Supplementary Materials for

Shared rules between planetary orbits existing in multi-planet systems and their functions

Yong Ma*, Xiumei Wen

*Corresponding author: Email: yma3916@sina.com

This PDF file includes:

Fig. S1 and S2

Tables S1-S77

Supplementary Figures:

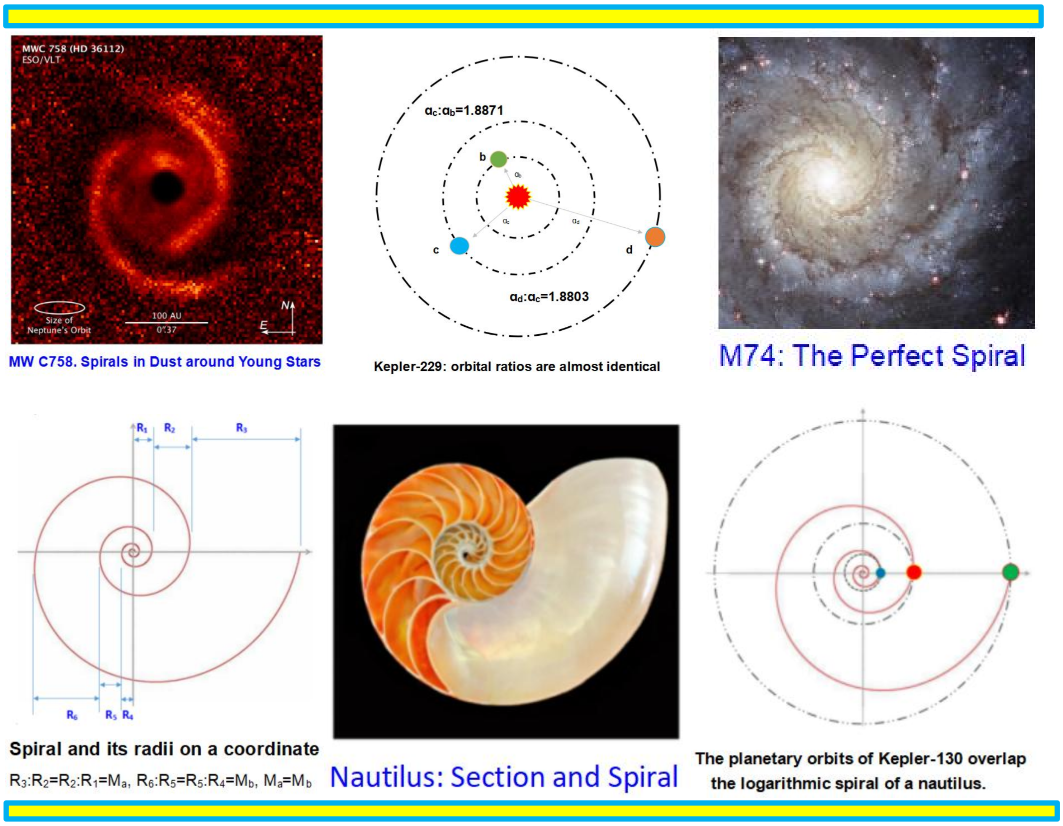

Fig. S1: The feature of logarithmic spiral existing in nature. The upper left image shows a protoplanetary disk around the young star MWC 758 (Benisty et al. 2015). The disk has two spiral arms that extend over 10 billion miles from the star. The disk also shows the matter distribution displaying the feature of logarithmic. The spirals in dust around MW C758 may betray the presence of massive planets. The upper middle sketch shows the orbital relations between the planets of Kepler-229 (NASA Exoplanet Archive 2022). It reflects that the orbital ratios of adjacent planets are almost identical and their orbital ratio parameters are almost equal to 1 (Table S1). The orbital changes of adjacent planets present the mode of logarithmic spiral to a great extent. The upper right image is the spiral galaxy of M74. The grand design of M74 shows graceful spiral arms (Newman et al. 2013; Ban & Su 2019). The lower left sketch shows a perfect logarithmic spiral that comes from the nautilus. When the spiral is put on a coordinate, the ratios of adjacent radii on each axis are almost equal, showing $R_3;R_2 \approx R_2;R_1$ and $R_6;R_5 \approx R_5;R_4$, and meanwhile all ratios are identical. The lower middle image is a nautilus, who shows the beautiful spiral (Ban & Su 2019). The lower right sketch shows the orbits of the three planets of Kepler-130 (NASA Exoplanet Archive 2022). It also shows the semimajor-axis ratios of adjacent planets almost equal (Table S1). When the planetary orbits of Kepler-130 and the spiral of the Nautilus are put in the same coordinate, the orbits overlap the spiral. Comparison shows that both the radii of a logarithmic spiral and the orbits of adjacent planets vary with shared divisors (or multiples). Though the objects are different, they show the same feature.

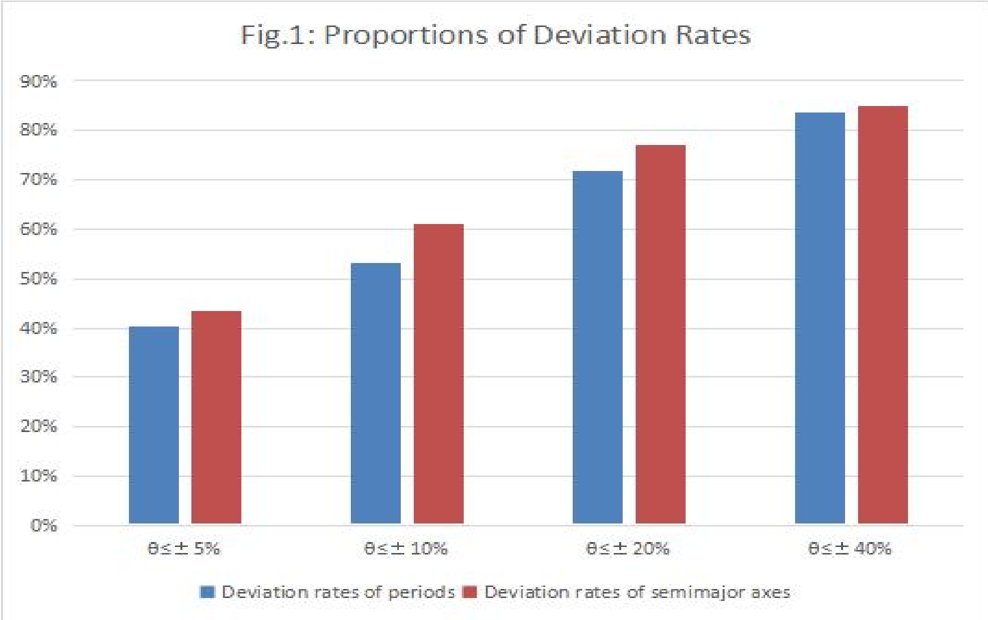

Fig. S2: Contrast of the proportions of the deviation rates of period and semimajor axis ratio parameters. It shows the proportions of different deviation rates of the exoplanets in the systems containing five and more exoplanets. Blue bars show the proportions of the deviation rates of period ratios in different ranges, red bars show that of semimajor axis ratios. According to statistics (Table S18), 53.80% of the 158 period ratios consisting of 194 exoplanets show the deviation rates with the absolute values smaller than 10% while 16.46 % show that larger than 40%. After considering all exoplanets that show semimajor axes (Table S20), 60.66% of the 122 semimajor axis ratios consisting of 154 exoplanets show the deviation rates with the absolute values smaller than 10% while 16.39 % show that bigger than 40%. According to the calculations with equation (11) and (12), the proportions of the deviation rates of semimajor axis ratios should be a little larger than that of period ratios. The results agree with the statistics.

Tables of Supplementary Data

Note: In all supplementary tables, if not specifically annotated, all planetary and stellar physical parameters come from the Confirmed Planets of NASA Exoplanet Archive (2022) by checking: https://exoplanetarchive.ipac.caltech.edu/.

Table S1. Calculations of orbital ratio parameters and deviation rates of adjacent planets of three-planet systems.

Period comparison of adjacent planets							Semimajor axis comparison of adjacent planets						
Kepler-271 (R_{star}=0.87R_{Sun}, Age=4.52±0.05Gyr)							Kepler-271 System (M_{star}=0.9M_{Sun},)						
Planet	**P (day)**	**Ratio of P**	**ω_P***	**θ_P* (%)**	**Radius ($R_{Jup.}$)**	**Ratio of R**	**Planet**	**α (AU)**	**Ratio of α**	**ω_α***	**θ_α* (%)**	**Mass ($M_{Earth.}$)**	**Ratio of M**
d	5.249725				0.059		d	0.558				0.219	
c	7.410863	1.4117	1.0013	0.13	0.0848	1.437	c	0.702	1.2581	1.0013	0.13	0.809	3.694
b	10.43548	1.4081	0.9987	-0.13	0.110	1.297	b	0.881	1.2550	0.9988	-0.12	2.04	2.522
average ratio $_{(P)}$=1.4099							average ratio$_{(\alpha)}$=1.2565				†ar_P=100.10%$(ar_\alpha)^{3/2}$		
Kepler-207 (R_{star}=1.588 R_{Sun}, Age=4.17±1Gyr)							Kepler-207 System (M_{star}=1.04 M_{Sun},)						
Planet	**P (day)**	**Ratio of P**	**ω_P***	**θ_P* (%)**	**Radius ($R_{Jup.}$)**	**Ratio of R**	**Planet**	**α (AU)**	**Ratio of α**	**ω_α***	**θ_α* (%)**	**Radius (R_{Earth})**	**Ratio of R**
b	1.611865				0.14		b	0.029				1.57	
c	3.071571	1.9056	0.9987	-0.13	0.134	0.957	c	0.044	1.5172	0.9908	-0.92	1.50	0.955
d	5.868075	1.9104	1.0013	0.13	0.295	2.202	d	0.068	1.5455	1.0092	0.92	3.31	2.207
average ratio $_{(P)}$=1.9080							average ratio$_{(\alpha)}$=1.5314				†ar_P=100.68%$(ar_\alpha)^{3/2}$		
Kepler-289 (R_{star}=1.0, Age=3.8±1.8Gyr)							Kepler-289 ($R_{Sun}M_{star}$=1.08M⊙)						
Planet	**P (day)**	**Ratio (P_n:P_{n-1})**	**ω_P***	**θ_P* (%)**	**Mass (M_{Jup})**	**Ratio (M_{-1})**	**Planet**	**α (AU)**	**Ratio of α**	**ω_α***	**θ_α* (%)**	**Mass ($M_{Jup.}$)**	**Ratio of M**
b	34.545				0.023		b	0.21				0.023	
d	66.064	1.9124	1.0019	0.19	0.013	0.565	d	0.33	1.5714	1.0083	0.83	0.013	0.565
c	125.852	1.9050	0.9981	-0.19	0.42	32.31	c	0.51	1.5455	0.9917	-0.83	0.42	32.31
average ratio$_{(P)}$=1.9087							average ratio$_{(\alpha)}$=1.55845				†ar_P=98.11%$(ar_\alpha)^{3/2}$		
Kepler-249 (R_{star}=0.482 R_{Sun}, Age=$3.09^{5.27}_{-1.67}$ Gyr)							Kepler-249 (M_{star}=0.510M_{Sun},)						
Planet	**P (day)**	**Ratio of P**	**ω_P***	**θ_P* (%)**	**Radius ($R_{Jup.}$)**	**Ratio of R**	**Planet**	**α (AU)**	**Ratio of α**	**ω_α***	**θ_α* (%)**	**Mass ($M_{Earth.}$)**	**Ratio of M**
b	3.30654				0.097		b	0.035				1.32	
c	7.11370	2.1514	0.9979	-0.21	0.135	1.392	c	0.058	1.6571	0.9954	-0.46	2.89	2.189
d	15.3685	2.1604	1.0021	0.21	0.140	1.037	d	0.097	1.6724	1.0046	0.46	3.09	1.069
average ratio$_{(P)}$=2.1559							average ratio$_{(\alpha)}$=1.6648				†ar_P=100.36%$(ar_\alpha)^{3/2}$		
Kepler-31 (R_{star}=1.22 R_{Sun}, Age=$4.17^{+1.38}_{-1.73}$ Gyr)							Kepler-31 System (M_{star}=1.21 M_{Sun})						
Planet	**P (day)**	**Ratio of P**	**ω_P***	**θ_P* (%)**	**Radius ($R_{Jup.}$)**	**Ratio of R**	**Planet**	**α (AU)**	**Ratio of α**	**ω_α***	**θ_α* (%)**	**Radius (R_{Earth})**	**Discovery**
b	20.8613				0.491		b	0.16				5.5	2011
c	42.6318	2.0436	0.9970	-0.3	0.473	0.963	c	0.26	1.6250	1.0273	2.73	5.3	2011
d	87.6451	2.0559	1.0030	0.3	0.35	0.74	d	0.4	1.5385	0.9726	-2.74	3.9	2014

average $\text{ratio}_{(P)}$=2.0498	average $\text{ratio}_{(\alpha)}$=1.5818 †ar_P=103.04%$(ar_\alpha)^{3/2}$

Kepler-398 (R_{star}=0.613 R_{Sun}, Age=$2.40^{2.52}_{-1.03}$ Gyr)							Kepler-398 (M_{star}=0.649 M_{Sun},)						
Planet	**P (day)**	**Ratio of**	**ω_P***	**θ_P* (%)**	**Radius ($R_{Jup.}$)**	**Discovery**	**Planet**	**α (AU)**	**Ratio of α**	**ω_α***	**θ_α* (%)**	**Mass ($M_{Earth.}$)**	**Ratio of M**
b	4.08142				0.083	2014	b	0.044				0.749	
d	6.83437	1.6745	1.0083	0.83	0.09	2016	d	0.062	1.4091	1.0021	0.21	1.01	1.3485
c	11.4194	1.6468	0.9917	-0.83	0.079	2014	c	0.087	1.4032	0.9979	-0.21	0.615	0.609
average ratio $_{(P)}$=1.6606							average $\text{ratio}_{(\alpha)}$=1.4062 †ar_P=99.59%$(ar_\alpha)^{3/2}$						

Kepler-332 (R_{star}=0.720 R_{Sun}, Age=$2.82^{+2.14}_{-1.22}$ Gyr)							Kepler-332 (M_{star}=0.80 M_{Sun})						
Planet	**P (day)**	**Ratio of P**	**ω_P***	**θ_P* (%)**	**Radius ($R_{Jup.}$)**	**Ratio of R**	**Planet**	**α (AU)**	**Ratio of α**	**ω_α***	**θ_α* (%)**	**Mass ($M_{Earth.}$)**	**Ratio of M**
b	7.626324				0.104		b	0.070				1.71	
c	15.99562	2.0974	0.9902	-0.98	0.097	0.933	c	0.114	1.6286	0.9911	-0.89	1.32	0.772
d	34.21154	2.1388	100.98	0.98	0.105	1.083	d	0.189	1.6579	1.0089	0.89	1.76	1.333
average ratio $_{(P)}$=2.1181							average $\text{ratio}_{(\alpha)}$=1.6432 †ar_P=100.55%$(ar_\alpha)^{3/2}$						

Kepler-130 (R_{star}=1.127 R_{Sun}, Age=5.89±1Gyr)							Kepler-130 System (M_{star}=0.965 M_{Sun})						
Planet	**P (day)**	**Ratio of P**	**ω_P***	**θ_P* (%)**	**Radius ($R_{Jup.}$)**	**Ratio of R**	**Planet**	**α (AU)**	**Ratio of α**	**ω_α***	**θ_α* (%)**	**Radius (R_{Earth})**	**Ratio of R**
b	8.457458				0.091		b	0.079				1.02	
c	27.50868	3.2526	1.0110	1.10	0.259	2.846	c	0.178	2.2532	1.0309	3.09	2.90	2.843
d	87.51791	3.1815	0.9890	-1.10	0.146	0.564	d	0.377	2.1180	0.9691	-3.09	1.64	0.566
average $\text{ratio}_{(P)}$=3.2170							average $\text{ratio}_{(\alpha)}$=2.1856 †ar_P=99.56%$(ar_\alpha)^{3/2}$						

HD 184010 (R_{star}=4.86R_{Sun}, Age=$2.76^{+2.24}_{-0.95}$ Gyr)							HD 184010 (M_{star}=1.35 M_{Sun})						
Planet	**P (day)**	**Ratio of P**	**ω_P***	**θ_P* (%)**	**Radius ($R_{Jup.}$)**	**Ratio of R**	**Planet**	**α (AU)**	**Ratio of α**	**ω_α***	**θ_α* (%)**	**Mass ($M_{Earth.}$)**	**Ratio of M**
b	286.6				1.08		b	0.940				99	
c	484.3	1.6898	0.9891	-1.09	1.06	0.982	c	1.334	1.4192	0.9930	-0.70	95	0.60
d	836.4	1.7270	1.0109	1.09	1.28	1.208	d	1.920	1.4393	1.0071	0.71	143	1.505
average $\text{ratio}_{(P)}$=1.7084							average $\text{ratio}_{(\alpha)}$=1.4292 †ar_P=99.99%$(ar_\alpha)^{3/2}$						

Kepler-1468 (R_{star}=1.05 R_{Sun}, Age=$3.55^{+2.63}_{-1.86}$ Gyr)							Kepler-1468 (M_{star}=1.04 M_{Sun},)					
Planet	**P (day)**	**Ratio of P**	**ω_P***	**θ_P* (%)**	**Radius ($R_{Jup.}$)**	**Ratio of R**	**α (AU)**	**Ratio of α**	**ω_α***	**θ_α* (%)**	**Mass ($M_{Earth.}$)**	**Ratio of M**
c	3.54553				0.103		0.0466				1.6	
b	8.23985	2.3240	0.9886	-1.14	0.154	1.495	0.0817	1.7532	0.9918	-0.82	3.64	2.275
d	19.5906	2.3775	1.0114	1.14	0.297	1.9286	0.1456	1.7821	1.0082	0.82	11.1	3.050
average $\text{ratio}_{(P)}$=2.3508							average $\text{ratio}_{(\alpha)}$=1.7677 †ar_P=100.02%$(ar_\alpha)^{3/2}$					

K2-219 (R_{star}=1.189232 R_{Sun} and M_{star}=1.021545 M_{Sun}, no age)							K2-219					
Planet	**P (day)**	**Ratio of P**	**ω_P***	**θ_P* (%)**	**Radius ($R_{Jup.}$)**	**Ratio of R**	**α (AU)**	**Ratio of α**	**ω_α***	**θ_α* (%)**	**Mass ($M_{Earth.}$)**	**Ratio of M**
b	3.90129				0.12023		--				2.38	
c	6.66767	1.7091	1.0115	1.15	0.12833	1.067	--				2.66	1.118
d	11.1373	1.6703	0.9885	-1.15	0.22985	1.791	--				7.16	2.692
average ratio $_{(P)}$=1.6897												

Kepler-1130 (R_{star}=0.842 R_{Sun}, Age=$1.580^{+0.776}_{-0.369}$ Gyr)							Kepler-1130 (M_{star}=0.944 M_{Sun})					
Planet	**P (day)**	**Ratio of P**	**ω_P***	**θ_P* (%)**	**Radius ($R_{Jup.}$)**	**Ratio of R**	**α (AU)**	**Ratio of α**	**ω_α***	**θ_α* (%)**	**Mass ($M_{Earth.}$)**	**Ratio of M**
c	3.26664				0.0707		0.0423				0.422	
d	4.27225	1.3078	1.0122	1.22	0.0575	0.813	0.0506	1.1962	1.0085	0.85	0.202	0.479
b	5.45298	1.2764	0.9879	-1.21	0.071	1.235	0.0595	1.1759	0.9914	-0.86	0.437	2.163
	average ratio$_{(P)}$=1.2921							average ratio$_{(\alpha)}$=1.1861			†ar_P=100.03%$(ar_\alpha)^{3/2}$	

Kepler-445 (R_{star}=0.42 R_{Sun}, Age=$6.61^{+5.50}_{-4.43}$ Gyr)							Kepler-445 (M_{star}=0.43 M_{Sun})					
Planet	**P (day)**	**Ratio of P**	**ω_P***	**θ_P* (%)**	**Radius ($R_{Jup.}$)**	**Ratio of R**	**α (AU)**	**Ratio of α**	**ω_α***	**θ_α* (%)**	**Mass ($M_{Earth.}$)**	**Ratio of M**
b	2.98415				0.14		0.0217				3.12	
c	4.87123	1.6324	0.9875	-1.25	0.22	1.571	0.0301	1.3871	0.9750	-2.51	6.85	2.196
d	8.15275	1.6737	1.0125	1.25	0.11	0.50	0.0439	1.4585	1.0251	2.51	2.1	0.307
	average ratio$_{(P)}$=1.6530				1.6971			average ratio$_{(\alpha)}$=1.4228			†ar_P=97.40%$(ar_\alpha)^{3/2}$	

YZ Cet System (R_{star}=0.157 R_{Sun}, , Age=5.0±1.0 Gyr)							YZ Cet System (M_{star}=0.130 M_{Sun})					
Planet	**P (day)**	**Ratio of P**	**ω_P***	**θ_P* (%)**	**Radius ($R_{Jup.}$)**	**Ratio of R**	**α (AU)**	**Ratio (α_n:α_{n-1})**	**ω_α***	**θ_α* (%)**	**Mass (M_{Jup})**	**Ratio (M)**
b	1.96876				0.0814		0.01557				0.0022	
c	3.06008	1.5543	1.0106	1.06	0.0933	1.146	0.02090	1.3423	1.0074	0.74	0.0036	1.636
d	4.65627	1.5216	0.9894	-1.06	0.0921	0.987	0.02764	1.3225	0.9926	-0.74	0.0034	0.944
	average ratio$_{(P)}$=1.5380							average ratio$_{(\alpha)}$=1.3324			†ar_P=100%$(ar_\alpha)^{3/2}$	

Note 1. Symbol ω_P* represents the period ratio parameter and θ_P* is the deviation rate of ω_P*. Symbol ω_α* is semimajor-axis ratio parameter and θ_α* is the deviation rate of ω_α*. They are calculated with equations (1-4) and by the average ratio as benchmark ratio.

Note 2. † is the equation calculated according to equation(11), in which ar_P is the average ratio of all period ratios and ar_α is the average ratio of all semimajor axis ratios. Usually The ar_P should be 100% equal to ar_α. If not, the orbital observation data may be doubtful. For the Kepler-31 system, it shows ar_P=103.04%$(ar_\alpha)^{3/2}$. This may be related to the lower observation accuracy of the semi-major axes of the three planets,

Note 3. Each system reflects the features that the orbital ratios of adjacent planets are close, the orbital ratio parameters are close to 1, and their deviation rates are close to 0. It also shows that the orbital features are not affected by the mass (or radius) ratios of corresponding planets. Meanwhile, though the stellar physical parameters of the 15 systems are different, they all show the same feature. So, the stellar physical parameters do not affect the display of the features, too.Note 4 According to the general standard, they belong to normal systems. Since all the 15 systems show the absolute values of deviation rates smaller than 1.3%, they can be regarded to be the systems near to ideal systems.

Note 5. Kepler-31d was discovered three years later than other planets (NASA Exoplanet Archive 2022). The discovery made the system become an almost ideal system. Kepler-398d was discovered two years later than other planets (NASA Exoplanet Archive 2022). The discovery also made the system become an almost ideal system. Because some exoplanets can not be detected in time, they affect the display of the shared rules.

Table S2. Calculations of orbital ratio parameters and deviation rates of adjacent planets of four-planet systems.

Compare the periods of adjacent planets					Compare the semimajor axes of adjacent planets					
Kepler-85 (R_{star}=0.893 R_{Sun}, age=$4.37^{3.31}_{-2.43}$Gyr)										
Planet	**P (day)**	**Ratio of P_n:P_{n-1}**	**ω_P**	**θ_P (%)**	**α (AU)**	**Ratio of α_n:α_{n-1}**	**ω_α**	**θ_α (%)**	**Radius ($R_{jup.}$)**	**Ratio of R_n:R_{n-1}**
b	8.305992				0.0766				0.176	
c	12.51217	1.5064	1.0522	5.22	0.1007	1.3146	1.0183	1.83	0.194	1.1023
d	17.91323	1.4317	1	0	0.130	1.2910	1	0	0.107	0.5516
e	25.21675	1.4077	0.9832	-1.68	0.163	1.2539	0.9713	-2.87	0.113	1.0561
		benchmark ratio$_{(P)}$=1.4317			benchmark ratio$_{(\alpha)}$=1.2910				†br_P=101.97%$(br_\alpha)^{3/2}$	
Kepler-172 (R_{star}=1.085 R_{Sun}, age=$6.46^{1.10}_{-3.69}$Gyr)										
Planet	**P (day)**	**Ratio of P_n:P_{n-1}**	**ω_P**	**θ_P (%)**	**α (AU)**	**Ratio of α_n:α_{n-1}**	**ω_α**	**θ_α (%)**	**Radius (R_{jup})**	**Ratio of R_n:R_{n-1}**
b	2.940309				0.040				0.21	
c	6.388996	2.1729	0.9491	-5.09	0.068	1.70	0.9797	-2.03	0.255	1.2143
d	14.62712	2.2894	1	0	0.118	1.7353	1	0	0.201	0.8933
e	35.11874	2.4009	1.0487	4.87	0.211	1.7881	1.0304	3.04	0.246	1.2239
	benchmark ratio$_{(P)}$=2.2894				benchmark ratio$_{(\alpha)}$=1.7353				†br_P=100.15%$(br_\alpha)^{3/2}$	
HR 8799 system (R_{star}=1.61M_{Sun}, age=0.03Gyr). The observation data are published by Extrasolar Planets Encyclopaedia (*11*).										
Planet	**P (day)**	**Ratio of P_n:P_{n-1}**	**ω_P**	**θ_P (%)**	**α (AU)**	**Ratio of α_n:α_{n-1}**	**ω_α**	**θ_α (%)**	**M ($M_{Jup.}$)**	**Mass of M_n:M_{n-1}**
e	18000(?)				16.4				9.6	
d	41054	2.2808	1.1399	13.99	27	1.6463	1.0361	3.61	8.3	0.8646
c	82145	2.0009	1	0	42.9	1.5889	1	0	8.3	1.0
b	164250	1.9996	0.9994	-0.06	68	1.5851	0.9976	-0.24	7	0.8434
	benchmark ratio$_{(P)}$ =2.0009				benchmark ratio$_{(\alpha)}$=1.5889				†br_P=99.91%$(br_\alpha)^{3/2}$	
Kepler-299 (R_{star}=0.97 R_{Sun} , age=$6.03^{3.05}_{-3.59}$Gyr)										
Planet	**P (day)**	**Ratio of P_n:P_{n-1}**	**ω_P**	**θ_P (%)**	**α (AU)**	**Ratio of α_n:α_{n-1}**	**ω_α**	**θ_α (%)**	**Radius (R_{jup})**	**Ratio of R_n:R_{n-1}**
b	2.927128				0.04				0.118	
c	6.885875	2.3524	1	0	0.07	1.750	1	0	0.236	2.0
d	15.054786	2.1863	0.9294	-7.06	0.118	1.6857	0.9633	-3.67	0.166	0.7034
e	38.285489	2.5431	1.0811	8.11	0.22	1.8644	1.0654	6.54	0.167	1.0060
	benchmark ratio$_{(P)}$=2.3524				benchmark ratio$_{(\alpha)}$=1.750				†br_P=101.62%$(br_\alpha)^{3/2}$	
Kepler-235 (R_{star}=0.53 R_{Sun}, age=$4.730^{0.019}_{-0.020}$Gyr)										
Planet	**P (day)**	**Ratio of P_n:P_{n-1}**	**ω_P**	**θ_P (%)**	**α (AU)**	**Ratio of α_n:α_{n-1}**	**ω_α**	**θ_α (%)**	**Radius (R_{jup})**	**Ratio of R_n:R_{n-1}**
b	3.340222				0.037				0.199	
c	7.824904	2.3426	1	0	0.065	1.7568	1	0	0.114	0.5729

d	20.060548	2.5637	1.0944	9.44	0.122	1.8769	1.0684	6.836	0.183	1.6053
e	46.183669	2.3022	0.9828	-1.73	0.213	1.7459	0.9938	-0.621	0.198	1.0820
	benchmark ratio$_{(P)}$=2.3426				benchmark ratio$_{(\alpha)}$=1.7568				†dr_P=100.60%$(dr_\alpha)^{3/2}$	

Kepler-1542 (R_{star}=0.99 R_{Sun}, age=$8.510^{3.290}_{-5.270}$Gyr)

Planet	**P (day)**	**Ratio of P_n:P_{n-1}**	**ω_P**	**θ_P (%)**	**Radius (R_{jup})**	**Ratio of R_n:R_{n-1}**	**α (AU)**	**Ratio of α_n:α_{n-1}**	**ω_α**	**θ_α (%)**
c	2.8922302				0.065		--			
b	3.9511688	1.36613	1.0582	5.82	0.078	1.20	--			
e	5.1011576	1.29105	1	0	0.068	0.8718	--			
d	5.9927374	1.17478	0.9099	-9.01	0.078	1.1471	--			
	benchmark ratio$_{(P)}$=1.29015									

Note 1. Symbol ω_P represents the period ratio parameter and θ_P is the deviation rate of ω_P*. Symbol ω_α is semimajor-axis ratio parameter and θ_α is the deviation rate of ω_α. They are calculated with equations (1-4) and by the benchmark ratio.

Note 2. † is the equation calculated according to equation(11), in which br_P is period benchmark ratio and br_α is semimajor axis benchmark ratio. The br_P usually should be 100% equal to br_α. If not, the orbital observation data may be a little doubtful.

Note 3. Though the system have four exoplanets, they reflect the same orbital features as the three-planet systems above (Table S1). The comparison also shows that the stellar masses and age, and the radius ratios of corresponding planets do not affect the appearance of the orbital features.

Note 4. According to the general standard, the six systems are normal systems.

Table S3. Calculation of orbital ratio parameters and equivalence rates of adjacent planets in multi-planet systems

Kepler-229 (M_{star}=0.8M☉,Age=$3.09^{6.09}_{-2.97}$ Gyr)							Kepler-229 System (R_{star}=0.728 R_{Sun})						
Planet	**P (day)**	**Ratio of P**	**ω_P***	**δ_P* (%)**	**Radius ($R_{Jup.}$)**	**Ratio (R_n:R_{n-1})**	**Planet**	**α (AU)**	**Ratio ($α_n$:$α_{n-1}$)**	**$\omega_α$***	**$\delta_α$* (%)**	**Mass (M_{Jup})**	**Ratio (M_n:M_{n-1})**
b	6.25297				0.196		b	0.062				5.48	
c	16.0686	2.5698	1.0012	100.12	0.439	2.240	c	0.117	1.8871	1.0018	100.18	21.5	3.923
d	41.1949	2.5637	0.9988	99.88	0.343	0.781	d	0.220	1.8803	0.9982	99.82	14.2	0.661
	average ratio$_{(P)}$=2.56675							average ratio$_{(α)}$=1.8837			arP=99.28%$(ar_α)^{3/2}$		

HD 158259 (M_{star}=1.08M☉)							Kepler-296 A (M_{star}=0.498M☉, Age=$4.37^{+5.60}_{-2.49}$Gyr)						
Planet	**P (day)**	**Ratio (P_n:P_{n-1})**	**ω_P***	**δ_P* (%)**	**Mass (M_{Jup})**	**Ratio (M_n:M_{n-1})**	**Planet**	**α (AU)**	**Ratio ($α_n$:$α_{n-1}$)**	**$\omega_α$***	**$\delta_α$* (%)**	**Mass (M_{Jup})**	**Ratio (M_n:M_{n-1})**
b	2.1780				0.0070		c	0.0521				0.0101	
c	3.4320	1.5758	1.0353	103.53	0.0176	2.514	b	0.079	1.5163	1.0100	101.0	0.0147	1.455
d	5.19808	1.5146	0.9951	99.51	0.0170	0.966	d	0.118	1.4937	0.9949	99.49	0.0158	1.075
e	7.9510	1.5296	1.0049	100.49	0.0191	1.124	e	0.169	1.4322	0.9540	95.40	0.0093	0.589
f	12.028	1.5128	0.9939	99.39	0.0193	1.011	f	0.255	1.5089	1.0051	100.51	0.0123	1.322
	benchmark ratio$_{(P)}$=1.5221							benchmark ratio$_{(α)}$=1.5013					

K2-138 (M_{star}=0.935M☉, Age=$2.8^{+3.8}_{-1.7}$ Gyr)							Kepler-20 (M_{star}=0.948M☉, Age=7.60±3.70Gyr)						
Planet	**P (day)**	**Ratio (P_n:P_{n-1})**	**ω_P***	**δ_P* (%)**	**Mass (M_{Jup})**	**Ratio (M_n:M_{n-1})**	**Planet**	**α (AU)**	**Ratio ($α_n$:$α_{n-1}$)**	**$\omega_α$***	**$\delta_α$* (%)**	**Radius (R_{jup})**	**Ratio (R_n:R_{n-1})**
b	2.35309				0.0098		b	0.0463				0.1667	
c	3.56004	1.5129	0.9886	98.86	0.0199	2.031	e	0.0639	1.3801	0.9350	93.50	0.0772	0.4631
d	5.40479	1.5182	0.9921	99.21	0.0249	1.251	c	0.0949	1.4851	1.0061	100.61	0.2718	3.5207
e	8.26146	1.5285	0.9988	99.88	0.0408	1.639	f	0.1396	1.4710	0.9966	99.66	0.0895	0.3292
f	12.7576	1.5442	1.0091	100.90	0.0051	0.125	g	0.2055	1.4721	0.9973	99.73	0.42	4.693
g	41.9680	3.2897	2.1497	214.97	0.0136	2.667	d	0.3506	1.7061	1.1558	115.58	0.2448	0.583
	benchmark ratio$_{(P)}$ =1.5303							benchmark ratio$_{(α)}$=1.4761					

Note 1. (A) Symbol ω_P represents period ratio parameter; δ_P is equivalence rate of ω_P; $\omega_α$, is semimajor-axis ratio parameter; and $\delta_α$, is deviation rate of $\omega_α$. They are calculated with equations (1), (2), (3-1) and (4-1) and by benchmark ratio. If the symbol ω_P or $\omega_α$ is marked with "*" it is calculated by average ratio. (B) Comparison displays a phenomenon that the orbital ratio parameters of most exoplanets are almost equal to 1 and their equivalence rates are very close to l00%.

Note 2. Explanation of the equivalence rate

To more directly and clearly reflect the degree to which all orbital ratios in a system are equal, the orbital ratio parameter can also be further interpreted with the equivalence rate and is expressed as percentage. Temporarily denoted by δ, the equivalence rate can be calculated by following equations.

$$\delta_{P[n:(n-1)]}(\%) = \{\omega_{P[n:(n-1)]}\} \times 100 \ , \quad (3b)$$

$$\delta_{a[n:(n-1)]}(\%) = \{\omega_{a[n:(n-1)]}\} \times 100 \ , \quad (4b)$$

It is named the equation of the period ratio parameter expressed as the equivalence rate and the equation of the semimajor-axis ratio parameter expressed as the equivalence rate. In the equations, $\omega_{P[n:(n-1)]}$ is the period ratio parameter of any two adjacent planets and $\omega_{\alpha[n:(n-1)]}$ is the semimajor-axis ratio parameter of any two adjacent planets.

The equivalence rate reflects the equivalence between the orbital ratio of a planet pair and that of the benchmark ratio. Being the reference, the latter is usually set as 100%. The equivalence rate can be 100%, smaller or larger than 100%, which reflects different meanings. If an equivalence rate is 100%, it usually reflects that the corresponding orbital ratio is taken as the reference value for all planets of a system or the corresponding orbital ratio of calculated planets is equal to the reference value. The latter implies that the internal connections exist between the orbits of the calculated planets and reference planets. If an equivalence rate is smaller than 100%, it usually suggests that one of the two planets composing the orbital ratio migrates inwards relatively to the orbit where it should be or another planet migrates to the region between the two planets that compose corresponding orbital ratio. If a equivalence rate is larger than 100%, it usually suggests that one planet migrates outwards relatively to the orbit where it should be or some planets between the two planets that compose corresponding ratio vanish or are not detected. In Table S1, the calculation with equations (3-1) and (4-1) displays the equivalence rates of the orbital ratio parameters of all planets of six systems in a clearer way. It can be seen that the first four systems show all equivalence rates very close to 100%. Still HD 158259 is taken as example. For Planet f and e, the calculation by $\theta_{P(f:e)}$=[(12.028:7.9510)÷1.5221]×100% shows that the equivalence rate of their period ratio is 99.39%. By the calculation, we can see that the period ratios of all planet pairs show equivalence rates very close to 100%, suggesting that all planets orbit the star in the same mode. In one intends to learn about the difference of the orbital ratio of a planet pair from another, the calculation can also reflect it. For example, the period ratio of Planet f to e is different from that of Planet e to e to the degree of 1.1% by |99.39%|-|100.49%|. The period ratio of Planet f to e is different from that of Planet c to b to the degree of 4.14% by |99.39%|-|103.53%|. The difference between the two pairs also implies the degree of the orbital change of one planet if the orbital ratio of one pair is taken as the benchmark ratio or leading ratio. For the K2-138 system. the calculations clearly show the degree to which each ratio is equal to that of other ratios, including MMR and non-MMR planets. It shows that the equivalence rate of the orbital ratio of Planet g to f is 214.97%. The degree that is so far from 100% implies planetary absence or migration. Since the analysis of extrasolar systems aims at the study of planetary migration and absence, or system evolution, we will use the deviation rate to analyse all systems observed.

Table S4. Calculations of the orbital ratio parameters and deviation rates of adjacent planets of the Kepler-20 system and display of later-discovered planet.

Analysis of the periods published in 2016 [1]					Analysis of the semimajor axes published in 2016 [2]					
Planet	**P (day)**	**Ratio P_n:P_{n-1}**	**ω_P**	**θ_P (%)**	**α (AU)**	**Ratio α_n:α_{n-1}**	**ω_α**	**θ_α (%)**	**Radius ($R_{Jup.}$)**	**Ratio of R_n:R_{n-1}**
b	3.69612		3.4440		0.0463				0.1667	
e	6.09852	1.6500	0.9221	-7.79	0.0639	1.3801	0.9350	-6.50	0.0772	0.4631
c	10.8541	1.7798	0.9946	-0.54	0.0949	1.4851	1.0061	0.61	0.2718	3.5207
f	19.5776	1.8037	1.0080	0.8	0.1396	1.4710	0.9966	-0.34	0.08948	0.3292
g	34.940	1.7847	0.9974	-0.26	0.2055	1.4721	0.9973	-0.27	0.0628 (M_{Jup})	
d	77.6113	2.2213	1.2414	24.1	0.3506	1.7061	1.1558	15.6	0.2448	--
	benchmark ratio$_{(P)}$ =1.7894				benchmark ratio$_{(\alpha)}$=1.4761					

Analysis of the periods published in 2012 [3]					Calculations based on the planets following the shared rules [4]				
Planet	**P (day)**	**Ratio P_n:P_{n-1}**	**ω_P**	**θ_P (%)**	**Planet**	**P (day)**	**Ratio P_n:P_{n-1}**	**ω_P**	**θ_P (%)**	**Analysis of planetary orbits**
b	3.69612				b	3.38076				Equal to 109.3%
e	6.09849	1.6500	0.921	-7.9	e	6.05765	1.7918	1	0	Equal to 100.7%
c	10.8541	1.7798	0.993	-0.7	c	10.8541	1.7918	1	0	Reference planet
f	19.5771	1.8037	1.007	0.7	f	19.4484	1.7918	1	0	Equal to 100.7%
d	77.6118	3.9644	2.213	121	x	34.8476	1.7918	1	0	100.27% equal to g
	benchmark ratio$_{(P)}$=1.7918				d	62.4399	1.7918	1	0	Equal to 124.3%
						leading ratio$_{(P)}$=1.7918				

Analysis of the periods published in 2012 [5]					Calculations based on the planets following the shared rules [6]					
Planet	**α (AU)**	**Ratio α_n:α_{n-1}**	**ω_α**	**θ_α (%)**	**Planet**	**α (AU)**	**Ratio α_n:α_{n-1}**	**ω_α**	**θ_α (%)**	**Analysis of planetary orbits**
b	0.0463				b	0.0434				Equal to 106.7%
e	0.0639	1.3801	0.934	-6.6	e	0.0642	1.47805	1	0	Equal to 99.53%
c	0.0949	1.4851	1.005	0.5	c	0.0949	1.47805	1	0	Reference planet
f	0.1396	1.4710	0.995	-0.5	f	0.1403	1.47805	1	0	Equal to 99.50%
d	0.3506	2.5115	1.699	69.9	x	0.2073	1.47805	1	0	99.13% equal to g
	benchmark ratio$_{(\alpha)}$=1.47805				d	0.3064	1.47805	1	0	Equal to 114.4%
						leading ratio$_{(\alpha)}$=1.47805				

Note 1. "ω_P" is the ratio parameter of periods and "θ_P" is the deviation rate of period ratio. "ω_α" is the ratio parameter of semimajor axes and "θ_α" is the deviation rate of semimajor axis ratio.

Note 2. Analysis of the planets discovered in 2016

With equation (1-4), we calculated the orbital ratio parameters of all planets and their deviation rates according to observation data published in 2016 (See [1] and [2]). The calculations show that the deviation rate of the period ratio of Planet g to f is -0.26% and its adjacent deviation rate of the period ratio of Planet f to c is 0.8% (See [1]). According to the trait (1), Planet f, e and d should follow the shared rules. According to principle (1), the three planets can be regarded as the planets following the shared rules. So, the shared rules should exist in the system according to principle (2). The calculation with equation (9) shows that Planet d should migrate outwards. The migration should be caused by the companion star of Kepler-20 B.

Note 3. Analysis of the planets discovered in 2012

In 2012, only five planets were discovered in the system. According to the calculation with equation (1-4), the system showed that the deviation rate of period ratio parameter of Planet d to f is 121% (See [3]) and that of semimajor axis is 69.9% (See [5]). Because the adjacent deviation rates of P_f:P_c=-0.5% and P_c:P_e=0.5%, Planet f, c and e should be the planets following the shared rules according to trait (1). So, the shared rules still existed in the system at that time according to principle (2). The big deviation rate suggested the absence of a planet, temporarily called Planet x. In this case, we calculated the periods of all possible planets with equation (9). The calculation by P_x=10.8541÷[$1.7918^{(4-5)}$] showed 34.8476 days, which did not belong to any observed planet. According to principle (4), the period should belong to an undetected planet. in 2016, Planet g was discovered. Its period is 34.940 days. It shows that the observed semimajor axis of Planet g is 100.27% equal to the predicted period of Planet x (See [4]). The calculation with equation (10) also provides 0.2073AU by α_x=(0.0949)÷[$1.47805^{(3-5)}$], suggesting a undetected planet between Planet d and f. Later-discovered Planet g shows that its semimajor axis is 0.2055 AU (Buchhave et al. 2016). The semimajor axis is 99.13% equal to the predicted semimajor axis of Planet x (See [6]). The discovery of Planet g further supports the shared rules and corresponding equations and principles.

