)]}$. It is equal to its observed period. According to principle (1), Planets A-f, A-c and A-b follow the shared rules. They are the planets following the shared rules before supposition. According to Principle (2), the shared rules exist in the system. Then, we calculated all periods with equation (9). By $P_{n(cal)}=(P_{A-c})\div\{(P_{A-c}):(P_{A-b})\}^{[sn(A-c)-(snn)]}$, the calculation shows 2.1780, 3.3386, 5.1181, 7.8461 and 12.028 days. The calculated periods of all planets are equal to their observed periods except Planet A-d*. If Planet A-b, A-e or A-f is taken as the reference planet, the calculations show the same results. Therefore, according to principle (1), Planets A-b, A-c, A-e and A-f can be judged to follow the shared rules. Because the properties of the four planets agree with that before supposition, the judgment is correct. The calculations demonstrate that principles (1) and (2) exist and hold in the system. However, the calculation also shows 5.1181 days by $P=3.3386\div1.5330^{(2-3)}$, which is 115% equal to the observed period of Planet A-d* by $(5.8858\div5.1181)\times100\%=115\%$. Since the calculation is based on equation (9) and the planets that follow the shared rules, it reflects the content of principle (3) and suggests that Planet A-d* should migrate outwards, equivalent to 15% of its previous period. Because the calculated period of Planet A-d* is equal to its period before supposition, the suggestion is correct and supports principle (3). However, if an improper reference planet or leading ratio is applied, such as P_{A-d^*} or $(P_{A-e}):(P_{A-d^*})$, the judgment will be wrong (Supplementary Table S9 [5] & [6]). By comparison, we can see the influence of the properties of reference planets on the judgment. The simulations further explain principles (1), (2) and (3). Moreover, the simulations also reflect that the application of corresponding principles and equations can reveal the change in planetary

configuration in a system.

The second simulation assumes an undetected planet (Table 2 [2]). By calculation, the simulated system shows deviation rates of 53.3%, 0 and 0. It is an abnormal system. Therefore, we recalculated the periods of the four planets with equation (9). By $P_{n(cal)} = (P_{A-f}) \div \{br_p\}^{[sn(A-f)-(snn)]}$, all calculated periods are equal to their observed periods. According to principle (1), the four planets follow the shared rules. Therefore, the shared rules exist in the system according to principle (2). However, because it shows a 53.3% deviation rate, it is not an ideal system. The calculations further explain principle (2). Meanwhile, the calculation also shows 3.3386 days by $P_x = 12.028 \div 1.533^{(5-2)}$, which does not belong to any observed planet. Since the calculation is based on the planets following the shared rules, the period should belong to an undetected planet according to principle (4). Because the calculated period is equal to the period of Planet A-c before supposition, the judgment is correct. The simulation reflects that principles (1), (2) and (4) are correct, the shared rules also exist in an abnormal system, the orbit of an undetected planet can be predicted, and the true planetary configuration can be reflected.

Meanwhile we performed eight other simulations based on the A-HD system (Supplementary Table S11 [2-9]). The simulations further explain the contents and functions of principle (1-4). They also explain why most multi-planet systems are not normal systems while many of them still show the shared rules. We also simulated the calculations for more natural systems, such as K2-138 and HD 158259 (Supplementary Table S5 Note 4 & S12). They show the same results. The simulation calculations to the semimajor axes of the planets of Kepler-444 and Kepler-85 also support principles (1-4) (Supplementary Table S6 Note 4 & S13 Note 3).

The discovery process of the Kepler-20 system shows the realistic meaning of principle (1-4). In 2012, Kepler-20 only showed five exoplanets (Gautier et al. 2012; Fressin et al. 2012; NASA Exoplanet Archive 2015). According to their orbits, the calculation with equation (4) displayed a 69.9% deviation rate (Supplementary Table S4 [5]). It was an abnormal system then. Therefore, we calculated the semimajor axes of the five planets with equation (10). By taking the benchmark ratio (1.47805) as the leading ratio and Planet c as the reference planet, the calculations by $\alpha_{n(cal)} = 0.0949 \div 1.47805^{(snc-snn)}$ showed 0.0434, 0.0642, 0.0949, 0.1403 and 0.3064 AU. Comparisons showed that the calculated semimajor axes of Planets e, c and f were 99.53%, 100% and 99.50% equal to their observed semimajor axes respectively, which satisfied criterion (N-A). Therefore, the three planets should follow the shared rules according to principle (1). At that time, the shared rules existed in the system according to principle (2). Meanwhile, the calculation showed 0.2073 AU by $\alpha_x = \alpha_c \div (br_a)^{(3-5)} = 0.0949 \div 1.47805^{(3-5)}$, which did not belong to the observed planets. Because the calculation was based on the planets following the shared rules, according to principle (4), it should belong to an undetected planet, temporarily called Planet x. In 2016, Prof. L. A. Buchhave (2016) discovered Planet g. The updated observation data of the system (NASA Exoplanet Archive 2020) show $\alpha_g = 0.2055$ AU, which is 99.13% equal to the semimajor axis of Planet x previously calculated (Supplementary Table S4 [6]). The calculation with equation (9) also provided the period of the undetected planet. The period (34.940 days) of Planet g is equal to 99.73% of the period (34.8476 days) of Planet x (Supplementary Table S4 [4]). The discovery of Kepler-20g objectively supports equations (1-4) and (9 & 10) and principles (1-4).

Some researchers have also tried to predict undetected exoplanets by using T-B-related formulas and provide their orbits. Although the calculations are based on different approaches, we still compared the calculated values with the observation data. Kepler-20g is taken as an example. It

was once predicted by the T-B-related formula, and its period was 40 ± 6 days, which is equal to approximately 87% of the later-observed period (Supplementary Table S4 Note 4). Comparisons show that the period calculated by equation (9) is more accurate than that calculated by the T-B-related formula.

To better reflect the function of principle (1-4), we will show the analysis of an actual system, HD 108236 (Table 4 [1]). Calculated with equation (3), a period ratio shows 45.45% deviation rate. It is an abnormal system. Therefore, we recalculated all possible periods with equation (9). We first tried to find the leading ratio. According to calculations, the average of $P_c:P_b$ and $P_e:P_d$ is 1.5081, which is 99.99% equal to $P_f:P_e$. Therefore, the latter is more likely to reflect the initial orbital relationship of most planets. For confirmation, we calculated the shared divisor (sd) of the periods from Planet e to b by $sd_{(p)} = (P_e \div P_b)^{1/(sne-snb)}$. It is 1.50723 by $sd_{(p)} = (19.590025 \div 3.795963)^{1/(5-1)}$, which is 99.95% equal to the ratio of $P_f:P_e$. The outcome once more reflects the content of the standard rule: the orbits of adjacent planets vary with a shared divisor. Therefore, $P_f:P_e$ can be taken as the leading ratio of equation (9). If taking the largest Planet e as the reference planet, the calculation by $P_{n(cal)} = P_e \div [P_f:P_e]^{(sne-snn)}$ presents six periods (Table 4 [2]). Comparisons show that the calculated period of Planet b, by $P_{b(cal)} = 19.590025 \div 1.5080^{(5-1)}$, is 100.2% equal to its observed period. Although there are several planets between Planets b and e, the period of Planet b calculated through Planet e is still almost 100% equal to its observed period. The result suggests that they revolve in the same orbital mode and their orbits have inner connections. According to principle (1), Planets b, e and f follow the shared rules well. Therefore, according to principle (2), the shared rules exist in the system. Based on Planets f and e, the calculated periods of Planets c and d are 108.6% and 109.1% equal to their observed periods. According to principle (3), they should migrate outwards relative to the orbits where they should be. In addition, the calculation also shows a period of 8.61455 days by $P_x = 19.590025 \div 1.5080^{(5-3)}$, which should belong to an undetected planet according to principle (4). The calculation with equation (9) and judgment according to principles (1-4) reflect the existence of the shared rules in the system and the change in planetary configuration. If one intends to study the evolution of the system, the calculations provide entry points. Therefore, the analysis of the change may become relatively easy (Supplementary Table S34 Note 3). Here, we offer a simple inference. Considering the mass relations of observed planets, orbital ratios of previous planets and effect of Kirkwood gap, mutual dragging forces between adjacent planets might first make Planet d migrate outwards and Planet x migrate inwards. Then, the interaction between Planets x and c caused Planet x to cross the orbit of Planet c and fall into the star. The exchange of kinetic energy with Planet x made Planet c migrate outwards, too. The orbital changes of the three planets agree with the law of kinetic energy conversion.

Table 4. Calculations of the period ratio parameters (ω_P) and deviation rates (θ_P) of adjacent planets in the HD 108236 system, and analysis of the change of its planetary configuration

Calculations of adjacent periods [1]						Calculations based on the planets following the shared rules [2]					
Pla	Observed	Ratio	ω_P	θ_P	Mass	Pla	Calculated	Ratio	ω_P	θ_P	Implication of
net	P (days)	($P_n:P_{n-1}$)		(%)	(M_{Earth})	net	P* (days)	($P_n:P_{n-1}$)		(%)	[($P:P^*$) $\times 100\%$]
b	3.795963				3.24	b	3.78817				Equal to 100.2%
c	6.203449	1.6342	1.0402	4.02	4.94	c	5.71256	1.5080	1	0	Equal to 108.6%

d	14.175685	2.2851	1.4545	45.45	6.98	x	8.61455	1.5080	1	0	Undetected planet
e	19.590025	1.3820	0.8796	-12.0	9.71	d	12.99073	1.5080	1	0	Equal to 109.1%
f	29.54115	1.5080	0.9598	-4.02	4.72	e	19.59003	1.5080	1	0	Equal to 100%
benchmark ratio _(P) =1.5711						f	29.54115	1.5080	1	0	Equal to 100%
						leading ratio _(P) =1.5080					

Note. The percentage of $[(P:P^*) \times 100\%]$ implies the degree to which the related planet migrates relative to the orbit where it should revolve. If it is equal to 100%, the related planet is taken as the reference planet or orbits the star in the same mode as reference planets. If it is larger or smaller than 100%, the related planet should migrate. For undetected planets, the orbits are not compared. According to principle (1), (3) and (4), one can judge the property of related planet. Because perturbations among planets are mutual, the degree of equality just provides a reference for qualitative analyses. The calculations with the equations of the shared rules can reflect the change of planetary configuration in the system.