Note 4. Comparison of the calculations of equation (9) and T-B-related formulas

According to the calculation and analysis with equation (9) and principle (1-4), there should be one undetected exoplanet between Planet d and f. Its period should be 34.8476 days, which is 99.736% equal to the period of the later-discovered Planet g by (34.8476÷34.94)×100%. T. Bovaird et al also analysed the Kepler-20 system with T-B-related formula and predicted one planet (Bovaird and Lineweaver 2013). The predicted period of the planet was 40±6 days. Even without considering the range of uncertainty in the predicted values, the predicted period is 115% equal to the period of the later-discovered Planet g. P. Lara et al also predicted the undetected planet with the formula of T-B relation (Lara et al. 2020). They obtained the period that coincided with the period of the later-discovered Kepler-20g within the accuracy of about 88%.

Table S5. Comparison of different benchmark ratios through the K2-138 system, analysis of undetected planets and proof of related principles by simulation

Comparison shows that the benchmark ratio is relatively more suitable than the average ratio in the analysis of a planetary system [1]

Period calculation with specific benchmark ratio [1-1]					Period calculation with average ratio [1-2]				Comparison		
Planet	**P (day)**	**Ratio of P_n:P_{n-1}**	**ω_P#**	**θ_P# (%)**	**P (day)**	**Ratio of P_n:P_{n-1}**	**ω_P***	**θ_P* (%)**	**Discrepancy‡**	**Mass (M_{Earth})**	**Ratio**
b	2.35309				2.35309					3.10	
c	3.56004	1.5129	0.9898	-1.02	3.56004	1.5129	0.8053	-19.47	18.45%	6.31	2.035
d	5.40479	1.5182	0.9933	-0.67	5.40479	1.5182	0.8081	-19.19	18.52%	7.92	1.255
e	8.26146	1.5285	1	0	8.26146	1.5285	0.8136	-18.64	18.64%	12.97	1.638
f	12.75758	1.5442	1.0103	1.03	12.75758	1.5442	0.8220	-17.8	16.77%	1.63	0.126
g	41.96797	3.2897	2.1522	115.2	41.96797	3.2897	1.7510	75.1	40.1%	4.32	2.650
	specific benchmark ratio$_{(P)}$=1.5285				average ratio$_{(P)}$=1.8787						

Period calculation with benchmark ratio [1-3]					Semimajor axis calculation with benchmark ratio [1-4]				
Planet	**P (day)**	**Ratio of P_n:P_{n-1}**	**ω_P**	**θ_P (%)**	**α (AU)**	**Ratio $α_n$:$α_{n-1}$**	**$\omega_α$**	**$\theta_α$ (%)**	**Degree of $\omega_P=(\omega_α)^{3/2}$** by the calculation with equation (12) and (11)
b	2.35309				0.03385				
c	3.56004	1.5129	0.9886	-1.14	0.04461	1.3179	0.9924	-0.76	09886=99.998%$(0.9924)^{3/2}$
d	5.40479	1.5182	0.9921	-0.79	0.05893	1.3210	0.9947	-0.53	0.9921=100.004%$(0.9947)^{3/2}$
e	8.26146	1.5285	0.9988	-0.12	0.07820	1.3270	0.9993	-0.07	0.9988=99.985%$(0.9993)^{3/2}$
f	12.75758	1.5442	1.0091	0.91	0.10447	1.3359	1.0060	0.60	1.0091=100.009%$(1.0060)^{3/2}$
g	41.96797	3.2897	2.1497	115	0.23109	2.2120	1.6657	66.6	2.1497=99.996%$(1.6657)^{3/2}$
	benchmark ratio$_{(P)}$=1.5303				benchmark ratio($_α$)=1.3280				†br_P=99.995%$(br_α)^{3/2}$

The calculation with equation (9) displays the undetected planets [2]

Calculation of observed planets [2-1]					Undetected planets revealed by observed planets showing the shared rules [2-2]					
Planet	**P (day)**	**Ratio of P_n:P_{n-1}**	**ω_P**	**θ_P (%)**	**Planet**	**P (day)**	**Ratio of P_n:P_{n-1}**	**ω_P**	**θ_P (%)**	**Remark**
b	2.35309				b	2.35309				Observed planet
c	3.56004	1.5129	0.9886	-1.14	c	3.56004	1.5129	1.0016	0.16	Observed planet
d	5.40479	1.5182	0.9921	-0.79	d	5.40479	1.5182	1.0051	0.51	Observed planet
e	8.26146	1.5285	0.9988	-0.12	e	8.26146	1.5285	1.0119	1.19	Observed planet
f	12.75758	1.5442	1.0091	0.91	f	12.75758	1.5442	1.0223	2.23	Observed planet
g	41.96797	3.2897	2.1497	115	x	18.9132	1.4825	0.9815	-1.85	Missing planet
	benchmark ratio$_{(P)}$=1.5303				y	28.7141	1.5182	1.0051	0.51	Missing planet
					g	41.96797	1.4616	0.9676	-3.2	Observed planet
						benchmark ratio$_{(P)}$=1.5105				

Simulation calculation of undetected planets proves that related principles hold true [3]

Supposing Planet e is not detected [3-1]					Calculations based on the planets following the shared rules [3-2]					
Planet	**Observed P (day)**	**Ratio of P_n:P_{n-1}**	**ω_P**	**θ_P (%)**	**Planet**	**Calculated P* (day)**	**Ratio of P_n:P_{n-1}**	**ω_P**	**θ_P (%)**	**Degree of equality of P and P***

b	2.35309				b	2.34488				100.35% equal
c	3.56004	1.5129	0.9965	-0.35	c	3.56004	1.5182	1	0	100% equal
d	5.40479	1.5182	1	0	d	5.40479	1.5182	1	0	Reference planet
f	12.75758	2.3604	1.5547	55.47	z	8.20555	1.5182	1	0	Undetected planet*
g	41.96797	3.2897	2.1746	117.5	f	12.4577	1.5182	1	0	102.41% equal
	Special benchmark ratio$_{(P)}$=1.5182				x	18.9132	1.5182	1	0	Missing planet
					y	28.7141	1.5182	1	0	Missing planet
					g	43.5937	1.5182	1	0	96.27% equal
						leading ratio$_{(P)}$=1.5182				

Note 1. The symbol ω_P# is the period ratio parameter calculated by specific benchmark ratio$_{(P)}$ and θ_P# is deviation rate of ω_P#. The symbol ω_P* is the period ratio parameter calculated by average ratio$_{(P)}$ and θ_P* is deviation rate of ω_P*. The symbol ω_P is the period ratio parameter calculated by benchmark ratio$_{(P)}$ and θ_P is deviation rate of ω_P.

Note 2. Comparison of the calculations by using different value as the benchmark ratio

For comparison, we calculated the orbital ratio parameters with the specific benchmark ratio, average ratio and benchmark ratio (See [1]). The specific benchmark ratio is more likely to represent the orbital relations of all or most planets, which also completely agrees with the current method of dive score calculation in the Olympic Games. Because the benchmark ratio is very close to the special benchmark ratio, the results calculated by the benchmark ratio are very close to that by the specific benchmark ratio. But the results calculated by the average ratio are obviously different from (See comparison of [1]). The degree of the discrepancy of the values of corresponding planets are shown by $|\theta_P\#|-|\theta_P*|$, expressed by percentage. It shows, for most planets, the calculations by the average ratio are relatively bigger. Because the orbital ratio of Planet g to f is obviously bigger than other ratios, the application of the average ratio can not objectively reflect the result that the ratio parameters of the inner five planets are almost equal to 1. In addition, according to equation (1), the ratio parameter of P_g:P_f is 2.1497 calculated by the special benchmark ratio, which can suggest the lack of two unseen planets between Planet g and f according to equation (7) (details in Section 3.3 of the article). However, the period ratio parameter of P_g:P_f is 1.7150 by the average ratio, which can not clearly reflect the absence of the two planets. So, the comparison of discrepancies suggest that the special benchmark ratio or benchmark ratio is more objective.

Note 3. The calculation with equation (9) displays the undetected planets.

Calculated with equation (11), the system shows the relation between the period benchmark ratio (br_P) and semimajor-axis benchmark ratios (br_a). They are almost 100% equal, showing †br_P=99.995%$(br_a)^{3/2}$ (See [1-4]). The calculations also show the relation between the period ratio parameter of any two adjacent planets and their semimajor axis ratio parameter. It shows that all ratio parameters are almost 100% equal (See [1-4]). The calculations demonstrate that equation (11) and (12) exists and the observation data are correct according to the degrees of the equality of all planets..

The very small deviation rates of the inner five planets suggest that they follow a kind of common orbital relation. According to trait (1), they are the planets that follow the shared rules. The very big deviation rate suggests the absence of two planets between Planet g and f according to equation (7). Calculated with equation (9), the periods of the two absent planets are 18.9132 and 28.7141 days by $P_{x(cal)}=P_d\div(P_d:P_c)^{(3-6)}$ and $P_{x(cal)}=P_d\div(P_d:P_c)^{(3-7)}$. If considering Planet x and y, the large deviation rate vanishes and all deviation rates are very small (See [2-2]).

Note 4. Simulation calculation of undetected planet proves that related principles hold true.

Supposing Planet e is not detected, the simulated system shows five planets (See [3]). Because the inner three planets show close period ratios, we will take the period ratio of Planet d to c as the specific benchmark ratio and calculated the ratio parameters and deviation rates with equation (1) and (3). The calculations are presented in the table (See [3-1]). Because of two big deviation rates, it is an abnormal system. So, we first recalculated the period of

Planet b with equation (9) by taking Planet d as reference planet and the period ratio of Planet d to c as leading ratio. The calculation show the calculated period is 2.36165 days by $P_{b(cal)}$=5.40479÷(1.5182)$^{(3-1)}$. It is 99.64% equal to its observed period. According to principle (1), Planet b, c and d follow the shared rules. According to principle (2), the shared rules exist in the system. In this case, we tried to calculate the periods of all possible planets. By $P_{n(cal)}=(P_d)\div[(P_d):(P_c)]^{(snd-snn)}$, the calculations show eight planets (See [3-2]). The calculations show three periods that do not belong to the observed planets. According to principle (4), they should belong to the orbits of undetected planets. Comparison also shows that the period of Planet z is 8.20555 days, which is 100.68% equal to the period of Planet e before supposition. The degree of equality states that principle (1), (2) and (4) exist and are suitable in an abnormal system. The calculations also support the three principles and demonstrate that they are correct. Meanwhile, the simulation calculations also reveal the lack of two planets between Planet f and g (See [3-2]).

Note 5. Although the inner five planets show close orbital ratios and a long chain of near MMRs, the mass ratios of corresponding planets are not close. So, the orbital relations can be studied independently. In addition, the stellar mass is 0.935 M_{Sun} and age is $2.8^{+3.8}_{-1.7}$ Gyr. Though the stellar mass and age of the system are different from that of the systems in Supplementary Table S1 and S2, it still shows that most orbital ratios are close. Therefore, the regular orbital relations usually are not affected by stellar mass and age.

Note 6. The K2-138 system shows that its five inner planets have the relations of orbital resonances, which are embodied as the shared rules. Because the simulated calculations by astronomers suggest the appearance of the orbital-resonance relations at the early time of the system (Morrison et al. 2020; MacDonald et al. 2022), it is possible that the shared rules appear along with the formation of planetary systems.

Table S6. Calculations of the ratio parameters and deviation rates of semimajor-axis ratios of adjacent planets of the Kepler-444 system, and simulation of the changes of its planetary configurations.

Kepler-444 system observed [1]					K-444SM1 system simulated [2]					K-444SM2 system simulated [3]				
Planet	α (AU)	Ratio of α	ω_α	θ_α (%)	Planet	α (AU)	Ratio of α	ω_α	θ_α (%)	Planet	α (AU)	Ratio of α	ω_α	θ_α (%)
b	0.04178				b-1	0.0418				b-2	0.04178			
c	0.04881	1.1683	1.0013	0.13	c-1	0.0600	1.4354	1.2319	23.19	c*-2	0.04393	1.0515	0.9044	-9.56
d	0.0600	1.2293	1.0536	5.36	e-1	0.0696	1.1600	0.9955	-0.45	d-2	0.0600	1.3658	1.1748	17.48
e	0.0696	1.1600	0.9942	-0.58	f-1	0.0811	1.1652	1	0	e-2	0.0696	1.1600	0.9978	-0.22
f	0.0811	1.1652	0.9986	-0.14		benchmark ratio$_{(\alpha)}$=1.1652				f-2	0.0811	1.1652	1.0022	0.22
	benchmark ratio$_{(\alpha)}$=1.1668										benchmark ratio$_{(\alpha)}$=1.1626			

Note 1. The symbol ω_α is the ratio parameter of semimajor axes and θ_α is the deviation rate of ω_α.

Note 2. Analysis of the system

The calculation with equation(2) and (4) quantifies the orbital relationships of the five planets. According to the general standard, it is a normal system. According to trait (1), the orbital ratio parameters of Planet f to e and e to d suggest that the three planets follow the shared rules. If calculated with equation (10), the calculated semimajor axis of Planet d is 0.05973AU by $\alpha_{d(cal)}=\alpha_f\div(\alpha_f:\alpha_e)^{(snf-snd)}$, which is 100.45% equal to its observed semimajor axis. The calculation also suggests that the Planet d, e and f follow the shared rules. Therefore, according to principle (2), the shared rules exist in the Kepler-444 system. Now it is a normal system showing the shared rules.

Note 3. Simulation calculation (1)

We will perform a simulation calculation with the Kepler-444 system. Supposing Planet d is not detected, the planetary configuration is presented as K-444SM1 (See [2]). The calculation of semimajor axes shows br_α=1.1652 and $\omega_{\alpha(e-1:c-1)}$=1.2319. According to equation (8), the two values show the equation of $\omega_{\alpha(e-1:c-1)}=105.7\%(br_\alpha)^1$, which reflects the absence of an undetected planet, temporarily called Planet x. As it is a natural system, the simulation can still suggest that equation (8) exists and holds true.

If the semimajor axis of the undetected planet is calculated with equation (10) by taking Planet f-1 and e-1 as reference planets, it shows 0.05973AU by $\alpha_{x(cal)}=\alpha_{f-1}\div(\alpha_{f-1}:\alpha_{e-1})[^{(snf-1)-(sne-1)}]$. The calculated orbit is 100.45% equal to the semimajor axis of Planet d before supposition. The degree of equality states that equation (10) is correct and holds true.

Note 4. Simulation calculation (2)

In the Kepler-444 system, supposing Planet c migrates inwards equivalent to 10% of its initial semimajor axis, the current semimajor axis becomes 0.04393 AU. The calculations with equation (2) and (4) shows the orbital ratio parameters and deviation rates (See [3]). According to trait (2), the deviation rates suggest the migration of Planet c*-2. According to trait (1), Planet f-2, e-2 and d-2 are the planets that follow the shared rules. In this case, based on the planets following the shared rules, the previous semimajor axis of Planet c*-2 can be calculated with equation (10).

If taking Planet f-1 as reference planet and the ratio of Planet f-1 to e-1as the leading ratio of equation (10), the calculated semimajor axis is of Planet c*-2 is 0.051265AU by $\alpha_{c*-2(cal)}=0.0811\div(1.1652)^{(5-2)}$. The calculated semimajor axis is 95.21% equal to the semimajor axis of Planet c before supposition. Because Planet c might have migrated, which causes the 5.36% deviation rate, the degree of equality should be acceptable. The simulation calculation also supports trait (1) and (2), and equation (10).

We may also calculate the the semimajor axis of all planets with equation (10). If taking Planet f-2 as reference planet and the ratio of Planet f-1 to e-1as the leading ratio of equation, the calculated semimajor axes are 0.0440, 0.051265, 0.05973, 0.0696 and 0.0811 AU by $\alpha_{n(cal)}=\alpha_{f-2}\div(\alpha_{f-2}:\alpha_{e-2})^{[(snf-2)-(sne-2)]}$. Comparison shows that the calculated semimajor axes of the five planets are 94.96%, 85.69% 100.45%, 100% and 100% equal to their observed semimajor axes. Since the degrees of equality for Planet f-2, e-2, d-2 and b-2 satisfy criterion (N-B), they should follow the shared rules. According to principle (1), the four planets that follow the shared rules suggest the existence of the shared rules in the system, which reflect the contents of principle (2). The calculation also reflects that the semimajor axis of Planet c*-2 is 0.051265AU by $\alpha_d=0.0811\div1.1652^{5-2}$. Compared with the observed semimajor axis of Planet c*-2 (0.04393AU), the calculation suggests that Planet c*-2 should migrate equivalent to 14.3% of its initial semimajor axis before supposition. Because, it is not an ideal system and shows the migration of Planet c to a certain degree (5.36%), the calculation basically agrees with the supposition. So, the simulation calculations also reflect that principle (1), (2) and (3) hold true.

Table S7. Calculations of the orbital ratio parameters and their deviation rates for the Kepler-32 system, and check of the observation data of planetary orbits.

	Comparison of periods [1]				Comparison of semimajor axes [2]				Check the data with equation (11-13) [3]	
Planet	P (day)	Ratio of $P_n:P_{n-1}$	ω_P	θ_P (%)	α (AU)	Ratio of $\alpha_n:\alpha_{n-1}$	ω_α	θ_α (%)	**Degree of the equality of $\omega_{P[n:(n:-1)]}$ and $\{\omega_{\alpha[n:(n:-1)]}\}^{3/2}$	β
f	0.74296				0.013					1.09257
e	2.8960	3.8979	1.6798	67.98	0.033	2.5385	1.5408	54.08	$\omega_{P(e:f)}$=87.83%$[(\omega_{\alpha(e:f)})^{3/2}]$	1.01492
b	5.90124	2.0377	0.8782	-12.18	0.05	1.5152	0.9141	8.59	$\omega_{P(b:e)}$=100.5%$[(\omega_{\alpha(b:e)})^{3/2}]$	1.21245
c	8.7522	1.4831	0.6391	36.09	0.09	1.80	1.0859	8.59	$\omega_{P(c:b)}$=56.48%$[(\omega_{\alpha(c:b)})^{3/2}]$	0.45694
d	22.7802	2.6028	1.1218	12.18	0.13	1.4444	0.8714	-12.86	$\omega_{P(d:c)}$=137.9%$[(\omega_{\alpha(d:c)})^{3/2}]$	1.02683
	benchmark ratio$_{(P)}$=2.3203				benchmark ratio$_{(\alpha)}$=1.6576				†† br_P=108.723%$(br_\alpha)^{3/2}$	

Note 1. The symbol ω_P is the ratio parameter of periods and θ_P is the deviation rate of period ratio. The symbol ω_α is the ratio parameter of semimajor axes and θ_α is the deviation rate of semimajor axis ratio.

Note 2. According to equation (1) and (2), we calculated the ratio parameters of period ratios of the five observed planets and provided their deviation rates (See 1]). According to equation (3) and (4), we also calculated the ratio parameters of semimajor axis ratios of the five observed planets and showed their deviation rates (See [2]). The calculations display the orbital relations of all observed planets, suggesting that it is an abnormal system.

Note 3. The sign "††" shows the result calculated according to equation (11), where br_P is the period benchmark ratio and br_α is semimajor axes benchmark ratio. In general, a relatively big deviation from 100% equality suggests that the orbital data are doubtful and need more check for some planetary orbits.

Note 4. The sign "**" shows the degree of the equality between the ratio parameters of the periods and semimajor axes of any two adjacent planets (See [3]). It is calculated with equation (12). The obvious deviation from 100% suggests the orbit of corresponding planet need checking. For example, Planet c and Planet b show $\omega_{P(c:b)}$=0.6391 and $\omega_{\alpha(c:b)}$=1.0859. The calculation by equation(16) shows $\omega_{P(c:b)}$=56.48%$[(\omega_{\alpha(c:b)})^{3/2}]$. Meanwhile, $\omega_{P(d:c)}$=137.9%$[(\omega_{\alpha(d:c)})^{3/2}]$. Because both equations relate to Planet c, the obvious differences of the two equations from 100% equation suggest that the orbit of Planet c has doubt and needs more observations.

Note 5. "β" is the gravitation-orbit parameter, which can be calculated through equation (9). On the basis of the stellar mass as M_{star}=0.58M_{Sun} and gravitational constant of G=6.6740×10^{-11}(kgs^2m^3), the β values of the five planets are calculated (See β in [3]). It shows that the β values of Planet c and Planet b obviously deviate from 1, indicating that they need more observations. Different methods show the same result. The analyses with equation (11, 12, 13) show the orbital doubt of Planet c. The period deviation rate of Planet c is a negative value while its semimajor axis deviation rate is a positive value. The result also suggests the doubt of the orbits of corresponding planets. So is Planet b because its β value is 1.21245.

Table S8. Calculations of the orbital ratio parameters and their deviation rates for the Kepler-62 system. Check of the observation data of planetary orbits. Simulation calculation to an inaccurate orbit.

	Analysis of periods [1]				Analysis of semimajor axes [2]				Check the data with equation (11-13) [3]	
Planet	P (day)	Ratio of P	ω_P	θ_P (%)	α (AU)	Ratio of α	ω_α	θ_α (%)	**Degree of the equality of $\omega_{P[n:(n:-1)]}$ and $\{\omega_{\alpha[n:(n:-1)]}\}^{3/2}$	β
b	5.71493				0.0553					0.99915
c	12.4417	2.1771	0.9984	-0.16	0.0929	1.6799	0.9995	-0.05	$\omega_{P(c:b)}$=99.915%[$(\omega_{\alpha(c:b)})^{3/2}$]	0.99884
d	18.1641	1.4599	0.6695	-33.05	0.120	1.2917	0.7686	-23.2	$\omega_{P(d:c)}$=99.357%[$(\omega_{\alpha(d:c)})^{3/2}$]	0.98779
e	122.384	6.7377	3.0900	209	0.427	3.5583	2.1172	111.7	$\omega_{P(e:d)}$=100.603%[$(\omega_{\alpha(e:d)})^{3/2}$]	0.99529
f	267.291	2.1840	1.0016	0.16	0.718	1.6815	1.0005	0.05	$\omega_{P(f:e)}$=100.085%[$(\omega_{\alpha(f:e)})^{3/2}$]	0.99857
	benchmark ratio$_{(P)}$=2.1805				benchmark ratio$_{(\alpha)}$=1.6807				† br_P=100.073%$(br_\alpha)^{3/2}$	

Simulation calculation: Supposing the observed semimajor axis of Planet d is 0.14 AU, the analysis is as follows.

	Analysis of periods [4]				Analysis of semimajor axes [5]				Check the data with equation (11-13) [6]	
Planet	P (day)	Ratio of P	ω_P	θ_P (%)	α (AU)	Ratio of α	ω_α	θ_α (%)	**Degree of the equality of $\omega_{P[n:(n:-1)]}$ and $\{\omega_{\alpha[n:(n:-1)]}\}^{3/2}$	β
b	5.71493				0.0553					0.99915
c	12.4417	2.1771	0.9984	-0.16	0.0929	1.6799	0.9995	-0.05	$\omega_{P(c:b)}$=99.915%[$(\omega_{\alpha(c:b)})^{3/2}$]	0.99884
d*	18.1641	1.4599	0.6695	-33.05	0.14*	1.5070	0.8967	-10.3	$\omega_{P(d^*:c)}$=78.85%[$(\omega_{\alpha(d^*:c)})^{3/2}$]	0.62199
e	122.384	6.7377	3.0900	209	0.427	3.050	1.8147	81.47	$\omega_{P(e:d^*)}$=126.4%[$(\omega_{\alpha(e:d^*)})^{3/2}$]	0.99529
f	267.291	2.1840	1.0016	0.16	0.718	1.6815	1.0005	0.05	$\omega_{P(f:e)}$=100.085%[$(\omega_{\alpha(f:e)})^{3/2}$]	0.99857
	benchmark ratio$_{(P)}$=2.1805				benchmark ratio$_{(\alpha)}$=1.6807				† br_P=100.073%$(br_\alpha)^{3/2}$	

Note 1. "ω_P" is the ratio parameter of periods and "θ_P" is the deviation rate of period ratio. "br_P" is the benchmark ratio of periods. "ω_α" is the ratio parameter of semimajor axes and "θ_α" is the deviation rate of semimajor axis ratio. "br_α" is the benchmark ratio of semimajor axes.

Note 2. According to (1) and (2), we calculated the ratio parameters of period ratios of the five observed planets and showed their deviation rates (See 1]). According to equation (3) and (4), we also calculated the ratio parameters of semimajor axis ratios of the five observed planets and showed their deviation rates (See [2]). The calculations reflect the orbital relations of the observed planets, suggesting that it is an abnormal system.

Note 3. "†" shows the result calculated according to equation(11). If the degree of equality between br_P and $(br_\alpha)^{3/2}$ is 100% equal or is equal within a certain range of precision, the observation data of the orbits of all planets in the system should be accurate. Kepler-62 shows br_P=100.073%$(br_\alpha)^{3/2}$. As the 100.073% is very close to 100%, the degree suggests that the orbits of all planets should be accurate. Although there are two very big deviation rates in the system, the two benchmark ratios are still very close (See [3]), which further reflect the correct orbital relations between period and semimajor axis.

Note 4. "**" shows the degree of the equivalence of $\omega_{P[n:(n:-1)]}$ and $\{\omega_{\alpha[n:(n:-1)]}\}^{3/2}$, which is calculated according to equation(12). If the degree of equality between $\omega_{P[n:(n:-1)]}$ and $\{\omega_{\alpha[n:(n:-1)]}\}^{3/2}$ is 100% equal or is equal within a certain range of precision, it suggests that the observation data of the orbits of the two planets should be accurate. For example (See [3]), Planet f and Planet e show $\omega_{P(f:e)}$=1.0016 and $\omega_{\alpha(f:e)}$=1.0005. The calculation by equation (12) shows $\omega_{P(f:e)}$=100.085%[$(\omega_{\alpha(f:e)})^{3/2}$]. The degree suggest that the orbits of Planet f and Planet e are accurate.

Note 5. "β" is the relation parameter of gravitation, mass and orbit, which is calculated according to equation (13). On the basis of the stellar mass of M_{star}=0.69M_{Sun} and gravitational constant of G=6.6740×10^{-11}(kgs^2m^3), the β values of the five planets are almost equal to 1, suggesting that the observation data of the five planets are accurate

(See β in the table). The calculations through the equation of the shared rules and equation of classical mechanics come to the same conclusion.

Note 6. Simulation calculation

Supposing the observation data of the semimajor axis of Planet d was not accurate and showed 0.14 AU instead of 0.12AU, the corresponding values changed at the same time (See [5]). Equation (12) can reflect the inaccuracy (See [6]). Meanwhile the β value can also reflect the inaccuracy.

Table S9. Calculations of period ratio parameters of an ideal system, and support of simulation calculations of the ideal system to principle (1)

Observed periods of HD 158259 [1]						Periods of corresponding planets of A-HD after adjustment [2]			
No.	**Planet**	**P (day)**	**Ratio of P**	ω_P	θ_P **(%)**	Planet	Periods before adjustment (day)	Percentage of adjustment	Periods after adjustment (day)
1	b	2.1780				A-b	2.1780	0	2.1780 by 2.1780 ×(100%+0)
2	c	3.4320	1.576	1.035	3.5	A-c	3.4320	-2.72%	3.3386 by 3.4320×[100%+(-2.72%)]
3	d	5.1981	1.515	0.995	-0.5	A-d	5.1981	-1.54%	5.1181 by 5.1981×[100%+(-1.54%)]
4	e	7.9510	1.530	1.005	0.50	A-e	7.9510	-1.32%	7.8461 by 7.9510×[100%+(-1.32%)]
5	f	12.028	1.513	0.994	-0.6	A-f	12.028	0	12.028 by 12.028×(100%+0)
benchmark ratio$_{(P)}$=1.5225									

Orbital relations between planets in an ideal system [3]						Some reason caused Planet A-d* to migrate [4]					
No.	Planet	P (day)	Ratio of $P_n:P_{n-1}$	ω_P	θ_P (%)	Planet	P (day)	Ratio of P	ω_P	θ_P (%)	Remark
1	A-b	2.1780				A-b	2.1780				Original planet
2	A-c	3.3386	1.5330	1	0	A-c	3.3386	1.533	1	0	Original planet
3	A-d	5.1181	1.5330	1	0	A-d*	5.8858	1.763	1.150	15.0	Migratory planet
4	A-e	7.8461	1.5330	1	0	A-e	7.8461	1.333	0.870	-13.0	Original planet
5	A-f	12.028	1.5330	1	0	A-f	12.028	1.533	1	0	Original planet
benchmark ratio$_{(P)}$=1.5330						benchmark ratio$_{(P)}$=1.5330					

Calculation with Planet A-d* as reference planet [5]						Calculation with period ratio of Planet A-e to A-d* as leading ratio [6]					
No.	Planet	Calculated P (day)	Ratio of $P_n:P_{n-1}$	ω_P	θ_P (%)	Planet	Calculated P (day)	Ratio of $P_n:P_{n-1}$	ω_P	θ_P (%)	Remark
1	A-b	2.5045				A-b	2.5045				Original planet
2	A-c	3.8394	1.533	1	0	A-c	3.3386	1.333	1	0	Original planet
3	A-d*	5.8858	1.533	1	0	A-d*	4.4504	1.333	1	0	Migrated planet
4	A-e	9.0229	1.533	1	0	A-e	5.9323	1.333	1	0	Original planet
5	A-f	13.832	1.533	1	0	A-f	7.9078	1.333	1	0	Original planet
leading ratio$_{(P)}$=1.5330						leading ratio$_{(P)}$=1.333					

Note 1. The symbol ω_P is period ratio parameter and θ_P is deviation rate of ω_P.

Note 2. An ideal system based on a normal system.

Although some systems shows that the absolute values of the deviation rates of the period ratio parameters of all planets are smaller than 1%, they do not show 0 because they are in natural universe. To better understand the orbital relations of all kinds of systems, we assumed an ideal system based on the HD 158259 system (See [1]). After adjusting the periods of the middle three planets of HD 158259, equivalent to -2.72%, -1.54% and -1.32% of their periods (See [2]), it shows an ideal system, named the A-HD system (See [3]). The adjustments are acceptable because the average ratio of all period ratios of the HD 158259 system (1.5335) is 99.97% equal to that of the A-HD system (1.5330).

Note 3. The calculations supporting principle (1)

Now we will calculate Planet A-b. By taking Planet A-e as reference planet and the period ratio of Planet A-f to A-e as leading ratio, the calculation by $P_{A\text{-}b(cal)}=(P_{A\text{-}e})\div\{(P_{A\text{-}f}):(P_{A\text{-}e})\}^{[sn(A\text{-}e)\text{-}(snA\text{-}e)]}$ provides 2.1780 days by

$P_{A\text{-}b(cal)}$=7.8461÷(1.5330)$^{(2-3)}$. The calculated period is also equal to its observed period (See [3]). Then, we calculated other planets by the same equation. All calculated periods are equal to their observed periods. If the calculations are based on other planets or the period ratios of other planets, the results are the same. The calculations reflect the contents of principle (1) and well support principle (1).

Note 4. Calculation of migratory planet and confirmation of planets following the shared rules

Supposing Planet A-d migrated outward, equivalent to 15% of its initial period, the system shows the periods of all planets (See [4]). In order to confirm which planet migrates or follow the shared rules, we calculated their periods with equation (9). By $P_{n(cal)}=(P_{A\text{-}c})\div\{(P_{A\text{-}c}):(P_{A\text{-}b})\}^{[sn(A\text{-}c)\text{-}(snn)]}$, the calculated periods are 2.1780, 3.3386, 5.1181, 7.8461 and 12.028 days. For example, Planet A-f shows $P_{A\text{-}f(cal)}$=3.3386÷1.533$^{2-5}$=12.028(day). Comparison shows that the calculated periods of all planets are equal to their observed periods except Planet A-d* (See [1]). The calculations by taking Planet A-b, A-e or A-f as reference planet show the same results. According to principle (1), Planet A-b, A-c, A-e and A-f can be judged to follow the shared rules. Because they were the planets that follow the shared rules before supposition, the judgement is correct. So, principle (1) is correct. Since the system has four planets that follow the shared rules, the shared rules can be judged to exist in the system according to principle (2). Besides, the calculation also shows 5.1181 days by P=3.3386÷1.5330$^{(2-3)}$, which is 115% equal to the observed period of Planet A-d* by (5.8858÷5.1181)×100%=115%. The value suggests that Planet A-d* should migrate outwards, equivalent to 15% of its previous period. Because the calculation is based on equation (9) and the planets that follow the shared rules, it reflects the contents of principle (3). Because the calculated period is equal to the period of Planet A-d before supposition, the suggestion is correct, stating that principle (3) is correct.

If taking Planet A-d* as reference planet and still the period ratio of Planet A-c to A-b as the leading ratio, the calculations by $P_{n(cal)}=(P_{A\text{-}d*})\div\{(P_{A\text{-}c}):(P_{A\text{-}b})\}^{[sn(A\text{-}d*)\text{-}(snn)]}$ show 2.5045, 3.8394, 5.8858, 9.0229 and 13.832 days. Comparison shows that the calculated periods of all planets are different from their observed periods except Planet A-d* (See [5]). If taking Planet A-c as reference planet and the period ratio of Planet A-e to A-d* as the leading ratio, the calculations by $P_{n(cal)}=(P_{A\text{-}c})\div\{(P_{A\text{-}e}):(P_{A\text{-}d*})\}^{[sn(A\text{-}d*)\text{-}(snn)]}$ show 2.5045, 3.3386, 4.4504, 5.9323, and 7.9078 days. Comparison shows that the calculated periods of all planets are different from their observed periods except Planet A-b (See [6]).

Table S10. Simulation calculations of the HD 158259 system shows the support to principle (1)

Period comparisons of current planets [1]					Calculation by taking Planet f as reference planet [2]					
Planet	Observed P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Planet	Calculated P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Compare observed P to calculated P
b	2.1780				b	2.24089				Equal 97.194%
c	3.4320	1.5758	1.0353	3.53	c	3.41086	1.5221	1	0	Equal 100.620%
d	5.19808	1.5146	0.9951	-0.49	d	5.19167	1.5221	1	0	Equal 100.123%
e	7.9510	1.5296	1.0049	0.49	e	7.90224	1.5221	1	0	Equal 100.617%
f	12.028	1.5128	0.9939	-0.61	f	12.028	1.5221	1	0	Reference planet
		benchmark ratio$_{(P)}$=1.5221				leading ratio$_{(P)}$=1.5221				

Calculation by taking Planet d as reference planet [3]						Calculation by taking Planet b as reference planet [4]				
Planet	Observed P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)		Calculated P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Compare observed P to calculated P
b	2.24366				To 97.07%	2.1780				Reference planet
c	3.41507	1.5221	1	0	To 100.50%	3.31513	1.5221	1	0	Equal to 103.53%
d	5.19808	1.5221	1	0	Reference	5.0460	1.5221	1	0	Equal to 103.01%
e	7.9120	1.5221	1	0	To 100.49%	7.68046	1.5221	1	0	Equal to 103.52%
f	12.0429	1.5221	1	0	To 98.88%	11.6904	1.5221	1	0	Equal to 102.89%
		leading ratio$_{(P)}$=1.5221				leading ratio$_{(P)}$=1.5221				

Calculation by taking Planet e as reference planet and the period ratio of Planet f to e as leading ratio [5]						Calculation by taking Planet d as reference planet and the period ratio of Planet e to d as leading ratio [6]				
Planet	Observed P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)		Calculated P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Compare observed P to calculated P
b	2.2966				To 94.84%	2.22171				Equal to 98.03%
c	3.4742	1.5128	1	0	To 98.79%	3.39833	1.5296	1	0	Equal to 100.99%
d	5.2558	1.5128	1	0	To 98.90%	5.19808	1.5296	1	0	Reference planet
e	7.9510	1.5128	1	0	Reference	7.9510	1.5296	1	0	Equal to 100%
f	12.028	1.5128	1	0	To 100%	12.1619	1.5296	1	0	Equal to 98.90%
		leading ratio$_{(P)}$=1.5128				leading ratio$_{(P)}$=1.5296				

Note 1. The symbol ω_P is the ratio parameter of periods and θ_P is the deviation rate of ω_P.They are calculated according to equation (1) and (2).

Note 2. The judgement of the planets that follow the shared rules

HD 158259 is a five-planet normal system (See [1]). So, all planets should reflect the property that they follow the shared rules. In other words, the five planets should reflect the same orbital relations. Here we will show the detailed calculations.

Based on the benchmark ratio as leading ratio and Planet f as reference planet, we calculated the periods of all planets according to equation (9), detailed as $P_{n(cal)}=P_f\div1.5221^{(sf-sn)}$. The calculations show corresponding periods (See [2]). Comparison shows that the calculated period of Planet b is equal to 97.194% of its observed period, Planet c equals to 100.62%, Planet d equals to100.12%, Planet e equals to 100.62% and Planet c equals to 100%. All five planets show that the degrees of equality are within the accuracy of 97-101% (See [2]). We also calculated the periods separately by Planet d and b as reference planets. The results are similar (See [3] & [4]). The calculated periods of all planets are equal to their observed periods within the range of high accuracy. Because it is a natural system, the calculated periods of corresponding planets are not equal to 100% of their observed periods as an ideal system. A certain range of difference should be acceptable. So, the five planets can be judged as the planets following the shared rules. The calculations reflect the contents of principle (1). Based on the five planets, HD 158259 can be regarded as a normal system according to principle (2).

Based on the deviation rates, Planet b should migrate a little. As a result, the calculations on the basis of Planet b as reference planet show a little difference than other planets for all planets (See [4]). The result also implies the orbital change of Planet b.

We also calculated the periods of all planets by taking the period ratio of two adjacent planets as leading ratio and some planet as reference planet. They reflect the same results (See [5] and [6]). All calculations reflect and explain the contents of principle (1) and (2).

Table S11. Calculations and comparisons of the changes of planetary configurations according to different simulations to the A-HD system.

Analysis of the periods of adjacent planets in the ideal system of A-HD (reference system for comparison) [1]						If two side planets migrated, the planetary configuration of the A-HD system changes as follows [2]					
No.	Planet	P (day)	Ratio of $P_n:P_{n-1}$	ω_P	θ_P (%)	Planet	P (day)	Ratio of $P_n:P_{n-1}$	ω_P	θ_P (%)	Supposition of orbital changes
1	b	2.1780				b*	1.8513				Migrate to -15%
2	c	3.3386	1.5330	1	0	c*	3.6725	1.9837	1.2940	29.4	Migrate to 10%
3	d	5.1181	1.5330	1	0	d	5.1181	1.3936	0.9091	9.1	Keep initial orbit
4	e	7.8461	1.5330	1	0	e	7.8461	1.5330	1	0	Keep initial orbit
5	f	12.028	1.5330	1	0	f	12.028	1.5330	1	0	Keep initial orbit
	benchmark ratio$_{(P)}$=1.5330					benchmark ratio$_{(P)}$=1.5330					

If one planet migrated and one planet is absent, the planetary configuration changes as follows [3-1]						Restore the previous planetary configuration through the calculation on the basis of the planets following the shared rules [3-2]					
No.	Planet	Observed P (day)	Ratio of $P_n:P_{n-1}$	ω_P	θ_P (%)	Planet	Calculated P (day)	Ratio of $P_n:P_{n-1}$	ω_P	θ_P (%)	Compare observed P to calculated P
1	b	2.1780				b	2.1780				Equal to 100%
2	c*	5.3740	2.4674	1.6095	60.95	x	3.3386	1.5330	1	0	Absent planet
3	d	7.8461	1.4600	0.9524	-4.76	c*	5.1181	1.5330	1	0	Equal to 105%
4	e	12.028	1.5330	1	0	d	7.8461	1.5330	1	0	Equal to100%
5	benchmark ratio$_{(P)}$=1.5330					e	12.028	1.5330	1	0	Reference planet
						leading ratio$_{(P)}$=1.5330					

If the innermost planet migrated, the planetary configuration changes as follows [4]						If two middle planets migrated, the planetary configuration changes as follows [5]					
No.	Planet	Observed P (day)	Ratio of $P_n:P_{n-1}$	ω_P	θ_P (%)	Planet	Observed P (day)	Ratio of $P_n:P_{n-1}$	ω_P	θ_P (%)	Supposition of orbital changes
1	b*	1.8939				b	2.1780				Keep initial orbit
2	c	3.3386	1.7628	1.150	15.0	c*	3.6725	1.6862	1.0717	7.17	Migrate to 10%
3	d	5.1181	1.5330	1	0	d*	4.8622	1.3240	0.8415	-15.85	Migrate to -5%
4	e	7.8461	1.5330	1	0	e	7.8461	1.6137	1.0256	2.56	Keep initial orbit
5	f	12.028	1.5330	1	0	f	12.028	1.5330	0.9743	-2.57	Keep initial orbit
	benchmark ratio$_{(P)}$=1.5330					benchmark ratio$_{(P)}$=1.5734					

If Planet c migrated to -10% and Planet e migrate to +10%, the planetary configuration changes as follows [6]						If two non-adjacent planets migrated, the planetary configuration changes as follows [7]					
No.	Planet	Observed P (day)	Ratio of $P_n:P_{n-1}$	ω_P	θ_P (%)	Planet	Observed P (day)	Ratio of $P_n:P_{n-1}$	ω_P	θ_P (%)	Supposition of orbital changes
1	b	2.1780				b*	2.6136				Migrate to 20%
2	c*	3.0047	1.3796	0.8958	-10.42	c	3.3386	1.2774	0.9091	-9.10	Keep initial orbit
3	d	5.1181	1.7034	1.1061	10.61	d	5.1181	1.5330	1.0910	9.10	Keep initial orbit
4	e*	8.6307	1.6863	109.50	9.50	e*	6.0355	1.1793	0.8392	-16.08	Migrate to -30%

5	f	12.028	1.3936	0.9050	-9.50	f	12.028	1.9929	1.4182	41.82	Keep initial orbit
benchmark ratio$_{(P)}$=1.540						benchmark ratio$_{(P)}$=1.4052					

If two adjacent planets migrated and one planet is absent, planetary configuration changes as follows [8]						If two non-adjacent planets migrated and Planet d disappeared, the planetary configuration changes as follows [9]					
No.	Planet	Calculated P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Planet	Calculated P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Supposition of orbital changes
1	b*	2.5047				b	2.1780				Keep initial orbit
2	c*	3.0047	1.1996	0.7825	-21.75	*c	4.0063	1.8394	1	0	Migrate to 20%
3	d	7.8461	2.6113	1.7034	70.34	*e	8.6307	2.1543	1.1712	17.12	Migrate to 10%
4	e	12.028	1.5330	1	0	f	12.028	1.3936	0.7577	-24.23	Keep initial orbit
benchmark ratio$_{(P)}$=1.5330						benchmark ratio$_{(P)}$=1.8349					

Note 1. The symbol ω_P is the ratio parameter of periods and θ_P is the deviation rate of ω_P.They are calculated according to equation (1) and (2).

Note 2. Calculation and analysis to different simulated systems

1. For convenience, we listed the ideal system of A-HD (See [1]) and provided the calculations with equation (3) and (4). It will play the role of the reference system during the comparisons of different simulated systems.

2. If two side planets migrated because of their mutual dragging forces, the A-HD system becomes a system with a new planetary configuration (See [2]). The deviation rates show a near normal system. Because the adjacent deviation rates composed by Planet f, e and d show the 0 value, the calculation with equation (9) shows that the three planets display the calculated periods equal to their observed periods. So, they can be considered to follow the shared rules according to principle (1). In this case, the calculations on the basis of the planets that follow the shared rules can reflect the migrations of Planet b* and Planet c* according to principle (3). For Planet b*, the calculated period is 2.1780 days by P_b=12.028÷1.5330$^{(5-1)}$, which is different from the observed period but equal to 100% of the period before supposition. The degree of migration is 15% by [(2.1780-1.8513)÷2.1780]×100%, which agrees with the supposition. So is Planet c*. The calculations reflect the contents of Principle (1), (2) and (3).

3. If one planet migrated and one planet is absent, the simulated system shows four planets now (See [3]). The 60.95% deviation rate indicates it is an abnormal system. In this case, we will take the period ratio of Planet e to d as the leading ratio and Planet e as reference planet, and then calculate the period of Planet b with equation (9). The calculation provides P_b=12.028÷1.5330$^{(5-1)}$=2.1780d. The calculated period equals to 100% of its observed period (See [3-1]), suggesting that Planet b, d and e are the planets following the shared rules according to principle (1). The calculations on the basis of Planet e and d also provide a period of 5.1181 days by P_c=12.028÷1.5330$^{(5-3)}$. The 5% difference suggests the migration of Planet c* according to principle (3). Another period is calculated at the same time, showing P_x=12.028÷1.5330$^{(5-2)}$=3.3368d. The period suggests the absence of one planet according to principle (4). Because the calculated periods of Planet c* and absent planet are equal to the periods of the corresponding planets before supposition, the calculation and judgement of planet migration and absence are correct. The calculations restore the previous planetary configuration, suggesting it once was an ideal system (See [3-1]). More importantly, the calculations reveal that the shared rules still exist in a system with a big deviation rate. The calculation makes the analysis of orbital changes meaningful in abnormal systems.

4. If the innermost planet migrated, the planetary configuration will changes (See [4]). According to trait (1), the 0 deviation rates of Planet c, d, e and f suggest that they are the planets following the shared rules. The calculation on the basis of the planets following the shared rules reflects the migration of Planet b and provides its previous period. If it is not considered that the three adjacent deviation rates with 0 value are accidental, the calculated period of Planet b* can be considered as its previous period.

5. If two middle planets (Planet d and c) migrated, the planetary configuration will changes (See [5]). The

calculation with equation (4) provides the deviation rates, showing a near normal system. Although the period ratio of Planet f to e is not regarded as the benchmark ratio, the average ratio of all planet (1.5392) are closer to the ratio of Planet f to e (1.5330). So, we may take the period ratio of Planet f to e as the leading ratio during the calculation with equation (9) to the periods of all planets. When we calculated the period of Planet b by taking Planet f as reference planet and the period ratio of Planet f to e as the leading ratio, the value is 2.1870 days, equal to the period before supposition. So, according to principle (1), Planet b, e and f are the planets following the shared rules. In this case, the calculated period of Planet c* should be its previous period since the calculation is based on the planets following the shared rules. The comparison shows that the calculated period of Planet c* equals the period before supposition. So is Planet d*. The calculations restored the previous planetary configuration of the system. Therefore, if the selection of the leading ratio and reference planet is right, the calculations may still reflect the previous planetary configuration.

Although we do not know which planets change their orbits, we may suppose that the outermost and innermost planet keep their initial orbit and the initial period of all planets vary with a shared divisor. So, we may adjust equation (9) and calculate the shared divisor of the system by $sd=(P_n \div P_m)^{1/(nsn-msn)}$. The detailed calculation shows $(12.028+2.1780)^{1/(5-1)}=1.5330$. The divisor is equal to the period ratio of Planet f to e. The calculation helps confirm the leading ratio of the system. So the period ratio of Planet f to e can be taken as the leading ratio and the calculations agree with the periods of the two migratory planets before supposition.

6. If Planet c migrated to -10% and Planet e migrate to +10%, the planetary configuration will changes (See [6]). At this moment, the direct look shows that the middle two ratios are close, which implies that they are the leading ratio. In this case, the conclusion of analyses would reflect the migrations of Planet A-b and A-f. However, it is difficult to explain why they migrated. But if taking the benchmark ratio as the leading ratio and Planet A-f as reference planet, the calculations can restore the initial planetary configuration. The calculations can also reflect the migrations of Planet A-c and A-e. Their migrations can be explained by the disturbance of Planet d. Although the calculations can restore the change of planetary configuration, it is not easy and not very convincing.

7. If two non-adjacent planets migrated, one may see such planetary configuration (See [7]). It is difficult to confirm which planets keep their initial orbits because the four ratios are different. In this case, there seems no way to find the changes of the planetary configuration because it is not easy to confirm which period ratio should be the leading ratio. If taking the benchmark ratio as the leading ratio, the calculated periods of all planets will be different from their observed periods except the reference planet. Even so, if one selects the period ratio of Planet d to c as leading ratio of equation (9) and take Planet c as reference planet, he can still restore the previous planetary configuration.

8. If two adjacent planets migrated and one planet is absent, the planetary configurations will change greatly (See [8]). Although the benchmark ratio is equal to the period ratio of Planet f to e, and one can calculated the possible periods of corresponding planets, the calculations are not convincing. Because the calculations only reflect that Planet f and e follow the shared rules, two planets can not suggest that the shared rules exist in the system according to principle (2) though they keep the initial orbital relation. So, if a system experienced such change of planetary configuration, the try of restoring the previous planetary configuration may not be accepted.

9. If two middle non-adjacent planets migrated and one planet disappeared for some reasons, all period ratios that reflect the initial orbital relations have vanished (See [9]). If it is an observed system, any calculations will not correctly reflect the change of the planetary configuration because there is no way to find the period ratio that reflects the initial orbital relation. In other words, the system fails to display the value of 1.5330, period ratio of the planets that follow the shared rules. Although we know that Planet b and f are the planets that follow the shared rules, there is no direct way to confirm them. So, there is no way to restore the previous planetary configuration of the system with equation (9). The simulation also explains why principle (2) emphasises at least three planets that

follow the shared rules. That is, if a system has three planets or more that follow the shared rules, usually it can be judged that the shared rules exists in the system. So, if a system experienced great disturbances, the initial orbital relations of a system will disappear and the restoration will be impossible.

Table S12. Calculations and comparisons of the changes of planetary configurations according to different simulations to the HD 158259 system.

Analysis of the periods of adjacent planets in the normal system of HD 158259 (reference system for comparison) [1]					If two side planets migrated, the HD 158259 system becomes a system with the planetary configuration as follows [2]					
Planet	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Planet	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Supposition of orbital changes
b	2.1780				b*	1.8939				Migrate to -15%
c	3.4320	1.5758	1.0353	3.53	c*	3.7752	1.9934	1.3104	31.04	Migrate to 10%
d	5.19808	1.5146	0.9951	-0.49	d	5.19808	1.3769	0.9051	-9.49	Keep initial orbit
e	7.9510	1.5296	1.0049	0.49	e	7.9510	1.5296	1.0055	0.55	Keep initial orbit
f	12.028	1.5128	0.9939	-0.61	f	12.028	1.5128	0.9945	-0.55	Keep initial orbit
		benchmark ratio$_{(P)}$=1.5221				benchmark ratio$_{(P)}$=1.5212				

If two non-adjacent planets migrated and one planet is absent, the planetary configuration changes as follows [3]						The try of restoring the previous planetary configuration by taking the benchmark ratio as leading ratio of equation (9)				
Planet	Observed P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Supposition	Calculated P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Compare observed P to calculated P
b	2.1780					2.1780				Reference planet
c*	4.0063	1.8394	1	0	Migrate	4.0063	1.8394	1	0	Equal to 100%
d*	Absent	↓	--	--	Missing	7.3692	1.8394	1	0	Equal to 117.1%
e*	8.6307	2.1543	1.1712	17.12	Migrate	13.555	1.8394	1	0	Equal to 88.7%
f	12.028	1.3936	0.7577	-24.23		leading ratio$_{(P)}$=1.8394				
		benchmark ratio$_{(P)}$=1.8394								

Note 1. The symbol ω_P is the ratio parameter of periods and θ_P is the deviation rate of ω_P.They are calculated according to equation (1) and (2).

Note 2. Calculation and analysis to different simulated systems

1. For convenience, we listed the normal system of A-HD (See [1]) and provided the calculations with equation (1) and (2). It will play the role of the reference system during the comparisons of different simulated systems.

2. If two side planets migrated, the HD 158259 system becomes a system with a new planetary configuration (See [2]). The deviation rates show a near normal system. Because the adjacent deviation rates composed by Planet f, e and d show very small deviation rates, the calculation with equation (9) shows that the three planets display the calculated periods almost equal to their observed periods. So, they can be considered to follow the shared rules according to principle (1). In this case, the calculations on the basis of the planets that follow the shared rules reflect the migrations of Planet b* and Planet c* according to principle (3). For Planet b*, the calculated period is 2.2217 days by P_b=7.9510÷$1.5296^{(4-1)}$, which is equal to 98% of the period before supposition. The degree of migration is 15% by [(2.2217-1.8513)÷1.8513]×100%, which agrees with the supposition. So is Planet c*. The calculations reflect the contents of Principle (1), (2) and (3). If only two side planets migrated, it is possible to restore the previous planetary configuration.

3. If two middle non-adjacent planets migrated and one planet disappeared, all traces of the previous planetary configuration have vanished (See [3]). In this case, any calculations will not correctly reflect the change of the planetary configuration because we do not know the planets that still follow the shared rules. In other words, the system can not display the period ratio of adjacent planets which keep their initial orbital relations because the system only has Planet b and f that keep the initial orbits. So, if a system experienced great disturbances, the

shared rules will disappear and it will be impossible to see the shared rules in the system even though it once existed in the system.

Table S13. Calculations of the ratio parameters and deviation rates of semimajor-axis ratios of adjacent planets of the Kepler-85 system. Simulation of the changes of planetary orbits in the Kepler-85 system.

The observed system [1]					The simulated system-1 [2]					The simulated system-2 [3]				
Planet	α (AU)	Ratio of α	ω_α	θ_α (%)	Planet	α (AU)	Ratio of α	ω_α	θ_α (%)	Planet	α (AU)	Ratio of α	ω_α	θ_α (%)
b	0.0766				b-1	0.0766				b-2	0.0766			
c	0.1007	1.3146	1.0183	1.83	c-1	0.1007	1.3146	0.896	-10.4	c-2	0.150	1.9582	1.286	28.6
d	0.130	1.2910	1	0	d-1	0.163	1.6187	1.104	10.4	d-2	0.163	1.0867	0.714	-28.6
e	0.163	1.2539	0.9713	-2.87		average $ratio_{(\alpha)}$=1.4666					average $ratio_{(\alpha)}$=1.5224			
	benchmark $ratio_{(\alpha)}$=1.2910													

Note 1. The symbol ω_α is the ratio parameter of semimajor axes and θ_α is the deviation rate of ω_α. They are calculated according to equation (2) and (4).

Note 2. The analysis of the observed system

The calculation to semimajor axes can also reflect the properties of the planets in a system and help judge which planet follow the shared rules and whether the shared rules exist in a multi-planet system. The Kepler-85 system is taken for example. The calculation with equation (4) provides the orbital relations of all planets and shows that it is a normal system (See [1]). Meanwhile, according to trait (1), all the four planets follow the shared rules.

We may judge the properties of the planets according to calculation. Based on the ratio of α_e:α_d as leading ratio and Planet e as reference planet, we calculated the semimajor axis of each planet according to equation (10), detailed as $\alpha_{n(cal)}=\alpha_e\div1.2910^{(se-sn)}$. They are 0.07800AU, 0.1007AU, 0.1300AU and 0.16783AU. The calculated semimajor axis of Planet b is equal to 97.44% of its observed semimajor, Planet c is equal to 100%, Planet d is equal to 100% and Planet e is equal to 97.12%. It shows that the degrees of the equality for all planets are bigger than 95%, satisfying criterion (N-A). They can be considered the planets following the shared rules according to principle (1). So, according to principle (2), the shared rules exist in the system. As all deviation rates are small, the system can be regarded as a normal system.

Note 3. Simulation of one absent planet

Supposing Planet d is not discovered or disappeared because of the impact of a stray planet, the system shows three planets, temporarily called the simulated system-1 (See [2]). Equation (4) shows that it is not a normal system. Because the semimajor axis ratio of Planet c-1 to b-1 is equal to the square of the semimajor axis ratio of Planet d-1 to c-1 to the degree of 93.66%, there should be an undetected planet between Planet c-1 and b-1 according to trait (3). In order to learn about the undetected planet, I will calculate the semimajor axes of possible planets with equation (10). By taking the semimajor-axis ratio of Planet c-1 to b-1 as the leading ratio and Planet dc1 as the reference planet, the calculated values are 0.17403AU, 0.13238AU, 0.1007AU and 0.0766AU. The comparison shows the calculated semimajor axis of Planet d-1 is equal to 93.66% of its observed semimajor axis, Planet c-1 is equal to its observed semimajor axis, and Planet b-1 is equal to its observed semimajor axis. The calculation satisfies criterion (N-B). So, the three observed planets can be considered as the planets that follow the shared rules. In this case, the calculated orbit where no planet is observed should be an undetected planet because it is calculated on the basis of the planets that follow the shared rules. The comparison also reflects that the calculated semimajor axis of the undetected planet is equal to 98.2% of the semimajor axis of Planet c before supposition. So, the calculations restore the previous planetary configuration and predict the undetected planet or show a disappeared planet, which may help study the evolution of the system. The calculations also reflect the contents of principle (1), (2) and (4), and support the principles.

Note 4. Simulation of one absent planet and one migratory planet

If Planet c disappeared and Planet d migrated at the same time, the simulated system-2 shows the orbits of the observed planets (See [3]). Although Planet b-2 and d-2 keep their initial orbits, there is not way to confirm that they are the planets following the shared rules because two non-adjacent planets can not reflect the relation of their initial orbits. In addition, the calculations of the three planets can not prove that the system has three planets that follow the shared rules. So, it is impossible to restore the previous planetary configuration and reveal that the shared rules once existed in the system.

Table S14. Analysis of the changes of the planetary configurations of two representative systems

Analysis of planet migration that occurs in the Kepler-299 System [1]										
Analysis with current planets [1-1]					Period of undetected planet, previous period of migratory planet [1-2]					
Planet	Observed P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Mass (M_{Earth})	Calculated P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Compare observed P to calculated P
b	2.927128				2.3	2.927128				Equal to 100%
c	6.885875	2.3524	1	0	7.51	6.885875	2.3524	1	0	Reference planet
d	15.054786	2.1863	0.9294	-7.06	4.12	16.19833	2.3524	1	0	Equal to 92.94%
e	38.285489	2.5431	1.0810	8.10	4.16	38.1050	2.3524	1.0047	0.47	Equal to 100.47%
benchmark $ratio_{(P)}$=2.3524					leading $ratio_{(P)}$=2.3524					

Analysis of the orbits of current planets in the Kepler-286 and orbital changes of some planets [2]											
Analysis with current planets [2-1]					Period of undetected planet, previous period of migratory planet [2-2]						
Planet	Observed P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	R (R_{Jup})	Planet	Calculated P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Compare observed P to calculated P
b	1.796302				0.111	b	2.03366				Equal to 88.33%
c	3.468095	1.93069	1	0	0.122	c	3.468095	1.70535	1	0	Reference planet
d	5.914323	1.70535	0.8833	-11.67	0.118	d	5.914323	1.70535	1	0	Equal to 100%
e	29.221289	4.94077	2.5591	155.9	0.168	x	10.0860	1.70535	1	0	Undetected
benchmark $ratio_{(P)}$=1.9307						y	17.2001	1.70535	1	0	Undetected
						e	29.221289	1.70535	1	0	Equal to 99.62%
						leading $ratio_{(P)}$ =1.70535					

Note 1. The symbol ω_P is the ratio parameter of periods and θ_P is the deviation rate of period ratio. They are calculated through equation (1) and (3).

Note 2. For the Kepler-299 system, according to trait (2), Planet d should migrate. Calculation with equation (9) shows that near 100% coincidence between the calculated and observed period of Planet e suggests the existence of the shared rules in the system. The calculation based on the periods of the planets that follow the shared rules reflects that Planet d migrated. The calculations agrees with trait (2).

Note 3. The analysis of the Kepler-286 system

Kepler-286 is a representative system. Equation (2) shows their deviation rates. Meanwhile, it shows (P_e:P_d)=99.62%(P_d:P_c)^3, suggesting two undetected planets according to trait (3). So, we first calculated Planet e by equation (9), showing $P_{e(cal)}=P_c \div [(P_d):(Pc)]^{(2-6)}$. It is 29.3323 days, 99.62% equal to its observed period. According to principle (1), Planet c, d and e follow the shared rules. Although it is an abnormal system, the shared rules still exist. If based on Planet d and c, the calculations show 10.0860 days by $P_x=3.468095 \div 1.70535^{(2-4)}$ and 17.2001 days by $P_y=3.468095 \div 1.70535^{(2-5)}$. They should belong to the undetected planets. The calculation also provides a 2.03366-day period by $P_{b(cal)}=P_c \div (P_d:P_c)^{(2-1)}$, suggesting Planet b migrated inwards. The calculations reveal the previous planetary configuration. If considering the physical parameters of the current planets, The dragging force may have ejected one undetected planets off the system and made the other fall into the star, or made them collide with each other.

Table S15. The 69 systems showing the shared rules

No.	System	Periods of the three planets (day)			Average of	Deviation rates (%)		Note
		longest	middle	shortest	two ratios	external	inner	
1	Kepler-229	41.20	16.069	6.253	2.567	-0.12	0.12	
2	Kepler-271	10.4355	7.4109	5.2497	1.410	-0.13	0.13	
3	Kepler-207	5.868	3.072	1.612	1.908	0.13	-0.13	
4	Kepler-289	125.852	66.0634	34.545	1.909	-0.19	0.19	
5	Kepler-249	15.37	7.114	3.307	2.156	0.20	-0.20	
6	Kepler-31	87.65	42.63	20.86	2.050	0.31	-0.31	
7	Kepler-398	11.42	6.834	4.081	1.661	-0.83	0.83	
8	YZ Cet	4.656	3.060	2.021	1.538	-1.06	1.06	
9	Kepler-332	34.21	16.00	7.626	2.116	1.07	-1.07	
10	Kepler-130	87.52	27.51	8.458	3.217	-1.10	1.10	Mixed system
11	HD 184010	836.4	484.3	286.6	1.7084	1.11	-1.11	
12	Kepler-1468	19.59	8.240	3.546	2.351	1.12	-1.12	
13	K2-219	11.14	6.668	3.901	1.690	-1.15	1.15	
14	Kepler-1130	5.453	4.272	3.267	1.2921	-1.20	1.20	Mixed system
15	Kepler-445	8.1528	4.8712	2.9842	1.6530	1.25	-1.25	
16	HD 39194	33.91	14.03	5.637	2.4530	-1.47	1.47	
17	K2-198	17.04	7.450	3.360	2.253	1.55	-1.55	
18	Kepler-106	51.26	23.67	11.43	2.119	2.22	-2.22	
19	Kepler-92	49.36	26.72	13.75	1.895	-2.55	2.55	
20	Kepler-206	23.44	13.14	7.782	1.736	2.79	-2.79	
21	Kepler-114	11.776	8.041	5.189	1.507	-2.82	2.82	
22	Kepler-394	12.13	8.005	5.614	1.471	3.02	-3.02	
23	Kepler-23	15.27	10.74	7.107	1.467	-3.07	3.07	
24	Kepler-60	11.90	8.919	7.133	1.2922	3.24	-3.24	
25	Kepler-191	17.74	9.940	5.945	1.728	3.28	-3.28	
26	Kepler-52	36.45	16.39	7.877	2.152	3.35	-3.35	
27	TOI-1749	9.050	4.492	2.388	1.948	3.41	-3.41	
28	Kepler-431	11.92	8.703	6.803	1.325	3.42	-3.42	
29	GJ 1061	13.03	6.689	3.204	2.018	-3.47	3.47	
30	Kepler-339	10.56	6.988	4.978	1.457	3.70	-3.70	
31	Kepler-288	56.63	19.31	6.097	3.050	-3.84	3.84	
32	Kepler-325	38.72	12.76	4.544	2.921	3.87	-3.87	
33	Kepler-295	33.88	21.53	12.65	1.638	-3.90	3.90	
34	Kepler-222	28.08	10.09	3.937	2.673	4.12	-4.12	
35	Kepler-350	26.14	17.85	11.19	1.528	-4.17	4.17	
36	K2-368	20.20	9.660	5.026	2.007	4.19	-4.19	
37	Kepler-83	20.09	9.770	5.170	1.973	4.22	-4.22	Mixed system
38	Kepler-53	38.56	18.65	9.752	1.981	4.37	-4.37	

39	Kepler-331	32.13	17.28	8.458	1.951	-4.69	4.69	
40	Kepler-244	20.05	9.767	4.312	2.159	-4.92	4.92	
41	Kepler-381	13.39	8.256	5.629	1.544	5.03	-5.03	
42	TOI-125	19.98	9.151	4.654	2.075	5.23	-5.23	
43	Kepler-450	28.46	15.41	7.515	1.949	-5.25	5.25	
44	Kepler-352	16.33	10.06	6.887	1.542	5.34	-5.34	
45	Kepler-226	8.109	5.350	3.941	1.437	5.50	-5.50	
46	K2-148	9.758	6.923	4.384	1.495	-5.69	5.69	Mixed system
47	Kepler-18	14.86	7.642	3.505	2.063	-5.74	5.74	
48	Kepler-357	49.50	16.86	6.475	2..770	5.99	-5.99	
49	HR 858	11.23	5.973	3.586	1.773	6.04	-6.04	Mixed system
50	Kepler-272	10.94	6.057	2.971	1.922	-6.06	6.06	
5	Kepler-374	5.028	3.283	1.898	1.631	-6.09	6.09	
052	K2-239	10.12	7.775	5.240	1.393	-6.60	6.60	
53	Kepler-446	5.149	3.036	1.565	1.818	-6.70	6.70	
54	Kepler-27	31.33	15.34	6.546	2.193	-6.84	6.84	
55	Kepler-54	21.00	12.07	8.011	1.623	7.16	-7.16	
56	Kepler-217	8.586	5.375	3.887	1.490	7.21	-7.21	
57	Kepler-30	143.3	60.32	29.33	2.216	7.23	-7.23	
58	K2-58	22.88	7.053	2.537	3.012	7.70	-7.70	
59	Kepler-81	20.84	12.04	5.955	1.876	-7.76	7.76	
60	Kepler-399	58.04	26.68	14.43	2.013	8.08	-8.08	
61	Kepler-196	122.1	47.43	20.74	2.381	8.10	-8.10	
62	Kepler-334	25.10	12.76	5.470	2.150	-8.50	8.50	
63	Kepler-1530	9.208	5.323	2.591	1.892	-8.56	8.56	
64	TOI-270	11.34	5.661	3.360	1.844	8.66	-8.66	
65	Kepler-1254	9.991	5.727	3.601	1.604	8.76	-8.76	
66	K2-381	26.80	16.03	7.939	1.846	-9.43	9.43	
67	K2-233	24.37	7.060	2.468	3.151	9.52	-9.52	
68	TOI-2076	35.13	21.02	10.36	1.851	-9.67	9.67	
69	Kepler-164	28.99	10.95	5.035	2.412	9.83	-9.83	
	Average	39.80	20.31	10.93	1.944	0.5576	-0.5576	

Note 1. Of the 191 three-planet systems discovered, the calculations with equation (1) and (3) show that 69 systems can be regarded as normal system according to the general standard, occupying about 36.13% of the 191 systems. If the standard is raised, 40 systems show the absolute values of deviation rates smaller than 5%. The planetary orbital relations of such systems should not be accidental, implying some inner connections between the orbits of the three planets of every system . Of them, 15 systems show the absolute values of deviation rates smaller than 1.5%, almost equal to the state of ideal systems. According to trait (1), the shared rules exist in the 69 systems. Here we only list the systems with the shared rules.

Note 2. The comparison of the period ratios reflects that some systems show continuously resonant orbits, even showing a chain of near MMRs. For example, The GJ 1061 system shows the continuously-resonant period ratios of near 2:1, or a chain of 1:2:4 period MMRs. Further comparisons suggest that the perturbations of side planets may make the middle planet migrate outwards, equivalent to about 2% of its initial period. The

Kepler-271 and Kepler-1130 systems also show the continuously-resonant period ratios, respectively embodying the chains of 2:3:4 and 3:4:5 period MMRs. Perhaps, both the middle planets may have migrated outwards, equivalent to about 5% of their original periods. For YZ Cet, the system show the near 4:6:9 period resonances. Because the perturbations between planets are mutual, the calculation of the orbital change for each planet should be complex. However, it is the gravitational perturbation that causes the shared rules becoming less pronounced.

Note 3. Of the 191 systems, 69 are normal systems, occupying 36%. Of the 191 systems, 17 are binary-star systems, occupying 8.9%. Of the 17 systems, 5 are binary-star systems, occupying 29%. So, the companion star has more or less some effects on its neighboring planetary system in a mixed system.

Note 4. The analysis of the average values of the 69 systems shows that most of the systems are discovered with the transit methods, the longest period of the average is 39.80 days, the average ratio of period ratios is the value close to 2, the absolute values of the average of all deviation rates are only 0.5576%. The outcomes of these orbital relations may bring more inspirations.

Table S16. Statistics of the deviation rates of the period ratios of adjacent planets of eight-planet, seven-planet and six-planet systems.

System Information					Number of period-ratio deviation rates (θ_P) of certain ranges					
No.	System	Number of planets	Number of ratios	br_P of the system	$\theta_P \le \pm 5\%$	$\pm 5 < \theta_P \le \pm 10\%$	$\pm 10 < \theta_P \le \pm 20\%$	$\pm 20 < \theta_P \le \pm 40\%$	$\theta_P > \pm 40\%$	System type
1	KOI-351	8	7	1.5631	2	2	1	1	1	abnormal
	Total	8	7		2	2	1	1	1	
	Proportion (%) of each range of θ_P				28.57	28.57	14.29	14.28	14.29	
1	TRAPPIST-1	7	6	1.5344	4	1	1	--	--	near-normal
	Total	7	6		4	1	1	--	--	
	Proportion (%) of each range of θ_P				66.67	16.67	16.66	--	--	
System Information					Number of period-ratio deviation rates (θ_P) of certain ranges					
No.	System	Number of planets	Number of ratios	br_P of the system	$\theta_P \le \pm 5\%$	$\pm 5 < \theta_P \le \pm 10\%$	$\pm 10 < \theta_P \le \pm 20\%$	$\pm 20 < \theta_P \le \pm 40\%$	$\theta_P > \pm 40\%$	System type
1	HD 10180	6	5	3.1760	1	--	2	1	1	abnormal
2	HD 191939	6	5	2.7275	2	--	1	--	2	abnormal
3	HD 219134	6	5	2.0103	2	1	--	--	2	abnormal
4	HD 34445	6	5	2.4570	1	--	--	3	1	abnormal
5	K2-138	6	5	1.530	4	--	--	--	1	abnormal
6	Kepler-11	6	5	1.4590	2	--	2	--	1	abnormal
7	Kepler-20**	6	5	1.7894	3	1	--	1	--	near-normal
8	Kepler-80	6	5	1.5227	3	--	1	--	1	abnormal
9	TOI-178	6	5	1.5799	2	1	1	1	--	near-normal
	Total	54	45	2.028/ave	20	3	7	6	9	
	Proportion (%) of each range of θ_P				44.44	6.67	15.56	13.33	20.0	

Note 1. The Symbol br_P is the benchmark ratio of periods and θ_P is the deviation rate of period ratio parameter.

Note 2. By October 2022, 1 eight-planet, 1 seven-planet and 9 six-planet extrasolar systems have been discovered according to the records published by NASA, by checking https://exoplanetarchive.ipac.caltech.edu/. The detailed calculations are presented in Supplementary Table S21-A31.

Note 3. The explanation of the proportion (%) of six-planet systems: The calculation of the proportion is based on 45 period ratios as the cardinal number. For example, 44.44% indicates the proportion of the exoplanets with the absolute values of deviation rates smaller than 5%, calculated according to (20÷45)×100%. For the KOI-35 and TRAPPIST-1 systems, the calculation is the same.

Note 4. ** expresses it is a muti-star system.

Table S17. Statistics of the deviation rates of the period ratios of adjacent planets of five-planet systems.

System information					Number of period-ratio deviation rates (θ_P) of certain ranges					
No.	System	Number of planets	Number of ratios	br_P of the system	$\theta_P \le \pm$ 5%	$\pm5<\theta_P$ $\le\pm10\%$	$\pm10<\theta_P$ $\le\pm20\%$	$\pm20<\theta_P$ $\le\pm40\%$	$\theta_P>\pm$ 40%	System type
1	55 Cnc A**	5	4	21.449	1	1	--	--	2	abnormal
2	GJ 667 C**	5	4	3.9081	1	1	--	--	2	abnormal
3	HD 108236	5	4	1.5711	2	--	1	--	1	abnormal
4	HD 158259	5	4	1.5332	4	--	--	--	--	normal
5	HD 23472	5	4	1.6124	2	1	1	--	--	near-normal
6	HD 40307	5	4	2.3819	--	2	1	---	1	abnormal
7	K2-268	5	4	1.8130	--	--	2	1	1	abnormal
8	K2-384	5	4	1.5234	2	1	--	1	--	near-normal
9	Kepler-102	5	4	1.5120	2	--	2	--	--	near-normal
10	Kepler-122	5	4	1.7375	2	--	1	1	--	near-normal
11	Kepler-150	5	4	2.3038	--	2	--	1	1	abnormal
12	Kepler-154	5	4	1.9787	2	--	1	1	--	near-normal
13	Kepler-169	5	4	1.7776	--	2	--	1	1	abnormal
14	Kepler-186	5	4	1.8529	2	1	--	--	1	abnormal
15	Kepler-238	5	4	2.1413	2	--	1	1	--	near-normal
16	Kepler-292	5	4	1.7185	2	--	2	--	--	near-normal
17	Kepler-296**	5	4	1.8411	3	1	--	--	--	normal
18	Kepler-32	5	4	2.3203	--	--	2	1	1	abnormal
19	Kepler-33	5	4	1.5562	--	2	1	--	1	abnormal
20	Kepler-444**	5	4	1.2604	3	1	--	--	--	normal
21	Kepler-55	5	4	2.1486	2	--	--	2	--	near-normal
22	Kepler-62	5	4	2.1806	2	--	--	1	1	abnormal
23	Kepler-82	5	4	2.2130	--	--	2	1	1	abnormal
24	Kepler-84	5	4	2.0976	2	--	--	2	--	near-normal
25	TOI-561	5	4	1.5420	2	--	--	--	2	abnormal
	Total	125	100	2.719/ave	38	15	17	14	16	
	Proportion (%) of each range of θ_P				38.0	15.0	17.0	14.0	16.0	

Note 1. The Symbol br_P is the benchmark ratio of periods and θ_P is the deviation rate of period ratio parameter.

Note 2. By October. 2022, twenty-five extrasolar systems with five confirmed planets have been discovered according to the records published by NASA, by checking https://exoplanetarchive.ipac.caltech.edu/. The statistics of the calculations of these systems with equation (4) are presented in the table. The details are offered in Supplementary Table S31-A56.

In addition, the systems of HD 20782 and HD 20781is taken as a four-planet system, which is analysed in the group of four-planet systems. Moreover, the HIP 41378 system shows the great uncertainty of the planetary orbits, which can not be analysed objectively.

Note 3. The explanation of the proportion (%): The calculation of the proportion is based on 100 period ratios as the cardinal number. For example, 38% indicates the proportion of the exoplanets with the absolute value of deviation rates that is smaller than 5%. It is calculated according to (38÷100)×100%.

Note 4. ** expresses it is a muti-star system.

Table S18. Statistics of the deviation rates of period ratios of adjacent planets in the systems with five or more planets and the statistics of the exoplanets following the shared rules.

System Information						Number of period ratios deviation rates (θ_P) of certain ranges					Planets with CCBPO†	
No.	System	Mass ($M_\odot$)	Age (Gyr)	Planet number	System br_P	$\theta_P \leq$ 5%	$5<\theta_P$ ≤10%	$10<\theta_P$ ≤20%	$20<\theta_P$ ≤40%	$\theta_P>$ 40%	Criterion (N-A)	Criterion (N-B)
1	KOI-351	1.2	--	8	1.5631	2	2	1	1	1	4	4
2	TRAPPIST-1	0.08	7.6±2.2	7	1.5344	4	1	1	--	--	3	5
3	HD 10180	1.06	4.3±0.5	6	3.1760	1	--	2	1	1	3	5
4	HD 191939	0.81	>8.7	6	2.7250	2	--	1	--	2	3	3
5	HD 219134	0.81	11±2.2	6	2.0103	2	1	--	--	2	3	3
6	HD 34445	1.07	8.5±2	6	2.4570	1	--	--	3	1	3	4
7	K2-138	0.935	$2.8^{+3.8}_{-1.7}$	6	1.530	4	-	--	--	1	5	5
8	Kepler-11	0.961	$6.92^{+1.7}_{-2.19}$	6	1.4590	2	--	2	--	1	4	5
9	Kepler-20	0.948	7.6±3.7	6	1.7894	3	1	--	1	--	4	5
10	Kepler-80	0.73	2±1	6	1.5227	3	--	1	--	1	3	4
11	TOI-178	0.65	$7.1^{+6.1}_{-5.3}$	6	1.5799	2	1	1	1	--	5	5
12	55 Cnc A	0.873	~9.5	5	21.449	1	1	--	--	2	3	3
13	GJ 667 C	0.33	>2	5	3.9081	1	1	--	--	2	3	3
14	HD 108236	0.869	$6.7^{+4}_{-5.1}$	5	1.5711	2	--	1	--	1	3	5
15	HD 158259	1.08	--	5	1.5332	4	--	--	--	--	5	5
16	HD 23472	0.67	--	5	1.6124	2	1	--	1	--	3	3
17	HD 40307	0.77	2.6/0	5	2.3819	--	2	1	---	1	3	4
18	K2-268	0.84	--	5	1.8130	--	--	2	1	1	4	4
19	K2-384	0.330	--	5	1.5234	2	1	--	1	--	4	4
20	Kepler-102	0.81	$4.07^{+2.66}_{-2.02}$	5	1.5120	2	--	2	--	--	3	3
21	Kepler-122	1.03	$3.89^{+2.66}_{-2.0}$	5	1.7375	2	--	1	1	--	3	3
22	Kepler-150	0.97	$4.57^{+3.23}_{-2.51}$	5	2.3038	--	2	--	1	1	--	3
23	Kepler-154	0.98	$4.47^{+2.58}_{-2.42}$	5	1.9787	2	--	1	1	--	3	4
24	Kepler-169	0.81	$3.72^{+2.71}_{-1.86}$	5	1.7776	--	2	--	1	1	4	4
25	Kepler-186	0.544	$3.89^{+2.71}_{-1.86}$	5	1.8529	2	1	--	--	1	3	3
26	Kepler-238	1.06	$6.76^{+2.05}_{-2.25}$	5	2.1413	2	--	1	1	--	4	4
27	Kepler-292	0.88	$5.13^{+3.72}_{-2.98}$	5	1.7185	2	--	2	--	--	3	4
28	Kepler-296	0.498	$4.37^{+5.6}_{-2.69}$	5	1.8411	3	1	--	--	--	4	5
29	Kepler-32	0.58	$2.69^{+3.46}_{-1.17}$	5	2.3203	--	--	2	1	1	--	4
30	Kepler-33	1.291	$4.37^{+1.37}_{-1.05}$	5	1.5562	--	2	1	--	1	3	4
31	Kepler-444	0.758	11.23±0.99	5	1.2604	3	1	--	--	--	3	5

32	Kepler-55	0.72	$2.34^{+2.54}_{-0.88}$	5	2.1486	2	--	--	2	--	3	4
33	Kepler-62	0.69	$2.34^{+2.15}_{-1.02}$	5	2.1806	2	--	--	1	1	4	4
34	Kepler-82	0.91	$4.68^{+3.55}_{-2.71}$	5	2.2130	--	--	2	1	1	--	--
35	Kepler-84	1.0	$3.98^{+1.83}_{-1.91}$	5	2.0976	2	--	--	2	--	--	4
36	TOI-561	0.785	10±3	5	1.5730	2	--	--	--	2	3	4
	Total	0.815/a	5.3436/a	194/158*	2.482/a	64	21	25	22	26	111	141
Proportion (%) of each range of θ_P for the 36 systems						40.51	13.29	15.82	13.92	16.46	32 systems	35 systems

Note 1. The symbol br_P is the benchmark ratio of periods and θ_P is the deviation rate of period ratio parameter.

Note 2. The explanation of 158**: The value represents the number of the period ratios of the 36 systems. For example. The TOI-561 system has five planets, which compose four period ratios of adjacent planets. In other words. Because the number of the period ratios of each system is one less than the number of the planets, the total number of the period ratios of the 36 systems is158 by 194-36=158, which is taken as the cardinal number in the calculation of proportions.

Note 3. The explanation of the proportion (%): The calculation of the proportion is based on 158 period ratios as the cardinal number. For example, 40.51% indicates the proportion of the exoplanets with the absolute value of deviation rates smaller than 5%, calculated according to (64÷158)×100%.

Note 3. By October. 2022, thirty-six extrasolar systems with more than four confirmed planets have been discovered according to the records published by NASA. The detailed calculations of the thirty-six systems are offered in Supplementary Table S21-A56respectively.

Note 4. Note 4. The explanation of "Planets following SRBPO†": SRBPO represents the shared rules between Planetary Orbits. The corresponding value shows the number of the planets with SRBPO, which are calculated according to Criterion (N-A) and (N-B) (explained in Section 3.4 of the paper). According to principle (2), they are the system with SRBPO if the number of the planets satisfying the criterion reaches three or more.

Table S19. Statistics of the deviation rates of the period ratios of adjacent planets of four-planet systems and statistics of the exoplanets following the shared rules.

System information				Number of period-ratio deviation rates (θ_P) of certain ranges					Remark	Planets following CCBPO#	
No.	System	Number of ratios	br_P of the system	dr≤5	5<dr ≤10	10<dr ≤20	20<dr ≤40	dr>40	System type	Criterion (N-A)	Criterion (N-B)
1	DMPP-1	3	1.9140	1	--	--	1	1	abnormal	3	3
2	GJ 676 A**	3	9.8929	1	--	--	1	1	abnormal	--	--
3	GJ 3293	3	2.3085	1	--	1	1	--	near-normal	--	3
4	GJ 876	3	2.0367	2	--	--	--	1	abnormal	3	3
5	HD 141399	3	4.6738	1	--	1	--	1	abnormal	--	3
6	HD 160691	3	6.5384	1	--	--	--	2	abnormal	--	--
7	HD 164922	3	3.3501	1	--	--	--	2	abnormal	3	3
8	HD 20781**	3	2.6142	1	--	2	--	--	near-normal	3	3
9	HD 20794	3	2.1902	2	--	--	1	--	near-normal	3	3
10	HD 215152	3	1.4920	1	--	1	1	--	near-normal	3	3
11	HD 3167	3	3.5203	2	--	--	--	1	abnormal	3	3
12	HR 8799	3	2.1667	2	1		--	--	normal (α)	3	3
13	K2-133	3	2.2648	1	1	--	1	--	near-normal	--	3
14	K2-187	3	2.4892	1	--	--	1	1	abnormal	--	--
15	K2-266**	3	1.8809	1	--	--	1	1	abnormal	--	3
16	K2-285	3	1.4648	2	--	--	--	1	abnormal	4	4
17	K2-32**	3	2.2979	1	--	1	1	--	near-normal	3	3
18	K2-72	3	1.5906	1	--	1	1	--	near-normal	--	3
19	KOI-94**	3	2.4312	1	--	2	--	--	near-normal	3	3
20	Kepler-106	3	1.8284	2	--	--	1	--	near-normal	3	3
21	Kepler-107	3	1.6237	1	1	1	--	--	near-normal	--	3
22	Kepler-132**	3	2.8076	1	--	--	--	2	abnormal	--	--
23	Kepler-1388	3	1.7957	2	--	--	1	--	near-normal	--	3
24	Kepler-1542	3	1.2911	1	2	--	--	--	normal	--	4
25	Kepler-167**	3	2.9440	1	--	--	--	2	abnormal	3	3
26	Kepler-172	3	2.2894	2	1	--	--	--	normal	3	4
27	Kepler-176	3	2.0182	2	--	1	--	--	near-normal	3	3
28	Kepler-197**	3	1.6080	1	1	1	--	--	near-normal	3	4
29	Kepler-208	3	1.4909	2	--	1	--	--	near-normal	3	3
30	Kepler-215	3	2.1043	2	--	--	1	--	near-normal	3	3
31	Kepler-220	3	2.1718	1	--	--	1	1	abnormal	--	3
32	Kepler-221	3	1.8294	2	--	1	--	--	near-normal	3	3
33	Kepler-223	3	1.3338	2	--	1	--	--	near-normal	3	3
34	Kepler-224	3	1.8912	2	--	1	--	--	near-normal	3	3

35	Kepler-235	3	2.3426	2	1	--	--	--	normal	3	3
36	Kepler-24	3	1.5404	2	--	--	1	--	near-normal	3	3
37	Kepler-245	3	2.3312	2	--	1	--	--	near-normal	3	3
38	Kepler-251	3	3.3067	2	--	--	--	1	abnormal	3	3
39	Kepler-256	3	1.8293	1	1	1	--	--	near-normal	--	4
40	Kepler-26	3	2.7135	1	--	--	1	1	abnormal	--	--
41	Kepler-265	3	2.4873	2	--	--	1	--	near-normal	3	3
42	Kepler-282	3	1.7878	2	--	1	--	--	near-normal	3	3
43	Kepler-286	3	1.9307	1	--	1	--	1	abnormal	3	3
44	Kepler-299	3	2.3524	1	2	--	--	--	normal	3	3
45	Kepler-304	3	1.8160	1	--	1	1	--	near-normal	--	3
46	Kepler-305	3	1.7118	1	--	2	--	--	near-normal	3	3
47	Kepler-306	3	2.3912	1	1	--	1	--	near-normal	--	3
48	Kepler-338	3	1.7710	2	--	1	--	--	near-normal	3	3
49	Kepler-341	3	1.5418	2	--	--	--	1	abnormal	3	3
50	Kepler-342	3	1.7293	1	--	1	--	1	abnormal	--	--
51	Kepler-37	3	1.5936	1	--	2	--	--	near-normal	--	3
52	Kepler-48	3	4.4342	1	--	--	--	2	abnormal	--	--
53	Kepler-49	3	1.7040	1	--	1	--	1	abnormal	--	--
55	Kepler-402	3	1.4566	2	--	1	--	--	near-normal	3	3
54	Kepler-411**	3	2.6070	1	--	--	1	1	abnormal	--	--
56	Kepler-65	3	2.7194	1	--	--	--	2	abnormal	3	3
57	Kepler-758	3	1.6926	2	--	1	--	--	near-normal	3	3
58	Kepler-79	3	1.9009	1	1	1	--	--	near-normal	--	3
59	Kepler-85	3	1.4317	2	1	--	--	--	normal	3	4
60	L 98-59	3	1.7174	2	--	1	--	--	near-normal	3	3
61	V1298 Tau	3	1.9462	1	--	--	2	--	near-normal	--	--
62	WASP-47	3	5.2674	1	--	--	--	2	abnormal	--	--
63	Tau Cet	3	3.2963	1	--	--	2	--	near-normal	3	3
64	TOI-1264	3	2.0327	1	--	--	1	1	abnormal	3	3
65	TOI-500	3	3.9534	1	--	--	--	2	abnormal	--	--
	Total	195	2.4228/av	91	14	32	25	33	n6/n34/a25	118	165
	Proportion (%) of each range of θ_P			46.67	7.18	16.41	12.82	16.92		39 systems	53 systems

Note 1. The Symbol br_P is the benchmark ratio of periods and θ_P is the deviation rate of period ratio parameter.