In addition, the discovery process of the HD 108236 system also supports the shared rules. In 2020, four exoplanets were discovered. If considering the material transition of the protoplanetary disk of HD 108236, there should be an undetected planet outside Planet e (Supplementary Table S34 Note 4). Therefore, by taking the shared divisor as the leading ratio of equation (9) and Planet e as the reference planet, we calculated its period by $P_y = 19.5917 \div 1.5073^{(5-6)} = 29.531(\text{day})$. In 2021, Prof. A. Bonfanti detected the fifth planet (Bonfanti et al. 2021; NASA Exoplanet Archive 2022). Its period is 29.54115 days, 99.97% equal to the period of the predicted planet.

3.5 Traits of reflecting planet properties

If a planet in an ideal (or normal) system migrates, is not detected or still follows the shared rules, it should leave some traits. The discovery of the traits will make the study of a system simpler. According to the calculation of simulated and observed systems, we show three traits.

Trait of a planet that follows the shared rules, trait (1): If the absolute values of two adjacent deviation rates in a system are equal to 0 within a certain precision (supposing 90%), the three planets composing the two rates usually follow the shared rules.

Trait of a migratory planet, trait (2): In a system with four or more planets, if two adjacent deviation rates show a positive value and a negative value and their absolute values are close, the planet associated with the two rates very possibly migrated to some extent.

Trait of an absent planet, trait (3): If an orbital ratio in a system is equal to the square of its adjacent orbital ratio within a certain precision, usually a planet should exist between the two planets that compose the larger ratio. If an orbital ratio is equal to the cube of its adjacent ratio within a certain precision, there should be two absent planets between them.

The systems in Table 1 show the content of trait (1). The simulated system of A-HD in Table 2 [2] shows trait (1) by $\theta_{P[(A-e):(A-d)]}=0$ and $\theta_{P[(A-f):(A-e)]}=0$, suggesting three planets following the shared rules, and shows trait (3) by $P_{(A-d)}:P_{(A-b)}=(P_{(A-e)}:P_{(A-d)})^2$, suggesting an undetected planet. In Table 2 [3], $\theta_{P[(A-d^*):(A-c)]}=15\%$ and $\theta_{P[(A-e):(A-d^*)]}=-13.04\%$ show the content of trait (2). They suggest the migration of Planet A-d*. Both suggestions agree with suppositions, reflecting that the three traits exist and are meaningful. The simulations of the HD 158259 system also show the three traits (Table 2 [4]-[6]). The traits played a role in analysing the Kepler-299 and Kepler-286 systems (Supplementary Table S14).

According to principles (1)-(4) and traits (1)-(3), including equations (7) & (8), one can objectively judge which exoplanets follow the shared rules, are not detected, have disappeared or have obviously migrated. With equations (9) and (10), one can also calculate the previous orbits of migratory exoplanets and possible orbits of absent planets. Based on the judgment and calculation, the explanation of system evolution will be more convincing.

The shared rules provide a different approach to analyze multi-planet systems. If the shared rules must be compared with MMRs, both can help research the changes in planetary orbits and the evolution of multi-planet systems. However, the approaches are different. In addition, research using the shared rules usually involves every planet in a system, whereas research using MMRs only involves planets with the features of MMRs. Meanwhile, some systems with the shared rules contain exoplanets showing MMRs chains, such as HD 158259 and K2-138 (Table 1). Therefore, one can also see the phenomenon of the continuous commensurabilities of planetary orbits in such systems. However, the exoplanets of most systems with the shared rules do not show MMRs chains, even the resonant orbits of small integer ratios, such as Kepler-130 and Kepler-172 (Supplementary Table S1 & S2). Therefore, MMRs only help analyze MMRs exoplanets in a system while the shared rules can help analyze all exoplanets in a system. In addition, the results that they aim at are different. If one intends to analyze the orbital changes of all planets of a system, such as HD 108236 (Table 4), the use of the shared rules may be more convenient, simple, accurate and founded. If the shared rules are compared with the T-B-related formulas, they are also different. The shared rules are based on the calculation of observations while T-B-related formulas are sometimes considered to be based on mathematical coincidence (Titius-Bode law [2021](#)). The contents reflected by T-B-related formulas are much less. If comparing the predicted orbits of undetected exoplanets, the equations of the shared rules usually can provide orbits much closer to observations than T-B-related formulas, such as Kepler-20g.

3.6 Statistics and analyses of the calculations of all multi-planet systems observed

By Oct. 30, 2022, 5190 confirmed exoplanets were discovered, composing 3877 systems (NASA Exoplanet Archive [2022](#)). They contain 3012 one-planet systems, 571 two-planet systems, 191 three-planet systems, 65 four-planet systems, 28 five-planet systems, 9 six-planet systems, 1 seven-planet system and 1 eight-planet system. Since the method of orbital ratio parameter analysis is only suitable for the analysis of systems with three or more planets, we calculated all suitable systems. According to the general standard, 69 of the 191 three-planet systems belong to normal systems, including 38 systems showing absolute values of deviation rates smaller than 5% (Supplementary Table S15). Because such a system has only three planets, it is either a normal or abnormal system. In addition, since disturbances are relatively simpler, 15 systems show absolute values of deviation rates smaller than 1.25%, which are very close to ideal systems (Table 1 and Supplementary Table S1). There are 101 systems containing four or more planets. They are composed of 454 confirmed planets, constituting 353 period ratios (Table 5). We first calculated the deviation rates of the period ratios of these systems. For convenient comparison, we divided the deviation rates of each system into different ranges, based on which we counted the numbers of period ratios showing corresponding deviation rates according to the systems with the same numbers of exoplanets (Table 5). We also counted the proportions of all period ratios showing each range of deviation rates in the related systems. For example, the 65 four-planet systems contain 260 planets, which constitute 195 period ratios. According to calculations and statistics, 105 period ratios show absolute values of deviation rates equal to or smaller than 10%, occupying

53.85% of the 195 period ratios; 57 period ratios show deviation rates between 10-40%, occupying 29.23% of the proportion; and 33 period ratios show deviation rates larger than 40%, occupying 16.92% of the proportion. We also performed the statistics for the 101 systems; 190 period ratios show absolute values of deviation rates equal to or smaller than 10%, occupying 53.83% of the 353 period ratios; 59 period ratios show deviation rates larger than 40%, occupying 16.71% of the total. It shows that most of the period ratios are relatively small.

Table 5. Statistics of the period ratios showing different ranges of deviation rates and their proportions in all period ratios of the systems with the same number of exoplanets

System with x planets	Number of systems	Number of planets	Number of period ratios	Number of period ratios showing different ranges of deviation rates and the proportion (prop.) of deviation rates of related period ratios (θ_p)					
				Ratios of $\theta_p \leq 10\%$	Prop. of $\theta_p \leq 10\%$	Ratios of $10 < \theta_p \leq 40\%$	Prop. of $10 < \theta_p \leq 40\%$	Ratios of $\theta_p > 40\%$	Ratios of $\theta_p > 40\%$
8-planet	1	8	7	4	57.14%	2	28.57%	1	14.29%
7-planet	1	7	6	5	83.33%	1	16.67%	--	--
6-planet	9	54	45	23	51.11%	13	28.89%	9	20%
5-planet	25	125	100	53	53%	31	31%	16	16%
4-planet	65	260	195	105	53.85%	57	29.23%	33	16.92%
Total	101	454	353	190		104		59	
Proportion (%) based on 353 period ratios					53.83%		29.46%		16.71%

Note. (A) We counted the period ratios according to the five ranges of the deviation rates of the period ratio parameters of the 101 systems. In the table, we showed the statistics according to three ranges. More ranges are presented in Supplementary Tables S16-S19. (B) The deviation rates for each system are calculated with equation (3), such as HD 158259 and K2-138 (Table 1).

We then analyzed the properties of the 454 exoplanets of the 101 systems according to the calculations with equation (9) and judgment with principle (1). We also analyzed the properties of these systems (Table 6). If judged according to criterion (N-A), 229 exoplanets are the planets following the shared rules. They come from 71 systems. According to principle (2), the shared rules exist in the 71 systems, which occupy 70.3% of the 101 systems. If judged according to criterion (N-B), 306 exoplanets follow the shared rules. They come from 88 systems. According to principle (2), it can still be considered that these systems show the shared rules. They occupy 87.13% of the 101 systems. Hence, most of these systems display the existence of the shared rules. For comparison, we also showed the statistics according to the systems with the same number of planets (Table 6). The statistics reflect that the fewer number of planets a system contains, the less likely the system is to show the shared rules (discussed in later sections).

Table 6. Statistics of exoplanets following the shared rules and extrasolar systems showing the shared rules

System with different numbers	Number of systems	Number of exoplanets	Number of the exoplanets following the shared rules	Number of the systems showing the shared rules
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of exoplanets			Criteria (N-A)	Criteria (N-B)	Criteria (N-A)	Criteria (N-B)
8-planet system	1	8	4	4	1	1
7-planet system	1	7	3	5	1	1
6-planet system	9	54	33	39	9	9
5-planet system	25	125	71	93	21	24
4-planet system	65	260	118	165	39	53
Total	101	454	229	306	71	88

Note. (A) Detailed calculations of each systems are based on Supplementary Tables S18 and S19. (B) The judgment of the planets that follow the shared rules is based on equation (9) and principle (1), such as the exoplanets of HD 158259 and HD 108236 (Table 1 and 4). (C) The judgment of the systems that show the shared rules is based on principle (2), such as HD 158259 and HD 108236 (Table 1 and 4).