Note 2. By October. 2022, sixty-five extrasolar systems with four confirmed planets have been discovered according to the records published by NASA, by checking https://exoplanetarchive.ipac.caltech.edu/. The calculations of these systems are based on equation (4), like the systems in Table S2.

Note 2. The explanation of the proportion (%): The calculation of the proportion is based on 186 period ratios as the cardinal number. For example, 46.67% indicates the proportion of the exoplanets with the absolute value of deviation rates smaller than 5%, calculated according to (91÷195)×100%.

Note 3. In "Number of period ratios with θ_P of certain ranges", we show the number of the deviation rates of the period ratios of each system for a certain range. Based on the deviation rates, the 65 systems can be classified into different types of systems. Normal systems show the absolute values of all deviation rates smaller than 10%. Near-normal systems show the absolute values of ore or two deviation rates between 10-40%. Abnormal systems show the absolute values of one or two deviation rates bigger than 40%. The classification of the systems may help analyse the changes of planetary configurations.

Note 4. The explanation of "Planets following SRBPO#": SRBPO represents the shared rules between Planetary Orbits. The corresponding value shows the number of the planets with SRBPO, which are calculated according to Criterion (N-A) and (N-B) (explained in Section 3.4 of the paper). According to principle (2), they are the system with SRBPO if the number of the planets satisfying the criterion reaches three or more.

Note 5. Limited by pages, we do not show the detailed calculations of all the 65 systems. We only showed part of the systems (Supplementary Table 2, 43-50, etc). The calculations of the deviation rates of the period ratios of adjacent planets of all 65 systems are the same as that of the Kepler-85 system (Supplementary Table 2) and Kepler-286 system (Supplementary Table 14).

Note 6. ** expresses that it is a muti-star system.

Table S20. Statistics of the deviation rates of the semimajor-axis ratios of adjacent planets of the systems with four and more planets.

System Information					Number of semimajor-axis ratio deviation rates (θ_P) of certain ranges					Relationship between br_P and br_α reflected by equation (11) †
No.	System	Number of Planets	Number of ratios	br_α of the system	$dr_\alpha \le \pm 5\%$	$\pm 5 < dr_\alpha \le \pm 10\%$	$\pm 10 < dr_\alpha \le \pm 20\%$	$\pm 20 < dr_\alpha \le \pm 40\%$	$dr_\alpha > \pm 40\%$	
1	KOI-351	8	6	1.3393	1	1	3	--	1	$br_P=100.84\%(br_\alpha)^{3/2}$
2	TRAPPIST-1	7	5	1.3333	3	2	--	--	--	$br_P=99.97\%(br_\alpha)^{3/2}$
3	HD 10180	6	5	2.1579	1	2	1	1	--	$br_P=100.2\%(br_\alpha)^{3/2}$
4	HD 191939	6	5	1.9521	2	--	1	--	2	$br_P=100\%(br_\alpha)^{3/2}$
5	HD 219134	6	5	1.6524	3	--	--	1	1	$br_P=98.24\%(br_\alpha)^{3/2}$
6	HD 34445	6	5	1.8107	1	--	2	1	1	$br_P=100.84\%(br_\alpha)^{3/2}$
7	K2-138	6	5	1.3280	4	--	--	--	1	$br_P=99.995\%(br_\alpha)^{3/2}$
8	Kepler-11	6	5	1.3296	1	2	1	--	1	$br_P=100.25\%(br_\alpha)^{3/2}$
9	Kepler-20	6	5	1.4761	3	1	1	--	--	$br_P=99.78\%(br_\alpha)^{3/2}$
10	Kepler-80	6	4	1.3300	2	1	--	--	1	$br_P=99.27\%(br_\alpha)^{3/2}$
11	TOI-178	6	5	1.3563	3	1	1	--	--	$br_P=100.01\%(br_\alpha)^{3/2}$
12	55 Cnc A	5	4	7.7283	2	--	--	--	2	$br_P=99.83\%(br_\alpha)^{3/2}$
13	GJ 667 C	5	4	2.4795	2	--	--		2	$br_P=100.10\%(br_\alpha)^{3/2}$
14	HD 108236	5	4	1.3278	3	--	--	1	--	$br_P=102.69\%(br_\alpha)^{3/2}$
15	**HD 23472**	**5**	**4**	**1.3745**	**2**	**1**	**1**	**--**	**--**	$\mathbf{br_P=100.06\%(br_\alpha)^{3/2}}$
16	HD 40307	5	4	1.7885	2	1	--	1	--	$br_P=99.59\%(br_\alpha)^{3/2}$
17	Kepler-122	5	3	1.4645	2	--	1	--	--	$br_P=99.31\%(br_\alpha)^{3/2}$
18	Kepler-150	5	4	1.7382	2	--	1	--	1	$br_P=100.54\%(br_\alpha)^{3/2}$
19	Kepler-169	5	4	1.4750	--	2	1	--	1	$br_P=98.92\%(br_\alpha)^{3/2}$
20	Kepler-186	5	4	1.5701	--	--	3	--	1	$br_P=94.18\%(br_\alpha)^{3/2}$
21	Kepler-292	5	4	1.4401	3	--	1	--	--	$br_P=98.92\%(br_\alpha)^{3/2}$
22	Kepler-296	5	4	1.5013	4	--	--	--	--	$br_P=100.08\%(br_\alpha)^{3/2}$
23	Kepler-32	5	4	1.6576	--	2	1	--	1	$br_P=108.72\%(br_\alpha)^{3/2}$
24	Kepler-33	5	4	1.3421	--	2	1	1	--	$br_P=100.1\%(br_\alpha)^{3/2}$
25	Kepler-444	5	4	1.1626	3	1	--	--	--	$br_P=100.03\%(br_\alpha)^{3/2}$
26	Kepler-62	5	4	1.6807	2	--	--	1	1	$br_P=100.07\%(br_\alpha)^{3/2}$
27	Kepler-82	5	4	1.7075	--	2	--	1	1	$br_P=99.184\%(br_\alpha)^{3/2}$
28	TOI-561	5	4	1.3365	2	--	--	--	2	$br_P=100.013\%(br_\alpha)^{3/2}$
	Total	154	122	1.780/*av*	53	21	20	8	20	
	Proportion (%) for each range br_α				43.45	17.21	16.39	6.56	16.39	

Note 1. "br_α" is the benchmark ratio of semimajor axes. "θ_α" is the deviation rate of period ratio parameter..

Note 2. By October. 2022, thirty-six extrasolar systems with more than four confirmed planets have been discovered. Of the systems, only twenty-eight completely show exoplanetary semimajor axes. So, the statistics show the results of the twenty-eight systems.

Note 3. The calculations of all system are based on equation (4). The detailed calculations of the systems are provided in Supplementary Table 56-89.

Note 4. The prompt "†" shows the relationship between the benchmark ratio of periods and that of semimajor axes in the corresponding system that is calculated according to equation (11). The values of br_P of the corresponding system is in Supplementary Table 22 and br_a is in Supplementary Table 28. For the detailed data of the corresponding system, please see Supplementary Table S21-A56. Of all systems, only the value of the calculations of the Kepler-32 System reaches 108.72% and Kepler-186 is 94.18%. The calculations suggest that the orbital observation data of all systems are correct to a high accuracy except two systems.

Remark: Table S21-A56 show the detailed calculations of the orbital ratio parameters and their deviation rates of the 36 systems with four and more planets. They are summed up in Table S18-20 and S22. Meanwhile, the tables provide the analyses of the completeness and evolution of corresponding systems.

Table S21. Calculations of the orbital ratio parameters and deviation rates of adjacent planets of the KOI-351 (Kepler-90) system and analysis of the change of the planetary configuration.

		Analysis of the periods of observed planets [1]				Analysis of the semimajor axes of observed planets [2]					
No.	Planet	P_{obs} (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	a_{obs} (AU)	Ratio of a_n:a_{n-1}	ω_a	θ_a (%)	Mass ($M_{Jup.}$)	Ratio of M_n:M_{n-1}
1	b	7.008151				0.074±0.016				0.117	
2	c	8.719375	1.2442	0.7960	-20.4	0.089±0.016	1.2027	0.8980	-10.2	0.106	0.9060
3	i	14.44912	1.6571	1.0601	6.01	---	↓			0.118	1.1132
4	d	59.73667	4.1343	2.6449	164.5	0.32±0.05	3.5955	2.685	169	0.256	2.1695
5	e	91.93913	1.5391	0.9846	-1.54	0.42±0.06	1.3125	0.980	-0.2	0.237	0.9258
6	f	124.9144	1.3587	0.8692	-13.1	0.48±0.09	1.1429	0.8534	-14.7	0.257	1.0844
7	g	210.6070	1.6860	1.0786	7.86	0.71±0.08	1.4792	1.1045	10.5	0.723	2.8132
8	h	331.6006	1.5745	1.0073	0.73	1.01±0.11	1.4225	1.0621	6.21	1.008	1.3942
		benchmark ratio$_{(P)}$=1.5631				benchmark ratio$_{(a)}$=1.3393				†br_P=100.85%$(br_a)^{3/2}$	

		Period calculations of all planets based on the planets that follow the shared rules [3]					Semimajor axis calculations of all planets based on the planets that follow the shared rules [4]				
No.	Planet	P_{cal} (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	##Compare P_{obs} to P_{cal}	a_{cal} (AU)	Ratio a_n:a_{n-1}	ω_a	θ_a (%)	Compare a_{obs} to a_{cal}
1	b	5.57611				Equal to 125.68%	0.0657				Equal to 112.67%
2	c	8.77959	1.5745	1	0	Equal to 99.314%	0.0890	1.3548	1	0	Equal to 100%
3	i	13.8235	1.5745	1	0	Equal to 104.53%	0.1206	1.3548	1	0	Predicted orbit
4	x	21.7650	1.5745	1	0	Undetected planet	0.1633	1.3548	1	0	Undetected planet
5	y	34.2690	1.5745	1	0	Undetected planet	0.2213	1.3548	1	0	Undetected planet
6	d	53.9566	1.5745	1	0	Equal to 110.71%	0.2998	1.3548	1	0	Equal to 106.74%
7	e	84.9547	1.5745	1	0	Equal to 108.22%	0.4062	1.3548	1	0	Equal to 103.40%
8	f	133.761	1.5745	1	0	Equal to 93.386%	0.5503	1.3548	1	0	Equal to 87.23%
9	g	210.6070	1.5745	1	0	Equal to 100%	0.7455	1.3548	1	0	Equal to 95.24%
10	h	331.6006	1.5745	1	0	Reference planet	1.01	1.3548	1	0	Reference planet
		leading ratio$_{(P)}$=1.5745				leading ratio$_{(a)}$=1.3548					

Note 1. The symbol ω_P is the ratio parameter of periods and θ_P is the deviation rate of ω_P. They are calculated according to equation (1) and (3). The symbol ω_a is the ratio parameter of semimajor axes and θ_a is the deviation rate of semimajor axis ratio. They are calculated according to equation (2) and (4). The symbol † is calculated according to equation (11) where br_P is benchmark ratio$_{(P)}$ and br_a is benchmark ratio$_{(a)}$.

Note 2. The symbol '##' reflects the degree to which the observed period of a planet is equal to its calculated period. For Planet b, "Equal to 125.68%" reflects that the calculated period is equal to 125.68% of its observed period by (7.008151:5.57611)×100%=125.68%.

Note 3. Analysis of the change of planetary configuration

The calculation here is to supplement the in-paper analysis of the KOI-351 system. The calculations show that the period ratio of Planet d to i is 4.1343 and the benchmark ratio is 1.5631. The two values reflect P_d:P_i=108.3%$(dr_P)^2$. Because the system has eight planets and some planets may have migrated, the equation does not show 100% equality. Even so, the equation may still suggest two undetected planets between Planet d and i according to equation (7). So we calculated the possible periods of all planets with equation (9). If taking the period ratio of Planet h to g as leading ratio and Planet h as reference planets, the calculations provide ten periods. Because Planet h, g and c should follow the shared rules, the values of 21.765 days by $P_x=P_h\div(P_h:P_g)^{(9-4)}$ and 34.269 days by $P_y=P_h\div(P_h:P_g)^{(9-5)}$ should be the periods of the two undetected planets. If considering the two undetected planets, the big deviation rate of the system would disappear (See [3]).

Meanwhile, we calculated the semimajor axes of all planets with equation (10). Because the semimajor axes of Planet h and g show a certain uncertainty, we took the shared divisor (sd) from Planet h to c as the leading ratio according to the contents of the standard rule according to $sd=(P_h\div P_c)^{1/(snh-snc)}$. By taking Planet h as reference planets, the calculated semimajor axes are presented in the table (See [4]). The calculations reflect the same results as the periods. In addition, since the semimajor axis of Kepler-90i is not published, we tried to calculate its value, showing 0.1206 AU by $\alpha_{i(cal)}=1.01\div(1.3548)^{(10-3)}$. Both calculations to the periods and semimajor axes reflect the changes of planetary configuration.

The analysis of the changes of the planetary configuration should be complex. Here we will just offer a qualitative inference based on observed physical parameters. Because the difference between the semimajor axes of Planet g and Planet f is only 0.23 AU (α_g-=0.71AU and α_f=0.48AU), the influence of the dragging force of Planet g on Planet f should be very great if considering $R_g=0.723R_{Jup}$. Although the mass of Planet g is not sure, if comparing the effect of Planet g on Planet f (g_g) with that of Jupiter on an asteroid (g_{Jup}) in asteroid belt (See table S23), it can still be judged that the influence of Planet g should be greater than that of Jupiter according to the calculations by equation (14) of $g_{(f)}:g_{(Jup)}=[(M_{(f)})\div(D_{a(f)-a(c)})^2]:[(M_{(Jup)})\div(D_{a(Jup)-a(ast)})^2]$. The effect of Kirkwood gap might first broke the stable motion of Planet f. Then, its dragging force made Planet f change its orbital shape and migrate inwards. As a result, Planet f got closer to Planet e, especially at its perihelion. Because Planet f and e have similar sizes, the mutual dragging forces made Planet f keep migrating inwards and Planet e to migrate outwards. Conductive perturbations perhaps occurred between planets, which led to the orbital changes of inner planets, embodying the outward migration of Planet d and inward migration of Planet y. Planet x might also disturbed. If Planet y or Planet x had an elliptical orbit, they would be very close at perihelion or aphelion. At some point, Planet y ejected Planet x off its orbit, making it disappear from the system. The ejection made Planet y lose kinetic energy again and fall towards the star. It quickly crossed the orbits of Planet i and c. When it fell to the orbital region of current Planet b, different forces might balance its motion and make Planet y settle there, revolving around the star at a new orbit. The current Planet b may be the previous Planet y. There is another possibility. Planet y interacted with Planet b, causing Planet b to migrate outward. As Planet y lost kinetic energy again, it fell into the star. The orbital changes of these planets comply with the contents of the law of kinetic energy conversion. Granados and Boley (*32*) also researched the KOI-351 system. They performed direct numerical simulations and found that the outer huge planet sometimes may cause system instability partly due to secular eccentricity resonances.

Note 4. The discovery of Kepler-90i supports the shared rules.

Before Kepler-90i was discovered, it was a seven-planet system. When I calculated the period of Planet c with equation (9) and by taking Planet g as reference planet and the period ratio of Planet h to g as leading ratio, its calculated period was 8.77959 days by $P_{c(cal)}=210.607\div1.5745^{(9-2)}$, equaling to 99.3% of its observed planet. According to principle (1), Planet h, g and c followed the shared rules. According to principle (2), the shared rules existed in the system at that time. In this case, the calculations based on Planet h and g predicted the periods of three undetected planets, showing $P_1=210.607\div1.5745^{(9-5)}$=34.269d, $P_2=210.607\div1.5745^{(9-4)}$=21.765d and

$P_3=210.607 \div 1.5745^{(9-3)}=13.8235d$. In Dec. 2017, Kepler-90i was discovered. The degree of the equality between the calculated and observed periods excesses 95% for Kepler-90i. The calculation showed the realistic meaning of the shared rules.

Note 5. A possibly-undetected exoplanet outside Planet h

In addition, KOI-351h is a Jupiter-sized planet, there should be a planet outside of its orbit if considering the matter transition in the protoplanetary disk. Its period is likely to be about 522 days by $P=331.6006 \div 1.5745^{(10-11)}$. If the observation time is not enough, we may not see its transit. If it orbits the star with an inclination a little different from the observed exoplanets, the transit may not occur on the planet in the Earth direction as its period is relatively too long. In other words, it is too far from the star. Perhaps a different method may detect its existence. More observations are necessary.

Table S22. Calculations of the orbital ratio parameters and deviation rates of adjacent planets of the TRAPPIST-1 system and analysis of the change of the planetary configuration.

Comparison of the periods of adjacent planets [1]						Comparison of the semimajor axes of adjacent planets [2]					
Planet	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	MMR Chains	α (AU)	Ratio of α_n:α_{n-1}	ω_α	θ_α (%)	Mass (R_{Earth})	Ratio of M_n:M_{n-1}
b	1.510826					0.01154				1.374	
c	2.421937	1.6031	1.0446	4.46	Near 5:3	0.01580	1.3692	1.0292	2.92	1.308	0.952
d	4.049219	1.6719	1.0894	8.94	Near 5:3	0.02227	1.4095	1.0595	5.95	0.388	0.297
e	6.101013	1.5067	0.9818	-1.82	Near 3:2	0.02925	1.3134	0.9873	-1.27	0.692	1.784
f	9.207540	1.5092	0.9834	-1.66	Near 3:2	0.03849	1.3169	0.9899	-1.01	1.039	1.501
g	12.35245	1.3416	0.8742	-12.58	--	0.04683	1.2167	0.9146	-8.54	1.321	1.271
h	18.77287	1.5198	0.9903	-0.97	--	0.06189	1.3216	0.9935	-0.65	0.326	0.2468
		benchmark ratio$_{(P)}$ =1.5347					benchmark ratio$_{(\alpha)}$=1.3303			†br_P=100.02%$(br_\alpha)^{3/2}$	

Period calculations of all planets based on the planets that follow the shared rules [3]						Semimajor axis calculations of all planets based on the planets that follow the shared rules [4]				
Planet	Calculated P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Compare observed P to calculated P	α (AU)	Ratio of α_n:α_{n-1}	ω_α	θ_α (%)	Compare observed P to calculated P
b	1.77485				Equal to 85.12%	0.01281				Equal to 90.09%
c	2.67860	1.5092	1	0	Equal to 90.42%	0.01687	1.3169	1	0	Equal to 93.66%
d	4.04255	1.5092	1	0	Equal to 100.17%	0.02221	1.3169	1	0	Equal to 100.27%
e	6.101013	1.5092	1	0	Reference Planet	0.02925	1.3169	1	0	Reference Planet
f	9.20754	1.5092	1	0	Equal to 100%	0.03849	1.3169	1	0	Equal to 100%
g	13.8962	1.5092	1	0	Equal to 88.89%	0.05069	1.3169	1	0	Equal to 92.39%
h	20.9721	1.5092	1	0	Equal to 89.51%	0.06675	1.3169	1	0	Equal to 92.72%
		leading ratio$_{(P)}$=1.5092					leading ratio$_{(\alpha)}$=1.3169			

Note 1. The symbol ω_P is the period ratio parameter and θ_P is the deviation rate of ω_P. They are calculated according to equation (1) and (3). The symbol ω_α is the semimajor axis ratio parameter and θ_α is the deviation rate of ω_α. They are calculated according to equation (2) and (4). The symbol † is calculated according to equation (11) where br_P is benchmark ratio of period ratios and br_α is benchmark ratio of semimajor axis ratios.

Note 2.The judgement of the planets that follow the shared rules

Since the calculation with equation (1) and (3) show a 12.58% deviation rate of period ratios, it should be a near-normal system according to the general standard. However, if judged according to trait (1), the period ratios of Planet f, e and d are the planets following the shared rules. According to principle (2), the shared rules still exist in the system. We can also judge the properties of all planets with corresponding equations and principles. Calculated with equation (9) and by taking Planet e as reference planet and the ratio of Planet f to e as the leading ratio, the calculated periods are presented in the table (See [3]). The calculation of Planet d shows an almost 100% equality, suggesting that Planet d, e and f follow the shared rules according to principle (1). Therefore, the shared rules exist in the system according to principle (2), which shows the same result as trait (1). Since the calculations are based on the planets that follow the shared rules, they also suggest that Planet b, c, g and h should migrate according to principle (3). The calculation to semimajor axes with equation (10) and the judgment according to principle (1), (2) and (3) show the same results (See [4]).

As to the reason of the migrations of the four planets, if considering the masses of the seven planets and the orbital relations among the middle three planets, it is possible that a relatively big stray planet passed through the system. The gravitational disturbance of the stray planet affected the orbiting speeds of the four planets of TRAPPIST-1, causing them to changed their orbits and embodying that they migrate outward relatively.The way that the stray planet passed through the system decided its influences on the motion of the planets of TRAPPIST-1, which might lead to a different result. Our analysis and inference are just a suggestion. More studies are required.

Note 3. The planetary physical parameter references are based on Agol et al. (2021) and published by NASA EXOPLANET ARCHIVE on November 17, 2022.

Table S23. Calculations of the period ratio parameters and deviation rates of adjacent planets of the HD 10180 system and analysis of the change of the planetary configuration.

Comparison of the periods of adjacent planets					Comparison of the semimajor axes of adjacent planets					
Planet	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	α (AU)	Ratio of α_n:α_{n-1}	ω_α	θ_α (%)	Mass ($M_{Jup.}$)	Ratio of M_n:M_{n-1}
c	5.75969				0.06412				0.0416	
d	16.357	2.8399	0.8942	-10.58	0.12859	2.0055	0.9294	-7.06	0.0378	0.9087
e	49.748	3.0414	0.9576	-4.24	0.2699	2.0989	0.9727	-2.73	0.0805	2.1296
f	122.744	2.4673	0.7769	-22.31	0.4929	1.8262	0.8463	-15.37	0.0722	0.8969
g	604.67	4.9263	1.5511	55.11	1.427	2.8951	1.3416	34.16	0.0732	1.0139
h	2205.0	3.6466	1.1482	14.82	3.381	2.3693	1.0980	9.80	0.2066	2.8224
	benchmark ratio$_{(P)}$=3.1760				benchmark ratio$_{(\alpha)}$=2.1579		†br_P=100.2%$(br_\alpha)^{3/2}$			

Period analysis of observed planets [1]					Period calculations of all planets based on the planets that follow the shared rules [2]					
Planet	Observed P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Planet	Calculated P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Compare observed P to calculated P
c	5.75969				c	6.3153				Equal to 91.20%
d	16.3570	2.8399	0.8942	-10.58	d	16.758	2.6536	1	0	Equal to 97.61%
e	49.748	3.0414	0.9576	-4.24	e	44.470	2.6536	1	0	Equal to 111.87%
f	122.744	2.4673	0.7769	-22.31	f	118.01	2.6536	1	0	Equal to 104.0%
g	604.67	4.9263	1.5511	55.11	g	313.14	2.6536	1	0	Equal to 193.1%
h	2205±105	3.6466	1.1482	14.82	x	830.95	2.6536	1	0	Undetected planet
	benchmark ratio$_{(P)}$=3.1760				h	2205.0	2.6536	1	0	Reference planet
						leading ratio$_{(P)}$=2.6536				

Note 1. The symbol ω_P is the ratio parameter of periods and θ_P is the deviation rate of ω_P. They are calculated according to equation (1) and (3). The symbol ω_α is the ratio parameter of semimajor axes and θ_α is the deviation rate of semimajor axis ratio. They are calculated according to equation (2) and (4). The symbol † is calculated according to equation (11) where br_P is benchmark ratio$_{(P)}$ and br_α is benchmark ratio$_{(\alpha)}$.

Note 2. Analysis of the system

According to the calculation with equation (2), the system shows a 55.11% deviation rate, suggesting it is an abnormal system. Because all deviation rates are obviously different, the orbits of some planets should have changed greatly. In this case, we tried to calculate the periods of all possible with equation (9). Because none of the period ratios may represent the initial orbital relations of most planets, we took the average of the period ratios of P_f:P_e and P_d:P_c as the leading ratio. The value is lr_P=(2.4673+2.8399)÷2=2.6536. In addition, Planet h has the most mass and may most possibly keep its initial orbit. So we took the period of Planet h for reference in the calculation. According to $P_{n(cal)}=P_h÷(2.6536)^{(7-n)}$, the calculations are shown in the table (See [2]). For example, Planet d show 16.758 day by $P_{d(cal)}=2205÷(2.6946)^{(7-2)}$, equal to 97.61% of its observed period. Because the degrees of the equality for Planet c, d, f and h reach 91.20, 97.61, 104.0 and 100%, for a natural system, they can be considered as the planets that follow the shared rules according to principle (1). So, according to principle (2), the shared rules still exist in the system. In this case, the calculated periods of Planet e and g suggest that they have migrated obviously according to principle (3). The calculation also provide a period of 830.95 days, which do not belong to any observed planet. According to principle (4), it should be the period of an undetected planet. The calculations with the equations and principles of the shared rules restore the possible planetary configuration of the system.

According to the calculations, three planets should have changed their orbits obviously. Because of the mutual gravitational influence between adjacent planets, other planets may also change their orbits more or less. So, the analyses of the changes of planetary configuration, or the evolution of the system, should be complex. To help understand the changes, we will just show a simple inference. Since Planet h is very big, its dragging force should have an obvious influence on the motion of its inner Planet x, first causing the change of its orbital shape. The change made Planet x get close to Planet g at its perihelion, When Planet x and g got too near to each other, the mutual dragging force caused the two planets to have changed their positions. In other words, the dragging force of Planet x made inner Planet g increase its orbiting speed and came to the orbit of 604.67 days from the orbit of 313.14 days. Meanwhile the dragging force of Planet g made outer Planet x decrease its orbiting speed and fall toward the star. It may also be thought that Planet x lost kinetic energy. During its falling, Planet x affected the motion of Planet e, causing Planet e to migrate obviously. Perhaps, falling Planet x may also affect the motion of Planet c before it fell into the star.

Table S24. Calculations of the period ratio parameters and deviation rates of adjacent planets of the HD 191939 system and analysis of the change of the planetary configuration.

Comparison of the periods of adjacent planets [1]					Comparison of the semimajor axes of adjacent planets [2]					
Planet	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	α (AU)	Ratio of α_n:α_{n-1}	ω_α	θ_α (%)	Mass (R_{Earth})	Mass of M_n:M_{n-1}
b	8.880326				0.0804				10.0	
c	28.57974	3.2184	1.1800	18.0	0.1752	2.1791	1.1163	11.63	8.0	0.80
d	38.35304	1.3419	0.4920	-50.80	0.2132	1.2169	0.6234	-37.66	2.80	0.35
e	101.12±0.13	2.6465	0.9703	-2.97	0.407±0.012	1.9090	0.9779	-2.21	112.2	40.07
g	284^{10}_{-8}	2.8085	1.0297	2.97	0.812±0.028	1.9951	1.0220	2.20	13.5	0.1203
f	>2200	>7.7465	>2.840	>184	>3.2	>3.941	>2.109	>109	660	44.44
	benchmark ratio$_{(P))}$=2.7275				benchmark ratio$_{(\alpha)}$=1.9521				†br_P=100%$(br_\alpha)^{3/2}$	

Note 1. The symbol ω_P is the ratio parameter of periods and θ_P is the deviation rate of ω_P. They are calculated according to equation (1) and (3). The symbol ω_α is the ratio parameter of semimajor axes and θ_α is the deviation rate of semimajor axis ratio. They are calculated according to equation (2) and (4). The symbol † is calculated according to equation (11) where br_P is benchmark ratio$_{(P)}$ and br_α is benchmark ratio$_{(\alpha)}$.

Note 2. Analysis of the planetary configuration of the system

Since the orbit of Planet f is not certain, we mainly analysed the system according to the orbits of the inner five planets. The calculations with equation (1-4) provide the deviation rates, reflecting that it is not a normal system (See [1] & [2]). According to trait (1), Planet g, e and d should follow the share rules. The calculations with equation (9) and (10) also suggest that they follow the share rules. So, it is an abnormal system showing the shared rules.

Because huge Planet f has a remote and uncertain orbit and its mass excesses 2 M_{Jup}, it may play the role of a companion of the star. Before Planet g was discovered, the published data of Planet f is α=4450±2750day and M=6.497±4.515$_{Jup}$, implying that it is more likely to be a companion star. The huge mass of Planet f must have great influence on the formation and motion of the inner five planets. In addition, the mass of Planet g is also relatively much bigger than its adjacent planets, which also affects the stable motion of them and causes the appearance of a large negative deviation rate. Moreover, there may be undetected planets between Planet f and g if considering the transitional matter in the protoplanetary disk. Therefore, the analysis of the change of planetary configuration may be very complex. More observations are required.

Table S25. Calculations of the period ratio parameters and deviation rates of adjacent planets of the HD 219134 system and analysis of the change of the planetary configuration.

Comparison of the periods of adjacent planets						Comparison of the semimajor axes of adjacent planets					
Planet	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	A chain of near MMRs	α (AU)	Ratio of α_n:α_{n-1}	ω_α	θ_α (%)	Mass ($M_{Jup.}$)	Ratio of M_n:M_{n-1}
b	3.0929					0.03876				0.0149	
c	6.7646	2.1871	1.0481	4.81	--	0.06530	1.6847	1.0186	1.96	0.0137	0.920
f	22.717	3.3582	1.6093	60.93	--	0.1463	2.2404	1.3559	35.59	0.0230	1.679
d	46.859	2.0627	0.9885	-1.15	Near 2:1	0.2370	1.6200	0.9804	-1.96	0.05088	2.212
g	94.2	2.0103	0.9634	-3.66	Near 2:1	0.3753	1.5835	0.9583	-4.17	0.034	0.668
h	2247±51	23.854	11.431	1043.1	--	3.11±0.04	8.2867	5.0150	401.5	0.34	10
	benchmark ratio$_{(P)}$ =2.0867					benchmark ratio$_{(\alpha)}$=1.6524			†br_P=98.24%$(br_\alpha)^{3/2}$		

Period analysis of observed planets [1]					Period calculations of all planets based on the planets that follow the shared rules [2]					
Planet	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Planet	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Compare observed P to calculated P
b	3.0929				b	2.5885				Equal to119.5%
c	6.7646	2.1871	1.0603	6.03	c	5.3392	2.0627	1	0	Equal to126.7%
f	22.717	3.3582	1.6281	62.81	x	11.012	2.0627	1	0	Undetected
d	46.859	2.0627	1	0	f	22.717	2.0627	1	0	Equal to 100%
g	94.2	2.0103	0.9746	-2.54	d	46.859	2.0627	1	0	Reference planet
h	2247±51	23.854	11.565	1056.5	g	96.656	2.0627	1	0	Equal to 97.46%
	benchmark ratio$_{(P)}$=2.0627				h	2247±51	23.247	11.71	1071	Not analyse
						leading ratio$_{(P)}$=2.0627				

Note 1. The symbol ω_P is the ratio parameter of periods and θ_P is the deviation rate of ω_P. They are calculated according to equation (1) and (3). The symbol ω_α is the ratio parameter of semimajor axes and θ_α is the deviation rate of semimajor axis ratio. They are calculated according to equation (2) and (4). The symbol † is calculated according to equation (11) where br_P is benchmark ratio$_{(P)}$ and br_α is benchmark ratio$_{(\alpha)}$.

Note 2. Because Planet h is too far from the star, the ratio of Planet h to Planet g will affect the true reflection of the relations of the inner four planets. So, the benchmark ratios are calculated on basis of the inner five planets.

Note 3. Analysis of the system

Comparisons show that Planet h is very far from its adjacent planet and is ten times the mass of its adjacent planet. In this case, we will first analyse the inner five planets first. We selected the period ratio of Plane d to f a representative period ratio and analyse the periods of all planets of the system (See [1]).

The calculations with equation (1) and (2) reflect the orbital relations (See [1]). According to trait (1), Planet g, d and f should follow the shared rules because the two adjacent deviation rates of the three planets are close to 0. Meanwhile, the 62.81% deviation rate suggests the lack of an undetected planet between Planet f and c. In order to better understand the change of planetary configuration, we calculated the periods of all possible planets with equation (9) by taking the period ratio of Planet d to f as the leading ratio and Planet d as reference planet. The calculations are shown in the table (See [2]). When we calculated the period of Planet g, the calculated period is 96.656 days by $P=46.859 \div 2.0627^{(5-6)}$, equal to 97.46% of its observed period. For a natural system, the calculation suggests that Planet f, d and g are the planets that follow the shared rules according to principle (1).

So, the shared rules still exist in the system according to principle (2). In this case, the period of 11.012 days, calculated by $P=46.859 \div 2.0627^{(5-3)}$, should be the period of an undetected planet according to principle (4). Meanwhile, according to principle (3), the calculations propose the migrations of Planet b and c (See [2]) because the calculated periods of Planet b and c are somewhat different from their observed periods. The calculations reflect the configuration change of the five planets. I will show a possibility of the change.

Because of disturbances of the mutual interaction between Planet x and Planet c, Planet x may have disappeared. As the orbits of Planet x and c were very close, the gravitational disturbances between them were huge. Some reason might first make Planet x or c change its orbital shape, leading to the closer distance between them at respective perihelion and aphelion. The mutual dragging force increased the orbiting speed of Planet c, causing it to migrate outward relatively. Because of the exchange of kinetic energy, the dragging force decreased the orbiting speed of Planet x, causing it to migrate inward and cross the orbit of Planet c. They changed their positions. There are two possibilities for Planet x. One is that Planet x flew toward the star. During the fall of Planet x, it dragged Planet b to the outer area, embodying the relatively outward migration. After the second interaction, Planet x lost more energy and fell into the star. The second possibility is that Planet x balanced different forces and settled at the orbital area of current Planet b. The migration of Planet c also implies that there should b a missing planet outside of its orbit.

As to Planet h, because its mass is about ten times bigger than other planets and is too far from other planets, it implies that the system formed in a greatly-disturbed disc. Before the system formed, a little part of matter of the air mass was thrown out and formed a body much smaller than the main star. Planet h can be considered as a companion planet. So, its current orbit may have no relation with other planets. In this case, it is more proper to analyse the five inner planets independently. The environments of the system formation should be complex. We will not discuss it as observations are limited.

In addition, Extrasolar Planets Encyclopaedia (check http://www.exoplanet.eu/catalog/) shows the existence of a planet with the period of 1842 days. Being as a transitional planet for the two rings, it should be possible.

Table S26. Calculations of the period ratio parameters and deviation rates of adjacent planets of the HD 34445 system system and analysis of the change of the planetary configuration.

Comparison of the periods of adjacent planets					Comparison of the semimajor axes of adjacent planets					
Planet	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	α (AU)	Ratio of α_n:α_{n-1}	ω_α	θ_α (%)	Radius ($R_{Jup.}$)	Ratio of R_n:R_{n-1}
e	49.175				0.2687				0.0529	
d	117.87	2.3969	0.9755	-2.446	0.4817	1.7927	0.8925	-10.75	0.097	1.8337
c	214.67	1.8212	0.7412	-25.88	0.7181	1.4908	0.8234	-17.66	0.168	1.7320
f	676.80±7.9	3.1528	1.2832	28.32	1.543	2.1487	1.1867	18.67	0.119	0.7083
b	1056.7±4.7	1.5613	0.6355	-36.45	2.075	1.3448	0.7427	-25.73	0.82	6.8908
g	5700±1500	5.3977	2.1969	119.7	6.36	3.0651	1.6928	69.28	0.38	0.4634
	benchmark ratio$_{(P)}$=2.4570				benchmark ratio$_{(\alpha)}$=1.8107		†br_P=100.84%$(br_\alpha)^{3/2}$			

Period analysis of observed planets [1]					Period calculations of all planets based on the planets that follow the shared rules [2]					
Planet	Observed P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Planet	Calculated P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Compare observed P to calculated P
e	49.175±0.05				e	46.725				Equal to 105.3%
d	117.87±0.18	2.3969	0.9755	-2.446	--	--	--			No planet
c	214.67±0.45	1.8212	0.7412	-25.88	d	113.90	1.5613	1	0	Equal to 103.5%
f	676.80±7.9	3.1528	1.2832	28.32	c	177.83	1.5613	1	0	Equal to 120.7%
b	1056.7±4.7	1.5613	0.6355	-36.45	x	277.65	1.5613	1	0	Migrated to e orbit
g	5700±1500	5.3977	2.1969	119.69	y	433.49	1.5613	1	0	Missing
	benchmark ratio$_{(P)}$=2.4570				f	676.80	1.5613	1	0	Equal to 100%
					b	1056.7	1.5613	1	0	Reference planet
					z	1650	1.5613	1	0	Missing
					g	2576	1.5613	1	0	Migrate from 2576 days to current orbit
						leading ratio$_{(P)}$=1.5613				

Note 1. The symbol ω_P is the ratio parameter of periods and θ_P is the deviation rate of ω_P. They are calculated according to equation (1) and (3). The symbol ω_α is the ratio parameter of semimajor axes and θ_α is the deviation rate of semimajor axis ratio. They are calculated according to equation (2) and (4). The symbol † is calculated according to equation (11) where br_P is benchmark ratio$_{(P)}$ and br_α is benchmark ratio$_{(\alpha)}$.

Note 2. Analysis of the system

Direct comparisons of the period ratios of the six current planets reflect the disordered planetary configuration (See [1]), which implies that the system may have been greatly disturbed and some planets may have migrated greatly or disappeared. Plus the uncertain orbit of Planet g, it is not easy to restore the initial planetary configuration of the system. Even so, we will try to analyse the orbital changes of the planets.

The calculation with equation (2) shows the deviation rates of all observed planets (See [1]). It is an abnormal system because it shows a big deviation rate. So we tried to calculate the periods of the observed planets with equation (9). After different tries, we will take the period ratio of Planet b to f as the leading ratio of equation (9) and the biggest Planet b as reference planet. The calculated periods of all possible planets are presented in the table (See [2]). It shows that the calculated periods of Planet b, f and d are equal to 100, 100 and 103.5% of their observed periods separately. According to principle (1) and criterion (N-A), the three planets should keep their initial orbital relations and follow the shared rules. According to principle (2), the shared rules still exist in the system. In this case, the calculations should be meaningful.

Because the system may have experienced great disturbances and some planets may have changed their orbits greatly, it is impossible to explain the change of planetary configuration with a few sentences. We will just offer a simple but possible inference. Some external force or the effect of Kirkwood gap may first disturbed the stable motion of Planet z or g. The perturbation of unbalanced dragging force then changed the orbital shape of Planet g and z, causing them get very near at their perihelion and aphelion. At some moment, the great dragging force made Planet g fly to the outer space and settle at the current orbit. The exchange of kinetic energy caused Planet z lost energy and fell toward the star. As Planet z fell very fast, it crossed the orbit of Planet b and f. When Planet z fell to the orbit of Planet y, it got near to Planet y and disturbed the motion of Planet y although it at last fell into the star. Because of the disturbance of Planet z, Planet y changed its orbit and came to the area nearer to Planet x. The mutual perturbation of the two planets caused Planet y to be ejected off the system and Planet x to fall toward the star. The result agreed with the the law of kinetic energy conversion. During its falling, Planet x interacted with Planet c, which made Planet c migrate outwards and accelerated the falling speed of Planet x. After Planet x crossed Planet d, it reached the area where the stellar gravitation and its centrifugal force came to a relative balance. As a result, it became a planet which kept orbiting the star at the orbit of planet e. During the interaction, Planet g and z might change their positions, Planet x and Planet y might change their positions. The change of position did not affect the result.

In addition, observations show that the mass of the star is $1.07M_{\odot}$. In this case, it is possible that Planet f in the habitable zone may produce the environment of the appearance of life for it has the similar size to the Earth. Before Planet y was ejected off the system, it was also in the habitable zone and enjoyed more suitable environments for it was located in the orbit similar as the Earth. If life once appeared on the surface of Planet y, it has vanished with the disappearance of the planet.

Though the inference complies with the law of kinetic energy conversion, it needs more observations and studies. The change of planetary configuration should be complex. My inference is just a qualitative analysis and not a conclusion. There might be other possibilities. Nevertheless, the change of planetary configuration reflected by the shared rules provides the entry points for the analysis of the evolution of the system.

Table S27. Calculations of the orbital ratio parameters and deviation rates of adjacent planets of the K2-138 system.