If the 101 systems are classified according to the general standard (Sect. 2.1), 9 systems are normal systems, 46 systems are near-normal systems, and 46 are abnormal systems. Since the normal systems only occupy approximately 9% of the 101 systems, it appears that there is a contradiction between the proportions of the normal systems and the systems showing the shared rules. The contradiction is reasonable. A normal system has a different concept from a system showing the shared rules. The latter contains the former. The latter also includes the planets showing deviation rates larger than 10%. In this case, the systems with the shared rules may be normal, near-normal or abnormal systems. For example, K2-138 is an abnormal system because it shows a 115% deviation rate. However, it contains six planets following the shared rules (Supplementary Table S4 [1] & [6]). Therefore, the shared rules still exist in the system according to principle (2). It is an abnormal system with the shared rules. The HD 108236 system offers another clearer display (Table 4). The statistics reflect that more than half of the exoplanets show relatively small deviation rates, stating that they maintain initial orbital relationships to a great extent, which agrees with the number of exoplanets showing the shared rules. In addition, the statistics also reflect that a small part of the exoplanets in the 101 systems show relatively large deviation rates. Large deviation rates suggest the influences of different factors, implying that corresponding planets have migrated obviously, disappeared or are not discovered. In this case, the systems with large deviation rates are classified as abnormal systems. Although most systems are near-normal or abnormal systems (Supplementary Tables S16-S19), they do not affect the proportion of the systems with the shared rules. The number of the planets that follow the shared rules determines the property of a system. The classification of the systems can help learn about the actual planetary configurations of these systems and study their evolution, especially the systems with large deviation rates.

For comparison, we also calculated the deviation rates of the semimajor axis ratios of the exoplanets of the systems containing five and more exoplanets (Supplementary Tables S21-S56). The statistics show that the proportions of different deviation rates of semimajor axis ratios are very close to those of period ratios (Supplementary Tables S18 and S20). To more intuitively show the proportions of the deviation rates of all these planets, we made a contrast figure of periods and semimajor axes (Supplementary Fig. S2).

4. Main reasons for large deviation rates

The statistics of deviation rates reflect that many systems show large deviation rates. Large deviation rates may cause some researchers to doubt the existence of the shared rules, which should be understandable. Although large deviation rates are mainly related to planet migration or absence, some other reasons may also cause large deviation rates. In addition, planet migration and absence may be caused by different factors. By studying the influences of different factors on planet motion, comparing the physical parameters of observed planets, and analyzing the results calculated with the equations of the shared rules, we summarized five main reasons that cause large deviation rates. Based on the reasons, one may better understand why all systems fail to perfectly show the shared rules, why so many systems show large deviation rates, and why most systems are abnormal systems. Starting with a large deviation rate, one may more easily, objectively and accurately analyze what happens in a system.

4.1 Perturbation of dragging force between adjacent planets

While exoplanets orbit their host stars, they are affected by different disturbances. One of them is the perturbation of the dragging force of neighboring planets. The mutual dragging forces of two neighboring exoplanets may change their normal orbiting speeds, causing changes in their semimajor axes. If it is a normal system, the changes may make the system no longer a normal system. The mutual dragging forces may also change the orbital shapes of two neighboring exoplanets of a system, embodying the continuous increases in their eccentricities. In this case, the two exoplanets sometimes may get very close to each other when they move in elliptic orbits. As a result, they may migrate obviously, collide with each other, exchange their positions, fall into their host stars, or be ejected off the system. During the evolution of a multi-planet system, some exoplanets may develop into orbital resonances. The resonances of some ratios may further enhance the effect of dragging force and become one of the inducements of the orbital changes of some planets, similar to the effects of the Kirkwood gap. If an exoplanet is much smaller than its neighbours, the effect is asymmetric. The smaller one may migrate greatly or be ejected off the system, while the larger one may only migrate slightly. The results of perturbations are usually reflected as increases in the absolute values of the deviation rates of some exoplanets in a system, showing large deviation rates. Because different systems have different physical conditions, the results of perturbations are different. However, the orbital changes of related planets caused by dragging force follow the law of conservation of momentum. The influence of the dragging force has also been extensively researched (Murray & Dermott 1999; Chatterjee & Ford 2015) and attested by computer simulations (Hardy & Gong 2017).

The KOI-94 system well displays the influence of dragging force. Now, its four exoplanets show $\alpha_b=0.05119$ AU, $\alpha_c=0.1013$ AU, $\alpha_d=0.1684$ AU and $\alpha_e=0.3064$ AU. From the current orbits, it is not easy to directly judge which exoplanet was obviously disturbed or changed in orbit. However, after their orbital ratios are quantified with equation (4), the judgment becomes easy. The calculations show $\theta_{a(c:b)}=8.76\%$, $\theta_{a(d:c)}=-8.63\%$ and $\theta_{a(e:d)}=0$ (Supplementary Table S57 [1]). According to trait (2), Planet c should have migrated. To confirm its migration, we first calculated the semimajor axes of Planet b with equation (10). By $\alpha_{b(cal)}=\alpha_d \div [\alpha_e:\alpha_d]^{(snd-snb)}$, the calculation provides 0.050867 AU, which is 100.6% equal to its observed semimajor axis. The almost 100% equality suggests that Planets b, d and e follow the shared rules well according to principle (1).

Therefore, the shared rules can be judged to exist in the system according to principle (2). In this case, we recalculated the orbit of Planet c by $\alpha_{c(cal)} = \alpha_d \div [\alpha_e : \alpha_d]^{(snd-snc)}$. It is 0.092552 AU, 109.45% equal to its observed semimajor axis (Supplementary Table S57 [2]). According to principle (3), Planet c has migrated outwards, equivalent to approximately 9.45% of its previous semimajor axis. The period calculation also shows the migration of Planet c (Supplementary Table S57 [3] & [4]). The calculations reflect the previous planetary configuration. Before Planet c migrated, it once was a near ideal system. Regarding migration, we will provide a brief analysis. Observations show that the mass of Planet d is $0.334M_{Jup}$, while that of Planet c is $0.049M_{Jup}$. Since the difference of their orbits is only 0.0671 AU, the dragging force of Planet d should be great and greatly perturb the motion of Planet c. For intuitive understanding, we compared the gravitational influence of Planet d on its adjacent planet and that of Jupiter on a testing asteroid in the asteroid belt. Each celestial body has its own gravitational field. Comparisons show that the effect of the gravitational field of Planet d ($g_{(d)} = F_{(d)}/m_{(c)}$) on Planet c is approximately 463 times that of Jupiter ($g_{(Jup)} = F_{(Jup)}/m_{(ast)}$) on the asteroid if calculated by the following equation.

$$g_{(d)} : g_{(Jup)} = [(M_{(d)}) \div (D_{a(d)-a(c)})^2] : [(M_{(Jup)}) \div (D_{a(Jup)-a(ast)})^2], \quad (14)$$

where M_d is the mass of Planet d ($0.334 M_{Jup}$), $D_{a(d)-a(c)}$ is the difference in the semimajor axes of Planets d and c, showing $0.1684-0.1013=0.0671$ AU, M_{Jup} is the mass of Jupiter, and $D_{a(Jup)-a(ast)}$ is the difference in the semimajor axes of Jupiter and a testing asteroid in the asteroid belt, showing $5.2-2.7=2.5$ AU. The derivation and explanation are presented in Note 3 of Supplementary Table S57. Since it is widely accepted that the dragging force of Jupiter greatly affects the motion of asteroids, the dragging force of Planet d should also greatly affect the motion of Planet c, embodying its outward migration. Before Planet c migrated, all periods were in a chain of near 2:5 period resonances. Perhaps the effect of Kirkwood gap first produced an influence on Planet c. Because the mass of Planet d is 6.8 times that of Planet c, the exchange of kinetic energy made Planet c migrate more obviously. After the migration of Planet c, the stable relations between planetary orbits are restored again. In addition, the eccentricity of Planet c is 0.43 ± 0.23 , also implying that it was greatly perturbed. As a qualitative analysis, the dragging force explains why Planet c migrates and why the system fails to show the shared rules well. Conversely, the relatively large deviation rate reflects the influence of the dragging force.

We will discuss more examples of exoplanet migration and disappearance caused by dragging force in Sect. 5. Nevertheless, a large deviation rate plays the role of identifying the party in the study of system evolution.

4.2 Impact of stray celestial bodies

During the formation and evolution of planetary systems, some planetesimals, planets and companion stars may leave their host systems and become stray celestial bodies, such as runaway stars (SDSS J1128), Mar-sized stray planets (OGLE-2016-BLG-1928), and interstellar interlopers 1I/Oumuamua and 2I/Borisov (Jewitt et al. 2020). If a sufficiently large stray body roves into a normal system, it may affect the stable motion of some exoplanets, regardless of whether it collides with them or just passes through them. The impact may cause corresponding exoplanets to migrate or vanish, making the related deviation rates change and the shared rules vague or disappear from the system. Moro-Martín et al. (2009) calculated and inferred the total mass density of interstellar material, showing $M_{total} = 2.2 \times 10^{-7} M_{\odot} pc^{-3}$. The density should imply that some interstellar objects with Earth mass or bigger move in interstellar space. Other astronomers

also believe in the existence of stray planets, even Jupiter-massed planets, and intend to hunt them with JWST in star-forming clusters and then learn about their density (Scholz et al [2022](#)). Nevertheless, if the time is billions of years, the possibility that rapidly-moving stray planets cross or pass through a planetary system should be high. Therefore, a certain proportion of exoplanets should be impacted by stray objects throughout their lifetimes.

The planetary configuration of Kepler-299 is likely to be affected by a stray object. It has four planets, showing $P_b=2.927128d$, $P_c=6.885875d$, $P_d=15.054786d$ and $P_e=38.285489d$. Equation (3) shows $\theta_{P(c:b)}=0$, $\theta_{P(d:c)}=-7.06\%$ and $\theta_{P(e:d)}=8.1\%$ (Supplementary Table S14 [1-1]). According to trait (2), the deviation rates suggest that Planet d migrated. Therefore, we first calculated the period of Planet e with equation (9). By $P_{e(cal)}=P_c \div (P_c:P_b)^{(2-4)}$, the calculated period is 38.1050 days, 100.47% equal to its observed period. According to principle (1), the almost 100% equality suggests that Planets e, c and b follow the shared rules. According to principle (2), the shared rules exist in the system. Therefore, based on $P_{d(cal)}=P_c \div (P_c:P_b)^{(2-3)}$, we recalculated the period of Planet d. It is 16.19833 days, 92.94% equal to its observed period. According to principle (3), it should migrate inwards, equivalent to approximately 7.06% of its previous period. Before it migrated, the planetary configuration of Kepler-299 was almost the same as an ideal system (Supplementary Table S14 [1-2]) If the dragging force caused the migration, the exchange of kinetic energy should make a neighboring planet migrate reciprocally because Planets c, d and e have close masses. Since adjacent planets showed deviation rates close to 0, the migration of Planet d is more likely to be caused by a stray planet.

4.3 Disturbance of companion stars

Multi-planet systems usually form in some particular conditions (Wei & Kai [2014](#); MacGregor [2020](#)). A molecular air mass with appropriate matter may collapse into a protoplanetary disk under the influence of different forces, including the Coriolis force. If the disk is circular or near circular, it may develop into a multi-planet system. If the system is not greatly disturbed during its formation and evolution, the shared rules may appear in the system (discussed later). Sometimes, an air mass may develop into a multi-star system or an open cluster of stars because of some disturbance. If an initial star in a multi-star system contains suitable matter, it may also develop into its own protoplanetary disk and form its own multi-planet system. A multi-star system with a multi-planet system is temporarily called a mixed system. In a mixed system, the companion star may affect its neighboring planetary system. The motion of heavenly bodies in such a system is complex. Here we will only discuss the influence of companion stars.

Usually, the distance and mass of a companion star determine the extent to which its neighboring planetary system is disturbed. If a large companion star is relatively near, its gravitational pull may first affect the matter distribution of its neighboring protoplanetary disk, then interfere in the formation of the planetary system, and finally disturb the motion of some planets in the planetary system. Therefore, the planetary system in a mixed system often shows unusual deviation rates and sometimes very large mass ratios. For example, GJ 676 is a binary-star system. GJ 676 A currently has four planets. The disturbance of GJ 676 B ($R_B=0.31R_\odot$) makes the planetary system show deviation rates of 0, 203.8% and -29.37% and mass ratios of 1.371, 194.5 and 1.841. The calculations do not reflect the Share Rules in the GJ 676 A system (Supplementary Table S58). However, sometimes, the shared rules still appear in a mixed system. 55 Cnc is a famous binary-star system. 55 Cnc A ($R=0.96R_\odot$) has five planets now, showing $\alpha_e=0.01544$ AU, $\alpha_b=0.1134$ AU, $\alpha_c=0.2373$ AU, $\alpha_f=0.7708$ AU and $\alpha_d=0.5957$ AU. Their deviation rates are -4.96%,

-72.92%, -57.87% and 0. Their mass ratios are 33.09, 0.194, 0.933 and 20.76 (Supplementary Table S32). The unusual values should be caused by the gravitational disturbance of 55 Cnc B ($R=0.29R_{\odot}$) because it is only approximately 1065 AU away from 55 Cnc A (Extrasolar Planets Encyclopaedia 2021). Although it is an abnormal system, the shared rules still exist in the system because the calculations by $\alpha_{n(cal)}=\alpha_d \div [I r_{\alpha}]^{(snd-sne)}$ show that the calculated semimajor axes of Planets e, b and d are 98.66, 100 and 100% equal to their observed semimajor axes. According to principle (1), they follow the shared rules. The calculations also show two missing planets ($\alpha_x=0.04213$ AU and $\alpha_y=2.213$ AU). They should be transitional planets and may have vanished because of the perturbations of huge Planets b and d. The GJ 667 C system is in a triple-star system. Its five exoplanets show deviation rates of 0, -64.51%, -59.19% and 5.33%. Now it is an abnormal system. However, the calculation with equation (9) of $P_{g(cal)}=P_e \div (P_e \cdot P_f)^{(5-8)}$ shows that the calculated period of Planet g is 100% equal to its observed period, implying that Planets g, e and f follow the shared rules (Supplementary Table S33). Based on the planets following the shared rules, the calculations with equation (9) also suggest two undetected planets with $P_x=99.2d$ and $P_y=158.3d$. Cuartas-Restrepo et al. also proposed similar planetary configuration by analysing the evolution of the GJ 667 C system (Cuartas-Restrepo 2016). The Kepler-167 system has a companion star. Although the disturbance made a Jupiter-sized planet form in the remote area, the shared rules still exist in the abnormal system (Supplementary Table S59). Kepler-20 was a near-normal system in a mixed system. Although the disturbance of Kepler-20 B made the outermost Planet d migrate outwards and show a 15.6% deviation rate of the semimajor axis ratio, the shared rules still exist (Table 1). If the companion star in a mixed system is very far away or relatively small, it will not show obvious disturbances. Such planetary systems still display the shared rules well, similar to the Kepler-296 A system (Table 1).

4.4 Limitations of observations

Limited by different factors, such as observation angles from Earth, distances between exoplanets and host stars, orbital inclinations of exoplanets, influences of other celestial bodies, and observation methods, some exoplanets of discovered systems may not be detected simultaneously or never be detected. If a normal system fails to display all its exoplanets, it will display large deviation rates. At this moment, related deviation rates will not truly reflect orbital relationships and will influence the display of the shared rules. Kepler-20 offers a real example. In 2012, only five exoplanets were discovered. It showed a 121.3% deviation rate (Supplementary Table S4 [3]-[6]). In 2016, Planet g was observed. The discovery erased and explained the large deviation rate (Sect. 2.4). Because of some limitations, Kepler-31 and Kepler-398 only show two planets respectively. However, after Kepler-31d and Kepler-398d were discovered, the systems clearly show the shared rules (Supplementary Table S1 Note 5).

Statistics also reflect that some exoplanets may not be detected because of the limitation of the observation method. Of the 101 systems with four and more planets, 81 are discovered mainly by transit and 20 are mainly by radial velocity or imaging (NASA Exoplanet Archive 2022). Although the 81 systems contain 356 transit-discovered exoplanets, only about 3% of the exoplanets show periods longer than 120 days. The 20 systems contain about 97 exoplanets. However, about 80% show the periods longer than 120 days. The statistics imply that transit-discovered systems should contain more undetected exoplanets, especially long-period exoplanets. Some limitations may affect the transits of some exoplanets. Prof. T. O. Hands and R. D. Alexander (2016) researched the influence of observation methods by performing the

simulations of the assembly of compact systems and found that some Kepler exoplanetary systems may host undetected giant planets with orbital periods longer than one year, but they are not shown because of the limit of observation methods. Therefore, different limitations affect the completeness of observations for some systems and the display of the shared rules of these systems.

4.5 Influence of inaccurate observation data

Affected by various factors, some observation data may not be very accurate temporarily, which may also affect the correct calculation of deviation rates. For example, according to the observation data of the Kepler-296 A system published in 2014 (NASA Exoplanet Archive [2022](#)), the deviation rates of the period ratios of its five exoplanets were -9.76%, 90.09%, -3.78% and 3.78% (Supplementary Table S48). In 2015, Prof. T. Barclay et al. ([2015](#)) researched and renewed the orbital data. Based on the updated data (NASA Exoplanet Archive [2022](#)), the deviation rates are 1.02%, -0.76%, -6.58% and 0.76%. It is a normal system now.

There are some other reasons that may cause large deviation rates. Sometimes, multiple reasons may cause a large deviation rate. Sometimes it is difficult to find or confirm the reason. However, the five reasons can not only explain most of the large deviation rates objectively, but also help understand why the shared rules are not obvious in some systems and why the number of planets that follow the shared rules is smaller than hoped.

5. Analyses of all multi-planet systems discovered

Since the shared rules and related equations, principles and traits are derived according to observations, they should be suitable for all or most systems. To better understand their realistic meanings, we calculated all multi-planet systems discovered. With corresponding equations, principles and traits, we also judged the types and properties of these systems, researched the actual planetary configurations and their changes, and studied the reasons for their evolution. The calculations support the shared rules well.

5.1 Analysis of the systems containing five and more exoplanets

By October 2022, astronomers discovered 36 extrasolar systems with five and more confirmed planets (NASA Exoplanet Archive [2022](#)). They contain 1 eight-planet, 1 seven-planet, 9 six-planet and 25 five-planet systems. We quantified the orbital relations with equations (1-4). According to the general standard, 3 are normal systems, 12 are near-normal systems, and 21 are abnormal systems (Supplementary Table S16 & S17). We also calculated the orbits with equations (9) and (10) and analysed the calculations with principles (1-4). If judged according to criterion (N-B), 35 are the systems showing the shared rules. If according to the criterion (N-A), 32 belong to the systems showing the shared rules (Table 6 and Supplementary Table S18). Although 21 are abnormal systems, 20 still show the shared rules. Because the shared rules exist in the 35 systems, we further analyzed these systems, providing the suggestions of migratory, disappeared and undetected planets. We also calculated the orbits of corresponding planets and studied the evolution of related systems. Detailed analyses are as follows.

KOI-351 (Kepler-90) has eight confirmed exoplanets now, showing $P_b=7.008151d$, $P_c=8.719375d$, $P_i=14.44912d$, $P_d=59.73667d$, $P_e=91.93913d$, $P_f=124.9144d$, $P_g=210.6070d$, and $P_h=331.6006d$. The calculation with equations (1) and (3) quantifies their period relations and shows a 164.5% deviation rate, suggesting that it is an abnormal system (Supplementary Table

S21 [1]). With equation (9), we recalculated the periods of all possible planets by $P_{n(cal)} = P_h \div (P_h : P_g)^{(snh-snn)}$. The calculations show ten periods (Supplementary Table S21 [3]). For example, the calculated period of Planet c is 8.77959 days by $P_{c(cal)} = 331.6006 \div (331.6006 : 210.6070)^{(10-2)}$, which is 99.314% equal to its observed period. According to principle (1), Planets c, g and h follow the shared rules. Therefore, the shared rules exist in the system according to principle (2). The calculations also reflect two periods that do not belong to observed planets, showing $P_x = 331.6006 \div (331.6006 : 210.6070)^{(10-4)} = 21.765(d)$ and $P_y = 331.6006 \div (331.6006 : 210.6070)^{(10-5)} = 34.269(d)$. According to principle (4), they should belong to undetected planets. In addition, according to principle (3), Planets b, d, e and f should have migrated, equivalent to approximately 25.68, 10.71, 8.22 and -6.62% of their previous periods (Supplementary Table S21 [3]). The calculations of the semimajor axes also show the absence and migration of the same planets (Supplementary Table S21 [2] & [4]). The calculations display the actual planetary configuration of the system and its change.