	Comparison of the periods of adjacent planets				Comparison of the semimajor axes of adjacent planets					
Planet	P ① (day)	Ratio of (P_n:P_{n-1})	ω_P	θ_P (%)	α (AU)	Ratio of α	ω_α	θ_α (%)	Radius (R_{jup})	Ratio of R_n:R_{n-1}
b	2.35309				0.03385				0.1347	
c	3.56004	1.5129	0.9886	-1.14	0.04461	1.3179	0.9924	-0.76	0.2051	1.5226
d	5.40479	1.5182	0.9921	-0.79	0.05893	1.3210	0.9947	-0.53	0.2132	1.0395
e	8.26146	1.5285	0.9988	-0.12	0.07820	1.3270	0.9993	-0.07	0.3024	1.4184
f	12.75758	1.5442	1.0091	0.91	0.10447	1.3359	1.0060	0.60	0.2591	0.8568
g	41.96797	3.2897	2.1497	115	0.23109	2.2120	1.6657	66.57	0.2688	1.0374
	benchmark ratio$_{(P)}$ =1.5303				benchmark ratio$_{(\alpha)}$=1.3280		†br_P=99.995%$(br_\alpha)^{3/2}$			

Note. The symbol ω_P is the ratio parameter of periods and θ_P is the deviation rate of ω_P. They are calculated according to equation (1) and (3). The symbol ω_α is the ratio parameter of semimajor axes and θ_α is the deviation rate of semimajor axis ratio. They are calculated according to equation (2) and (4). The symbol † is calculated according to equation (11) where br_P is benchmark ratio$_{(P)}$ and br_α is benchmark ratio$_{(\alpha)}$.

Table S28. Calculations of the orbital ratio parameters and deviation rates of adjacent planets of the Kepler-11 system and analysis of the change of the planetary configuration.

Comparison of the periods of adjacent planets [1]					Comparison of the semimajor axes of adjacent planets [2]					
Planet	P (day)	Ratio of $P_n:P_{n-1}$	ω_P#	θ_P# (%)	α (AU)	Ratio of $\alpha_n:\alpha_{n-1}$	ω_α#	θ_α# (%)	Radius ($R_{Jup.}$)	Ratio of $R_n:R_{n-1}$
b	10.304				0.091				0.161	
c	13.024	1.2640	0.8643	-13.4	0.107	1.1758	0.9171	-8.29	0.256	1.5901
d	22.685	1.7418	1.1938	19.4	0.155	1.4486	1.1299	12.99	0.278	1.0859
e	32.000	1.4106	0.9668	-3.3	0.195	1.2581	0.9813	-1.87	0.374	1.3453
f	46.689	1.4590	1	0	0.250	1.2821	1	0	0.272	0.7273
g	118.38	2.5355	1.7378	73.8	0.466	1.8640	1.4539	45.39	0.297	1.0919
	specific benchmark ratio$_{(P)}$ =1.4590					specific benchmark ratio$_{(\alpha)}$=1.2821			†sbr$_P$=100.50%(sbr$_\alpha$)$^{3/2}$	

Analysis and contrast of the calculations of planetary periods with different reference ratios [3]

Analysis with representative ratio [3-1]					Analysis with dominate ratio [3-2]				Analysis with average ratio [3-3]			
Planet	P (day)	Ratio of $P_n:P_{n-1}$	ω_P	θ_P (%)	P (day)	Ratio of $P_n:P_{n-1}$	ω_P#	θ_P# (%)	P (day)	Ratio of $P_n:P_{n-1}$	ω_P*	θ_P* (%)
b	10.304				10.304				10.304			
c	13.024	1.2640	0.8223	-17.8	13.024	1.2640	0.8643	-13.4	13.024	1.2640	0.7514	-24.86
d	22.685	1.7418	1.1332	13.3	22.685	1.7418	1.1938	19.4	22.685	1.7418	1.0354	3.54
e	32.000	1.4106	0.9177	-8.23	32.000	1.4106	0.9668	-3.3	32.000	1.4106	0.8385	-16.15
f	46.689	1.4590	0.9492	-5.08	46.689	1.4590	1	0	46.689	1.4590	0.8673	-13.27
g	118.38	2.5355	1.6495	65.0	118.38	2.5355	1.7378	73.8	118.38	2.5355	1.5072	50.75
	benchmark ratio$_{(P)}$ =1.5371					specific benchmark ratio$_{(P)}$ =1.4590				average ratio$_{(P)}$=1.6822		

Possible change of planetary configuration calculated according to corresponding equations and principles

Calculations based on the planets with shared rules [4]						Analysis of the changes of planetary distribution [5]	
Planet	Observed P (day)	Calculated P@ (day)	Ratio of $P_n:P_{n-1}$	ω_P	θ_P (%)	Compare observed P to calculated P@	The properties of corresponding planets
b	10.304	10.3035				P is equal to ~100% of p@.	Planet with shared rules
c	13.024	15.033	1.4590	1	0	P is equal to 86.6% of p@.	Inward-migrating planet
d	22.685	21.933	1.4590	1	0	P is equal to 103.4% of p@.	Planet with shared rules
e	32.000	32.000	1.4590	1	0	It is the reference planet.	Planet with shared rules
f	46.689	46.689	1.4590	1	0	P is equal to 100% of p@.	Planet with shared rules
x	--	68.12	1.4590	1	0	It should be an undetected planet.	Missing planet
g	118.38	99.386	1.4590	1	0	P is equal to 119.1% of p@.	Outward-migrating planet
		leading ratio$_{(P)}$=1.4590					

Analysis of semimajor axes [6]						Semimajor axis calculations based on the planets following shared rules [7]					
Planet	Observed α (AU)	Ratio of $\alpha_n:\alpha_{n-1}$	ω_α#	θ_α# (%)	Mass (M_{Earth})	Planet	Calculated α (AU)	Ratio of $\alpha_n:\alpha_{n-1}$	ω_α	θ_α (%)	Compare observed α to calculated α
b	0.091				$1.9^{+1.4}_{-1}$	b	0.0925				Equal to 98.4%
c	0.107	1.1758	0.9171	-8.29	$2.9^{+2.9}_{-1.6}$	c	0.1186	1.2821	1	0	Equal to 90.2%
d	0.155	1.4486	1.1299	12.99	$7.3^{+0.8}_{-1.6}$	d	0.1521	1.2821	1	0	Equal to 101.9%
e	0.195	1.2581	0.9813	-1.87	$8.0^{+1.5}_{-2.1}$	e	0.195	1.2821	1	0	Reference planet
f	0.250	1.2821	1	0	$2.0^{+0.8}_{-0.9}$	f	0.250	1.2821	1	0	Equal to100%

g	0.466	1.8640	1.4539	45.39	<25	x	0.3205	1.2821	1	0	Undetected planet
specific benchmark ratio$_{(a)}$=1.2821						g	0.4110	1.2821	1	0	Equal to 113.4%
						leading ratio$_{(a)}$=1.2821					

Note 1. Symbol ω_P# is the period ratio parameter calculated by specific benchmark ratio and θ_P# is the deviation rate of ω_P#. The symbol ω_a# is the semimajor-axis ratio parameter calculated by specific benchmark ratio and θ_a# is the deviation rate of ω_a#. Symbol ω_P is the period ratio parameter calculated by benchmark ratio and θ_P is the deviation rate of ω_P. Symbol ω_P* is the period ratio parameter calculated by average ratio and θ_P* is the deviation rate of ω_P*. The orbital ratio parameters and deviation rates of the system are calculated with equation (1), (2), (3) and (4) by using different reference ratio separately.

Note 2. The symbol † is calculated according to equation (11) where sbr_P is specific benchmark ratio$_{(P)}$ and sbr_a is specific benchmark ratio$_{(a)}$.

Note 3. We also separately calculated the period ratio parameters and deviation rates by the specific benchmark ratios, benchmark ratio and average ratio (See [3-1], [3-2] and [3-3] in the table). Comparisons can show the discrepancies between the deviation rates of orbital ratios of any two adjacent planets. Since the three benchmark ratios show that the deviation rate of the period ratio of Planet g to f is very big, Kepler-11 is an abnormal system. It also shows that the specific benchmark ratio can relatively more clearly and objectively reflect the true configuration of all planets, especially the planets that keep initial orbits. In this case, the orbital relationships of the system are calculated by the benchmark ratio (See [1] and [2]).

Note 4. Analysis of the change of the planetary configuration of the system based on the shared rules

According to the calculation with the representative ratio, the system clearly shows the orbital relations between all planets. The 73.8% deviation rate states that it is an abnormal system. In order to learn about whether the shared rules still exist in the system, we calculated the periods of all observed planets with equation (9). Comparison shows that the average period ratios of the inner five planets is 1.4688, which is 99.33% equal to the period ratio of Planet f to e (1.4590). The coincidence suggests that the period ratio of Planet f to e may represent the orbital relations of the previous planetary configuration. So, the period ratio of Planet f to e can be taken as the leading ratio of equation (9). To further confirm the orbital relations, we calculated the shared divisor of the system according to the contents of the standard rule. Calculated by $sd=(P_f \div P_b)^{1/(5-1)}$, the shared divisor of the periods from Planet f to b is 1.4590 by $(46.689 \div 10.304)^{1/(5-1)}$. The value is completely equal to the period ratio of Planet f to e. In this case, we took 1.4590 as the leading ratio of equation (9) and Planet e as reference planet, The calculations provide seven periods (See [4]). For example, the computational period of Planet b is 10.3035 days by $P_{b(cal)}=32.0 \div 1.4590^{(4-1)}$, equal to 100% of its observed period. According to principle (1), Planet b, e and f should be the planets following the shared rules. According to principle (2), the shared rules still exist in the system. In this case, Planet c and d should be the migratory planets according to Principle (3) because the computational periods are calculated on the basis of the planets following the shared rules and the computational periods are obviously different from their observed periods. In other words, the Planet c migrated inwards equivalent to 13.4% of its observed period while Planet d migrated outwards equivalent to about 3.4% of its observed period. Because Planet c migrated inwards while Planet d migrates outwards, it is more possible that the opposite migrations are caused by the mutual dragging force of the two planets. As we know, every planet has its own kinetic energy while it orbits its host. If its orbit changes, its kinetic energy will change at the same time. The calculations show that the change of the kinetic energy caused by the migration of Planet c is almost equal to that of Planet d (See Note 5).

More importantly, the calculation also shows a period of $P_x=32.0 \div 1.4590^{(4-6)}=68.12$day, which does not belong to any observed planet. Since it is calculated according to the orbits of the planets following the shared rules, the value should be the period of an undetected planet according to principle (4). According to the calculation on the basis of the periods of Planet f and e, the period of Planet g is 99.39 days by $P_{g(cal)}=32.0 \div 1.4590^{(4-7)}$, which is equal to 119.1% of its observed period. The value suggests that Planet g migrated outwards according to Principle (3). Because of some disturbance, Planet g perhaps first changed its orbital shape, which offered the chance that Planet g got near to the undetected planet at its perihelion. The interaction between them further changed their orbits. As a result, the dragging force of Planet g ejected the undetected planet off the system. In return, Planet g was forced to migrate outwards. Scientists have also research the planetary configuration of the Kepler-11 system (*34*). They performed simulation calculations and proposed that an undetected relatively-bigger planet perhaps orbits the star outside Planet g in a different inclination, which affected the motion of Planet g. If a planet revolves around the star at the orbit outside of Planet g and its orbit has a different inclination, it may not be discovered with the transit method. The 100% equality between calculation and observation for Planet b should reflect the existence of the shared rules in the system, the migration of some planets, the absence of a planet and the change of planetary configuration.

Note 5. Calculation of the exchange of kinetic energy

As we know, every planet has its kinetic energy at its specific orbit. The value is equivalent to the gravitational potential energy. So, its migration naturally embodies the change of its kinetic energy. If the opposite migrations of Planet d and c are caused by their mutual dragging force, the kinetic energy decreased by Planet c should be equal or close to the kinetic energy increased by Planet d. In this case, we can calculate the change of their kinetic energy and compare the changes. We will analyse the energy changes with the most basic equations. Usually, the kinetic energy of an orbiting planet is $E=\frac{1}{2}mv^2$ or $E=2m\pi^2(R/T)^2$, based on $v=2\pi R/T$. For Planet c, its current kinetic energy is $E_{c(1)}$ and previous energy is $E_{c(2)}$. The change of their energy or decreased energy is $\Delta E_c = E_{c(2)} - E_{c(1)}$. If the change of the energy is expressed with the planetary mass, semimajor axis and period, it is $\Delta E_c = E_{c(2)} - E_{c(1)} = 2m_c\pi^2[(a_{c(2)}/P_{c(2)})^2 - (a_{c(1)}/P_{c(1)})^2]$. For Planet d, its current kinetic energy is $E_{d(1)}$ and previous energy is $E_{d(2)}$. The change of its energy or increased energy is $\Delta E_d = E_{d(2)} - E_{d(1)}$ or $\Delta E_d = E_{d(2)} - E_{d(1)} = 2m_d\pi^2[(a_{d(2)}/P_{d(2)})^2 - (a_{d(1)}/P_{d(1)})^2]$. The comparison of the changes of their kinetic energy will show the degree of the exchange of their kinetic energy. Because all necessary data have been provided by observations, we can see the results after the relevant data are brought into the formulas.

For Planet c: $M=2.9M_{Earth}$, $\alpha_1=0.107AU$, $\alpha_2=0.1186AU$, $P_1=13.024d$ and $P_2=15.033d$.

For Planet d: $M=7.3M_{Earth}$, $\alpha_1=0.155AU$, $\alpha_2=0.1521AU$, $P_1=22.685d$ and $P_2=21.933d$.

The calculations based on the above data show $\Delta E_c = 5.386\times10^{33} kg\cdot m\cdot \sec^2$ and $\Delta E_d = 3.629\times10^{33} kg\cdot m\cdot \sec^2$. For comparison, $\Delta E_c : \Delta E_d = 1.4842:1$ or $\Delta E_d = 0.6738\Delta E_c$, suggesting that the energy decreased by Planet c is about 48.42% more than that increased by Planet d. If considering the mass uncertainty of the two planets, showing $M_c=2.9^{+2.9}_{-1.6}M_{Earth}$ and Planet d $M_d=7.3^{+0.8}_{-1.6}M_{Earth}$, the 48.57% difference is acceptable for a qualitative analysis. In other words, if the mass of Planet c is $1.96M_{Earth}$, comparison shows $\Delta E_c = \Delta E_d$. So, the kinetic energy decreased by Planet c is almost equal to that increased by Planet d. The result agrees with the contents of the energy conservation law and is a support to the above analysis. For the Kepler-11 system, because the period ratios of all planets were close to the resonance of 3:2, the effect of Kirkwood gap might play a role, which might lead to the unstable motion and initial migrations of some planets.

Note 6. The stellar mass of Kepler-11 is 0.99 M_{Sun} and the age is $6.92^{+1.70}_{-2.19}$ Gyr.

Table S29. Calculations of the orbital ratio parameters and deviation rates of adjacent planets of the Kepler-20 system.

Comparison of the periods of adjacent planets [1]					Comparison of the semimajor axes of adjacent planets [2]					
Planet	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	α (AU)	Ratio of α_n:α_{n-1}	ω_α	θ_α (%)	Radius ($R_{Jup.}$)	Ratio of R_n:R_{n-1}
b	3.69612		3.4440		0.0463				0.1667	
e	6.09852	1.6500	0.9221	-7.79	0.0639	1.3801	0.9350	-6.50	0.0772	0.4631
c	10.8541	1.7798	0.9946	-0.54	0.0949	1.4851	1.0061	0.61	0.2718	3.5207
f	19.5776	1.8037	1.0080	0.8	0.1396	1.4710	0.9966	-0.34	0.08948	0.3292
g	34.940	1.7847	0.9974	-0.26	0.2055	1.4721	0.9973	-0.27	0.0628 (M_{Jup})	
d	77.6113	2.2213	1.2414	24.1	0.3506	1.7061	1.1558	15.6	0.2448	--
	benchmark ratio$_{(P)}$=1.7894				benchmark ratio$_{(\alpha)}$=1.4761		†br_P=99.78%$(br_\alpha)^{3/2}$			

Note 1. The symbol ω_P is the ratio parameter of periods and θ_P is the deviation rate of ω_P. They are calculated according to equation (1) and (3). The symbol ω_α is the ratio parameter of semimajor axes and θ_α is the deviation rate of semimajor axis ratio. They are calculated according to equation (2) and (4). The symbol † is calculated according to equation (11) where br_P is benchmark ratio$_{(P)}$ and br_α is benchmark ratio$_{(\alpha)}$.

Table S30. Calculations of the period ratio parameters and deviation rates of adjacent planets of the Kepler-80 system and analysis of the change of the planetary configuration.

Comparison of the periods of adjacent planets [1]					Comparison of the semimajor axes of adjacent planets [2]					
Planet	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	α (AU)	Ratio of $α_n$:$α_{n-1}$	$\omega_α$	$\theta_α$ (%)	Radius ($R_{Jup.}$)	Ratio of R_n:R_{n-1}
f	0.98679				0.0175				0.108	
d	3.07222	3.1134	2.045	104.5	0.0372	2.1257	1.5983	59.83	0.136	1.265
e	4.64489	1.5119	0.993	-0.7	0.0491	1.3199	0.9924	-0.76	0.143	1.046
b	7.05246	1.5183	0.997	-0.3	0.0648	1.3401	1.0076	0.76	0.238	1.669
c	9.52355	1.3504	0.887	-11.3	0.0792	1.2037	0.9050	-9.5	0.244	1.026
g	14.6456	1.5378	1.010	1.0	---				0.101	0.412
	benchmark ratio$_{(P)}$ =1.5227				benchmark ratio$_{(α)}$=1.330		†br_P=99.27%$(br_α)^{3/2}$			

Period analysis of observed planets [1]							Period calculations of all planets based on the planets that follow the shared rules [3]					
No.	Planet	Observed P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Radius ($R_{Jup.}$)	Planet	Calculated P (day)**	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Remark
1	f	0.98679				0.108	f	0.88896				Equal to 111.0%
2	d	3.07222	3.1134	2.0447	104.5	0.136	x	1.3440	1.5119	1	0	Missing
3	e	4.64489	1.5119	0.9929	-0.71	0.143	y	2.0320	1.5119	1	0	Missing
4	b	7.05246	1.5183	0.9971	-0.29	0.238	d	3.07222	1.5119	1	0	Equal to 100%
5	c	9.52355	1.3504	0.8869	-11.3	0.244	e	4.64489	1.5119	1	0	Reference
6	g	14.6456	1.5378	1.099	0.99	0.101	b	7.02261	1.5119	1	0	Equal to 100.4%
7		benchmark ratio$_{(P)}$=1.5227					c	10.6175	1.5119	1	0	Equal to 89.70%
8							g	16.0526	1.5119	1	0	Equal to 91.24%
								leading ratio$_{(P)}$=1.5119				

Note 1. The symbol ω_P is the ratio parameter of periods and θ_P is the deviation rate of ω_P. They are calculated according to equation (1) and (3). The symbol $\omega_α$ is the ratio parameter of semimajor axes and $\theta_α$ is the deviation rate of semimajor axis ratio. They are calculated according to equation (2) and (4). The symbol † is calculated according to equation (11) where br_P is benchmark ratio$_{(P)}$ and $br_α$ is benchmark ratio$_{(α)}$.

Note 2. The analysis of the system

Kepler-80 has six planets now. The calculation with equation (1) and (2) shows the ratio parameters and deviation rates of observed planets. The 104.5% deviation rate suggests that it is an abnormal system (See [1]). According to trait (1), Planet b, e and d should follow the shared rules. When we calculated the period of Planet b with equation (9) and on the basis of the period ratio of Planet e to d as the leading ratio and Planet e as reference planets, the calculated period shows 7.02261 days by $P_{(cal)}$=4.64489÷$1.5119^{(5-6)}$, which equals to 100.42% of its observation period. The calculations also suggest that Planet b, e and d are the planets well following the shared rules according to principle (1). So, the shared rules still exist in the system according to principle (2). The calculations based on Planet e and d reflect two periods that do not belong to any observed planets (See [2]). According to principle (4), they should be the planets that are not discovered or have disappeared. The calculation also reflected that Planet f migrated outwards according to its calculated period by $P_{(cal)}$=4.64489÷$1.5119^{(5-1)}$=0.88896(day).

According to the calculations, we will offer a brief inference of the evolution of the system. Because the two undetected planets are very near, they might have been ejected off the system by each other under the influence of the mutual dragging force. One of the missing planet might disturb the motion of Planet f during its falling to the star. There is another possibility that Planet f once was at the orbit of Planet x or y and migrated to the current orbit after the interaction with the missing planet. If considering the missing planet and orbital change of Planet f, the big deviation rate can be explained objectively. Based on the calculation with equation (9), Planet g and c should also migrate inwards. The migrations may be caused by a stray planet that passed by the system.

Table S31. Calculations of the orbital ratio parameters and deviation rates of adjacent planets of the TOI-178 system, and analysis of the possible change of planetary configuration of the system.

Comparison of the periods of adjacent planets					Comparison of the semimajor axes of adjacent planets					
Planet	Observed P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	ɑ (AU)	Ratio of $ɑ_n$:$ɑ_{n-1}$	$\omega_ɑ$	$\theta_ɑ$ (%)	Mass (M_{Jup})	Ratio of M_n:M_{-1}
b	1.914558				0.02607				0.00472	
c	3.23845	1.6915	1.0706	7.06	0.0370	1.4193	1.0465	4.645	0.0150	3.5129
d	6.5577	2.0250	1.2817	28.17	0.0592	1.600	1.1797	17.97	0.00947	0.6313
e	9.961881	1.5191	0.9615	-3.85	0.0783	1.3226	0.9752	-2.48	0.0121	1.2777
f	15.231915	1.5290	0.9678	-3.22	0.1039	1.3270	0.9784	-2.16	0.0243	2.0083
g	20.7095	1.3596	0.8606	-13.94	0.1275	1.2271	0.9047	-9.53	0.0124	0.5104
	benchmark ratio$_{(P)}$ =1.5799				benchmark ratio$_{(ɑ)}$=1.3563		†br_P=100.013%$(br_ɑ)^{3/2}$			

Analysis of the change of planetary configuration of the TOI-178							
No.	Planet	Calculated P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Compare observed P to calculated P	Analysis of the properties of all planets
1	b	1.87067				Equal to 102.35%	Planet with shared rules
2	c	2.84173	1.5191	1	0	Equal to 119.1%	Migrated outward
3	x	4.31687	1.5191	1	0	Missing planet	Missing planet
4	d	6.5577	1.5191	1	0	Equal to 100%	Planet with shared rules
5	e	9.961881	1.5191	1	0	Reference planet	Planet with shared rules
6	f	15.1331	1.5191	1	0	Equal to 100.65%,	Planet with shared rules
7	g	22.9887	1.5191	1	0	Equal to 90.09%	Migrated inward
		leading ratio$_{(P)}$=15191					

Note 1. The symbol ω_P is the ratio parameter of periods and θ_P is the deviation rate of ω_P. They are calculated according to equation (1) and (3). The symbol $\omega_ɑ$ is the ratio parameter of semimajor axes and $\theta_ɑ$ is the deviation rate of semimajor axis ratio. They are calculated according to equation (2) and (4). The symbol † is calculated according to equation (11) where br_P is benchmark ratio$_{(P)}$ and $br_ɑ$ is benchmark ratio$_{(ɑ)}$.

Note 3. The period ratio of Planet f to e is equal to 100.65% of that of Planet e to d. According to the trait of the shared rules, the almost 100% equality suggests that Planet f, e and d should follow the shared rules. According to principle (1), the three planets can be regarded as the planets following the shared rules. So, the shared rules should exist in the system according to principle (2).

Note 4. Although it shows 28.17% deviation rate, the calculation based on Planet e and d reflects that there is an undetected planet according to principle (4). Under the influence of the interaction of the dragging force between Planet c and undetected planet, the undetected planet should be ejected off the system by Planet c. In return, Planet c migrated outwards.

Table S32. Calculations of the orbital ratio parameters and deviation rates of adjacent planets of the 55 Cnc A system, and analysis of the possible change of planetary configuration of the system.

Period analysis of observed planets [1]						Semimajor axis analysis of observed planets [2]					
No.	Planet	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	α (AU)	Ratio of α_n:α_{n-1}	ω_α	θ_α (%)	$M_{Jup.}$ ($M_{Jup.}$)	Ratio of M_n:M_{n-1}
1	e	0.736547				0.01544				0.0251	
2	b	14.6516	19.893	0.9275	-7.25	0.1134	7.3446	0.9504	-4.96	0.8306	33.092
3	c	44.3989	3.0303	0.1413	-85.87	0.2373	2.0926	0.2708	-72.92	0.1611	0.1940
4	f	259.88	5.8559	0.2730	-72.70	0.7708	3.2482	0.4203	-57.87	0.1503	0.9330
5	d	5574.2	21.449	1	0	5.957	7.7283	1	0	3.12	20.758
special benchmark ratio$_{(P)}$ =21.449						special benchmark ratio$_{(\alpha)}$=7.7283				†br_P=99.83%$(br_\alpha)^{3/2}$	

Period analysis of observed planets [3]						Analysis of the system after considering unobserved planets [4]					
No.	Planet	Observed p (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Planet	Calculated p (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Compare observed P to calculated P
1	e	0.736547				e	0.75113				Equal to 97.41%
2	b	14.6516	19.893	4.4773	347.73	x	3.31735	4.4165	1	0	Disappeared planet
3	c	44.3989	3.0303	0.6820	-31.80	b	14.6516	4.4165	1	0	Equal to 100%
4	f	259.88	5.8559	1.3180	31.80	c	64.7065	4.4165	1	0	Equal to 68.60%
5	d	5574.2	21.449	4.8275	382.75	f	285.78	4.4165	1	0	Equal to 90.92%
representative ratio$_{(P)}$=4.4431						y	1262.13	4.4165	1	0	Disappeared planet
						d	5574.2	4.4165	1	0	Reference planet
						leading ratio$_{(P)}$=4.4160					

Period analysis of observed planets [5]						Analysis of the system after considering unobserved planets [6]					
No.	Planet	α (AU)	Ratio of α_n:α_{n-1}	ω_α	θ_α (%)	Planet	α (AU)	Ratio of α_n:α_{n-1}	ω_α	θ_α (%)	Compare observed α to calculated α
1	e	0.01544				e	0.01565				Equal to 98.66%
2	b	0.1134	7.3446	0.9504	-4.96	x	0.04213	2.6921	1	0	Disappeared planet
3	c	0.2373	2.0926	0.2708	-72.92	b	0.1134	2.6921	1	0	Equal to 100%
4	f	0.7708	3.2482	0.4203	-57.87	c	0.3053	2.6921	1	0	Equal to 77.73%
5	d	5.957	7.7283	1	0	f	0.8220	2.6921	1	0	Equal to 93.77%
special benchmark ratio$_{(\alpha)}$=7.7283						y	2.213	2.6921	1	0	Disappeared planet
						d	5.957	2.6921	1	0	Reference planet
						leading ratio$_{(\alpha)}$=2.6921					

Note 1. The symbol ω_P is the ratio parameter of periods and θ_P is the deviation rate of ω_P. They are calculated according to equation (1) and (3). The symbol ω_α is the ratio parameter of semimajor axes and θ_α is the deviation rate of semimajor axis ratio. They are calculated according to equation (2) and (4). The symbol † is calculated according to equation (11) where br_P is benchmark ratio$_{(P)}$ and br_α is benchmark ratio$_{(\alpha)}$.

Note 2. The ratio parameters are calculated according to the representative ratio because the period ratios of adjacent planets show too big difference. Since Planet d has the biggest mass, we temporarily take the period ratio of Planet d to f as the representative ratio of the system. Based on the calculations, the calculations provide the deviation rates of all observed planets (See [1] & [2]). It is an abnormal system although the innermost period ratio is close to the outermost ratio, implying some similar orbital relation. So are the calculations of semimajor axis ratios.

Note 3. The possible evolution of the system

The system is a binary-star system. Perhaps, the companion star affected the formation of the planetary system of 55 Cnc A. The gravitational pull of the companion star might make the initial protoplanetary disc become an elliptical disc. As a result, the matter of the elliptical disc in the apical region of the long axis formed the super-Jupiter Planet d. The influence of the companion star at last makes the planets of the system show very big orbital ratios and very big mass ratios.

Although it is a mix system, we still tried to analyses the possible evolution of the system according to the orbital relations and mass relations of the current planets. In general, the period ratios of the planets in a multi-planet system are between 1.3 and 4.0. Although 55 Cnc A shows the period ratios composed by the two huge planets, the period ratios composed by the relatively two small planets are 3.03 and 5.86, which are close to the period ratios of normal planets. So, we took the average of the period ratios of P_f:P_c and P_c:P_b as the representative ratio and analysed the system with equation (1) and (2) (See [3]). Even though all deviation rates are still big, the system can be analysed with the corresponding equations and principles of the shared rules. Comparisons show that Planet c seems to migrate greatly according to trait (2). According to equation (7), there should be an undetected planet between Planet d and f by $\omega_{P(d:f)}$=108.6%$(rr_P)^2$, and an undetected planet between Planet b and e by $\omega_{P(f:c)}$=100.08%$(dr_P)^2$. The calculations reflect the possibly real planetary configuration. Since Planet d and b are huge, they should keep the initial orbital relations and do not change their orbits for special reasons. So, the shared divisor of the periods from Planet d to Planet b are supposed to be an identical value. According to the calculation with the adjusted equation (9), showing sd=$(P_d \div P_b)^{1/(7-3)}$, the shared divisor is 4.4165 by sd=$(5574.2 \div 14.6516)^{1/(7-3)}$. It can be seen that the shared divisor is 99.4% equal to the average of the period ratios of P_f:P_c and P_c:P_b. The degree of equality state the above analyses seems reasonable. In this case, according to the concept of the standard rule, we took the shared divisor as the leading ratio and Planet d as reference planet. The calculations with equation (9) show five planets between Planet d and e, seven planets totally in the system. We will not discuss whether there are planets outside the orbit of Planet c temporarily though there might be. The calculations reflect that the calculated period of Planet e, showing 0.75113 days by $P_{e(cal)}$=5574.2÷$4.4165^{(7-1)}$, is 97.41% equal to its observed period. If considering Planet d and b, which calculated periods are equal to their observed planets, Planet b, c and e can be judged to still follow the shared rules according to principle (1). Therefore, the shared rules should exist in the system according to principle (2). In this case, the two calculated periods that are not displayed by the undetected planets should belong to undetected planets according to principle (4). Since the calculated periods of Planet c and f are different from their observed periods obviously, they should migrate in the past and be migratory planets according to principle (3). The calculations reflect the previous planetary distribution. Although 55 Cnc A is in a mixed system and is an abnormal system, the shared rules still exist in the system. With the application of corresponding equations and principles, one can still research the evolution of such a system.

Because it is in a mixed system, the formation and evolution of the 55 Cnc A system should be complex. Especially the influence of the two huge planets may cause some planets to have disappeared or migrated greatly. Now that the calculations proposed the possible change of planetary configuration, we will try to briefly explain the undetected planets and migratory planets. Because Planet d is huge, it must have great perturbation on the motion of its inner planets. So, Planet y might be ejected of the system by the dragging force of Planet d. For Planet x, the dragging force of Planet b may also eject it off the system. Planet x and y may have disappeared from the system. If comparing the 55 Cnc A system with the Solar System, the relation of Planet d and y is something like that of Jupiter and an asteroid. Since Planet d is bigger than Jupiter, Planet y should be disturbed more greatly than an asteroid. So, it is more possible that Planet y was ejected off the system by Planet d. So is the relation ob

Planet b and Planet x. As to the migrations of Planet c and f, the perturbations of huge Planet c and b may play important roles. Because Planet c and f are much smaller than Planet b and d, they migrated inwards under the influence of the dragging forces of Planet d and b. But meanwhile, Planet y should have affected the motion of Planet f and c. As Planet d is a super-Jupiter planet, there might be another planet outside of its orbit. If considering the missing planets, the explanation of the formation of the system may be more objective. Supposing the protoplanetary disk of 55 Cnc A was elliptic under the influence of 55 Cnc B, the matter distribution in the disk should be relatively even and continuous. The missing planets played the roles of matter transition between planets. For the evolution of the mixed system, more studies are required.

Table S33. Calculations of the period ratio parameters and deviation rates of adjacent planets of the GJ 667 C system and analysis of the change of the planetary configuration.

Comparison of the periods of adjacent planets [1]					Comparison of the semimajor axes of adjacent planets [2]					
Planet	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	α (AU)	Ratio of α_n:α_{n-1}	ω_α	θ_α (%)	$M_{Jup.}$ ($M_{Jup.}$)	Ratio of M_n:M_{n-1}
b	7.2004				0.0505				0.018	
c	28.14	3.9081	1	0	0.125	2.4795	1	0	0.012	0.667
f	39.026	1.3869	0.3549	-64.51	0.156	1.6354	0.6596	-34.04	0.008	0.667
e	62.24	1.5948	0.4081	-59.19	0.213	1.3664	0.5511	-44.89	0.008	1
g	$256.2^{+13.8}_{-7.9}$	4.1163	1.0533	5.33	0.549	2.5775	1.0395		0.014	1.75
	representative ratio$_{(P)}$=3.9081				representative ratio$_{(\alpha)}$=2.4795				†br_P=100.1%$(br_\alpha)^{3/2}$	

Analysis of current planets [3]					Period of undetected planet, previous periods of migratory planets [4]					
Planet	Observed P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Planet	Calculated P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Compare observed to calculated period
b	7.2004				b	9.6215				Equal 75% or
c	28.14	3.9081	1	0	z	15.344	1.5948	1	0	Undetected planet
f	39.026	1.3869	0.3549	-64.51	c	24.47	1.5948	1	0	Equal 115%
e	62.24	1.5948	0.4081	-59.19	f	39.026	1.5948	1	0	Equal 100%
g	256.2	4.1163	1.0533	5.33	e	62.24	1.5948	1	0	Reference planet
	representative ratio$_{(P)}$=3.9081				x	99.23	1.5948	1	0	Undetected planet
					y	158.3	1.5948	1	0	Undetected planet
					g	252.46	1.5948	1	0	Equal ~100%
						leading ratio$_{(P)}$=1.5948				

Note 1. The symbol ω_P is the ratio parameter of periods and θ_P is the deviation rate of ω_P. They are calculated according to equation (1) and (3). The symbol ω_α is the ratio parameter of semimajor axes and θ_α is the deviation rate of semimajor axis ratio. They are calculated according to equation (2) and (4). The symbol † is calculated according to equation (11) where br_P is benchmark ratio$_{(P)}$ and br_α is benchmark ratio$_{(\alpha)}$.

Note 2. The analysis of the system

It is a mixed system. A direct comparison does not reflect the signs of the shared rules. But we still tried to analyse the system with the shared rules. We first analysed the periods of all observed planets with equation (9). By taking the period ratio of Planet e to f as the leading ratio and Planet e as reference, the calculations are shown in the table (See[4]). For Planet g, the calculated period is 252.46 days by $P_{g(cal)}=P_e\div[P_e:P_f]^{(5-8)}$, almost 100% equal to its observed period. If the equality is not by chance, it should reflect the specific orbital relations among Planet g, e and c, suggesting that the shared rules exist among them. So, according to principle (1), the three planets should be the planets that follow the shared rules though it is an abnormal planetary system of a mixed system. Since Planet e and f follow the shared rules, the calculations based on the orbits of the two planets should reflect the orbital changes of corresponding planets. So, the results that the calculated periods of Planet b and c are separately equal to their observed periods to the degrees of 75 and 115% suggest that they have migrated obviously. The calculation also provide a period of 15.344 days by $P_z=62.24\div1.5948^{(5-2)}$, which does not belong to any observed planet. According to principle (4), it should belong to an undetected planet between Planet c and b. The calculations meanwhile show two periods, which are not the periods of observed planets. According to principle (4), it is very possible that they belong to the periods of two undetected planets between Planet g and e. According to the calculations and analyses, GJ 667 C is likely to be a normal system in the past (See [4]).

As to the change of planetary configuration, there may be a few possibilities according to the limited observations. We will first analyse the migrations of Planet b and c. Because Planet b and c reflect relatively big deviation rates, it is more possible that they were disturbed greatly. The calculation shows an undetected planet between them. So, the disappearance of the planet may be the reason that caused their migrations. Some inducement, perhaps the effect of Kirkwood gap of near 3:2 resonance, may first change the orbital shape of Planet c a little and then the dragging force of its adjacent Planet z enlarged the change. Because of the increase of the eccentricity of Planet c, it became close to Planet z at its perihelion. Under the influence of Planet c, the eccentricity of Planet x also increased. So, when Planet c moved to its perihelion while Planet z moved to its aphelion, they should be very close, which made the effect of dragging forces more obvious. When the two planets were very close at some region, Planet c was bounced to the orbit farther to the star while Planet z was pushed toward the star because of the exchange of kinetic energy. During the fall of Planet z with an elliptical orbit, it got close to Planet b and dragged Planet b to the star, embodying the inward migration of Planet b. The interaction between Planet z and b made Planet z change its orbit again. As a result, the system shows that Planet c migrates outwards equivalent to about 15% of its previous period, Planet b migrates inwards equivalent to about 25% of its previous period, and Planet z fails to appear in the area where it should orbit the star. For the two undetected planets between Planet g and e, it is more possible that they have collided with each other. Because of some disturbance, perhaps a stray planet, Planet x or y changed its orbit first and got near to its adjacent planet. Some day, they were too close to each other and collided with each other or were ejected off their initial orbits by themselves, and disappeared from the system. So, the two calculated orbits between Planet g and e do not display the two planets.

There is another possibility. The original system had six planet, Planet g, y, x, e, f and c. The interaction of Planet x and y caused them to migrate to different directions. Planet x flew out of the system and Planet y fell toward the star, or visa versa. Planet y crossed the orbits of Planet e and f with a fast speed. When it passed to the orbit of Planet c, they came near, which caused them to be affected by the mutual dragging forces obviously. As a result, Planet c migrated outwards while Planet y kept falling. When the orbiting speed of Planet y was balanced by the gravitation of the star, it stopped to fall and orbit the star at the orbit of current Planet b. As we mainly discuss the shared rules, we will not discuss the system evolution more.

Table S34. Calculations of the orbital ratio parameters and deviation rates of adjacent planets of the HD 108236 system. Prediction of Planet f based on the orbits of observed planets.

Analysis of the planetary orbits of the system based on the observation data published in 2021

Comparison of the periods of adjacent planets [1] — Comparison of the semimajor axes of adjacent planets [2]

Planet	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	α (AU)	Ratio of α_n:α_{n-1}	ω_α	θ_α (%)	Radius (R_{jup})	Ratio of R_n:R_{n-1}
b	3.795963				0.04527±0.0009				0.1441	
c	6.203449	1.6342	1.0402	4.02	0.0620±0.0012	1.3696	1.0315	3.15	0.1848	1.282
d	14.175685	2.2851	1.4545	45.45	0.1074±0.0023	1.7323	1.3046	30.46	0.2265	1.226
e	19.590025	1.3820	0.8796	-12.04	0.1367±0.0021	1.2728	0.9586	-4.14	0.2750	1.214
f	29.54115	1.5080	0.9598	-4.02	0.1758±0.0040	1.2860	0.9685	-3.15	0.1799	0.654
	benchmark ratio$_{(P)}$=1.5711				benchmark ratio$_{(\alpha)}$=1.3278		†br_P=102.69%$(br_\alpha)^{3/2}$			

Analysis of observed planets shows the change of planetary configuration

Comparison of current semimajor axes [3] — Calculations according to the planets with the shared rules [4]

Planet	Calculated P (days)	Ratio P_n:P_{n-1}	ω_P	θ_P (%)	Compare of periods	α (AU)	Ratio of α_n:α_{n-1}	ω_α	θ_α (%)	Compare observed α to calculated α
b	3.78817				Equal to 100.2%	0.04527				Equal to 100%
c	5.71256	1.5080	1	0	Equal to 108.6%	0.05968	1.31823	1	0	Equal to 103.9%
x	8.61455	1.5080	1	0	undetected planet	0.07867	1.31823	1	0	undetected planet
d	12.99073	1.5080	1	0	Equal to 109.1%	0.1037	1.31823	1	0	Equal to 103.6%
e	19.59003	1.5080	1	0	Reference planet	0.1367	1.31823	1	0	Reference planet
f	29.54115	1.5080	1	0	100%	0.1802	1.31823	1	0	Equal to 97.6%
	leading ratio$_{(P)}$=1.5080					leading ratio$_{(\alpha)}$=1.31823				

Analysis of the planetary orbits of the system based on the observation data published in 2020

Comparison of the periods of adjacent planets [5] — Calculations according to the planets with the shared rules [6]

Planet	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Planet	Calculated P (days)	Ratio P_n:P_{n-1}	ω_P	θ_P (%)	Compare observed P to calculated P
b	3.79523				b	3.79523				Equal to 100%
c	6.20370	1.6346	1	0	c	5.7207	1.5073	1	0	Equal to 108.44%
d	14.17555	2.2850	1.3979	39.79	x	8.6231	1.5073	1	0	Undetected planet
e	19.5917	1.3821	0.8455	-15.45	d	12.9978	1.5073	1	0	Equal to 109.06%
	benchmark ratio$_{(P)}$=1.6346				e	19.5917	1.5073	1	0	Reference planet
					y	29.531	1.5073	1	0	Predicted planet
						leading ratio$_{(P)}$=1.5073				

Note 1. "ω_P" is the ratio parameter of periods and "θ_P" is the deviation rate of period ratio. "ω_α" is the ratio parameter of semimajor axes and "θ_α" is the deviation rate of semimajor axis ratio. They are calculated according to equation (1-4). The symbol † is calculated according to equation (11) where br_P is benchmark ratio$_{(P)}$ and br_α is benchmark ratio$_{(\alpha)}$.

Note 2. Calculated according to the data published in 2021 (*20,10*), we show the orbital relations of the five planets (See [1] and [2]). It shows a 45.45% deviation rate. The analysis of the period ratios reflected an undetected planet between Planet d and c (Sect. 2.4 in the paper). In the same way, we calculated the semimajor axes with equation (10). The calculations also reflected the undetected planet and the migrations of Planet d and c

(See [3] and [4]).

Note 3. Analysis of the change of planetary configuration

We will try to offer a simple inference. Since the calculations reflect an undetected and tow migratory planets, we can take them as the entry point to study the evolution of the system. As super Earths, the perturbations of dragging force between Planet d, x and c should be great since their previous orbits were relatively very near. Perhaps they were once in the orbital resonances of near 3:2 period ratios. The obviously-increased mutual dragging forces might first break the stable motion of Planet d and x and then change their orbital shapes. When they reached the region of perihelion and aphelion separately, they got relatively closer, which made the perturbations more obvious. The interaction between them made them migrate to opposite directions, showing that Planet d migrated outwards while Planet x migrated inwards. In this case, Planet x got near to Planet c. Perhaps the gravitation between them caused them to have changed their position, showing that Planet c migrated outwards while Planet x crossed the orbit of Planet c. As Planet c increased its kinetic energy during the interaction, Planet x naturally lost kinetic energy. As a result, Planet x flew toward the star. It quickly got across the orbit of Planet b and fell into the star. The inference complies with the law of kinetic energy conversion. More studies are required.