The analysis of the change should be complex. We tried to offer a simple inference (Supplementary Table S21 Note 3). Because Planet g is very large ($R_g = 0.723R_{Jup}$) and very close to adjacent Planet f (0.71-0.48=0.23AU), the influence of its dragging force on Planet f should be very great. If comparing the gravitational influence of Planet g on Planet f with that of Jupiter on an asteroid of the asteroid belt, the calculation with equation (14) shows that the former is greater than the latter. Therefore, Planet g might first perturb the steady motion of Planet f, and then gravitational perturbations occurred among its inner planets in a conduction mode. As a result, Planets f, e and d migrated successively, including Planet y. As Planet y changed its orbit, it might get close to Planet x and eject it off the system. The exchange of kinetic energy with Planet x made Planet y fall toward the star. It crossed the orbit of Planet c and finally settled at the orbit of the current Planet b. The unbalanced perturbations among all planets in the system lead to the current planetary configuration. Because the dragging forces are mutual, the orbital changes should be related to all planets. We only provided a qualitative analysis. Because the analysis with the shared rules shows which planets may have changed their orbits, it becomes relatively easy and objective. Prof. A. Granados Contreras and A. Boley (2018) also researched the KOI-351 system. They performed many simulations and found that the outer huge planet may cause changes in planetary orbits in the system.

In addition, Kepler-90i was discovered four years later than other planets (Shallue & Vanderburg 2018). The calculation with equation (9) could predict its existence before it was discovered. By $P_i^* = P_h \div (P_h : P_g)^{(snh-sni^*)}$ or $P_i^* = 331.6006 \div (331.6006 : 210.6070)^{(10-3)}$, its calculated period was 13.8235 days, which is 104.52% equal to the period of later-discovered Kepler-90i ((Supplementary Table S21 Note 4). The discovery supports the shared rules again. The calculation also reflected $\alpha_i^* = 0.1206(\pm 0.012)AU$ by $\alpha_i^* = 1.01 \div (1.3548)^{(10-3)}$. Moreover, because KOI-351h is a Jupiter-sized planet, there should be a planet outside it if considering the matter transition in the protoplanetary disk. By $P = 331.6006 \div (331.6006 : 210.6070)^{(10-11)}$, its period may be approximately 522 days. Perhaps the observation time or angle may affect the transit of the planet (Supplementary Table S21 Note 5).

TRAPPIST-1 is a seven-planet system. Since it shows a 12.6% deviation rate of the period ratio, it is a near-normal system (Supplementary Table S22). According to trait (1) or based on the calculations with equation (9), Planets b, g and h follow the shared rules well. According to principle (2), it is a system showing the shared rules. According to the planetary physical

parameters, a stray planet possibly caused the migrations of Planets b, c, g and h (Supplementary Table S22 [4] and Note 4). In addition, because the semimajor axis of Planet h was not announced in 2021, we tried to calculate it with equation (12) of $(P_n:P_{n-1})=(\alpha_n:\alpha_{n-1})^{3/2}$ according to NASA Exoplanet Archive (2021), showing $\alpha_h=\alpha_g(P_h:P_g)^{2/3}=0.0451\times(18.767\div12.3529)^{2/3}=0.0596012$ AU. It is 96.3% equal to 0.06189 AU (Agol et al. 2021), published on Nov. 17, 2022 by NASA.

There are 9 six-planet systems now. Kepler-20 and TOI-178 are near-normal systems while K2-138, Kepler-11, HD 219134, HD 34445, HD 191939, HD 10180 and Kepler-80 are abnormal systems (Supplementary Table S16). According to principle (2), they all show the shared rules. Since we have analyzed the K2-138 and Kepler-20 systems, to reduce reading, we presented the analyses of other seven systems in Supplementary Tables S23-S26, S28, S30 & S31. In the analysis, we offered the proposals of migratory and absent planets, calculated their orbits, and suggested the reasons that cause these systems to be abnormal.

Kepler-11 is an interesting system, showing many calculations in agreement with observations. We will show some calculations (Supplementary Table S28). The calculation shows $[(P_c:P_b)+(P_d:P_c)+(P_e:P_d)+(P_f:P_e)]\div4=100.68\%(P_f:P_e)$, suggesting connections among the five ratios. If taking the period ratio of Planets f and e as the leading ratio of equation (9) and Planet e as the reference planet, the period of Planet b, calculated by $(P_{b(cal)}=P_e\div[P_f:P_e]^{(sne-snb)})=10.3035(d)$, is 100% equal to its observed period (10.304 days). The result reflects that the shared rules well exist in the system. Further calculations reflect that Planet c migrated inwards equivalent to 13.4% of its observed period, Planet d migrated outwards to 3.4%, and Planet g migrated outwards to 19.1%. The calculation also reflects an undetected planet between Planets g and f, whose period should be 68.12 days. The calculations show the change in planetary configuration. Based on the planetary physical parameters, we first analyzed the migrations of Planets c and d. Observations show that the mass ratio of Planets d to c is approximately 2.5:1 according to $M_d=7.3^{+0.8}_{-1.5} M_\square$ and $M_c=2.9^{+2.9}_{-1.6} M_\square$, while the difference in their semimajor axes is only 0.048 AU according to $\alpha_d=0.115$ AU and $\alpha_c=0.107$ AU. In this case, the gravitational perturbation between them should be great. Perhaps the Kirkwood gap is the inducement. The mutual dragging forces caused their opposite migrations, which should be embodied in their exchange of kinetic energy. Since the mass of Planet d is much larger than that of Planet c, the migration of Planet c should be more obvious, which agrees with observations (Supplementary Table S28 [5] & [7]). If their orbital relations followed the shared rules before migration and they were not obviously disturbed by other factors at that time, relative to their previous orbits, the kinetic energy increased by Planet d (ΔE_d) should be equal or close to that decreased by Planet c (ΔE_c). In other words, $(\Delta E_d):(\Delta E_c)$ should be equal to or close to 1. Therefore, we calculated the changes in their kinetic energy by

$$\Delta E_n = E_{n(2)} - E_{n(1)} = 2m_n\pi^2[(a_{n(2)}:P_{n(2)})^2 - (a_{n(1)}:P_{n(1)})^2] .$$

The comparison of the calculations shows $(\Delta E_d):(\Delta E_c)=0.6738$ (Supplementary Table S28 Note 5). If considering the uncertainty of their masses and the possible tiny influence of reference planets, it is very possible that $(\Delta E_d):(\Delta E_c)\approx1$. The calculations objectively explain the orbital changes of Planets c and d. In addition, we also analyzed the migration of Planet g and the absence of Planet x. Because some reason may first disturb the stable motion of Planet g, it changed its orbital shape and approached inner Planet x at its perihelion. The mutual dragging forces then made one planet migrate obviously and the other fall to the star or fly out of the system. The disappearance of Planet x can objectively explain the migration of Planet g. Astronomers also researched the Kepler-11 system

(Hands and Alexander 2016). They performed numerous simulations and suggested that Kepler-11 may host an undetected giant planet, which may make its inner planet change its orbit. Prof. L. Esteves et al. (2020) researched the origins of nearly coplanar, non-resonant systems by dynamic simulations and concluded that the systems such as Kepler-11 are a natural outcome of the migration model. Different studies come to similar conclusions.

There are 25 five-planet systems discovered. The calculations with equation (3) show their types (Supplementary Table S17). Three are normal systems, including HD 158259, Kepler-296 and Kepler-444. Nine are near-normal systems, including HD 23472, K2-384, Kepler-102, Kepler-122, Kepler-154, Kepler-238, Kepler-292, Kepler-55, and Kepler-84. Thirteen are abnormal systems, including 55 Cnc A, GJ 667 C, HD 108236, HD 40307, K2-286, Kepler-150, Kepler-169, Kepler-186, Kepler-32, Kepler-33, Kepler-62, Kepler-82 and TOI-561. All 26 systems show the shared rules except Kepler-82 (Supplementary Table S17). We analyzed the systems of HD 158259, Kepler-444, HD 108236, 55 Cnc A, GJ 667 C and Kepler-296 (Supplementary Tables S32-S35, S48 & S51). To reduce reading, we presented the analyses of the remaining systems in Supplementary Tables S36-S47, S49, S50, S52-S56. In the tables, we show the planets that follow the shared rules, the planets that possibly migrate or are absent, the orbits of related planets and the analyses of the changes in planetary configurations.

5.2 Analyses of four-planet systems

By Oct. 30, 2022, NASA published the observation data of 65 four-planet systems (NASA Exoplanet Archive 2022). According to the general standard, 6 systems are normal systems, 34 are near-normal systems, and 25 are abnormal systems. If judged according to criterion (N-A), 39 systems show the shared rules. If judged according to criterion (N-B), 53 systems show the shared rules (Supplementary Table S19). Because the 53 systems contain planets that follow the shared rules, the real planetary configurations and their changes can be studied more objectively. In addition, since the four-planet systems discovered reach 65, they may show some similarity, based on which the study may be simpler. By comparison, we found that the planetary configurations of most systems present some specific orbital patterns, similar to those shown by the moon systems of Jupiter, Saturn or Uranus. Therefore, we classified the 65 systems into four groups (Table 7) and analyzed them.

No.	System	System brp	Deviation rates of period ratio (%)			Orbital pattern of planet configuration	Possible reasons of large deviation rates
			$\theta_{(P2:P1)}$	$\theta_{(P3:P2)}$	$\theta_{(P4:P3)}$		
1	GJ 876	2.0367	652	0	0.09	†Jupiter pattern	Planet d migrated greatly.
2	HD 20794	2.1902	0	2.79	-25.67	Jupiter pattern	Planet e migrates inwards.
3	HD 3167	3.5203	150.97	0	-2.83	Jupiter pattern	A planet is absent
4	HR 8799	1.5889	3.61	0	-0.24	Jupiter pattern	Based on semimajor axes
5↓	K2-285	1.4648	40.36	0	-3.615	Jupiter pattern	A planet is not detected.
65	WASP-47	5.2674	0	-58.78	1137	Disordered pattern	It was disturbed greatly.
Note. The entire list of the 65 systems are presented in Supplementary Table S60. †Jupiter pattern refers to the							

systems with the orbital pattern of Jupiter's moons. Because each of such systems shows that the absolute values of two adjacent deviation rates of period ratio parameters ($\theta_{(p)}$) are smaller than 10%, it can be considered a system with the shared rules according to criterion (N-B). Uranus pattern refers to...
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5.2.1 Extrasolar systems with the orbital pattern of Jupiter's moons

The first group contains 28 systems (Table 7 or Supplementary Table S60, No.1-28). In such a system, the absolute values of two adjacent deviation rates of period ratio parameters are smaller than 10% while one deviation rate is relatively larger. According to trait (1), the three planets composing the two smaller deviation rates follow the shared rules. Therefore, according to principle (2), the shared rules exist in the system, although a larger deviation rate may make it not a normal system. Based on the planets following the shared rules, the calculation with equation (9) can reveal migratory or absent planets and provide their possible orbits.