Note 4. Prediction of the fifth planet

Only four planets were discovered in 2020 (*10*). However, the calculation with corresponding equations could reflect the real planetary configuration of the system at that time . We will show the explanations.

According to the calculation with equation (2), the system showed a 39.79% deviation rate, implying the absence of a planets between Planet d and c (See [5]). In order to learn about the possible period of the absent planet, we tried to calculate its orbit with equation (9). Because the three period ratios are obviously different, neither of the ratios may reflect the leading ratio. So, we supposed that the innermost and the outermost planet keep the initial orbit. In this case, the periods of all planets should vary with a shared divisor if the planets followed the shared rules. Therefore, the shared divisor from Planet e to b should be 1.50733, by $(19.5917\div3.79523)^{1/(5-1)}$. When we calculated the average of P_e:P_d and P_c:P_b, it is 1.50835. The two values are almost equal. So, the calculated shared divisor should be possible for the system. By taking 1.5073 as the leading ratio and Planet e as reference planet, the calculations provide the period of the absent planet, showing $P_x=19.5917\div1.5073^{(5-3)}=8.6231$(day). Meanwhile, the calculations shows the migrations of Planet c and d. Their migrations should be caused by the dragging forces between Planet c, x and d. Perhaps, some reason may first caused the orbital changes of Planet d and x. The mutual dragging force made Planet d migrate outwards. Because of the exchange of kinetic energy, Planet x migrated inwards and got nearer to the orbit of Planet c. The gravitational interaction between them caused Planet c to migrate outwards and Planet x to fall into the star or orbits the star in an obvious inclination.

Because Planet e is a Neptune-sized planet, it might not consume away all matter in the outer space of the protoplanetary disk. At the area outside the orbit of Planet e, there might be a transitional planet smaller than Planet e. So, it is possible that an undetected planet may orbit the star, temporarily called Planet y. Based on the inference, we calculated the possible period of Planet y according to the orbital relations of the discovered four planets. Still by taking Planet e as reference planet and the shared divisor as the leading ratio, its period should be 29.531 days by $P_y=19.5917\div1.5073^{(5-6)}$ (See [4]). One year later, Bonfanti, A. et al. (*20*) declared the discovery of a fifth exoplanet in the HD 108236 system and named it as Planet f. Comparison shows that the period of Planet y is 99.97% equal to the period of Planet f by $(29.531\div29.54115)\times100\%$ (See [1] and [6]). If taking the average of P_e:P_d and P_c:P_b as the leading ratio of equation (9), the calculated period of Planet y is 29.550 days, almost the same result. The discovery of Planet f is also a nice support to the existence of the shared rules, reflecting the realistic meaning of the relation equations between planetary orbits.

Table S35. Calculations of the period ratio parameters and deviation rates of adjacent planets of the HD 158259 system.

	Comparison of the periods of adjacent planets				Comparison of the semimajor axes of adjacent planets					
Planet	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	ɑ (AU)	Ratio of $ɑ_n$:$ɑ_{n-1}$	$\omega_ɑ$	$\theta_ɑ$ (%)	Mass ($M_{Jup.}$)	Ratio of M_n:M_{n-1}
b	2.17800				--				0.00698	
c	3.43200	1.5758	1.0278	2.78	--				0.0176	2.522
d	5.198081	1.5146	0.9879	-1.21	--				0.0170	0.966
e	7.9510	1.5296	0.9977	-0.23	--				0.0191	1.124
f	12.028	1.5128	0.9867	-1.33	--				0.0193	1.010
	benchmark $ratio_{(P)}$=1.5332									

Note 1. "ω_P" is the ratio parameter of periods and "θ_P" is the deviation rate of period ratio. They calculated according to equation (1) and (3).

Note 2.The semimajor axes of the five planets are not published now.

Table S36. Calculations of the orbits ratio parameters and deviation rates of adjacent planets of the HD 23472 system.

Comparison of the periods of adjacent planets [1]					Comparison of the semimajor axes of adjacent planets [2]					
Planet	Observed P (day)	Ratio of $P_n:P_{n-1}$	ω_P	θ_P (%)	Observed α (AU)	Ratio of $α_n:α_{n-1}$	$\omega_α$	$\theta_α$ (%)	Radius (R_{jup})	Ratio of $R_n:R_{n-1}$
d	3.976640				0.04298				0.0669	
e	7.90754	1.9885	1.2333	23.33	0.0680	1.5821	1.1510	15.1	0.0730	1.091
f	12.16218	1.5381	0.9539	-4.60	0.0906	1.3324	0.9694	-3.06	0.1014	1.389
b	17.66709	1.4526	0.9009	-9.91	0.1162	1.2826	0.9331	-6.69	0.178	1.7554
c	29.79749	1.6866	1.0460	4.60	0.1646	1.4165	1.0306	3.06	0.167	0.938
benchmark ratio$_{(P)}$ =1.6124					benchmark ratio$_{(α)}$=1.3745				†br_P=100.06%$(br_α)^{3/2}$	

Calculations based on the planets following the shared rules [3]						Calculations based on the planets following the shared rules [4]				
Planet	Calculated P* (day)	Ratio of $P_n:P_{n-1}$	ω_P	θ_P (%)	Comparison of P:P*	Calculated P* (day)	Ratio of $P_n:P_{n-1}$	ω_P	θ_P (%)	Comparison of P:P*
d	3.976640				100%	3.86435				102.91%
e	6.5793	1.6545	1	0	120.2%	6.41366	1.6597	1	0	123.29%
f	10.8855	1.6545	1	0	111.7%	10.6448	1.6597	1	0	114.25%
b	18.010	1.6545	1	0	98.10%	17.66709	1.6597	1	0	Reference
c	29.79749	1.6545	1	0	Reference	29.3221	1.6597	1	0	101.62%
leading ratio$_{(P)}$ =1.6545						leading ratio$_{(P)}$ =1.6597				

Note 1.The symbol ω_P is the ratio parameter of periods and θ_P is the deviation rate of ω_P. They are calculated according to equation (1) and (3). The symbol $\omega_α$ is the ratio parameter of semimajor axes and $\theta_α$ is the deviation rate of semimajor axis ratio. They are calculated according to equation (2) and (4). The symbol † is calculated according to equation (11) where br_P is benchmark ratio$_{(P)}$ and $br_α$ is benchmark ratio$_{(α)}$.

Note 2. Analysis of the system

The calculations with equation (1-4) suggest a near-normal system. Because of the interaction of mutual dragging forces, all planets might migrate more or less. In the analysis, we used two values as the leading ratio of equation (9). The first is the shared divisor for all planets, which is calculated from Planet c to d, showing 1.6545 by sd=$(P_c÷P_d)^{1/(5-1)}$ or sd=$(29.79749÷3.976640)^{1/(5-1)}$.The second is the average of the inner three period ratios, showing 1.6597 by 1.9885+1.5381+1.4526)/3. The results are similar (See [3] and [4]), suggesting that Planet f and e migrate obviously. For the cause of the migrations, we will not discuss more because they may not be caused by the interaction of mutual dragging forces.

Table S37. Calculations of the period ratio parameters and deviation rates of adjacent planets of the HD 40307 system and analysis of the change of the planetary configuration.

Comparison of the periods of adjacent planets [1]					Comparison of the semimajor axes of adjacent planets [2]					
Planet	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	α (AU)	Ratio of α_n:α_{n-1}	ω_α	θ_α (%)	Mass ($M_{Jup.}$)	Ratio of M_n:M_{n-1}
b	4.3123				0.0468				0.0126	
c	9.6184	2.2305	0.9364	-6.36	0.0799	1.7073	0.9546	-4.54	0.0208	1.6508
d	20.432	2.1243	0.8919	-10.8	0.1321	1.6533	0.9244	-7.56	0.0299	1.4375
f	51.76	2.5333	1.0636	6.36	0.247	1.8698	1.0455	4.54	0.0164	0.5485
g	197.8	3.8215	1.6044	60.44	0.600	2.4292	1.3582	35.82	0.0223	1.3598
	benchmark ratio$_{(P)}$=2.3819				benchmark ratio$_{(\alpha)}$=1.7885		†br_P=99.59%$(br_\alpha)^{3/2}$			

Period analysis of observed planets [3]					Period calculations of all planets based on the planets that follow the shared rules [4]					
Planet	Observed P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Planet	Calculated P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Compare observed P to calculated P
b	4.3123				b	4.5278				Equal to 95.24%
c	9.6184	2.2305	0.9364	-6.36	c	9.6184	2.1243	1	0	Equal to 100%
d	20.432	2.1243	0.8919	-10.8	d	20.432	2.1243	1	0	Reference planet
f	51.76	2.5333	1.0636	6.36	f	43.4	2.1243	1	0	Equal to 119.3%
g	197.8	3.8215	1.6044	60.44	x	92.2	2.1243	1	0	Undetected planet
	benchmark ratio$_{(P)}$=2.3819				g	195.87	2.1243	1	0	Equal to 100.98%
						leading ratio$_{(P)}$=2.1243				

Note 1.The symbol ω_P is the ratio parameter of periods and θ_P is the deviation rate of ω_P. They are calculated according to equation (1) and (3). The symbol ω_α is the ratio parameter of semimajor axes and θ_α is the deviation rate of semimajor axis ratio. They are calculated according to equation (2) and (4). The symbol † is calculated according to equation (11) where br_P is benchmark ratio$_{(P)}$ and br_α is benchmark ratio$_{(\alpha)}$.

Note 2. Analysis of the system

Calculated according to equation (1) and (2), the system shows the ratio parameters and deviation rates of observed five planets. The 60.44% deviation rate suggests that it is an abnormal system now (See [3]). In this case, we tried to calculate the periods of all observed planets with equation (9). Considering that Planet d has the biggest mass and its mass is close to the mass of Planet c, we took their period ratio as the leading ratio and Planet d as reference. The calculation provides a set of periods (See [4]). It shows that the calculated period of Planet g is equal to 100.98% of its observed period by $P_{g(cal)}=P_d\div[P_d:P_c]^{(3-6)}$. According to principle (1), Planet g, d and c are the planets that follow the shared rules. So, according to principle (2), the shared rules should still exist in the system. The calculation also shows 92.2 days by $P_x=P_d\div[P_d:P_c]^{(3-5)}$, a period that does not belong to any observed planet. Since the calculated period is based on the planets following the shared rules, it should belong to an undetected planet according to principle (4), temporarily called Planet x. Comparison shows that the calculated period of Planet f is equal to its observed period to the degrees of 119.3%. According to principle (3), the calculated period of 43.4 days should be the previous period of Planet f before it migrated. Because the calculated period of Planet b is equal to 95.24% of its observed period, it can be considered as a planet following the shared rules although it may migrate a little. The calculations restored the previous planetary configuration (See [4]).

The calculations reflect the migration of Planet f and absence of a planet between Planet f and g. For the orbital change of Planet f and Planet x, there are two possibilities. The first possibility is that the interaction of mutual dragging forces caused Planet x to be ejected off the system by Planet f. The exchange of kinetic energy also caused Planet f to migrate outwards. The second possibility is that Planet x has not been discovered for some reason and still orbits the star at an undetected orbit because of the perturbation of Planet f. More observations are needed.

Note 3. Uncertain planet in the system

According to the NASA Exoplanet Archive (2021), five exoplanets are published. But according to Extrasolar Planets Encyclopaedia (*11*), six exoplanets are published, including Planet e with P=34.62(day). Tuomi et al. (2013) proposed six exoplanets in the system while Diaz et al.(2016) had different proposals for the system. After comparison, the calculations with the equations of the shared rules show something similar as the researches performed by astronomers (Tuomi et al. 2013, Diaz et al. 2016). When we read the research article of Prof. Tuomi M. et al., we carefully researched the RV signals in the article reflected by the possible planets of HD 40307 and noticed that in the figures of RV signals, especially the 4^{th} signals of Fig 1, 2 and 8, a clear peak simultaneously points at the period of 80-100 days, which may be reflected by a planet. In this case, more observation are required.

Table S38. Calculations of the orbits ratio parameters and deviation rates of adjacent planets of the K2-268 system.

Comparison of the periods of adjacent planets [1]					Comparison of the semimajor axes of adjacent planets [2]					
Planet	Observed P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	α (AU)	Ratio of α_n:α_{n-1}	ω_α	θ_α (%)	Radius (R_{jup})	Ratio of R_n:R_{n-1}
b	2.151676				0.0308				0.134	
d	4.528598	2.1047	1.1609	16.09	0.0819				0.133	0.993
e	6.131243	1.3529	0.7462	-25.38	--				0.119	0.895
c	9.327527	1.5213	0.8391	-16.09	--				0.240	2.017
f	26.27057	2.8166	1.5536	55.36	--				0.199	0.829
	benchmark ratio$_{(P)}$=1.8130									

Calculations based on the planets following the shared rules [2]						Calculations based on the planets following the shared rules [3]				
Planet	P* (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	**Comparison (P:P*)**	P* (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	**Comparison (P:P*)**
b	2.32419				92.58%	2.1514				Equal to ~100%
d	3.77495	1.6242	1	0	119.96%	3.5486	1.6495	1	0	Equal to 127.6%
e	6.13128	1.6242	1	0	100%	5.8535	1.6495	1	0	Equal to 104.7%
c	9.95842	1.6242	1	0	93.67%	9.6553	1.6495	1	0	Equal to 96.61%
x	16.1745	1.6242	1	0	Missing	15.926	1.6495	1	0	Missing planet
f	26.27057	1.6242	1	0	Reference	26.27057	1.6495	1	0	Reference planet
	leading ratio$_{(P)}$=1.6242					leading ratio$_{(P)}$=1.6495				

Note 1.The symbol ω_P is the ratio parameter of periods and θ_P is the deviation rate of ω_P. They are calculated according to equation (1) and (3).

Note 2. The semimajor axes of the five exoplanets are not published by NASA. The stellar mass is M_{star}=0.84Mʘ

Note 3. Analysis of the system

According to the calculation with equation (1) and (2), it is an abnormal system for the large deviation rate of 55.36%. The large deviation rate usually implies the absence of one or two planets. So, we calculated the periods of all possible planets with equation (9). Because the period ratios of observed planets are obviously different and do not show any connections, we calculated the shared divisor (sd) of the periods of all planets according to the standard rule between planetary orbits. It shows 1.6495 by sd=$(P_f \div P_b)^{1/(6-1)}$ or sd=$(26.27057 \div 2.151676)^{1/(6-1)}$. Then we took the shared divisor as the leading ratio of equation (9) and Planet f as reference planet. The calculations are presented in the table (See [3]). Comparison shows that Planet b, e, c and f should be the planets that follow the shared rules according to criterion (N-A). Therefore, the shared rules still exist in the system according to principle (2). In this case, the calculated period of 15.926 days should belong to the absent planet according to principle (4). The calculations reflect the true planetary configuration.

As to the absent planet, it is likely to still orbits the star with an inclination obviously different from that of other planets. The calculations only reflect the possibly undetected exoplanet. Because the mutual perturbations of all planets, every planets may have changed its orbit more or less. So, the obvious migration of Planet d may be the result of the influence of the dragging forces of all planets. There is another possibility that a stray planet passed by the system. The stray planet is the inducement of the orbital changes of Planet x and Planet d. More observations and analyses are needed.

The orbital change of planetary configuration should be complex because all planets are perturbed with each other. The perturbations must cause every planets to change their orbits more or less, so as to keep the balance of dynamical stability and the equivalence of the exchange of kinetic energy. Since we do not know how the system evolves, we have to propose the most possible inference based on the hypothesis that some planets may keep their initial orbits. A different hypothesis may lead to a different inference, such the calculation in the table (See [2]). Therefore, it is easy to learn about the change of planetary configuration, but it is not easy to analyse the cause of the change. More observations are necessary.

There is another analysis We can also analyse the system according to the standard rule by taking the shared divisor (sd) of the planetary periods from Planet e to b. It show sd=$(37.9216 \div 4.51512)^{1/(4-1)}$=2.06485. The calculations are shown in the table.

Table S39. Calculations of the period ratio parameters and deviation rates of adjacent planets of the K2-384 system and analysis of the possible change of planetary configuration of the system

Calculation of adjacent periods [1]						Calculation based on the planets following shared rules [2]					
Planet	Observed P (days)	Ratio P_n:P_{n-1}	ω_P	θ_P (%)	Radius (R_{Earth})	Planet	Calculated P (days)	Ratio P_n:P_{n-1}	ω_P	θ_P (%)	Compare observed P to calculated P
b	2.231527				1.076	b	2.29014				Equal to 97.44%
c	4.194766	1.8798	1.2340	23.4	1.191	x	3.27170	1.4286	1	0	Disappeared planet
d	6.679582	1.5924	1.0453	4.53	1.392	c	4.67395	1.4286	1	0	Equal to 89.75%
e	9.715043	1.4544	0.9547	-4.53	1.345	d	6.67720	1.4286	1	0	Equal to 100.04%
f	13.62749	1.4027	0.9208	-7.92	2.222	e	9.53905	1.4286	1	0	Equal to 101.85%
	benchmark ratio$_{(P))}$=1.5234					f	13.62749	1.4286	1	0	Reference planet
							leading ratio$_{(P))}$=1.4286				

Note 1.The symbol ω_P is the ratio parameter of periods and θ_P is the deviation rate of ω_P. They are calculated according to equation (1) and (3).

Note 2. The semimajor axes of the planets are not published.

Note 3. Analysis of the change of planetary configuration of K2-384

The calculation with equation (4) reflects that it is not a normal system. In order to learn about whether the shared rules still exist in the system, we calculated the periods of all planets with equation (9). By taking the average of P_f:P_e and Pe:P_d as the leading ratio and the biggest Planet f as reference planet, the calculations by $P_{n(cal)}=P_f\div[lr_P]^{(sf-sn)}$, are presented in the table. The calculations show that the calculated periods of Planet b, d, e and f are equal to 97.44, 100.04, 101.85 and 100% of their observed periods separately. According to principle (1), they follow the shared rules. The four planets suggest the existence of the shared rules in the system according to principle (2). The calculation also reveals an undetected planet with the period of 3.2717 days by $P_{d(cal)}=13.62749\div1.4286^{(6-2)}$. In addition, the calculated period of Planet c is about 10% from its observed period. Because the calculated period is based on the planets that follow the shared rules, according to principle (3), Planet c should be a migratory planet. It is more possible that the dragging force of Planet c ejected Planet x off the system. The exchange of kinetic energy caused Planet c to migrate inwards, equivalent to about 10% of its previous period.

Table S40. Calculations of the period ratio parameters and deviation rates of adjacent planets of the Kepler-102 system and analysis of the possible change of planetary configuration of the system.

Comparison of the periods of adjacent planets						Calculation based on the planet with the shared rules					
Planet	Observed P (days)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Radius (R_{jup})	Ratio of R_n:R_{n-1}	Calculated P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Compare observed P to calculated P
b	5.28696				0.042		4.67091				Equal to 113.2%
c	7.07142	1.3375	0.8846	-11.54	0.052	1.2381	7.0624	1.5120	1	0	Equal to 100.1%
d	10.3117	1.4582	0.9644	-3.56	0.105	2.0192	10.6784	1.5120	1	0	Equal to 96.57%
e	16.1457	1.5658	1.0356	3.56	0.198	1.8857	16.1457	1.5120	1	0	Reference
f	27.4536	1.7004	1.1246	12.46	0.079	0.3990	24.4123	1.5120	1	0	Equal to 112.5%
	benchmark ratio$_{(P)}$=1.5120						leading ratio$_{(P)}$=1.5120				

Note 1.The symbol ω_P is the ratio parameter of periods and θ_P is the deviation rate of ω_P. They are calculated according to equation (1) and (3).

Note 3. Observation data do not provide the semimajor axes of the planets.

Note 4. It is a near-normal system. In order to confirm whether the shared rules still exist in the system, we recalculated the periods of all planets with equation (9). By taking the benchmark ratio as the leading ratio and the biggest Planet e as reference planet, the calculations are presented in the table. Comparisons show that the calculated periods of Planet c, d and e are equal to their observed periods within criterion (N-A). According to principle (1), they follow the shared rules. So the shared rules exist in the system.

Table S41. Calculations of the orbital ratio parameters and deviation rates of adjacent planets of the Kepler-122 system and judgement of the planets following the shared rules.

Comparison of the periods of adjacent planets					Comparison of the semimajor axes of adjacent planets					
Planet	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	α (AU)	Ratio of α_n:α_{n-1}	ω_α	θ_α (%)	Radius (R_{jup})	Ratio of R_n:R_{n-1}
b	5.766193				0.064				0.209	
c	12.46598	2.1619	1.2443	24.43	0.108	1.6875	1.1523	15.23	0.524	2.5072
d	21.587475	1.7150	0.9871	-1.29	0.155	1.4352	0.980	2	0.196	0.3740
e	37.993273	1.7600	1.0130	1.3	0.227	1.4645	1	0	0.232	1.1837
f	56.268	1.4810	0.8524	-14.76	---				0.156	0.6724
benchmark ratio$_{(P)}$=1.7375					benchmark ratio$_{(\alpha)}$=1.4645		†br_P=99.31%$(br_\alpha)^{3/2}$			

The judgement of the planets that follow the shared rules

Comparison of the periods of adjacent planets [3]					Calculations based on the planets with shared rules [4]				
Planet	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Calculated P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Compare observed P to calculated P
b	5.766193				6.96909				Equal to 82.74%
c	12.46598	2.1619	1.2443	24.43	12.2656	1.7600	1	0	Equal to 101.6%
d	21.587475	1.7150	0.9871	-1.29	21.587475	1.7600	1	0	Reference planet
e	37.993273	1.7600	1.0130	1.3	37.993273	1.7600	1	0	Equal to 100%
f	56.268	1.4810	0.8524	-14.76	66.868	1.7600	1	0	Equal to 84.2%
benchmark ratio$_{(P)}$=1.7375					leading ratio$_{(P)}$=1.7600				

Note 1.The symbol ω_P is the ratio parameter of periods and θ_P is the deviation rate of ω_P. They are calculated according to equation (1) and (3). The symbol ω_α is the ratio parameter of semimajor axes and θ_α is the deviation rate of semimajor axis ratio. They are calculated according to equation (2) and (4). The symbol † is calculated according to equation (11) where br_P is benchmark ratio$_{(P)}$ and br_α is benchmark ratio$_{(\alpha)}$.

Note 2. Because the observation does not show the semimajor axis of Planet f, the period dominant radio is based on the inner four planets. So, the calculation with equation(1) shows benchmark ratio$_{(P)}$ =1.7600, without considering P_f. According to 1.7600, the calculation of "†" shows, dr_P=99.31%$(dr_\alpha)^{3/2}$.

Note 3. According to trait (1), Planet e, d and c should be the planets that follow the shared rules. The calculation with equation (9) also shows the same results (See [4]). In this case, the shared rules still exists in the system according to principle (2).

Table S42. Calculations of the period ratio parameters and deviation rates of adjacent planets of the Kepler-150 system and analysis of the change of the planetary configuration.

Comparison of the periods of adjacent planets [1]					Comparison of the semimajor axes of adjacent planets [2]					
Planet	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	α (AU)	Ratio of α_n:α_{n-1}	ω_α	θ_α (%)	Radius (R_{jup})	Ratio of R_n:R_{n-1}
b	3.428054				0.044				0.112	
c	7.381998	2.1534	0.9347	-6.53%	0.073	1.65909	0.9545	-4.55	0.329	2.938
d	12.56093	1.7016	0.7386	-26.14%	0.104	1.42466	0.8196	-18.04	0.249	0.757
e	30.82656	2.4542	1.0653	6.53%	0.189	1.81730	1.0455	4.55	0.278	1.117
f	637.2093	20.671	8.9726	797%	1.24	6.56085	3.7745	277.45	0.325	1.169
	benchmark ratio$_{(P)}$=2.3038				benchmark ratio$_{(\alpha)}$=1.7382		†br_P=100.54%$(br_\alpha)^{3/2}$			

Period analysis of observed planets [3]					Period calculations of all planets based on the planets that follow the shared rules [4]					
Planet	Observed P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Planet	Calculated P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Compare observed P to calculated P
b	3.428054				b	3.428054				Equal to 100%
c	7.381998	2.1534	0.9347	-6.53%	c	7.381998	2.1534	1	0	Reference planet
d	12.56093	1.7016	0.7386	-26.1%	d	15.8964	2.1534	1	0	Equal to 79.02%
e	30.82656	2.4542	1.0653	6.53%	e	34.2313	2.1534	1	0	Equal to 90.05%
f	637.2093	20.671	8.9726	797%	f	73.7	2.1534	1	0	Previous orbit, migrated
	benchmark ratio$_{(P)}$=2.3038					leading ratio$_{(P)}$=2.1534				

Note 1.The symbol ω_P is the ratio parameter of periods and θ_P is the deviation rate of ω_P. They are calculated according to equation (1) and (3). The symbol ω_α is the ratio parameter of semimajor axes and θ_α is the deviation rate of semimajor axis ratio. They are calculated according to equation (2) and (4). The symbol † is calculated according to equation (11) where br_P is benchmark ratio$_{(P)}$ and br_α is benchmark ratio$_{(\alpha)}$.

Note 2. Analysis of the system

The planetary configuration is in disorder because great orbital changes may have occurred in the system. So, based on the period ratios, it is not easy to see the traces that reflects the initial orbital relation. Although the period ratio of Planet f to e is very big, about ten times bigger than other ratios, it has the similar radius as other planets. It may form in the same protoplanetary disk as other planets but later migrated greatly. So, its migration leaves a big gap area. In this case, I will analyse the inner four planets independently.

The representative period ratio of the inner four planets is period ratio of Planet c to b. So I will take the ratio as the leading ratio of equation (9) and recalculate the periods of the four planets. If taking Planet c as reference planet, the calculations show their periods (See [4]). It shows that the calculated period of Planet e is equal to 90.05% of its observed period. According to principle (1), Planet e, c and b can be considered the planets that follow the shared rules. Although the equation is not quite satisfactory, it can still be considered that the shared rules exist in the system. The calculation also reflects that Planet d should migrate inwards according to principle (3). Because Planet f is too far from other planets, it is possible that it greatly migrated outward to the current orbit for some reason.

There might be two possibilities that caused the current planetary configuration. The first possibility is that a planet (Planet x) once existed between Planet f and e. The mutual interactions among the external four planets made Planet x disappear and other planets f migrate greatly. The second possibility is that a stray planet passed by the system and caused the orbital changes of the external three planets. Because the shared rules are not quite clear, we

will not infer the orbital changes of the planets that might have migrated. More observations are necessary.

Table S43. Calculations of the period ratio parameters and deviation rates of adjacent planets of the Kepler-154 system and analysis of the possible change of planetary configuration of the system

Comparison of the periods of adjacent planets							Calculation based on the planet with shared rules				
Planet	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Radius (R_{jup})	Ratio of R_n:R_{n-1}	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Compare two periods
e	3.9328				0.134		5.0913				Equal to 77.25%
f	9.9194	2.5222	1.2747	27.47	0.134	1	9.4448	1.8551	1	0	Equal to 105.2%
d	20.550	2.0717	1.0470	4.7	0.343	2.560	17.521	1.8551	1	0	Reference
b	33.041	1.6078	0.8126	-18.74	0.202	0.589	32.503	1.8551	1	0	Equal to 101.7%
c	62.303	1.8857	0.9530	-4.7	0.263	1.302	60.297	1.8551	1	0	Equal to 103.3%
	benchmark ratio$_{(P)}$=1.9787						leading ratio$_{(P)}$=1.8551				

Note 1.The symbol ω_P is the ratio parameter of periods and θ_P is the deviation rate of ω_P. They are calculated according to equation (1) and (3). The symbol ω_a is the ratio parameter of semimajor axes and θ_a is the deviation rate of semimajor axis ratio. They are calculated according to equation (2) and (4).

Note 2. Observation data do not provide the semimajor axes of the planets.

Note 3. Though it is a near-normal system, the calculation with equation (9) reflects that the shared rules still exist in the system. According to calculations with the average of the period ratios of the external four planets as the leading ratio, three planets shows the calculated periods that equal to 100-105.2% of their observed planets. According to principle (1), they are the planets that follow the shared rules. Meanwhile, the calculation also suggests one absent planet and one obviously-migratory planet.

Table S44. Calculations of the period ratio parameters and deviation rates of adjacent planets of the Kepler-169 system and analysis of the change of the planetary configuration.

Comparison of the periods of adjacent planets [1]					Comparison of the semimajor axes of adjacent planets [2]					
Planet	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	α (AU)	Ratio of α_n:α_{n-1}	ω_α	θ_α (%)	Radius (R_{jup})	Ratio of R_n:R_{n-1}
b	3.2506				0.040				0.101	
c	6.1955	1.9060	1.0722	7.22	0.062	1.550	1.0509	5.09	0.108	1.069
d	8.3481	1.3475	0.7580	-24.20	0.075	1.2097	0.8201	-17.99	0.112	1.037
e	13.767	1.6491	0.9277	-7.23	0.105	1.40	0.9492	-5.08	0.169	1.509
f	87.090	6.3260	3.5587	257.0	0.325	3.0952	2.0984	109.84	0.230	1.361
	benchmark ratio$_{(P)}$ =1.7776				benchmark ratio$_{(\alpha)}$=1.4750		†br_P=98.92%$(br_\alpha)^{3/2}$			

Period analysis of observed planets [3]					Period calculations based on the planets that follow the shared rules [4]					
Planet	Observed P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Planet	Calculated P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Compare observed P to calculated P
b	3.2506				b	3.1545		1	0	Equal to 103.05%
c	6.1955	1.9060	1.0722	7.22	c	5.1550	1.6342	1	0	Equal to 120.18%
d	8.3481	1.3475	0.7580	-24.20	d	8.4243	1.6342	1	0	Equal to 99.10%
e	13.767	1.6491	0.9277	-7.23	e	13.767	1.6342	1	0	Reference planet
f	87.090	6.3260	3.5587	257.0	x	22.498	1.6342	1	0	Missing planet
	benchmark ratio$_{(P)}$ =1.7776				f	60.083	1.6342	1	0	Migrated greatly
						leading ratio$_{(P)}$=1.5895				

Note 1.The symbol ω_P is the ratio parameter of periods and θ_P is the deviation rate of ω_P. They are calculated according to equation (1) and (3). The symbol ω_α is the ratio parameter of semimajor axes and θ_α is the deviation rate of semimajor axis ratio. They are calculated according to equation (2) and (4). The symbol † is calculated according to equation (11) where br_P is benchmark ratio$_{(P)}$ and br_α is benchmark ratio$_{(\alpha)}$.

Note 2. Analysis of the system with the shared rules

Kepler-169 is a five-planet system now. It does not directly reflect any orbital regularity, MMR feature or connection between planetary orbits (See [1]). Even so, the analysis with the shared rules still shows the existence of the Share Rules. The calculation with equation (2) reflects a 257% deviation rate, stating it is an abnormal system. As all period ratios are obviously different, we have to try different divisors to calculate planetary periods through equation (9). By comparison, the average of the period ratio of the inner four planets is more suitable for the leading ratio of equation (9). So we took the average ratio as the leading ratio and Planet e as reference, and calculated all possible periods (See [4]). The comparisons further show that the calculated periods of Planet b, d and e are very close to their observed periods, satisfying criterion (N-A). So, they should be the planets following the shared rules according to principle (1). The shared rules can be considered to exist in the system according to principle (2). Based on the calculation, Planet c should migrate outwards. The calculations also show that there should be some undetected planets between Planet e and f. If considering the relatively bigger orbit of Planet f, there might be one undetected planet. The interaction of mutual dragging force between Planet f and Planet x ejected Planet f to the current orbit, showing a very big outward migration. For the exchange of kinetic energy, Planet x lost energy and dropped into the star. During its falling, Planet x might disturb the motion of Planet c, causing the outward migration of Planet c. The interaction between Planet x and c increased the falling speed of Planet x. As a result, Planet x quickly crossed the orbit of Planet b and fell into the star. Perhaps, Planet f and x changed their

positions during interaction. Planet f once was the planet inside the orbit of Planet x.

Table S45. Calculations of the period ratio parameters and deviation rates of adjacent planets of the Kepler-186 system and analysis of the change of the planetary configuration.

Comparison of the periods of adjacent planets [1]					Comparison of the semimajor axes of adjacent planets [2]					
Planet	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	α (AU)	Ratio of α_n:α_{n-1}	ω_α	θ_α (%)	Radius (R_{jup})	Ratio of R_n:R_{n-1}
b	3.88679				0.0343±0.0046				0.095	
c	7.26730	1.8697	1.0091	0.91%	0.0451±0.007	1.3149	0.8375	-16.25	0.112	1.179
d	13.3430	1.8360	0.9909	-0.91%	0.0781±0.01	1.7317	1.1029	10.29	0.125	1.116
e	22.4077	1.6794	0.9064	-9.36%	0.110±0.015	1.4085	0.8971	-10.29	0.113	0.904
f	129.944	5.7988	3.1296	213%	0.432±0.053	3.9273	2.5013	150.13	0.104	0.920
	benchmark ratio$_{(P)}$=1.8529				benchmark ratio$_{(\alpha)}$=1.5701		†br_P=94.18%$(br_\alpha)^{3/2}$			

Period analysis of observed planets [3]						Calculations of planetary periods based on the planets that follow the shared rules [4]					
Planet	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Radius (R_{jup})	Planet	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Compare observed P to calculated P
b	3.88679				0.095	b	3.88679				Equal to 100%
c	7.26730	1.8697	1.0091	0.91	0.112	c	7.26730	1.8697	1	0	Reference planet
d	13.3430	1.8360	0.9909	-0.91	0.125	d	13.5877	1.8697	1	0	Equal to 98.20%
e	22.4077	1.6794	0.9064	-9.36	0.113	e	25.405	1.8697	1	0	Equal to 88.20%
f	129.944	5.7988	3.1296	213.0	0.104	x	47.50	1.8697	1	0	missing Planet
	benchmark ratio$_{(P)}$=1.8529					f*	88.81	1.8697	1	0	Equal to 146%
							leading ratio$_{(P)}$=1.8697				

Note 1.The symbol ω_P is the ratio parameter of periods and θ_P is the deviation rate of ω_P. They are calculated according to equation (1) and (3). The symbol ω_α is the ratio parameter of semimajor axes and θ_α is the deviation rate of semimajor axis ratio. They are calculated according to equation (2) and (4). The symbol † is calculated according to equation (11) where br_P is benchmark ratio$_{(P)}$ and br_α is benchmark ratio$_{(\alpha)}$.

Note 2. Analysis of the system

The calculation with equation (4) quantitatively shows the orbital relations of current planets. The 213% deviation rate states that it is an abnormal system (See [1]). According to trait (1), Planet b, c and d should follow the shared rules. So the shared rules still exist in the system. In this case, we calculated the periods of possible planets by taking the period ratio of Planet c to b as leading ratio and Planet c as reference planet. The calculations are shown in the table (See [4]). Comparisons show that the calculated periods of Planet d, c and b are equal to their observed periods to the degrees of 98.2, 100 and 100%. They reflect a specific orbital regularity among them and can be considered the planets that follow the shared rules according to principle (1). The calculations agree with trait (1). The calculations also reflect that Planet f and e should migrate according to principle (3). Meanwhile, the calculations provide a period of P=47.50 days by $P_x=7.2673 \div 1.8697^{(2-5)}$. It does not belong to any observed planet. According to principle (4), it should be the period of an undetected planet between Planet f and e.

If considering the current orbits of Planet e and f, it is possible that there once was a planet between Planet e and f*. Some disturbance may first change the orbits or orbital shapes of Planet e, x or f*.

Supposing Planet x or Planet e interacted with each other because of the influence of dragging force. As a result, Planet e migrated inwards and Planet x was pushed to the orbit farther from the star. In this case, Planet x and Planet f* had the chance to get near, especially at perihelion and aphelion. The mutual dragging forces of the two planets made one planet settle at the current orbit of Planet f and the other fly out of the system or fall into the star . The inferences can be explained by the current planetary configuration and agrees with the exchange of kinetic energy. The current planetary configuration reflects the evolution of the system. Although some planets changed their initial orbits, the shared rules still exist in the system. More observations and studies are required.

Table S46. Calculations of the period ratio parameters and deviation rates of adjacent planets of the Kepler-238 system and analysis of the planetary configuration of the system

Comparison of the periods of adjacent planets							Remark
Planet	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Radius (R_{jup})	Ratio of R_n:R_{n-1}	Observations do not offer the semimajor axes of the five planets.
b	2.0909				0.154		
c	6.1556	2.9327	1.3696	36.96	0.213	1.383	
d	13.234	2.1499	1.0040	0.4	0.274	1.286	The period ratio is 100.8% equal to that of f to e.
e	23.654	1.7874	0.8347	-16.53	0.50	1.825	
f	50.447	2.1327	0.9960	-0.4	0.178	0.356	The period ratio is 99.2% equal to that of d to c.
	benchmark ratio$_{(P)}$=2.1413						

Note 1. The symbol ω_P is the ratio parameter of periods and θ_P is the deviation rate of ω_P. The value of ω_P is calculated according to equation (1) and θ_P is according to equation (3).

Note 2. Observation data do not provide the semimajor axes of the planets.

Note 3. Comparison shows that the period ratio of Planet f to e is 99.2% equal to that of Planet d to c. The orbital relations also reflect that the four corresponding planets orbit the star according to the common mode, suggesting that the shared rules should exist in the system. The different ratio of Planet e to d may be related to the formation of the system or caused by planet migration. In addition, the size of Planet e is much bigger than its adjacent planets, which may be one reason. More researches are needed.

Table S47. Calculations of the period ratio parameters and deviation rates of adjacent planets of the Kepler-292 system and judgement of the planets following the shared rules.

Comparison of the periods of adjacent planets [1]					Comparison of the semimajor axes of adjacent planets [2]					
Planet	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	α (AU)	Ratio of $α_n$:$α_{n-1}$	$\omega_α$	$\theta_α$ (%)	Radius (R_{jup})	Ratio of R_n:R_{n-1}
b	2.5808				0.035				0.118	
c	3.7153	1.4396	0.8377	-16.23	0.045	1.2857	0.8928	-10.72	0.131	1.110
d	7.0557	1.8991	1.1051	10.51	0.068	1.5111	1.0493	4.93	0.199	1.519
e	11.979	1.6978	0.9880	-1.15	0.097	1.4265	0.9905	-0.94	0.238	1.196
f	20.834	1.7392	1.0121	1.21	0.141	1.4536	1.0094	0.94	0.210	0.882
	benchmark ratio$_{(P)}$=1.7185				benchmark ratio$_{(α)}$=1.4401		†br_P=98.92%$(br_α)^{3/2}$			

Calculation by referring to the planets that follow the shared rules [3]						
Planet	Calculated P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Compare observed P to calculated P	Note
b	2.4477				Equal to 105.44%	The exchange of kinetic energy caused the opposite migrations of the two planets.
c	4.1557	1.6978	1	0	Equal to 89.40%	
d	7.0557	1.6978	1	0	Equal to 100%	Planet that follows the shared rules
e	11.979	1.6978	1	0	Reference planet	Planet that follows the shared rules
f	20.338	1.6978	1	0	Equal to 102.44%	Planet that follows the shared rules
	benchmark ratio$_{(P)}$=1.6978					

Note 1.The symbol ω_P is the ratio parameter of periods and θ_P is the deviation rate of ω_P. They are calculated according to equation (1) and (3). The symbol $\omega_α$ is the ratio parameter of semimajor axes and $\theta_α$ is the deviation rate of semimajor axis ratio. They are calculated according to equation (2) and (4). The symbol † is calculated according to equation (11) where br_P is benchmark ratio$_{(P)}$ and $br_α$ is benchmark ratio$_{(α)}$.

Note 2. The period ratio of Planet f to e is equal to 102.44% of that of Planet e to d. According to the trait of the shared rules, the almost 100% equality suggests that Planet f, e and d should follow the shared rules. According to principle (1), the three planets can be regarded as the planets following the shared rules. So, the shared rules should exist in the system according to principle (2).

Table S48. Calculations of the period ratio parameters and deviation rates of adjacent planets of the Kepler-296 system and influence of inaccurate observation data on deviation rates

Analysis of the periods published in 2015 (R.1)					Analysis of the semimajor axes published in 2015					
Planet	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	α (AU)	Ratio of α_n:α_{n-1}	ω_α	θ_α (%)	Radius (R_{Earth})	Ratio of R_n:R_{n-1}
c	5.841637				0.0521				1.61	
b	10.86438	1.8598	1.0102	1.02	0.079	1.5163	1.0100	1.0	2.0	1.2422
d	19.85029	1.8271	0.9924	-0.76	0.118	1.4937	0.9949	-0.51	2.09	1.0450
e	34.14211	1.7200	0.9342	-6.58	0.169	1.4322	0.9540	-4.6	1.53	0.7321
f	63.33627	1.8551	1.0076	0.76	0.255	1.5089	1.0051	0.51	1.80	1.1765
	benchmark ratio$_{(P)}$=1.8411				benchmark ratio$_{(\alpha)}$=1.5013		†br_P=100.08%$(br_\alpha)^{3/2}$			

Orbital relations of the planets of the Kepler-296 System in 2014

Analysis of the periods published in 2014 (R.2)					Analysis of the semimajor axes published in 2014					
Planet	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	α (AU)	Ratio of α_n:α_{n-1}	ω_α	θ_α (%)	Radius (R_{Earth})	Ratio of R_n:R_{n-1}
b	3.621457				0.039				0.94	
c	5.841648	1.6131	0.9024	-9.76	0.054	1.3846	0.9426	5.74	2.15	2.2872
d	19.85024	3.3981	1.9009	90.09	0.122	2.2593	1.5381	53.8	2.28	1.0605
e	34.14189	1.7200	0.9622	-3.78	0.174	1.4262	0.9709	-2.90	1.75	0.7675
f	63.33588	1.8551	1.0378	3.78	0.263	1.5115	1.0290	2.90	1.79	1.0229
	benchmark ratio$_{(P)}$=1.7876				benchmark ratio$_{(\alpha)}$=1.4689					

Note 1.The symbol ω_P is the ratio parameter of periods and θ_P is the deviation rate of ω_P. They are calculated according to equation (1) and (3). The symbol ω_α is the ratio parameter of semimajor axes and θ_α is the deviation rate of semimajor axis ratio. They are calculated according to equation (2) and (4). The symbol † is calculated according to equation (11) where br_P is benchmark ratio$_{(P)}$ and br_α is benchmark ratio$_{(\alpha)}$.

Note 2. In the upper part of the table, the analysis is based on the observation data published in 2015.