For example, the four planets of HD 20794 show periods of 18.315, 40.114, 90.309 and 147.02 days. The calculations with equation (3) show $\theta_{P(c:b)}=0$, $\theta_{P(d:c)}=2.79\%$ and $\theta_{P(e:d)}=-25.67\%$ (Supplementary Table S61 [1]). Sometimes, the orbital pattern presents other styles, such as the GJ 876 system, which shows deviation rates of 651%, -0.09% and 0; Kepler-282, which shows -17.26%, 1.73% and 0; and Kepler-107, which shows -5.07%, 0 and 14.14%. Although the four systems show large deviation rates at different positions, they can be considered systems with the same orbital pattern (Supplementary Table S61 [2], [3] and [4]). When we calculated the period ratios of the Galilean moons of Jupiter with equation (3), they show deviation rates of -0.37%, 0 and 15.78% (Supplementary Table S61 [5]). The four moons present a similar orbital pattern as the exoplanets of the four systems. For convenient analysis, such systems can be called systems with the orbital pattern of Jupiter's moons. Galilean moons have been widely researched, which may help understand the orbital pattern of such extrasolar systems. Such orbital patterns should be caused mainly by planetary migration. Still HD 20794 is taken as the example. Further analyses show that Planets b, c and d follow the shared rules according to trait (1). According to principle (2), the shared rules exist in the system. If calculated with equation (9), Planet e should migrate inwards according to principle (3). Because the mutual dragging forces for two neighboring planets with close masses ($M_e:M_d=0.994:1$) usually do not cause only one planet to migrate, the migration of Planet e is likely to be caused by a stray planet. Moreover, the eccentricity of Planet e is $0.29^{+0.14}_{-0.18}$, also implying that it was greatly disturbed. Astronomers also analyzed exoplanets with such orbital patterns, especially MMR exoplanets. They proposed that the orbital pattern should originate from different migrations (Cimerman et al. [2018](#)). Sometimes, a very large deviation rate may appear in such a system, which might be caused by planet absence. K2-285 shows deviation rates of $\theta_{P(c:b)}=40.36\%$, $\theta_{P(d:c)}=0$ and $\theta_{P(e:d)}=3.615\%$ (Supplementary Table S62). The large deviation rate is likely caused by a missing or undetected planet of $P=5.245$ days.

We calculated and compared the deviation rates of the period ratios of all planets of the 65 systems. If the absolute values of two adjacent deviation rates of a system are limited to be smaller than 5%, 21 systems present such orbital patterns, including GJ 876, HD 20794, HD 3167, HR 8799, K2-285, Kepler-106, Kepler-107, Kepler-1388, Kepler-172, Kepler-176, Kepler-208, Kepler-215, Kepler-221, Kepler-224, Kepler-24, Kepler-245, Kepler-265, Kepler-282, Kepler-338, Kepler-402 and Kepler-85. If the absolute values are limited to be smaller than 10%, 28 of the 65 systems present such orbital patterns, adding K2-133, Kepler-1542, Kepler-197, Kepler-256, Kepler-299, Kepler-306 and Kepler-79 (Supplementary Table S60). Although most of them are not

normal systems now, they used to be.

HR 8799 is considered to be a benchmark to test the formation and evolution models of planetary systems (Geiler et al. 2019). Newly updated observation data (NASA Exoplanet Archive 2022; Extrasolar Planets Encyclopaedia 2022) show $\alpha_e=16.4$ AU, $\alpha_d=27$ AU, $\alpha_c=42.9$ AU, and $\alpha_b=68$ AU. According to equation (4), the deviation rates are 3.61, 0 and -0.24% (Supplementary Table S63). It is a normal system. Because the system has the age of 0.06 ± 0.03 Gyr and clearly presents the planetary configuration with the shared rules, it hints that the shared rules appear along with the formation of planetary systems. In addition, a planet may not be discovered if considering the distance between the star and Planet e and the mass of Planet e. By referring to observed planets, its orbit should be 10.72 AU by $\alpha_x=68\div1.5870^{(5-1)}$. Observations show a debris disk in the area (Geiler et al 2019; Faramaz 2021). Since the disk displays a clear edge at its outer side, it is possibly carved by the forming planet. If the chain of Laplace resonance exists between planets, an undetected planet is possible. Wahhaj et al. (2021) tried to find more planets of HR 8799 by different methods. Read et al. (2018) also suggested an undetected planet with a semimajor axis of 138 AU. The orbit agrees with the calculation with equation (10), showing $\alpha_y=68\div1.5870^{(5-6)}=138$ AU. The study of HR 8799 may bring more surprises.

5.2.2 Extrasolar systems with the orbital pattern of Saturn's moons

The second group contains 19 systems (Supplementary Table S60, No.29-47). Such a system usually shows two deviation rates with absolute values larger than 10% and a 0 deviation rate as a benchmark reference. Although such a system does not directly show the shared rules, the calculations with equation (9) or (10) still reflect that the calculated orbits of three planets can satisfy criterion (N-A) or (N-B). According to principle (1), the three planets follow the shared rules. Therefore, according to principle (2), such a system still shows the shared rules although it is a near-normal or abnormal system. Based on the planets following the shared rules, migratory or absent planets can be reflected and their orbits can be calculated.

We will explain the pattern with a few representative examples. Kepler-286 shows $P_b=1.796302d$, $P_c=3.468095d$, $P_d=5.914323d$ and $P_e=29.22129d$. According to equation (3), their deviation rates are $\theta_{P(c:b)}=0$, $\theta_{P(d:c)}=-11.67\%$ and $\theta_{P(e:d)}=155.9\%$ (Supplementary Table S14 [2]). It is an abnormal system. According to trait (3), the equation $(P_e:P_d)=99.62\%(P_d:P_c)^3$ suggests two undetected planets. In this case, we first calculated Planet e with equation (9). $P_{e(cal)}=P_c\div[(P_d):(P_c)]^{(2-6)}$ shows 29.3323 days, 99.62% equal to its observed period. According to principle (1), Planets c, d and e follow the shared rules. According to principle (2), the shared rules still exist in the system. We then calculated the two undetected planets with equation (9). The calculations show 10.0860 days by $P_x=P_c\div[(P_d):(P_c)]^{(2-4)}$ and 17.2001 days by $P_y=P_c\div[(P_d):(P_c)]^{(2-5)}$. They should belong to two undetected planets. The calculation also shows a 2.03366-day period by $P_{b(cal)}=P_c\div(P_d:P_c)^{(2-1)}$, suggesting that Planet b migrated inwards. The calculations reflect the actual Kepler-286 planetary configuration. KOI-94 is also such a system. Although the system shows $\theta_{P(c:b)}=14.54\%$, $\theta_{P(d:c)}=-11.80\%$ and $\theta_{P(e:d)}=0$, it clearly displays the shared rules (discussed in 3.1). The four planets of Kepler-65 show $\theta_{P(c:b)}=0$, $\theta_{P(d:c)}=-49\%$ and $\theta_{P(e:d)}=1070\%$ (Supplementary Table S64). TOI-1264 shows -32.57%, 55.45% and 0 (Supplementary Table S65). However, the calculations still reflect that the shared rules exist in the systems.

Saturn has dozens of moons. But it only has five big moons. If the remote Iapetus is not considered temporarily, the calculation with equation (3) reveals that the four largest moons

(Saturn III, IV, V and VI) show deviation rates of -12.17%, 0 and 129.56% (Supplementary Table S66). They present a similar orbital pattern as the above extrasolar systems. For convenient analysis, such systems can be called systems with the orbital pattern of Saturn's moons. If calculated with equation (9) and judged according to criterion (N-B), 19 of the 65 systems display such orbital patterns. They are HD 164922, HD 20781, HD 215152, K2-32, KOI-94, Kepler-167, Kepler-286, Kepler-305, Kepler-65, Tau Cet, TOI-1264, DMPP-1, GJ 3293, HD 141399, K2-72, K2-266, Kepler-220, Kepler-304 and Kepler-37. Of them, the first 11 systems satisfy criterion (N-A).

Though the equations of the shared rules are derived from observation data, they are also supported by computer simulations. When we researched astronomer studies, we found that the orbits of some exoplanets calculated by the equations of the shared rules agree with those calculated by computer simulations. For example, HD 141399 now shows four planets. Equation (4) reflects an abnormal system (Supplementary Table S67). The calculation with equation (10) shows that Planets b, c and d follow the shared rules. Therefore, according to principle (2), the shared rules exist in the system. Since the calculation with equation (8) suggests an undetected planet between Planets d and c, we tried to calculate its possible orbit with equation (10). The calculation provides 1.2 AU by $\alpha_x = 0.689 \div 1.7417^{(2-3)}$. In 2019, Prof. R. Dvorak et al. (2019) deeply researched the HD 141399 system. They performed extensive numerical integrations with different initial conditions and completed numerous computer simulations. In their simulations, they found that most planets formed in the area $0.8 \text{ AU} < a < 1.6 \text{ AU}$, and the largest one is observed at 1.15 AU. They conclude that it is possible that a terrestrial planet forms in the habitable zone of the system. Different studies come to the same conclusion. In addition, the published orbit of HD 141399e ($a_e = 5.0 \pm 1.5$) is still uncertain (Vogt et al. 2014; NASA Exoplanet Archive 2022). According to the calculation with equation (10) and comparison of the RV signals, two planets most likely revolve with orbits of approximately 3.64 and 6.34 AU (Supplementary Table S67).

5.2.3 Extrasolar systems with the orbital pattern of Uranus's moons

The third group contains 6 systems (Supplementary Table S60, No.48-53). Such a system shows two nonadjacent deviation rates with absolute values close to 0 and a middle deviation rate with a large absolute value. The Kepler-223 system is a representative, showing $\theta_{(P_c:P_b)} = -0.4\%$, $\theta_{(P_d:P_c)} = 12.66\%$, and $\theta_{(P_e:P_d)} = 0$. Although the two deviation rates are not neighbors, the almost equal rates still reflect that planet operations show the shared rules in a different way. Such systems also contain Kepler-235, Kepler-251, Kepler-758, L 98-59 and Kepler-341 (Supplementary Table S68 [1]-[6]).

When we calculated the periods of the four major moons of Uranus (Ariel, Umbriel, Titania and Oberon) with equation (3), their deviation rates are 0, 27.76% and -5.95% (Supplementary Table S68 [9]). The four moons present a similar orbital pattern as the exoplanets of the six systems above. Therefore, such systems can be called systems with the orbital pattern of Uranus's moons. Scientists have explained such configurations of celestial bodies (Dermott 1968; Mills et al. 2016; Pichierri et al. 2019; Morrison et al. 2020). Although such orbital patterns might originate in system formation, they may also be caused by planet absence. For example, Kepler-341 shows $P_b = 5.195528\text{d}$, $P_c = 8.01041\text{d}$, $P_d = 27.66631\text{d}$ and $P_e = 42.47327\text{d}$. Their deviation rates are 0, 124% and -0.47% (Supplementary Table S68 [6] and Note 5). It also shows $(P_d:P_c) = 95.46\%(P_e:P_d)^3$, which suggests two undetected planets between Planets d and c according to trait (3). Therefore,

we recalculated the periods of the observed planets with equation (9). They are 5.195528, 7.9089, 27.901 and 42.47327 days by $P_{n(cal)}=P_e \div [sd]^{(sne-snn)}$. The calculated periods are equal to 100, 101.28, 99.16 and 100% of their observed periods. The values satisfy criterion (N-A), stating that the four planets follow the shared rules. The calculations also show 12.04 days by $P_x=42.47327 \div [1.5223]^{(6-3)}$ and 18.33 days by $P_x=42.47327 \div [1.5223]^{(6-3)}$. They should belong to two undetected planets according to principle (4). If considering the two planets, all deviation rates were smaller than 1%, and all planets showed a complete chain of near 2:3 MMRs (Supplementary Table S68 [7] & [8]). They might have disappeared because of mutual perturbations or the disturbance of a stray planet.

5.2.4 Extrasolar systems with disordered orbital patterns

The fourth group contains 12 systems (Supplementary Table S60, No.54-65). Such a system displays two deviation rates larger than 10%. Because the calculations cannot show three planets following the shared rules, it is a system without the shared rules. GJ 676 A is representative, showing deviation rates of 0, 203.8% and -29.37% (Supplementary Table S58). Kepler-49 is another typical system, showing deviation rates of 64.08%, -11.1% and 0 (Supplementary Table S69). Such systems also contain HD 160691, Kepler-132, K2-187, Kepler-26, Kepler-342, Kepler-411, Kepler-48, TOI-500, V1298 Tau and WASP-47. They are temporarily regarded as systems with disordered orbital patterns. Because the attempt to restore the planetary configuration of such a system is not convincing, we will not further discuss it.

5.3 Analysis of the systems containing three and fewer exoplanets

NASA pronounced that 191 systems with three confirmed planets were discovered by October, 2022 (NASA Exoplanet Archive 2022). The calculation with equations (1) and (3) shows that 69 of the 191 systems are normal systems according to the general standard (Supplementary Table S15). If raising the standard, 40 systems still show absolute values of deviation rates smaller than 5%. So many systems that show such small deviation rates further demonstrate that the shared rules exist objectively. Moreover, 15 systems show that the absolute values of deviation rates are smaller than 1.5%, reflecting that the orbital relations are almost the same as those of ideal systems. In addition, some systems also display continuously resonant orbits, even the chains of near MMRs. For example, the period ratios of the three planets of GJ 1061 display the continuous resonances of near 2:1, presenting a chain of near 1:2:4 MMRs. The systems of Kepler-271, Kepler-1130 and YZ Cet also show continuous resonances of period ratios, embodying the chains of near 2:3:4, 3:4:5 and 4:6:9 MMRs, respectively. The calculations reflect that the systems showing the shared rules also contain the systems showing the chains of near MMRs. The comparisons may help better understand the shared rules and MMRs (Supplementary Table S15 Note 3 & 4).

Because a three-planet system shows fewer planets, the migration or absence of one planet will make the shared rules disappear from a normal system and it is difficult to convincingly judge which planet changes its orbit. We showed explanations with the actual system of EPIC 206024342 and the simulated system of Kepler-289 (Supplementary Table S70). Therefore, we will not analyze more. However, if one or two planets are absent from a four-planet or five-planet system that used to be a normal system, the calculations sometimes can reflect the existence of the shared rules and planet absence. For example, K2-136 now shows three planets with $P_b=7.979252d$, $P_c=17.307137d$ and $P_d=25.575065d$ (Supplementary Table S71). The calculation with equation (3) shows $\theta_{P(c:b)}=18.98\%$ and $\theta_{P(d:c)}=-18.98\%$. It is not a normal system. The

calculation also shows $(P_c:P_b)=99.38\%(P_d:P_c)^2$, implying an undetected planet between Planets c and b according to trait (3). If calculated with equation (9), the calculated period of Planet b is 7.925989 days by $P_{b(cal)}=P_c\div(P_d:P_c)^{(3-1)}$, 100.62% equal to its observed period. According to principle (1), Planets b, c and d follow the shared rules. According to principle (2), the shared rules still exist in the system. Therefore, the period of 11.7122 days calculated by $P_x=P_c\div(P_d:P_c)^{(3-2)}$ should belong to the undetected planet according to principle (4). If considering the physical parameters of neighboring planets, the undetected planet is likely to vanish or orbit in a different orbital inclination because of the impact of a stray planet. The K2-80 system shows deviation rates of 38.51% and -38.51% (Supplementary Table S72). The calculation shows $(P_c:P_b)=98.54\%(P_d:P_c)^2$, which suggests the absence of two planets according to trait (3). The calculation with equation (9) shows that the calculated period of Planet b is equal to 101.48% of its observed period by $P_{b(cal)}=P_d\div(P_d:P_c)^{(snc-snb)}$. Therefore, the three observed planets should follow the shared rules according to principle (1). According to principle (2), the system shows the shared rules. If calculated by referring to the planets following the shared rules, the orbits of the two undetected planets should be 8.3511 days and 12.629 days. They more likely have disappeared under the influence of mutual dragging forces. Kepler-267 shows -33.73% and 33.73% deviation rates. Calculated with $P_{d(cal)}=P_c\div(P_c:P_b)^{(snc-snd)}$, the three observed planets follow the shared rules. Further calculations reflect an undetected planet of $P=14.1036(\text{day})$, which may orbit the star at a different inclination (Supplementary Table S73).

Currently, nearly 600 two-planet systems have been discovered (NASA Exoplanet Archive 2022). Because they cannot be directly calculated by the equations of the shared rules, it is impossible to judge whether the shared rules exist in a two-planet system. However, if a two-planet system contains undetected planets, the shared rules may appear in the system when undetected planets are discovered, such as Kepler-31d and Kepler-398d (Supplementary Table S1 Note 5). Now, GJ 887 shows two confirmed planets with $P_b=9.262(\text{day})$ and $P_c=21.789(\text{day})$. Prof. S. V. Jeffers et al. (2020) also detected an unconfirmed signal with a period of ~50 days. Based on the confirmed planets, equation (9) provides its period, showing 51.26 days (Supplementary Table S74). If it is confirmed someday, the confirmation will once more support the shared rules.

Since the shared rules appear in extrasolar systems with different numbers of planets, they should be considered to exist naturally instead of accidentally. Since one-third of the systems containing five or more planets show that the degrees to which the calculated periods of at least three planets in every system are equal to their observed periods exceed 99%, the equations of the shared rules should be considered to have reference significance in system analysis. The systems contain KOI-351, TRAPPIST-1, K2-138, Kepler-11, Kepler-20, TOI-178, GJ 667 C, HD 40307, HD 108236, HD 158259, Kepler-444, Kepler-55 and Kepler-80. Since the observed orbits of many exoplanets that are discovered later agree well with the orbits that were previously calculated according to the equations of the shared rules, such as Kepler-20g and Kepler-31d, the shared rules should be considered to have practical meaning in exoplanet observation. Although varying disturbances and observation limitations affect the clear display of the shared rules, the systems of Kepler-229, Kepler-271, Kepler-207, Kepler-289, Kepler-249, Kepler-31, Kepler-398, Kepler-332, HD 184010 and YZ Cet still show orbital relationships between planets that are almost 99% the same as ideal systems. Thus, the shared rules are real, widespread and significant in multi-planet systems.

6 Possible appearance time of the shared rules

The analyses of all multi-planet systems further confirmed the existence of the shared rules. Meanwhile, the comparisons of observations also hint that the shared rules prefer to originate in the formation rather than subsequent evolution of multi-planet systems. Here, we will show some intimations or observational supports.

The first intimation is the planetary configuration of the young HR 8799 system, which clearly shows that all planets follow shared rules well (Supplementary Table S63). Since the shared rules exist in the young system, the initial planetary configuration of HR 8799 objectively supports the above judgment. The second intimation is the comparison of the matter distributions of protoplanetary disks and the planet distributions of multi-planet systems. Scattered light imaging studies have detected tens of spiral arm systems in circumstellar disks that show matter distributions with traces of logarithmic spirals (Benisty et al. 2015; Shuai et al. 2022). Some other protoplanetary disks also show the matter distributions in the pattern of logarithmic spirals. Meanwhile, the orbital relationships of adjacent planets of normal systems also present the feature of logarithmic spirals (Table 1, Supplementary Fig. S1 and S2). However, the orbital relationships become indistinct or disappear in some systems because of varying disturbances. Observations reflect that the shared rules are likely to appear in system formation and slowly become unclear or disappear in later evolution. We also noticed the third intimation. When we compared the physical parameters of different systems that show the shared rules, we found that some system pairs display similar physical parameters. The similarity of related parameters also suggests that the shared rules appear along with system formation. Here we list four pairs of such systems (Table 8) and show detailed analyses.