Note 3. In the lower part of the table, the analysis is based on the observation data published in 2014.

Table S49. Calculations of the period ratio parameters and deviation rates of adjacent planets of the Kepler-32 system and analysis of the change of the planetary configuration.

Comparison of the periods of adjacent planets [1]					Comparison of the semimajor axes of adjacent planets [2]					
Planet	P (day)	Ratio of $P_n:P_{n-1}$	ω_P	θ_P (%)	α (AU)	Ratio of $\alpha_n:\alpha_{n-1}$	ω_α	θ_α (%)	Radius (R_{Jup})	Ratio of $R_n:R_{n-1}$
f	0.7430				0.013				0.073	
e	2.8960	3.8977	1.680	68.0%	0.033	2.5385	1.5408	54.08	0.13	1.7808
b	5.9012	2.0377	0.878	-12.2%	0.05	1.5151	0.9140	8.60	0.196	1.5077
c	8.7522	1.4831	0.639	36.1%	0.09	1.80	1.0859	8.59	0.178	0.9082
d	22.781	2.6029	1.122	12.2%	0.13	1.4444	0.8714	-12.86	0.24	1.3483
benchmark ratio$_{(P)}$=2.3203					benchmark ratio$_{(\alpha)}$=1.6576			†br_P=108.72%$(br_\alpha)^{3/2}$		

Period analysis of observed planets [3]						Period calculations of all planets based on the planets that follow the shared rules [4]					
Planet	Observed P (day)	Ratio of $P_n:P_{n-1}$	ω_P	θ_P (%)	R (R_{Jup})	Planet	Calculated P (day)	Ratio of $P_n:P_{n-1}$	ω_P	θ_P (%)	Compare observed P to calculated P
f	0.74296				0.073	f	0.69747				Equal to 106.5%
e	2.8960	3.8979	1.6798	67.98	0.13	x	1.42122	2.0377	1	0	Undetected planet
b	5.90124	2.0377	0.8782	-12.18	0.196	e	2.8960	2.0377	1	0	Equal to 100%
c	8.7522	1.4831	0.6391	36.09	0.178	b	5.90124	2.0377	1	0	Reference planet
d	22.7802	2.6028	1.1218	12.18	0.24	c	12.0250	2.0377	1	0	Equal to 72.78%
benchmark ratio$_{(P)}$=2.3203						d	24.5033	2.0377	1	0	Equal to 92.97%
						leading ratio$_{(P)}$=2.0377					

Note 1.The symbol ω_P is the ratio parameter of periods and θ_P is the deviation rate of ω_P. They are calculated according to equation (1) and (3). The symbol ω_α is the ratio parameter of semimajor axes and θ_α is the deviation rate of semimajor axis ratio. They are calculated according to equation (2) and (4). The symbol † is calculated according to equation (11) where br_P is benchmark ratio$_{(P)}$ and br_α is benchmark ratio$_{(\alpha)}$.

Note 2. Analysis of the planetary configuration

Kepler-32 is five-planet system now (Confirmed Planets 2021). Although the orbits of some planets are doubtful (See Section 3 and Supplementary Table 8), the system can still be analysed with the shared rules. The 67.98% deviation rate states that it is an abnormal system (Table 16 [3]). According to the the calculation with equation (12) and (13), the periods of Planet b and e should be relatively accurate. So, I will try to calculate the periods of all planets with equation (9) and by taking the period ratio of Planet b to e as the leading ratio and Planet e as reference planet. The calculations are shown in the table See [4]). It shows that the calculated periods of Planet b and d are equal to 106.5% and 92.97% of their observed periods. For a natural system, the four planets can be considered the planets that follow the shared rules. The calculation also provides a period of 1.42122 days by P_x=5.90124÷2.0377$^{(4-2)}$. Because the leading ratio is calculated by two planets that follow the shared rules, the period should belong to an undetected planet according to principle (4). The calculation also shows that the calculated period of Planet c is equal to 72.28% of its observed period. According to principle (3), Planet c should migrated obviously.

According to the calculations, equation (12), the system reflects $\omega_{P(c:b)}$=56.48%$[(\omega_{\alpha(c:b)})^{3/2}]$ and $\omega_{P(d:c)}$=137.9%$[(\omega_{\alpha(d:c)})^{3/2}]$ in the Kepler-32 system (table S7 [3]). Because the two equations are related to Planet c, the orbital data of Planet c should be doubtful. If checked by equation (13), it shows β_c=0.45694, also suggesting a doubtful orbit. In this case, the migration of Planet c implies that the observed period may need more observations.The application of the shared rules restores the previous planetary configuration, displaying the previous planetary configuration with a continuous chain of near 2:1 Laplace resonances. Because the orbit of Planet c has some doubt, we will not analyse the evolution of the system temporarily.

Table S50. Calculations of the period ratio parameters and deviation rates of adjacent planets of the Kepler-33 system and analysis of the change of the planetary configuration.

Comparison of the periods of adjacent planets [1]					Comparison of the semimajor axes of adjacent planets [2]					
Planet	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	ɑ (AU)	Ratio of $ɑ_n$:$ɑ_{n-1}$	$\omega_ɑ$	$\theta_ɑ$ (%)	Radius (R_{jup})	Ratio of R_n:R_{n-1}
b	5.6679				0.0677				0.16	
c	13.176	2.3248	1.4939	49.39	0.1189	1.7826	1.3282	32.82	0.29	1.813
d	21.776	1.6527	1.0620	6.20	0.1662	1.3978	1.0415	4.15	0.48	1.655
e	31.784	1.4596	0.9379	-6.21	0.2138	1.2864	0.9203	-4.15	0.36	0.75
f	41.029	1.2909	0.8295	-17.1	0.2535	1.1857	0.9218	-11.65	0.4	1.111
benchmark ratio$_{(P)}$=1.5562					benchmark ratio$_{(ɑ)}$=1.3421		†br_P=100.1%$bdr_ɑ)^{3/2}$			

Period analysis of observed planets [3]					Period calculations of all planets based on the planets that follow the shared rules [4]					
Planet	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Planet	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Compare observed P to calculated P
b	5.6679				b	5.4407				Equal to 104.2%
c	13.176	2.3248	1.4939	49.4	x	8.4668	1.5562	1	0	undetected
d	21.776	1.6527	1.0620	6.20	c	13.176	1.5562	1	0	Reference planet
e	31.784	1.4596	0.9379	-6.21	d	20.505	1.5562	1	0	Equal to 105.9%
f	41.029	1.2909	0.8295	-17.1	e	31.909	1.5562	1	0	Equal to 99.61%
benchmark ratio$_{(P)}$=1.5562					f	49.567	1.5562	1	0	Equal to 82.78%
					leading ratio$_{(P)}$=1.5562					

Note 1.The symbol ω_P is the ratio parameter of periods and θ_P is the deviation rate of ω_P. They are calculated according to equation (1) and (3). The symbol $\omega_ɑ$ is the ratio parameter of semimajor axes and $\theta_ɑ$ is the deviation rate of semimajor axis ratio. They are calculated according to equation (2) and (4). The symbol † is calculated according to equation (11) where br_P is benchmark ratio$_{(P)}$ and $br_ɑ$ is benchmark ratio$_{(ɑ)}$.

Note 2. Analysis of the system

The calculation with equation (2) shows the deviation rates of all observed planets (See [1]). It is an abnormal system for a 49.39% deviation rate. In order to study the change of planetary configuration, we calculated the periods of all possible planets with equation (9). By taking Planet c as reference planet and the benchmark ratio as the leading ratio, the calculations are shown in the table (See [2]). The calculations show that the calculated periods of Planet b, c, d and e are equal to their observed periods to the degrees of 104.2%, 100%, 105.9% and 99.61% separately. For a natural system, they should follow the shared rules according to principle (1). So the shared rules still exist in the system. The calculation also provides 8.4668 days by $P_{x(cal)}$=13.176÷1.5562$^{(3-2)}$, which does not belong to any observed planet. Because the calculation is based on the periods of the planets that follow the shared rules, the period should belong to an undetected planet according to principle (4). The 17.22% difference between the calculated period and observed period of Planet f suggests its inward migration according to principle (3). Perhaps some disturbance broke the stable motion of the outer three planets, causing their migrations more or less. As to the undetected planet, the interaction with Planet b may cause it to fall into the star or orbit the star in an inclination obviously different from other planets. If taking Planet d as reference planet, the degrees of equality for different planets may be a little different, showing 98.09%, undetected, 94.16%, 100%(reference), 93.79% and 77.8% by $P_{n(cal)}$=P_d÷1.5562$^{(snd-snn)}$. The shared rules still exist in the system. More observations are necessary.

Table S51. Calculations of the orbital ratio parameters and deviation rates of adjacent planets of the Kepler-444 system.

	Comparison of the periods of adjacent planets				Comparison of the semimajor axes of adjacent planets					
Planet	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	α (AU)	Ratio of α_n:α_{n-1}	ω_α	θ_α (%)	Radius (R_{jup})	Ratio of R_n:R_{n-1}
b	3.6000				0.04178				0.036	
c	4.5459	1.2628	1.0019	0.19	0.04881	1.1683	0.9895	-1.05	0.044	1.2222
d	6.1894	1.3615	1.0802	8.02	0.0600	1.2293	1.0412	4.12	0.047	1.0682
e	7.7435	1.2511	0.9926	-0.74	0.0696	1.1600	0.9825	-1.75	0.049	1.0425
f	9.7405	1.2579	0.9980	-0.2	0.0811	1.1652	0.9869	-0.13	0.066	1.3469
	benchmark ratio$_{(P)}$=1.2604				benchmark ratio$_{(\alpha)}$=1.1807		†br_P=100.03%$(br_\alpha)^{3/2}$			

Note 1.The symbol ω_P is the ratio parameter of periods and θ_P is the deviation rate of ω_P. They are calculated according to equation (1) and (3). The symbol ω_α is the ratio parameter of semimajor axes and θ_α is the deviation rate of semimajor axis ratio. They are calculated according to equation (2) and (4). The symbol † is calculated according to equation (11) where br_P is benchmark ratio$_{(P)}$ and br_α is benchmark ratio$_{(\alpha)}$.

Table S52. Calculations of the period ratio parameters and deviation rates of adjacent planets of the Kepler-55 system and judgement of the planets following the shared rules.

Comparison of the periods of adjacent planets										
Planet	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	α (AU)	Ratio of α_n:α_{n-1}	ω_α	θ_α (%)	Radius (R_{jup})	Ratio of R_n:R_{n-1}
d	2.21110				--				0.142	
e	4.61753	2.0883	0.9719	-2.81	--				0.138	0.972
f	10.1985	2.2088	1.0280	2.80	--				0.142	1.029
b	27.9481	2.7403	1.2754	27.54	--				0.217	1.5282
c	42.1516	1.5082	0.7020	-30.8	--				0.197	0.908
	benchmark ratio$_{(P)}$=2.1486									

The judgement of migratory planets and planets that follow the shared rules

Planet	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Compare observed P to calculated P	
d	2.21110				Equal to 100%	
e	4.61753	2.0883	1	0	Reference planet	
f	9.64279	2.0883	1	0	Equal to 105.6%	
b	20.137	2.0883	1	0	Equal to 138.8%	A stray planet made it migrate outwards
c	42.052	2.0883	1	0	Equal to 100.24%	
	leading ratio$_{(P)}$=2.0883					

Note 1. The symbol ω_P is the ratio parameter of periods and θ_P is the deviation rate of ω_P. They are calculated according to equation (1) and (3)

Note 2. Although it is not a normal system, the shared rules still exist in the system. When we calculated the period of Planet c with equation (9), its calculated period is equal to 100.24% of its observed period if the calculation is based on Planet e as reference planet and the period ratio of Planet e to d as leading ratio. So Planet c, d and e are the planets following the shared rules according to principle (1). In this case, the shared rules should exist in the system according to criterion (N-A).

Note 3. According to trait (2), Planet b should migrate outwards. According to the sizes of Planet b and f, the migration may be caused by an alien body.

Table S53. Calculations of the orbital ratio parameters and deviation rates of adjacent planets of the Kepler-62 system and analysis of the change of the planetary configuration.

Comparison of the periods of adjacent planets [1]					Comparison of the semimajor axes of adjacent planets [2]					
Planet	P (day)	Ratio of $P_n:P_{n-1}$	ω_P	θ_P (%)	α (AU)	Ratio of $α_n:α_{n-1}$	$\omega_α$	$\theta_α$ (%)	Radius (R_{jup})	Ratio of $R_n:R_{n-1}$
b	5.714932				0.0553				0.117	
c	12.4417	2.1771	0.9984	-0.16	0.0929	1.6799	0.9995	-0.05	0.048	0.410
d	18.16406	1.4599	0.6695	-33.05	0.120	1.2917	0.7686	-23.2	0.174	3.625
e	122.384	6.7377	3.0900	209	0.427	3.5583	2.1172	111.7	0.144	0.828
f	267.291	2.1840	1.0016	0.16	0.718	1.6815	1.0005	0.05	0.126	0.875
	benchmark ratio$_{(P)}$=2.1805				benchmark ratio$_{(α)}$=1.6807		†br_P=100.07%$(br_α)^{3/2}$			

Period analysis of observed planets [3]					Period calculations of all planets based on the planets that follow the shared rules [4]					
Planet	Observed P (day)	Ratio of $P_n:P_{n-1}$	ω_P	θ_P (%)	Planet	Calculated P (day)	Ratio of $P_n:P_{n-1}$	ω_P	θ_P (%)	Compare observed P to calculated P
b	5.71493				b	5.71493				Reference planet
c	12.4417	2.1771	0.9984	-0.16	c	12.4417	2.1771	1	0	Equal 100%
d	18.1641	1.4599	0.6695	-33.1	d	27.0868	2.1771	1	0	Equal 67.06%
e	122.384	6.7377	3.0900	209	x	58.9707	2.1771	1	0	Undetected planet
f	267.291	2.1840	1.0016	0.16	e	128.385	2.1771	1	0	Equal 95.33%
	benchmark ratio$_{(P)}$=2.1806				f	279.507	2.1771	1	0	Equal 95.63%
						leading ratio$_{(P)}$=2.1771				

Calculation to the semimajor axes [3]					Calculation according to the planets following shared rules [4]					
Planet	Observed α (AU)	Ratio of $α_n:α_{n-1}$	$\omega_α$	$\theta_α$ (%)	Planet	Calculated α (AU)	Ratio of $α_n:α_{n-1}$	$\omega_α$	$\theta_α$ (%)	Compare observed α to calculated α
b	0.0553				b	0.0553				Reference planet
c	0.0929	1.6799	1	0	c	0.0929	1.6799	1	0	Equal 100%
d	0.120	1.2917	0.7689	-23.11	d	0.1561	1.6799	1	0	Equal 76.87%
e	0.427	3.5583	2.1182	111.8	x	0.2622	1.6799	1	0	Undetected planet
f	0.718	1.6815	1.0010	0.10	e	0.4404	1.6799	1	0	Equal 96.96%
	benchmark ratio$_{(α)}$=1.6799				f	0.7399	1.6799	1	0	Equal 97.04%
						leading ratio$_{(α)}$=1.6799				

Note 1.The symbol ω_P is the ratio parameter of periods and θ_P is the deviation rate of ω_P. They are calculated according to equation (1) and (3). The symbol $\omega_α$ is the ratio parameter of semimajor axes and $\theta_α$ is the deviation rate of semimajor axis ratio. They are calculated according to equation (2) and (4). The symbol † is calculated according to equation (11) where br_P is benchmark ratio$_{(P)}$ and $br_α$ is benchmark ratio$_{(α)}$.

Note 2. Analysis of the system

Kepler-62 has five planets now. The calculation with equation(1) and (2) displays a 209% and a -33.1% deviation rate, suggesting it is an abnormal system (See [1]). The 209% deviation rate suggests planetary absence, suggesting that some changes of planetary configuration occurred in the system. In order to learn about the change, we tried to calculate the periods of all observed planets with equation (9). First we take the period ratio of Planet c to b as the leading ratio and Planet b as reference planet, the calculations are shown in the table [2]. It shows that the computational period of Planet e is 95.33% equal to its observed period. Meanwhile, Planet f shows a 95.63% equality between calculation and observation. According to principle (1), Planet b, c, e and f should follow the shared rules. According to principle (2), the shared rules can be considered to still exist in the system. In this case, 58.9707 days calculated on the basis of Planet b and c, showing $P_x=5.71493\div2.1771^{(1-4)}$., should be the period of an undetected planet according to principle (4). Because the computational period of Planet d, showing $P_{d(cal)}=5.71493\div2.1771^{(1-3)}$, is much bigger than its observed period, Planet d should migrate to a certain degree according to principle (3). The calculations with equation (9) restored the previous planetary configuration, and suggests that it once was a normal system.

Now, we will analyse the absence of Planet x and migration of Planet d. The orbital changes of the two planets should be caused by the interaction of dragging force between them. Perhaps, some reason first changed the orbital shape of Planet x. Under the influence of the dragging force, the eccentricity of Planet x kept increasing. When it got very close to Planet d at its perihelion some day, super-Earth Planet d ejected Planet x off its orbit to the external space. In return, the exchange of the kinetic energy made Planet d migrate inwards, equivalent to about 33% of its previous period. On its way to the external space, it may have disturbed the orbits of Planet e and f, making them migrate a little.

The calculation to the semimajor axes of the observed planets show the same result (See [3] and [4]).

Table S54. Calculations of the period ratio parameters and deviation rates of adjacent planets of the Kepler-82 system and analysis of the change of the planetary configuration.

Comparison of the periods of adjacent planets [1]					Comparison of the semimajor axes of adjacent planets [2]					
Planet	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	ɑ (AU)	Ratio of $ɑ_n$:$ɑ_{n-1}$	$\omega_ɑ$	$\theta_ɑ$ (%)	Radius (R_{jup})	Ratio of R_n:R_{n-1}
d	2.382961				0.034				0.158	
e	5.902206	2.4769	1.1193	11.93	0.063	1.8529	1.0852	8.52	0.220	1.3924
b	26.4438	4.4803	2.0245	102.45	0.1683	2.6825	1.5710	57.1	0.375	1.7045
c	51.538	1.9490	0.8807	-11.93	0.2626	1.5621	0.9149	-8.52	0.477	1.272
f	75.732	1.4694	0.6640	-33.6	0.3395	1.2928	0.7571	-24.3	--	
	benchmark ratio$_{(P)}$=2.2130				benchmark ratio$_{(ɑ)}$=1.7075			†br_P=99.184%$(br_ɑ)^{3/2}$		

Period analysis of observed planets [3]					Period calculations of all planets based on the planets that follow the shared rules [4]					
Planet	Observed P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Planet	Calculated P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Compare observed P to calculated P
d	2.382961				d	3.57181				Equal to 66.7%
e	5.902206	2.4769	1.1193	11.93	e	6.96146	1.9490	1	0	Equal to 84.8%
b	26.4438	4.4803	2.0245	102.45	x	13.5679	1.9490	1	0	Undetected planet
c	51.538	1.9490	0.8807	-11.93	b	26.4438	1.9490	1	0	Equal to 100%
f	75.732	1.4694	0.6640	-33.6	c	51.538	1.9490	1	0	Reference planet
	benchmark ratio$_{(P)}$=2.2130				f	100.45	1.9490	1	0	Equal to 75.4%
						leading ratio$_{(P)}$=1.9490				

Note 1.The symbol ω_P is the ratio parameter of periods and θ_P is the deviation rate of ω_P. They are calculated according to equation (1) and (3). The symbol $\omega_ɑ$ is the ratio parameter of semimajor axes and $\theta_ɑ$ is the deviation rate of semimajor axis ratio. They are calculated according to equation (2) and (4). The symbol † is calculated according to equation (11) where br_P is benchmark ratio$_{(P)}$ and $br_ɑ$ is benchmark ratio$_{(ɑ)}$.

Note 2. Analysis of the system

The calculation shows a 102.45% deviation rate, indicating it is an abnormal system (See [3]). If calculating the periods of all possible planets with equation (9) by taking the period ratio of Planet c to b and Planet c as reference planet, the calculations are shown in the table (See [4]). For example, Planet e shows It shows P_e=51.538÷1.9490$^{(5-2)}$=6.96146d. It shows that only two planets reflect the periods that are equal to their observed periods to the degrees of bigger than 90%. According to principle (1), only two planets may be the planets that follow the shared rules. According to principle (2), the shared rules should not be considered in the system. Although the calculations display an undetected planet and three planets migrated greatly, they may not be very convincing. So, we will not analyse the system more.

Table S55. Calculations of the period ratio parameters and deviation rates of adjacent planets of the Kepler-84 system and corresponding analysis.

Comparison of the periods of adjacent planets					Calculation based on the planet with shared rules					
Planet	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Radius (R_{jup})	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	**Compare observed P to calculated P**
d	4.2245				0.123	4.2245	1.8021	1	0	Equal to 100%
b	8.7259	2.0656	0.9847	-1.52	0.199	7.6126	1.8021	1	0	Equal to114.62%
c	12.883	1.4764	0.7039	-29.6	0.211	13.719	1.8021	1	0	Equal to 93.54%
e	27.434	2.1295	1.0152	1.52	0.232	24.722	1.8021	1	0	Equal to 110.97%
f	44.552	1.6240	0.7742	-22.58	0.196	44.552	1.8021	1	0	Reference planet
	representative ratio$_{(P)}$=2.0976					leading ratio$_{(P)}$=1.8021				

Note 1.The symbol ω_P is the ratio parameter of periods and θ_P is the deviation rate of ω_P. They are calculated according to equation (1) and (3).

Note 2. Observation data do not provide the semimajor axes of the planets.

Note 3. The calculations reflect the orbital relations. It is a near-normal system. So, we recalculated the periods with equation (9). As the average of the four period ratios are 1.8239, equal to 1.8021, shared ratio from Planet f to Planet d by sd=$(44.552\div4.2245)^{1/(5-1)}$=1.8021, we took the shared ratio as the leading ratio and Planet f as reference. The calculations are presented in the table. It shows that Planet f, c and d should be the planets following the shared rules according to principle (1). The shared rules still exist in the system according to principle (2). Perhaps, a stray planet passed between Planet e and c, causing that Planet e migrated outwards and Planet c changed its orbital shape . When Planet c got near to Planet b at its perihelion, the interaction of mutual dragging forces made the two planets migrate to opposite directions, showing the current planetary configuration. Although some disturbances caused the middle three planets to migrate to different directions, they finally settled at the orbits which show the orbital relations of the shared rules.

There is another inference.Observations show that it is a particular system, showing that two spaced period ratios are very close. The results should not be accidental. The two very close ratios suggest that the planets are affected by the same mode of gravitation, suggesting that they operate according to the common mode. The planetary configuration implies that the system formed in a particular environment, especially the ratios of P_b:P_d and P_e:P_c. The result that different planet pairs show almost the same period ratios suggest that a specific orbital relation exists between corresponding planets. In other words, the relation reflects the shared rules in a different way. So, it can be considered that the shared rules still exist in the system. Because of the particular environment of the formation, the system presents such a planetary configuration. As to the reason, more observations and researches are needed. Here I just showed the result of the planetary distribution.

Table S56. Calculations of the period ratio parameters and deviation rates of adjacent planets of the TOI-561 system and analysis of the change of the planetary configuration.

Comparison of the periods of adjacent planets [1]					Comparison of the semimajor axes of adjacent planets [2]					
Planet	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	α (AU)	Ratio of α_n:α_{n-1}	ω_α	θ_α (%)	Mass (M_{jup})	Ratio of M_n:M_{n-1}
b	0.446578				0.01055				0.0050	
c	10.779	24.137	15.345	1535	0.08809	8.3498	6.2475	524.8	0.0170	3.40
f	16.287	1.5110	0.9606	-3.94	0.1174	1.3327	0.9972	-0.28	0.0094	0.553
d	25.62	1.5730	1	0	0.1569	1.3365	1	0	0.0376	4.0
e	77.23	3.0144	1.9163	91.6	0.3274	2.0867	1.5613	56.13	0.0503	1.338
	benchmark ratio$_{(P)}$=1.5730				benchmark ratio$_{(\alpha)}$=1.3365		†br_P=100.013%$(br_\alpha)^{3/2}$			

Period analysis of observed planets [3]					Period calculations of all planets based on the planets that follow the shared rules [4]					
Planet	Observed P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Planet	Calculated P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Compare observed P to calculated P
					b (x)	No planet				Planet x migrated here
b	0.446578				c	10.779	1.5110	1	0	Equal to 100%
c	10.779	24.137	15.345	1535	f	16.287	1.5110	1	0	Reference planet
f	16.287	1.5110	0.9606	-3.94	d	24.610	1.5110	1	0	Equal to 104.1%
d	25.62	1.5730	1	0	x(b)	37.185	1.5110	1	0	It flied to the orbit of Planet b
e	77.23	3.0144	1.9163	91.6	e	56.187	1.5110	1	0	Equal to 137.45%
	benchmark ratio$_{(P)}$=1.5730					leading ratio$_{(P)}$=1.5110				

Note 1.The symbol ω_P is the ratio parameter of periods and θ_P is the deviation rate of ω_P. They are calculated according to equation (1) and (3). The symbol ω_α is the ratio parameter of semimajor axes and θ_α is the deviation rate of semimajor axis ratio. They are calculated according to equation (2) and (4). The symbol † is calculated according to equation (11) where br_P is benchmark ratio$_{(P)}$ and br_α is benchmark ratio$_{(\alpha)}$.

Note 2. Analysis of the system

The calculation shows the planetary distribution (See [1]). Because Planet b is too far from other planets, it may settle the current orbit for some incident. So, we will first analyse the external four planets. The calculation with equation (4) shows a 91.6% deviation rate, suggesting planet absence between Planet e and d. In order to learn about the change of the planetary configuration, We calculated the periods of all observed planets with equation (9). We took the period ratio of Planet f to c as the leading ratio of equation (9) and Planet f as reference. The calculations are shown in the table (See [2]). For example, Planet d shows 24.610 days by P=16.287÷$1.5110^{(3-4)}$, which is equal to 104.1% of its observed period. The calculation suggests that Planet c, f and d are the planets that follow the shared rules according to principle (1). Based on the planets that follow the shared rules, the calculations reflect that there should be an undetected planets and Planet e migrated outwards obviously.

If considering the orbit of Planet b and the undetected planet, the planetary configuration should have changed greatly. Because of limited conditions, it is not easy to offer a satisfactory analysis of the evolution of the system. So, here we will just try to offer a possible inference according to the physical parameters of all observed planets. Some disturbance may first affect the motion of Planet e, causing it to get near to Planet x. At the perihelion of Planet e and aphelion of Planet x, the dragging force of Planet x increased the orbiting speed of Planet e, causing Planet e to migrate to the outer space and settle at the current orbit. In return, Planet x lost kinetic energy, embodying the decrease of its orbiting speed. As a result, Planet x fell toward the star. Planet x crossed the orbits of

Planet d, f and c. When Planet x fell to a certain area, all forces were balanced and Planet x began to orbit the star at the current orbit of Planet b. Perhaps, Planet x and e changed their positions during the gravitational interaction between them. If considering the mass of current Planet b, it is possible that it comes from the area near Planet e. The inference is logical and the exchange of the kinetic energy makes the orbital change possible. However, more observations are acquired.

Table S57. One planet migration causes the simultaneous changes of two adjacent deviation rates in the KOI-94 system.

Semimajor axis analysis of current planets [1]					Semimajor axis calculation with the planets following shared rules [2]				
Planet	Observed α (AU)	Ratio $α_n$:$α_{n-1}$	$ω_α$	$θ_α$ (%)	Calculated α (AU)	Ratio $α_n$:$α_{n-1}$	$ω_α$	$θ_α$ (%)	Compare observed P to calculated P
b	0.05119				0.050867				Equal to 100.635%
c	0.1013	1.9789	1.0876	8.76	0.092553	1.8195	1	0	Equal to 109.451%
d	0.1684	1.6624	0.9137	-8.63	0.1684	1.8195	1	0	Reference planet
e	0.3064	1.8195	1	0	0.3064	1.8195	1	0	Equal to 100%
	benchmark ratio$_{(α)}$=1.8195				leading ratio$_{(α)}$=1.8195				

Period analysis of current planets [3]					Period calculation with the planets following shared rules [4]					
Planet	Observed P (day)	Ratio P_n:P_{n-1}	$ω_P$	$θ_P$ (%)	Calculated P (day)	Ratio P_n:P_{n-1}	$ω_P$	$θ_P$ (%)	Mass ($M_{Jup.}$)	Eccentricity
b	3.743208				3.78007				0.033	0.25±0.17
c	10.423648	2.7847	1.1454	14.54	9.19011	2.4312	1	0	0.049	0.43±0.23
d	22.342989	2.1434	0.8816	-11.8	22.342989	2.4312	1	0	0.334	0.022±0.04
e	54.32031	2.4312	1	0	54.32031	2.4312	1	0	0.110	0.019±0.23
	benchmark ratio$_{(P)}$=2.4312				leading ratio$_{(P)}$=2.4312					

Note 1. "$ω_P$" is the ratio parameter of periods and "$θ_P$" is the deviation rate of $ω_P$. "$ω_α$" is the ratio parameter of semimajor axes and "$θ_α$" is the deviation rate of $ω_α$.

Note 2. Usually, all planets of a system slowly fall towards the star during their life time. Because of the great dragging force of Planet d, the falling speed of Planet c may become relatively slow, showing the outward migration.

Note 3. Calculation of the effect of gravitational field

As we know, each celestial body has its own gravitational field. Expressed by equation, it is g=F/m, where F is the gravitational force and m is the mass of test particle (checking by https://encyclopedia.thefreedictionary.com/gravitational+field). For Planet d, its gravitational field is $g_{(d)}=(GM_{(d)})\div(D_{a(d)-a(c)})^2$, where M_d is the mass of Planet d and $R_{d\to c}$ is the difference of the semimajor axes of Planet d and c, showing 0.1684-0.1013=0.0671AU. The value of g_d reflects the effect of the gravitational field of Planet d on Planet c when they reached the nearest distance. For Jupiter, its gravitational field is $g_{(Jup)}=(GM_{(Jup)})\div(D_{(Jup)-a(ast)})^2$, where M is the mass of Jupiter and $R_{Jup\to ast}$ is the difference of the semimajor axes of Jupiter and a testing asteroid in asteroid belt, showing 5.2-2.7AU=2.5AU. The value of g_{Jup} reflects the effect of the gravitational field of Jupiter on the asteroid when they reached the nearest distance. If comparing the effects of the two fields, we obtain:

$$g_{(d)}:g_{(Jup)}=[(M_{(d)})\div(D_{a(d)-a(c)})^2]:[(M_{(Jup)})\div(D_{a(Jup)-a(ast)})^2], \quad (14)$$

When we brought related observation data into equation (14), the ratio shows g_d:g_{Jup}=463:1. According to the ratio, the influence of Planet d on Planet c is much greater than that of Jupiter on the asteroid.

Table S58. Calculations of the period ratio parameters and deviation rates of adjacent planets of the GJ 676 A system and analysis of the change of the planetary configuration.

Comparison of the periods of adjacent planets					Calculation with equation (9) based on the two biggest planets						
Planet	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Planet	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Compare observed P to calculated P	Mass ($M_{Earth.}$)
d	3.6005				d	3.1580				Equal to 114.0%	4.4
e	35.39	9.8292	1	0	e	21.925	6.9427	1	0	Equal to 161.4%	8.1
b	1056.8±2.8	29.86	3.0379	203.8	x	152.2	6.9427	1	0	transitional planet	--
c	7337±95	6.9427	0.7063	-29.37	b	1056.8	6.9427	1	0	Reference planet	1576
	benchmark $ratio_{(P)}$ =9.8292				c	7337	6.9427	1	0	Equal to 100%	2161
						leading $ratio_{(\alpha)}$=6.9427					

Note 1. The symbol ω_P is the ratio parameter of periods, and θ_P is the deviation rate of ω_P. They are calculated with equation (1) and (2).

Note 2. The calculation with equation (2) provides the deviation rates. The planets are in disorder. If calculated with equation (9), the calculations reflect that the calculated periods of only two planets are equal to their observed periods within the accuracy of smaller than 10%. So, it is not convincing to regard that the shared rules exist in the system. So, the system is regarded as an abnormal system without the shared rules.

Note 3. Possibly reason of why the system fails to display the shared rules

The GJ 676 is a binary-star system. The main star shows R=0.73 R_{Sun} and the companion star shows R=0.31 R_{Sun}. Because the original air mass of GJ 676 possibly was in a disturbed environment, it might have developed into an asymmetric dumbbell-shaped air mass and then formed a binary-star system. Because of the disturbance of the companion star, the protoplanetary disc of GJ 676 A perhaps presented the eccentric shape or an eccentric disc. Because the matter distribution in an elliptical disc may be a little different from that in a round disc, two super-Jupiter planets formed at the external part of the disc while some super-Earth planets formed at the area near to the star. Perhaps, some transitional planets once existed between Planet e and b or between Planet b and c. The huge Planet b and c might eject them off their original orbits or make them collide with each other. Some of the transitional planets might change the orbit of Planet e or d. Nevertheless, the initial unbalanced gravitational pull makes the system present the planetary configuration in disorder now, failing to display the shared rules.

Table S59. Calculations of the period ratio parameters and deviation rates of adjacent planets of the Kepler-167 system and analysis of the change of the planetary configuration.

Comparison of the periods of observed planets					Calculation based on the planets following the shared rules						
Planet	Calculated P (day)	Ratio of $P_n:P_{n-1}$	ω_P	θ_P (%)	Radius ($R_{Jup.}$)	Planet	P (day)	Ratio of P	ω_P	θ_P (%)	Compare calculated and observed P
b	4.3931632				0.1441	b	4.393163				Equal to 100%
c	7.406114	1.6858	0.5726	-42.74	0.1381	c	7.406114	1.6858	1	0	Reference planet
d	21.803855	2.9440	1	0	0.1065	x	12.485	1.6858	1	0	Undetected planet
e	1071.2323	49.130	16.688	1569	0.9055	d	21.0476	1.6858	1	0	Equal to 103.6%
benchmark ratio$_{(P)}$=2.9440						e	1071.232	49.130	---	---	Not analyse
						leading ratio$_{(\alpha)}$=1.6858					

Note 1. The symbol ω_P is period ratio parameter and θ_P is the deviation rate of ω_P.

Note 2. Analysis of the system

The calculations according to equations (1) and (2) reflect the orbital relations of the planets of the system and show a 1569% deviation rate. As Planet e is too far from Planet d, it is not only an abnormal system, but also implies that the formation of the system may be different from most systems.

Because it is a binary-star system, the companion star may affect the formation of the Kepler-167 system. Influence by the companion star, the protoplanetary disc of Kepler-167 may change its shape. Some materials were dragged out of the disc by the companion, which formed an independent Jupiter-sized planet orbiting initial Kepler-167. Some materials formed a protoplanetary disc around Kepler-167, which developed into a few super-Earth planets. In this case, we just analysed the orbital relations of the inner three planets. We calculated their periods with equation (9). If the calculation is based on Planet c and b, it shows that the calculated period of Planet d is 21.0476 days by $P_{d(cal)}=P_c÷[P_c:P_b]^{(2-4)}$, equal to 103.6% of its observed period. According to principle (1), Planet b, c and d can be considered to follow the shared rules since it is a natural system. Although the companion star affected the formation of the planetary system, the host star still dominates the inner ring disc. So, the planets forming in the disc still display the shared rules. In addition, the calculation also provides a period which does not belong to any observed planet. According to principle (4), the period should belong to an undetected planet. Because the period ratio of Planet d to c is almost equal to the square of the period ratio of Planet c to b, showing trait (3) by $P_d:P_c=103.6\%(P_c:P_b)^2$, there should be an undetected planet between Planet d and c according to trait (3). The trait agrees with the calculation with equation (9). In this case, 12.485 days is likely to be the period of an undetected planet. It is a special planetary system. Although the companion star affected the formation of its adjacent planetary system, the host star still show that the configuration of the inner planets present the orbital pattern with the shared rules.

Table S60. Classification of 62 four-planet systems based on the orbital pattern of planetary configurations.

No.	System	System br_P	Period-ratio deviation rates (%) $\theta_{(P2::P1)}$	$\theta_{(P3::P2)}$	$\theta_{(P4::P3)}$	Orbital pattern of planet configuration	Possible reason of big deviation rates
1	GJ 876	2.0367	652	0	0.09	Jupiter pattern	Planet d is pushed here.
2	HD 20794	2.1902	0	2.79	-25.67	Jupiter pattern	Planet e migrates inwards.
3	HD 3167	3.5203	150.97	0	-2.83	Jupiter pattern	A planet is absent
4	HR 8799	1.5889	3.61	0	-0.24	Jupiter pattern	Based on semimajor axes
5	K2-285	1.4648	40.36	0	-3.615	Jupiter pattern	A planet is not detected.
6	Kepler-106	1.8284	20.4	-3.33	0	Jupiter pattern	Planet b migrated inwards.
7	Kepler-107	1.6237	-5.075	0	14.14	Jupiter pattern	Planet e migrated outwards.
8	Kepler-1388	1.7957	23.58	-5.0	0	Jupiter pattern	Opposite migration of b and c
9	Kepler-172	2.2894	-5.09	0	4.87	Jupiter pattern	Normal system
10	Kepler-176	2.0182	16.37	0	-1.55	Jupiter pattern	Planet b migrated inwards.
11	Kepler-208	1.4909	18.43	0	-2.03	Jupiter pattern	Planet b migrated inwards.
12	Kepler-215	2.1043	-25.54	0	4.95	Jupiter pattern	Planet b migrated outwards.
13	Kepler-221	1.8294	11.26	-3.54	0	Jupiter pattern	Planet b migrated inwards.
14	Kepler-224	1.8912	0	1.285	-13.14	Jupiter pattern	Planet e migrated inwards.
15	Kepler-24	1.5404	24.59	-1.7	0	Jupiter pattern	Planet b migrated inwards.
16	Kepler-245	2.3312	2.66	0	-10.88	Jupiter pattern	Planet d migrated inwards.
17	Kepler-265	2.4873	0	1.83	-36.77	Jupiter pattern	Planet e migrated inwards.
18	Kepler-282	1.7878	-17.26	1.734	0	Jupiter pattern	Planet b migrated outwards.
19	Kepler-338	1.7710	-17.02	0	3.195	Jupiter pattern	Planet b migrated outwards.
20	Kepler-402	1.4566	4.38	0	-13.48	Jupiter pattern	Planet e migrated inwards.
21	Kepler-85	1.4317	5.22	0	-1.68	Jupiter pattern	Normal system
22	K2-133	2.2648	-18.36	0	6.47	Jupiter pattern	Planet b migrated outwards.
23	Kepler-1542	1.2911	5.82	0	-9.01	Jupiter pattern	Normal system
24	Kepler-197**	1.6080	14.95	-5.80	0	Jupiter pattern	Opposite migration of b and c
25	Kepler-256	1.8293	14.29	-5.78	0	Jupiter pattern	Opposite migration of b and c
26	Kepler-299	2.3524	0	-7.06	8.10	Jupiter pattern	Normal system
27	Kepler-306	2.3912	-34.88	0	8.14	Jupiter pattern	Planet b migrated outwards
28	Kepler-79	1.9009	6.91	0	-18.13	Jupiter pattern	Planet e migrated inwards
29	HD 164922	3.3501	0	-45.86	376	*Saturn pattern	A missing and a companion planet
30	HD 215152	1.4920	-15.26	0	55.4	Saturn pattern	One vanished, b migrated
31	K2-32**	2.2979	0	11.13	-25.77	Saturn pattern	One undetected, e migrated
32	KOI-94**	2.4312	14.54	-11.8	0	Saturn pattern	Planet c migrated
33	Kepler-167**	2.9440	-42.74	0	1569	*Saturn pattern	Influence of companion star
34	Kepler-286	1.9307	0	-11.67	155.9	Saturn pattern	Two undetected planets
35	Kepler-305	1.7118	0	-11.73	11.79	Saturn pattern	Opposite migration of d and e
36	Kepler-65	2.7194	0	-48.97	1070	*Saturn pattern	two missing & a companion planet
37	Tau Cet	3.2963	-25.05	0	20.89	Saturn pattern	One missing, b migrated

38	TOI-1264	2.0327	-32.57	55.45	0	Saturn pattern	Planet c migrated obviously
39	DMPP-1	1.9140	0	-32.58	47.36	Saturn pattern	Opposite migrations
40	GJ 3293	2.3085	0	-31.86	10.34	Saturn pattern	Opposite migrations
41	HD 141399	4.6738	-54.2	13.3	0	Saturn pattern	Need more observations
42	HD 20781	2.6142	0	-19.7	12.18	Saturn pattern	Disturbed in evolution
43	K2-72	1.5906	-12.52	23.06	0	Saturn pattern	Planet d migrated
44	K2-266**	1.8809	530	0	-29.52	Saturn pattern	External disturbance
45	Kepler-220	2.1718	0	43.33	-24.84	Saturn pattern	Planet d migrated outwards
46	Kepler-304	1.8160	21.06	-11.08	0	Saturn pattern	Opposite migration of b and c
47	Kepler-37	1.5936	0	17.22	-19.27	Saturn pattern	Opposite migration of d and e
48	Kepler-223	1.3338	-0.4	12.66	0	Uranus pattern	Disturbed during formation
49	Kepler-235	2.3426	0	9.438	-1.725	Uranus pattern	Normal system
50	Kepler-251	3.3067	0.47	-44.82	0	Uranus pattern	Disturbed during formation
51	Kepler-341	1.5418	0	124	-0.427	Uranus pattern	A planet collision occurred
52	Kepler-758	1.6926	1.74	-12.68	0	Uranus pattern	Disturbed during formation
53	L 98-59	1.7174	-4.62	17.55	0	Uranus pattern	Disturbed by a stray planet
54	GJ 676 A**	9.8929	0	203.8	-29.37	Disordered pattern	Disturbed by companion star
55	HD 160691	6.5384	394	-68.32	0	Disordered pattern	Planet c is a companion planet
56	K2-187	2.4892	49.09	0	23.52	Disordered pattern	Specific orbital relations
57	Kepler-132**	2.8076	-63.02	0	181.1	Disordered pattern	Two planets may collide soon
58	Kepler-26	2.7135	27.7	-48.31	0	Disordered pattern	Planet d migrated
59	Kepler-342	1.7293	433.6	0	-13.02	Disordered pattern	Opposite migration of b and c
60	Kepler-411**	2.6070	0	54.28	-29.37	Disordered pattern	Disturbed by companion star
61	Kepler-48	4.4342	-54.34	0	416.3	Disordered pattern	Planet e is a companion planet
62	Kepler-49	1.7040	64.08	-11.1	0	Disordered pattern	Opposite migration of d and b
63	TOI-500	3.9534	206.2	0	-40.89	Disordered pattern	Planet c and b changed positions
64	V1298 Tau	1.9462	-22.75	0	27.7	Disordered pattern	Uncertain and especial orbits
65	WASP-47	5.2674	0	-58.78	1137	Disordered pattern	It was disturbed greatly.