Table 8. Four pairs of systems showing similar stellar and planetary physical parameters

System	Stellar radius (R_{Sun})	Tempe rature (K)	Stellar age (Gyr)	1 st planet		2 nd planet		3 rd planet		4 th planet		Bench mark ratio
				P (day)	R (R_{Jup})	P (day)	R (R_{Jup})	P (day)	R (R_{Jup})	P (day)	R (R_{Jup})	
Kepler-221	0.821	5243	$4.57^{+3.67}_{-2.7}$	2.7959	0.153	5.691	0.261	10.04	0.244	18.37	0.235	1.8294
Kepler-305	0.827	4974	$4.37^{+3.26}_{-2.4}$	3.2054	0.160	5.487	0.321	8.291	0.294	16.74	0.242	1.7118
Kepler-172	1.085	5526	$6.46^{+3.14}_{-3.7}$	2.9403	0.210	6.389	0.255	14.63	0.201	35.12	0.246	2.2894
Kepler-299	1.032	5617	$6.03^{+3.05}_{-3.6}$	2.9271	0.118	6.886	0.236	15.06	0.166	38.29	0.167	2.3524
Kepler-244	0.803	5554	$4.79^{+6.15}_{-3.1}$	4.312	0.246	9.767	0.183	20.05	0.206	--	--	2.1590
TOI-125	0.848	5320	$6.80^{+4.60}_{-4.2}$	4.654	0.243	9.151	0.246	19.98	0.261	--	--	2.0749
Kepler-60	1.257	5905	$5.13^{+1.0}_{-1.69}$	7.133	0.153	8.919	0.170	11.90	0.178	--	--	1.2922
Kepler-431	1.092	6004	$4.27^{+0.9}_{-0.96}$	6.803	0.068	8.703	0.060	11.92	0.10	--	--	1.3246

Note. For every pair of systems, the two compared values of related physical parameters are very close, which implies that the system pair form and evolve in similar conditions.. As all systems are in the natural universe, a little difference between two compared values should be acceptable (Supplementary Tables S75-S77), which does not

If two planetary systems show the same stellar physical parameters, including mass (radius), temperature and age, and the same planetary physical parameters, including number, mass (radius), period (semimajor axis) and benchmark ratio, the most likely possibility is that the two systems formed in two protoplanetary disks that had the identical physical conditions and the same physical environments. Meanwhile, the two systems have not been influenced by external disturbances and perturbations between planets since they formed. If both systems simultaneously show some regular orbital relations, such as the shared rules, it is more possible that the relations appear in the formation of the systems instead of subsequent evolution. Therefore, if the shared rules exist in a pair of systems with the same physical parameters, it can be judged that the shared rules appear along with the formation of the systems. However, because all systems form and evolve in the natural universe, varying disturbances may more or less influence system formation and evolution. Therefore, the compared values of the related parameters of a system pair are more likely to be close instead of identical, similar to the system pairs in Table 8. Even so, such a system pair can still state that the shared rules appear along with system formation. If one system pair may be considered accidental, several system pairs can well support the above judgment. The discovery of more systems will show more pairs of systems with closer parameters.

Because of varying disturbances in the natural universe, all systems evolve, embodying that some planets change their orbits or disappear from some systems. For one system, the results of disturbances may be uncertain. For a pair of systems, some results of disturbances should be certain. In general, it is impossible for disturbances to cause a pair of systems with different physical parameters to evolve into systems with identical physical parameters. Conversely, it is possible. Besides, if a system pair shows some regularity of planetary distribution, disturbances or evolution are more likely to break the regularity that appeared at system formation rather than make irregular distribution become regular distribution. Therefore, it is almost impossible for the shared rules reflecting orbital regularity to gradually appear in system evolution. In addition, the shared rules are different from MMRs. If taking all planets of a system as a whole, the evolution of the system reflects that all planets tend to migrate toward the star. The convergent migrations may result in progressively smaller orbital ratios. At some times, some planets may become planet pairs showing the features of MMRs. Astronomers have performed numerous studies and proposed that most exoplanets prefer convergent migrations (Fabrycky et al. [2014](#); Hands & Alexander [2016](#); Izidoro et al. [2017](#)); Esteves et al. [2020](#)). Therefore, MMRs may appear in a system during both system formation and evolution. Some disturbance or perturbation may even cause some planets to develop into the MMR state. However, because convergent migrations may cause orbital resonances, the latter may influence the stable motion of some planets. The influence may also make the shared rules become indistinct or disappear in some systems. Assuming the shared rules appeared in all systems during their formation, evolution inevitably makes the number of the systems showing the shared rules smaller over time. The assumption is consistent with the observations. If judged according to criterion (N-A), about 70% of the observed 101 systems containing four or more planets now show the shared rules (Table 6). If only taking the normal systems into account, the proportion is much lower (Supplementary Tables S16-S19).

The results of computer simulations may also serve as an intimation. Astronomers have conducted many computer simulations and calculations of system formation (Cresswell & Nelson

2006; Pierens et al. 2013; Mills et al. 2016; Izidoro et al. 2017). Simulations show that at the early time of system formation, protoplanets or planets tend to prefer convergent or inward migration, which causes them to form a series of MMRs or resonant chains. Prof. M. MacDonald et al. (2022) carefully researched the 2:3 resonance chain of K2-138. They ran numerous simulations by modeling the system. After analyzing the resulting simulations, they found that nearly all of the simulations show a long chain of 2:3 resonances. Because the planets following shared rules contain the planets with the feature of MMR chains, the computer simulations also support the appearance of the shared rules at the early time of system formation. Therefore, we propose that the shared rules are more likely to originate from the formation of a system rather than from evolution. More observations are required.

Discussion

Based on exoplanetary observations, we find the shared rules between planetary orbits and corresponding equations, principles and traits, which provide a new approach to studying the planetary configurations of multi-planet systems and the evolution of most systems. Because the shared rules are the natural result that first need to be displayed by a great deal of computation of observation data, instead of that reflected by theoretical hypotheses or computer simulations, we mainly focus on the computations and comparisons to confirm the result. We also provide main reasons that cause the shared rules not to be clearly displayed in some systems. Meanwhile, we proposed the appearance time of the shared rules. However, we have not explained much about why and how the formation of multi-planet systems produces the shared rules between planetary orbits. Although explanations can help understand the process of the emergence of the result, they do not affect the display of the result. Therefore, we will temporarily have no more discussion. More interpretations of why and how the shared rules appear may be put forward for discussion. In addition, we only discussed the influence of orbital changes on the shared rules. We have not discussed the influence of stellar mass and age on the shared rules and the influence of planetary mass on the shared rules. Although they do not play a major role in affecting the emergence of the shared rules (Table 1), they may play an important role in affecting the size of the benchmark ratio of a system. The study of the influences may promote the understanding of system formation and evolution. Therefore, deeper and wider studies are required.

Since a system that displays the shared rules contains planets that are in the MMRs chain, the studies on MMRs and orbital resonances may help understand why and how the shared rules appear, such as research on the origin of orbital commensurabilities (Dermott 1968), the formation of resonant planetary systems (Hardy & Gong 2017), the origins of nearly coplanar but non-resonant systems (Esteves et al. 2020), the role of dissipative evolution for near-resonant extrasolar systems (Pichierri et al. 2019), the formation of the protoplanetary disks of spiral arm systems (Shuai et al. 2022), and the influence of planets in the chains of MMRs on system formation (Cresswell. & Nelson, 2006; Morrison et al. 2020; MacDonald et al. 2022).

The shared rules may not be exclusive to the planetary systems of our galaxy. The Kepler-444 system has the great possibility of an extragalactic origin (Campante et al. 2015) because its age is 11.2 ± 1.0 Gyr. However, it also displays the shared rules (Supplementary Table S51). In the moon systems of Jupiter, Saturn and Uranus, the traces of the shared rules also seem to be displayed (Supplementary Tables S61 [5], S66 and S68 [9]). Therefore, the shared rules may have a more general applicability in the study of the systems of more ranges and may help further study the

solar system and the moon systems of solar planets.

Conclusion

With the method of orbital ratio parameter analysis, we further quantified the orbital ratios of all adjacent planets of 292 systems containing three or more confirmed planets. According to the analyses of orbital ratio parameters and statistics of deviation rates, we summarized the shared rules between planetary orbits. From the shared rules, we derived the relation equations of planetary orbits, ascertained the principles of judging planet and system properties, and showed the traits of the properties of different planets. To facilitate study, we classified the 292 extrasolar systems into different types and properties. Of the 191 three-planet systems, 69 belong to normal systems according to the general standard and show the shared rules if referring to criterion (N-B). Moreover, 15 systems show planetary orbital relations nearly identical to those shown by ideal systems. Of the 101 systems containing four or more exoplanets, according to the general standard, 9 are normal systems, 46 are near-normal systems and 46 are abnormal systems. If referring to criterion (N-B), the shared rules exist in 88 systems. If referring to criterion (N-A), a higher criterion, 71 systems show the shared rules. We further analyzed the 88 systems and provided the suggestions of migratory and absent exoplanets. Meanwhile, we calculated the previous orbits of migratory exoplanets and inferred the possible orbits of absent exoplanets. The shared rules can help study most of the systems that have been discovered and will be helpful for the analysis of the systems that will be discovered. Because different factors cause the appearance of large deviation rates, some systems fail to display the shared rules clearly. We also summarized five main reasons, based on which most of the large deviation rates can be explained objectively.

The shared rules are the outcome of the influence of the central gravitation of the same property on the motion of all planets of different systems. As a characteristic of group behavior, the shared rules not only reflect the operation regularity of multi-planet systems, but also show the changes of planetary orbits in some systems. By using the equations and principles derived from the shared rules, one can judge planets that follow the shared rules. By referring to the planets that follow the shared rules, one can also judge planets that have migrated, disappeared or are not detected, and calculate the corresponding orbits of these planets. The discovery of the shared rules provides a new approach to analysing the completeness of multi-planet systems and the changes in planetary configurations. It also provides a relatively more objective approach to studying the evolution of planetary systems, as the shared rules originate from the computations of planetary systems themselves. Along with the observations of more extrasolar systems, the contents of the shared rules between planetary orbits will be enriched.

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Fig. S1 and S2

Table S1-S77

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