Note 1. The entire list of the 65 systems are presented in Supplementary Table S60. †Jupiter pattern refers to the systems with the orbital pattern of Jupiter's moons. Because each of such systems shows that the absolute values of two adjacent deviation rates of period ratios ($\theta_{(P)}$) smaller than 10%, it can be considered a system with the shared rules according to criterion (N-B). Uranus pattern refers to the system presenting the orbital pattern of Uranus's moons, which has two non-adjacent deviation rates smaller than 10% and also show the existence of the shared rules. Saturn pattern refers to the system presenting the orbital pattern of Saturn's moons, which shows two deviation rates bigger than 10% and indirectly show the existence of the shared rules. Disordered pattern refers to the system presenting the orbital pattern of disordered planets, which shows two deviation rates bigger than 10%, the shared rules can not be seen in such systems.

Note 2. Limited by pages, I have not shown the calculations of all 65 systems. The deviation rates of adjacent planets in all systems are calculated with equation (2), the same as the KOI-94 system in the paper or Supplementary Table 24, also the systems in Supplementary Table 2. The judgement of a planet that follows the shared rules may refer to the calculation of the Kepler-286 system (Supplementary Table 17). The inference of undetected planets or migratory planets can also be based on the method that the system of Kepler-286 and

Kepler-65 are analysed (Supplementary Table 15 and 47). Supplementary Table 43-50 also show the calculations of 15 four-planet systems.

Note 3, Of the 65 systems with four planets, the shared rules exist in 53 systems, which can provide three planets with the shared rules. So, the shared rules still exist in these systems.

Table S61. Extrasolar systems showing the orbital pattern similar to that of Jupiter's moons.

The migration of one planet changes the property of the HD 20794 system (Planet e migrates inwards) [1]

Analysis of current planets						Period calculations of all planets based on the planets that follow the shared rules					
Planet	M (M_{Earth})	Observed P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Planet	Calculated P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Compare observed P to calculated P
b	2.7	18.315				b	18.315				Reference planet
c	2.4	40.114	2.1902	1	0	c	40.114	2.1902	1	0	Identical to 100%
d	4.8	90.309	2.2513	1.0279	2.79	d	87.858	2.1902	1	0	Identical to 102.8%
e	4.77	147.02	1.6280	0.7433	-25.67	e	192.42	2.1902	1	0	Identical to 76.4%
		benchmark ratio$_{(P)}$=2.1902					leading ratio$_{(P)}$=2.1902				

The migration of one planet changes the property of the GJ 876 system (Planet d migrates inwards) [3]

Analysis of current planets						Period calculations of all planets based on the planets that follow the shared rules					
Planet	M (M_{Earth})	Observed P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Planet	Calculated P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Compare observed P to calculated P
d	6.83	1.93778				d	14.7337				Identical to 13.15%
c	227	30.0081	15.486	7.6035	660	c	30.0081	2.0367	1	0	Identical to 100%
b	732.2	61.1166	2.0367	1	0	b	61.1166	2.0367	1	0	Reference planet
e	14.6	124.26	2.0332	0.9983	-0.17	e	124.476	2.0367	1	0	Identical to 99.83%
		benchmark ratio$_{(P)}$=2.0367					leading ratio$_{(P)}$=2.0367				

The migration of one planet changes the property of the Kepler-282 system (Planet b migrates outwards) [2]

Analysis of current planets						Period calculations of all planets based on the planets that follow the shared rules					
Planet	R (R_{Earth})	Observed P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Planet	Calculated P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Compare observed P to calculated P
b	1.01	9.220524				b	7.7610	1.7878	1	0	Identical to 118.8%
c	1.20	13.63872	1.4792	0.8274	-17.26	c	13.875	1.7878	1	0	Identical to 98.3%
d	2.46	24.806	1.8188	1.0173	1.73	d	24.806	1.7878	1	0	Identical to 100%
e	3.10	44.347	1.7878	1	0	e	44.347	1.7878	1	0	Reference planet
		benchmark ratio$_{(P)}$=1.7878					leading ratio$_{(P)}$==1.7878				

The migration of one planet changes the property of the Kepler-107system (Planet d migrates outwards) [4]

Analysis of current planets						Period calculations of all planets based on the planets that follow the shared rules					
Planet	R (R_{Earth})	Observed P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Planet	Calculated P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Compare observed P to calculated P
d	1.536	3.179997				d	3.179997				Equal 100%
c	1.597	4.901425	1.5413	0.9493	-5.07	c	4.901425	1.5413	1	0	Reference planet
b	0.866	7.95820	1.6237	1	0	b	7.554567	1.5413	1	0	Equal 105.3%
e	2.903	14.74905	1.8533	1.1414	14.14	e	11.64385	1.5413	1	0	Equal 126.7%
		benchmark ratio$_{(P)}$=1.6237					leading ratio$_{(P)}$=1.5413				

The migration of one moon breaks the Laplace resonances of the Jupiter System (Callisto migrates outwards) [5]

Analysis of current moons					Period calculations of all moons based on the period of the moons that follow the shared rules					
Planet	Observed P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Planet	Calculated P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Compare observed P to calculated P
Io	1.7691				Io	1.762635				Equal to 100.37%
Europa	3.5512	2.0073	0.9963	-0.37	Eur	3.55118104	2.0147	1	0	Reference planet
Ganymede	7.1546	2.0147	1	0	Gan	7.1545530	2.0147	1	0	Equal to 100%
Callisto	16.689	2.3326	1.1578	15.78	Cal	14.4143	2.0147	1	0	Equal to 115.78%
	benchmark ratio$_{(P)}$==2.0147					leading ratio$_{(P)}$=2.0147				

Note 1.Symbol ω_P is period ratio parameter and θ_P is the deviation rate of ω_P. They are calculated with equation (1) and (2).

Note 2. Each system above shows that the absolute values of any two adjacent deviation rates are smaller 5%. The orbital relations are also reflected by the moon system of Jupiter (The Free Dictionary by checking https://encyclopedia.thefreedictionary.com/Moons+of+Jupiter.), showing the deviation rates of -0.37%, 0 and 15.78% (See [5]). In other words, the three planets that have the adjacent deviation rates smaller than 5% should belong to the planets following the shared rules according to trait (1). Let's take the Jupiter moon system for example. If calculating the period of Io with equation (9) and by taking the period of Europe as reference moon and the period ratio of Ganymede to Europa as the leading ratio, the computational period is 1.762653 days by $P_{Io}=3.55118\div2.0147^{(2-1)}$, equal to its observed period to the degree of 100.37%. According to principle (1), Io, Europa and Ganymede follow the shared rules. According to principle (2), the shared rules should exist in the moon system. Because each planetary system in the table has three planets that follow the shared rules within the accuracy of 5%, they can be judged to be the systems with the shared rules. In this case, the calculation according to the three planets may reflects the degree of the migration of the fourth planet.

Table S62. Calculations of the period ratio parameters and deviation rates of adjacent planets of the K2-285 system and analysis of the change of the planetary configuration.

Analysis of current planets [1]						Period calculations of all planets based on the planets that follow the shared rules [2]					
Planet	Mass (M_{Jup})	Observed P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Planet	Calculated P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Compare observed P to calculated P
b	0.231	3.471745				b	3.714893				Equal to 93.46%
c	0.315	7.138048	2.0560	1.4036	40.36%	x	5.245057	1.4119	1	0	Undetected planet
d	0.221	10.45582	1.4648	1	0	c	7.405496	1.4119	1	0	Equal to 96.39%
e	0.174	14.76289	1.4119	0.9639	-3.615	d	10.45582	1.4119	1	0	Reference planet
		benchmark ratio$_{(P)}$=1.4648				e	14.76289	1.4119	1	0	Equal to 100%
							leading ratio$_{(P)}$=1.4119				

Analysis of current planets [1]						Even if only considering the undetected planet but not considering planetary migrations, it still used to be a normal system [3]					
Planet	R (R_{Jup})	Observed P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Planet	Calculated P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Judgment of planet property
b	0.231	3.471745				b	3.471745				Migratory planet
c	0.315	7.138048	2.0560	1.4036	40.36%	x	5.245057	1.5108	1.0503	5.03	Undetected planet
d	0.221	10.45582	1.4648	1	0	c	7.138048	1.3609	0.9461	-5.39	Migratory planet
e	0.174	14.76289	1.4119	0.9639	-3.615	d	10.45582	1.4648	1.0184	1.84	Reference planet
		benchmark ratio$_{(P)}$=1.4648				e	14.76289	1.4119	0.9816	-1.84	Reference planet
							leading ratio$_{(P)}$=1.4384				

Note 1.The symbol ω_P is the ratio parameter of periods and θ_P is the deviation rate of ω_P.

Note 2. Analysis of the system

The two adjacent small rates present the orbital pattern of Jupiter moons. Because of the 40.36% deviation rate, it should be an abnormal system. So, I calculated all planets with equation (9). The calculation provides five periods according to $P_n=P_d\div(P_e:P_d)^{(snd-snn)}$. The comparison shows that the calculated periods for the four detected planets are equal to their observed periods to the degrees of 93.46%, 96.39%, 100% and 100%, stating that the four planets follow characteristic. Because the calculation is based on the planets with the shared rules, 5.245057 days should be the period of an undetected planet. The current planetary configuration can be explained with the planetary physical parameters and deviation rates of observed planets. Because Planet x and b were relatively nearer, the mutual dragging force first pushed Planet x to the orbit closer to Planet c while Planet b migrated inwards. In this case, Planet x had the chance to get close to Planet c at their aphelion or perihelion. The huge dragging force of super-Neptune Planet c might eject Planet x off the system or make Planet x orbit in a different inclination with the period of a little larger than 5.245 days. Because of the exchange of kinetic energy, Planet c migrated to the orbit nearer to the star. Before Planet x disappeared, K2-285 was a normal system with small deviation rates. There is another possibility. A stray planet crossed the system. The stray planet might collide with Planet x or affect the motion of Planet b, causing Planet b to migrate to the current orbit. More observation and studies are needed.

Table S63. Calculations of the orbital ratio parameters and deviation rates of adjacent planets of the HR 8799 system. Analysis of the possibility of undetected planet.

Analysis of the planetary orbits according to the data published by Extrasolar Planets Encyclopaedia [1]

Planet	α (AU)	Ratio of α	ω_α	θ_α (%)	P (day)	Ratio of P	ω_P	θ_P (%)	M_{planet} ($M_{Jup.}$)	Eccentricity	Updated in
e	16.4				18000				9.6	0.15	2021
d	27	1.6463	1.0361	3.61	41054	2.2808	1.1399	13.99	8.3	0.1	2019
c	42.9	1.5889	1	0	82145	2.0009	1	0	8.3	--	2019
b	68	1.5851	0.9976	-0.24	164250	1.9996	0.9994	-0.06	7	--	2015
	benchmark ratio$_{(\alpha)}$=1.5889				benchmark ratio$_{(P)}$ =2.0009						

Analysis of the planetary orbits according to the data published by NASA) [2]

Planet	α (AU)	Ratio of α	ω_α	θ_α (%)	P (day)	Ratio of P	ω_P	θ_P (%)	Note
e	$16.4^{+2.1}_{-1.1}$				20815±1128				
d	26.97±0.73	1.6445	1.036	3.6	41627±2275	1.9999	1	0	2:1 resonance
c	42.81±1.16	1.5873	1	0	83255±4510	2	1	0	2:1 resonance
b	67.96±1.85	1.5875	1	0	166510±9021	2	1	0	2:1 resonance
	benchmark ratio$_{(\alpha)}$=1.5875				benchmark ratio$_{(P)}$ =2.0				

The possible orbits of Planet e and undetected planet calculated according to the orbits of Planet b, c and d [3]

Planet	α (AU)	Ratio of α	ω_α	θ_α (%)	P (day)	Ratio of P	ω_P	θ_P (%)	Note
x	10.7				10260				Possible orbit
e	17	1.5870	1	0	20524	2.0003	1	0	Possible orbit
d	27	1.5870	1	0	41054	2.0003	1	0	Current orbit
c	42.9	1.5889	1.0012	0.12	82145	2.0009	1.0003	0.03	Current orbit
b	68	1.5851	0.9988	-0.12%	164250	1.9996	0.9997	-0.03	Current orbit
	leading ratio$_{(\alpha)}$ =1.5870				leading ratio$_{(P)}$ =2.0003				

Note 1. The symbol ω_α is the ratio parameter of semimajor axes and θ_α is the deviation rate of ω_α. They are calculated with equation (3, 4). The symbol ω_P is the ratio parameter of periods and θ_P is the deviation rate of ω_P. They are calculated with equation (1, 2)

Note 2. Analyses of the system

The HR 8799 System was discovered in 2008 by direct imaging. Because the observation time is much shorter than the period of the four planets, astronomers still kept verifying the observation data of the four planets, especially Planet e (NASA Confirmed Planet). The latest announcement is made in 2019 by Gravity Collaboration in 2019. In this case, I analysed the orbits of the four planets according to two sets of present observation data (See [1] and [2]). In addition, the period of Planet e is still uncertain. We will mainly analyse the semimajor axes of the four planets and calculate the orbits of possibly-undetected planets on the basis of the semimajor axes observed.

The calculation with equation (2) and (4) shows that the deviation rates of all planets are very small, suggesting it is a normal system (See [1] and [2]). If the semimajor axis of Planet b is calculated with equation (10) and by taking Planet d as reference planet and the semimajor axis ratio of Planet c to d as the leading ratio, the calculated semimajor axis is almost 100% equal to its observed semimajor axis. So, Planet d, c and b are the planets following the shared rules according to principle (1). If the calculation is based on the planets following the shared rules, it may provide the orbit of an undetected planet or confirm the uncertain orbit of a planet. For example, the calculation shows 10.7 AU by α_x=27÷1.5870^{3-1}, which should belong to an undetected planet. The unconfirmed orbit of Planet e can also be calculated in this way, showing α_e=27÷1.5870^{3-2}=17.01 AU.

Table S64. Calculations of the period ratio parameters and deviation rates of adjacent planets of the Kepler-65 system and analysis of the change of the planetary configuration.

Analysis of current planets [1]						Period calculations of all planets based on the planets that follow the shared rules [2]					
Planet	M (M_{Earth})	Observed P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Planet	Calculated P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Compare observed P to calculated P
b	2.4±2.4	2.154910				b	2.19330				Equal to 98.25%
c	5.4	5.859944	2.7194	1	0	x	3.04343	1.3876	1	0	Undetected planet
d	4.14	8.13123	1.3876	0.5103	-49	y	4.22306	1.3876	1	0	Undetected planet
e	260	258.8	31.828	11.704	1070	c	5.859944	1.3876	1	0	Reference planet
	benchmark ratio$_{(P)}$=2.7194					d	8.13123	1.3876	1	0	Equal to 100%
						e	258.8	31.828	--	--	Non analysis
							leading ratio$_{(P)}$=1.3876				

Note 1. The symbol ω_P is the ratio parameter of periods, and θ_P is the deviation rate of ω_P. They are calculated with equation (1) and (2).

Note 2. Analysis of the system

The system seems different from other systems. Observations show that it has three super-Earth planets and a near Jupiter-sized planet which is very far from the inner three planets, suggesting that the four planet formed in a different way from most systems. Even so, we still analysed the four planets.

The the calculation with equation (4) provides the deviation rates of the four planets. The very big deviation rate of the period ratio of Planet e to d also reflects that Planet e formed in the remote region. The comparison also shows that P_c:P_b=2.7194 and P_d:P_c=1.3876, embodying P_d:P_c=101.8%(P_c:P_b)^3. According to trait (3), there should be two undetected planets between Planet c and b. So, I first calculated the period of Planet b with equation (9) and by taking Planet c as reference planet and the period ratio of Planet d to c as the leading ratio. The calculated period is 2.19330 days, which is 98.25% equal to its observed period. According to principle (1), the three planets should follow the shared rules. According to principle (2), the shared rules still exist in the system. In this case, I calculated the periods of the two undetected planets on the basis of the planets following the shared rules. The calculations show 3.04343 days by P_x=5.859944÷(1.3876)$^{4-2}$ and 4.22306 days by P_y=5.859944÷(1.3876)$^{4-3}$. According to principle (4), the values should be the periods of two undetected planets. The two undetected planets are more likely to have collided with each other unless they orbit the star with the inclinations obviously different from Planet c and b.

According to the physical parameters of observed planets, Planet e is likely be a small companion during the system formation. However, the inner three planets still clearly show the shared rules even though a planet collision might occur between Planet c and d. So, the shared rules will naturally appear when the conditions of system formation are suitable.

Table S65. Calculations of the period ratio parameters and deviation rates of adjacent planets of the TOI-1264 system and analysis of the change of the planetary configuration.

Period analysis of current planets [1]											
Planet	Observed P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	α (AU)	Ratio of $α_n$:$α_{n-1}$	$\omega_α$	$\theta_α$ (%)	Mass (M_{Earth})	Mass ($M_{Jup.}$)	Ratio of M_n:M_{n-1}
b	4.30744				0.049				8.1	0.025	
c	5.904144	1.3707	0.6743	-32.57	0.061	1.2449	0.7729	-22.71	8.8	0.028	1.120
d	18.65590	3.1598	1.5545	55.45	0.131	2.1475	1.3333	33.33	5.3	0.017	0.607
e	37.9216	2.0327	1	0	0.211	1.6107	1	0	14.8	0.0466	2.741
	benchmark ratio$_{(P)}$=2.0327				benchmark ratio$_{(α)}$=1.6107				†br_P=99.44%$(br_α)^{3/2}$		

Period calculations of the observed planets based on the planets that follow the shared rules in two ways									
Planet	Calculated P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Compare observed P to calculated P	Calculated P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Compare observed P to calculated P
b	4.51512				Equal to 95.40%	4.30744				Equal to 100%
c	9.17789	2.0327	1	0	Equal to 64.33%	8.89426	2.06485	1	0	Equal to 66.38%
d	18.65590	2.0327	1	0	Equal to100%	18.3653	2.06485	1	0	Equal to101.6%
e	37.9216	2.0327	1	0	Reference planet	37.9216	2.06485	1	0	Reference planet
	leading ratio$_{(P)}$=2.0327					leading ratio$_{(P)}$=2.06485				

Note 1. The symbol ω_P is the ratio parameter of periods and θ_P is the deviation rate of ω_P. They are calculated according to equation (1) and (3). The symbol $\omega_α$ is the ratio parameter of semimajor axes and $\theta_α$ is the deviation rate of semimajor axis ratio. They are calculated according to equation (2) and (4). The symbol † is calculated according to equation (11) where br_P is benchmark ratio$_{(P)}$ and $br_α$ is benchmark ratio$_{(α)}$.

Note 2. Analysis of the system

Although the system shows two large deviation rates, which suggest that it is an abnormal system. However, if calculated with equation (9), the calculated period of Planet b is 4.51512 day by $P_{d(cal)}$=37.9216÷$(2.0327)^{(4-1)}$, equal to 95.4% of its observed period, satisfying criterion (N-A). according to principle (1), Planet b, d and e should follow the shared rules. According to principle (3), Planet c should migrate obviously. The calculations restore the change of planetary distribution. We can also analyse the system according to the standard rule by taking the shared divisor (sd) of the planetary periods from Planet e to b. It show sd=$(37.9216÷4.51512)^{1/(4-1)}$=2.06485. The calculations are shown in the table. It shows the same results and still Planet c migrates obviously. For the migration of Planet c, the cause is likely to be the impact of a stray planet if considering the masses of its neighbouring planets. It is just an inference. More observations are necessary.

Table S66. Calculations of the orbital ratio parameters and deviation rates of adjacent moons of the moon system of Saturn.

Moon		Observed Period (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Diameter (km)	Mass (10^{20}kg)	Density (g/cm^3)	α (km)	Ratio of α_n:α_{n-1}
Saturn I	Mimas	0.942422				392	0.380			
Saturn II	Encecladus	1.370218	1.4539	1.0028	0.28	502	0.730	1.6096		
Saturn III	Tethys	1.887802	1.3777	0.8347	-16.5	1062	6.22	0.9735	294,619	
Saturn IV	Dione	2.736915	1.4498	0.8783	-12.17	1120	10.5	1.50	377,400	1.2810
Saturn V	Rhea	4.5175	1.6506	1	0	1529	24.9	1.233	527,108	1.3967
Saturn VI	Titan	15.94542	3.5297	2.1384	129.6	5150	1350	1.88	1,221,850	2.3180
Saturn VII	Hyperion	21.2766	1.3344	0.8084	-19.16	205//L	0.05584	0.5667		
Saturn VIII	Iapetus	79.3215	3.7281	2.2586	125.9	1469	18.05	1.088		
		specific benchmark ratio$_{(P)}$=1.6506								

Note 1. The symbol ω_P is the ratio parameter of periods and θ_P is the deviation rate of period ratio. They are calculated through equation (1) and (3).

Note 3. The analysis of the orbits of the moons of Saturn

Saturn has tens of moons. If considering their masses or sizes, most moon might not be the initial moons. Here we list the eight biggest moons and analyse their orbits with the concept of the shared rules. According to equation (1) and (3), we calculated their period ratio parameters and deviation rates according to current ratios. Considering the difference of masses and distances between moons, we used the period ratio of Rhea to Dione as the representative ratio in the equation. The calculation quantified the orbital relationships between the eight moons. Further comparison shows that Saturn I, Saturn II and Saturn VII are relatively small, and Saturn VIII is relatively far, which suggests that it might migrate greatly (discussed later). It is more possible that the middle four moons (Tethys, Dione, Rhea and Titan) keep or represent the orbital relations of the initial moon configuration. For a better analysis, we darkened the physical data of the four moons. Then, we independently calculated their periods with equation (3), as shown in the table.

The orbital pattern of the four moons is very similar to that of the planets of Kepler-286, showing 0, -11.67% and 155.9% (Table S17). More comparisons reflect that the planetary configurations of many four-planet extrasolar systems present the orbital pattern as that of the four moons of Saturn. In order to facilitate the study, such four-planet systems are temporarily called the systems with the orbital pattern of Saturn's moons.

The analyses of the main moons of Saturn may also imply that the shared rules exist in the system. Based on the equations and principles of the shared rules, the evolution of the Saturn system may be studied in a different but objective way. But because the paper is the discussion of extrasolar systems, we will discuss the system in later time.

Note 5. The observation data of the moons of Saturn are based on Baidu baike by checking https://baike.baidu.com/item/%E5%9C%9F%E6%98%9F%E5%8D%AB%E6%98%9F/4338243?fr=aladdin and The Free Dictionary by checking https://encyclopedia.thefreedictionary.com/Moons+of+saturn.

Table S67. Calculations of the period ratio parameters and deviation rates of adjacent planets of the HD 141399 system and analysis of the change of the planetary configuration.

Analysis of the semimajor axes of current planets [1]						Semimajor axis calculations of all planets based on the planets that follow the shared rules [2]						
Planet	Mass (M_{Jup})	Observed α (AU)	Ratio of $α_n$:$α_{n-1}$	$ω_α$	$θ_α$ (%)	Planet	Calculated α (AU)	Ratio of $α_n$:$α_{n-1}$	$ω_α$	$θ_α$ (%)	Compare observed α to calculated α
b	0.451	0.415±0.011				b	0.3956				Equal to 104.9%
c	1.33	0.689±0.02	1.6602	0.694	-30.6	c	0.689	1.7417	1	0	Reference planet
d	1.18	2.09±0.06	3.0334	1.268	26.8	x	1.20	1.7417	1	0	Undetected planet (1.15 AU by Dvorak)
e	0.66	5.0±1.5	2.3923	1	0	d	2.09	1.7417	1	0	Equal to 100%
		benchmark ratio$_{(α)}$=2.3923				y	3.64	1.7417	1	0	Unconfirmed planet
						e*	6.34	1.7417	1	0	Unconfirmed planet
							leading ratio$_{(α)}$=1.7417				

Analysis of the periods of current planets [3]						Period calculations of all planets based on the planets that follow the shared rules [4]					
Planet	Eccentricity	Observed P (days)	Ratio of P_n:P_{n-1}	$ω_P$	$θ_P$ (%)	Planet	Calculated P (day)	Ratio of P_n:P_{n-1}	$ω_P$	$θ_P$ (%)	Compare observed P to calculated P
b	0.04	94.44±0.05				b	87.79				Equal to 107.4%
c	0.048	201.99±0.08	2.1388	0.458	-54.2	c	201.99	2.3010	1	0	Reference planet
d	0.074	1069.8±6.7	5.2963	1.133	13.3	x	465	2.3010	1	0	Undetected planet
e	0.26±	5000^{+560}_{-2000}	4.6738	1	0	d	1069.8	2.3010	1	0	Equal to 100%
		benchmark ratio$_{(P)}$=4.6738				y	2462	2.3010	1	0	Unconfirmed planet (2600 days by RV)
						e*	5664 (5700)	2.3010	1	0	Unconfirmed planet (5700 days by RV)
							leading ratio$_{(P)}$=2.3010				

Note 1. The symbol $ω_α$ is semimajor-axis ratio parameter, $θ_α$ is the deviation rate of $ω_α$, $ω_P$ is period ratio parameter, $θ_P$ is the deviation rate of $ω_P$. They are calculated according to equation (3,4,1,2).

Note 2. Analysis of the system

HD 141399 shows four planets now . The calculations with equation (4) provides -30.6%, 26.8% and 0 deviation rates (See [1]). It is not a normal system. Because $α_d$:$α_c$=3.0334 and $α_c$:$α_b$=1.6602, according to trait (3), ($α_d$:$α_c$)=110%($α_c$:$α_b$)^2 suggests an undetected planet between Planet d and c, temporarily called Planet x. The 10% inequality should be caused by the about 5% migration of smaller Planet b (See [2]). To learn about the orbit of Planet x, we analysed the observed planets with equation (10). Because of the uncertain orbit of Planet e and undetected planet, we calculated the leading ratio according to the contents of the standard rule between planetary orbits. Because Planet d and c are super Jupiter, they are more likely to keep the initial orbits. So, the orbital relations from Planet d to c may keep the shared divisor of the system. If considering the undetected planet, the shared divisor (sd) can be calculated by the adjusted equation (10), showing sd=($α_n$÷$α_m$)$^{1/(nsn-msn)}$. The calculation shows sd=(2.09÷0.689)$^{1/(4-2)}$=1.7417. Therefore, by taking the divisor as leading ratio and the biggest Planet c as reference planet, we first calculated the semimajor axis of Planet b, showing $α_{b(cal)}$=0.689÷1.7417$^{(2-1)}$=0.3956AU. Comparisons reflect that the degree of the equality between the calculated and observed semimajor axis of Planet b is 95.33% by (0.3956÷0.415)×100%. Then we calculated Planet d. The value is $\alpha_{d(cal)}=0.689\div1.7417^{(2-4)}=2.09$AU, 100% equal to its observed orbit. Being the reference planet, Planet c shows the 100% equality. So, according to principle (1), Planet b, c and d can be judged to follow the shared rules. According to principle (2), the shared rules still exist in the system. Therefore, we calculated all possible semimajor axes by $\alpha_{n(cal)}=\alpha_c\div(lr_\alpha)^{(sc-sn)}$. They are 0.3956, 0.689, 1.20, 2.09, 3.64 and 6.34 AU (See [2]). Because the calculations are based on the planets that follow the shared rules, 1.2 AU, calculated by $\alpha_x=0.689\div1.7417^{(2-3)}$, should be the semimajor axis of an undetected planet according to principle (4). The calculation agrees with the above suggestion of trait (3). In addition, the calculations do not display the semimajor axis of Planet e, instead provide two other values, showing $\alpha_y=0.689\div1.7417^{(2-5)}=3.64$AU and $\alpha_{e*}=0.689\div1.7417^{(2-6)}=6.34$AU. The calculations imply two other planets, temporarily called Planet y and Planet e*. Meanwhile, we analysed the planetary periods of HD 141399. The calculations reflect the same judgement (See [3] & [4]). With equation (9), the calculation provides 465 days by $P_x=201.99\div2.3010^{(2-3)}$, possible period of Planet x. The calculations also fail to provide the period of Planet e, instead display 2462 days by $P_y=201.99\div2.3010^{(2-5)}$ and 5664 days by $P_{e*}=201.99\div2.3010^{(2-6)}$, also implying two other planets. The two calculated orbits seem to contradict the orbit of planet e. However, if considering Planet e has an uncertain orbit and the calculated orbits are close to the upper and lower limits of its orbit, the calculations may suggest more analyses to observations. In addition, Planet e has a 0.26±0.22 eccentricity, which should suggest two sets of RV signals for Planet e. One comes from its orbit at perihelion and the other at aphelion. If considered in a different way, the RV signals do not deny two planets with small eccentricities. More observations are required.

To better learn about the planetary configuration, we studied more articles about HD 141399. Prof. R. Dvorak et al. deeply researched the formation of habitable planets in the four-planet system HD 141399 (Dvorak et al. 2019). They performed extensive numerical integrations of the equations of motion in the pure Newtonian framework of small bodies with different initial conditions in the habitable zone and completed numerous computer simulations. The simulations show that planets form in the habitable zone connected to MMR, most Earth-mass planets appear in the area 0.8 AU $<\alpha<$ 1.6 AU, and the biggest one is observed at 1.15 AU. From the result of their simulations they concluded that the terrestrial like planets may exist in the HD 141399 system and it might even harbor life (Dvorak et al. 2019). Comparisons show that Planet x reflected by the calculation with the equations of the shared rules agrees with the planet observed in the computer simulations performed by Prof. R. Dvorak et al. Different methods come to the same conclusion. We also studied the article published by Prof. S. Vogt et al. In 2014, Vogt et al. (2014) discovered the HD 141399 system. Because the RV signals of Planet e are not quite certain, they just provided a period range, showing 3717.75±555.081 days. In the Lomb–Scargle periodograms of Figure 4 created by Prof. S. Vogt, the RV signals reflecting the periods of Planet b, c and d display three well-defined peaks While Planet e displays a half-elliptical wave of high amplitude with two humps, which reflects a range of period. When we calculated the two humps in the same way as other planets, we noticed that the two humps point at the periods of about 2600 and 5700 days. The periods of Planet y and Planet e* are very close to the periods reflected by the RV signals reflected in the periodograms (Vogt et al. 2014). More observations are required.

Table S68. Extrasolar systems showing the orbital patterns similar to that of Uranus's moons.

Kepler-223 System (Radius=1.72R_{Sun}) [1]

Planet	P (day)	Ratio of $P_n:P_{n-1}$	ω_P	θ_P (%)	R (R_{Jup})
b	7.38449				0.267
c	9.84564	1.3333	0.9996	-0.4	0.307
d	14.78869	1.5021	1.1266	12.66	0.467
e	19.72567	1.3338	1	0	0.410

benchmark $ratio_{(P)}$=1.3338

Kepler-235 System (Radius=0.554R_{Sun}) [2]

Planet	P (day)	Ratio of $P_n:P_{n-1}$	ω_P	θ_P (%)	R (R_{Jup})
b	3.340222				0.199
c	7.824904	2.3426	1	0	0.114
d	20.06055	2.5637	1.0944	9.438	0.183
e	46.18367	2.3022	0.9828	-1.725	0.198

benchmark $ratio_{(P)}$=2.3426

Kepler-251 System (Radius=0.890R_{Sun}) [3]

Planet	P (day)	Ratio of $P_n:P_{n-1}$	ω_P	θ_P (%)	R (R_{Jup})
b	4.790936				0.119
c	16.51404	3.3221	1.0047	0.47	0.247
d	30.13300	1.8245	0.5518	-44.82	0.247
e	99.64016	3.3067	1	0	0.247

benchmark $ratio_{(P)}$=3.3067

Kepler-758 System (Radius=1.420R_{Sun}) [4]

Planet	P (day)	Ratio of $P_n:P_{n-1}$	ω_P	θ_P (%)	R (R_{Jup})
c	4.757940				0.221
e	8.193472	1.7221	1.0174	1.74	0.151
b	12.10971	1.4780	0.8732	-12.68	0.189
d	20.49662	1.6926	1	0	0.136

benchmark $ratio_{(P)}$=1.6926

L 98-59 System (Radius=0.303R_{Sun}) [5]

Planet	P (day)	Ratio of $P_n:P_{n-1}$	ω_P	θ_P (%)	M ($M_\oplus$)
b	2.253114				0.40
c	3.690678	1.6380	0.9538	-4.62	2.22
d	7.450725	2.0188	1.1755	17.55	1.94
e	12.796	1.7174	1	0	3.06

benchmark $ratio_{(P)}$=1.7174

Kepler-341 System (Radius=1.024R_{Sun}) [6]

Planet	Observed P (day)	Ratio of $P_n:P_{n-1}$	ω_P	θ_P (%)	R (R_{Jup})
b	5.195528				0.105
c	8.01041	1.5418	1	0	0.152
d	27.66631	3.4538	2.240	124.0	0.165
e	42.47327	1.5352	0.9957	-0.427	0.178

benchmark $ratio_{(P)}$=1.5418

Orbital calculation of undetected planets of Kepler-341 [7]

Planet	P (day)	Ratio of $P_n:P_{n-1}$	ω_P	θ_P (%)	Near MMR
b	5.195528				
c	8.01041	1.5418	1.0128	1.28	2:3
x	12.040	1.5030	0.9873	-1.27	2:3
y	18.328	1.5223	1	0	2:3
d	27.66631	1.5095	0.9914	-0.86	2:3
e	42.47327	1.5352	1.0085	0.85	2:3

benchmark $ratio_{(P))}$=1.5223

Calculation based on Planet e and b of Kepler-341 [8]

Planet	Calculated P (days)	Ratio $P_n:P_{n-1}$	ω_P	θ_P (%)	Compare P_{obs} and P_{cal}
b	5.195528				100% equal
c	7.9089	1.5223	1	0	101.28% equal
x	12.040	1.5223	1	0	Disappeared
y	18.328	1.5223	1	0	Disappeared
d	27.901	1.5223	1	0	99.16% equal
e	42.47327	1.5223	1	0	100% equal

leading $ratio_{(P))}$=1.5223

Analysis of the orbits of the four major moons of Uranus by using ratio parameter and deviation rate [9]

Moon		Period (day)	Ratio of $P_n:P_{n-1}$	ω_P	θ_P (%)	α (km)	Ratio of $\alpha_n:\alpha_{n-1}$	Mass (10^{21} kg.)
Uranus I	Ariel	2.52038				191,020		1.353
Uranus II	Umbriel	4.14418	1.6443	1	0	266,300	1.3941	1.172
Uranus III	Titania	8.70587	2.1008	1.2776	27.76	435,910	1.6369	3.527

Uranus IV	Oberon	13.4632	1.5465	0.9405	-5.95	583,520	1.3386	3.014
			benchmark ratio$_{(P)}$=1.6443					

Note 1. The symbol ω_P is the ratio parameter of periods, and θ_P is the deviation rate of ω_P. They are calculated with equation (1) and (2). The symbol ω_a is the ratio parameter of semimajor axes and θ_a is the deviation rate of ω_a. They are calculated with equation (3) and (4).

Note 2. The comparison shows that each of the six four-planet systems displays the two non-adjacent period ratios that are almost equal (See [1]-[7]). In other words, each system shows two non-adjacent deviation rates almost equal to 0. The orbital pattern of the planets of such systems reflect that the orbital relations of corresponding planets should also reflect the existence of the shared rules.

Note 3. The orbital relationship between the four biggest moons of Uranus.

When we compared the period ratios of the major moons of Uranus (The Free Dictionary by checking https://encyclopedia.thefreedictionary.com/Moons+of+Uranus), we found that the four moons show the same pattern as above six planetary systems (See [9]). Because Miranda (Uranus V) has the mass of 0.0659×10^{21} kg, only about 4.87% the mass of Ariel, it is relatively too small and is not regarded as a main moon temporarily. By the way, Saturn I, II, III and IV also display the similar orbital pattern, showing the deviation rates 0, -4.98% and 0.28%.

Note 4. The possible reasons that lead to the orbital pattern

The systems of Kepler-223, Kepler-235, Kepler-758 and L 98-59 show almost the same orbital pattern. The very close absolute values of two deviation rates for each system reflect the existence of the shared rules in the systems although they are not adjacent ratios. Perhaps, such a system was disturbed during formation, which caused the middle deviation rate.

Note 5. Analysis of the big deviation rate in the system of Kepler-341

The calculation with equation (2) show two almost equal deviation rates and a 124% deviation rate, suggesting it is an abnormal system (See [7]). If calculating with equation (9) by $P_{n(cal)}=(P_e)\div(lr_P)^{[(se)-(sn)]}$, where lr_P is 1.5223 of the common divisor from Planet e to b according to the contents of the standard rule, the calculated periods are presented in the table (See [8]). Comparison shows that the calculated periods of all observed planets are equal to their observed periods to the degrees of 100, 101.28, 99.16 and 100%. According to principle (1), the four observed planets follow the shared rules. So, the shared rules exist in the system according to principle (2). In this case, the two calculated periods that do not belong to the observed planets should belong to the undetected planets according to principle (4). If considering the two disappeared planets, it once was an almost ideal system. Perhaps because of the perturbation of mutual dragging force, the two undetected planets are likely to be collide with each other or ejected off the system. The almost equality of two pairs of period ratios, the almost equality between the calculated and observed periods of all planets, and a complete chain of near 3:2 MMRs for all planets should not be by chance. They clearly reflect the existence of the shared rules and the disappearances of two planets for some reason.

Note 6. Analysis of the big deviation rate in the system of Kepler-251

The system of Kepler-251 shows a -44.82% deviation rate (See [3]). Perhaps, one of the two planet pairs with resonant orbits migrated together. Calculation reflects that the period ratios show $P_c:P_d$=99.54%($P_c:P_b$), $P_c:P_b$=99.78%$(P_d:P_c)^2$, $P_c:P_d$=99.33%$(P_d:P_c)^2$. The interesting orbital relations need more researches.

Note 7. Analysis of the L 98-59 system

A stray planet may pass through the orbits of Planet b and c of L 98-59, decreasing the orbiting speeds of the two planets (See [3]). As a result, their orbits show the inward migrations, equivalent to about 10% of the period of Planet b and 15% of the period of Planet c. The inferences are just the possibilities. More observations are needed.

Note 8. Although Uranus has tens of moons, the four main moon with the diameter of over 1000km should be the original moons. So we calculated the orbital relations between the moons with equation (1) and (2). The orbital pattern shows the same pattern as some extrasolar systems show(See [9]).

Table S69. Calculations of the period ratio parameters and deviation rates of adjacent planets of the Kepler-49 system and analysis of the change of the planetary configuration.

	Calculation to the periods of current planets [1]					Period calculations with supposed planets that follow the shared rules [2]				
Planet	R (R_{Jup})	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	α (AU)	Ratio of $α_n$:$α_{n-1}$	$\omega_α$	$\theta_α$ (%)	Compare observed P to calculated P
d	0.243	2.576549				3.75849				Equal to 68.55%
b	0.227	7.2037945	2.7959	1.6408	64.08	6.40447	1.7040	1	0	Equal to 112.48%
c	0.143	10.912934	1.5149	0.8890	-11.1	10.912934	1.7040	1	0	Reference planet
e	0.139	18.596108	1.7040	1	0	18.596108	1.7040	1	0	Equal to 100%
		benchmark ratio$_{(P)}$=1.7040				benchmark ratio$_{(P)}$=1.7040				

Note 1. The symbol ω_P is the ratio parameter of periods, and θ_P is the deviation rate of ω_P.

Note 2. Analysis of the system

The calculation with equation (1) and (2) provides the ratio parameters and deviation rates of the period ratios of the four planets (See [1]). Since only one deviation rate is smaller than 10%, the system is an abnormal system. If the periods of all planets are calculated with equation (9), the calculations show that the computational periods of only two planets are equal to their observed periods to the degrees bigger than 90% (See [2]). So the shared rules should not be considered to exist in the system now. In this case, the system is regarded as the system which does not reflect the shared rules.

However, the calculations also show the opposition migrations of Planet b and d. As we know, the mutual dragging force between two orbiting planets may cause the opposition migrations after the exchange of kinetic energy. If considering the masses of the two planets and the degrees of their migrations, the current planetary configuration of the system is likely to be caused by the interaction of the dragging force of Planet b and d. Before they migrated, the shared rules may still exist in the system.

Table S70. Analysis of orbital change of some planet based on the simulated system of Kepler-289 and based on the actual system of EPIC 206024342.

Supposing Planet d of Kepler-289 migrated equivalent to 20% of its period, the period analysis is as follows										
Analysis of the observed planetary configuration [1]						Supposing Planet d migrated outwards, the comparison is as follows [2].				
Planet	P (day)	Ratio of $P_n:P_{n-1}$	ω_P	θ_P (%)	Mass (M_{Earth})	P (day)	Ratio of $P_n:P_{n-1}$	ω_P	θ_P (%)	Remark
b	34.545				0.023	34.545				Initial period
c	66.0634	1.9124	1.0019	0.19	0.013	66.0634	1.9124	0.9110	-8.90	Initial period
d	125.852	1.9050	0.9981	-0.19	0.42	151.022	2.2860	1.0890	8.90	Migrated to 20%
	average ratio$_{(P)}$=1.9087					reference ratio$_{(P)}$=2.0992				
Analysis based on the period ratio of Planet c to b [3]						Analysis based on the period ratio of Planet d to c [4]				
Planet	P (day)	Ratio of $P_n:P_{n-1}$	ω_P	θ_P (%)	Comparison	P (day)	Ratio of $P_n:P_{n-1}$	ω_P	θ_P (%)	Comparison
b	34.545				100%	28.899				Equal to 119.53%
c	66.0634	1.9124	1	0	Reference	66.0634	2.2860	1	0	Reference planet
d	126.34	1.9124	1	0	99.6%	151.022	2.2860	1	0	Equal to100%
	leading ratio$_{(P)}$=1.9124					leading ratio$_{(P)}$=1.6901				
Analysis of the observed planetary configuration of the EPIC 206024342 system										
Analysis of the observed planetary configuration [5]						Analysis based on period ratio of Planet c to b as leading ratio [6]				
Planet	P (day)	Ratio of $P_n:P_{n-1}$	ω_P	θ_P (%)	R (R_{Jup})	P (day)	Ratio of $P_n:P_{n-1}$	ω_P	θ_P (%)	The first result of calculation
c	0.91162				1.16	0.91162				Equal to 100%
b	4.50717	4.9441	1.2069	20.69	1.44	4.50717	4.9441	1	0	Reference planet
d	14.6435	3.2489	0.7931	-20.69	2.52	22.284	4.9441	1	0	Equal to 65.7%
	average ratio$_{(P)}$=4.0965					leading ratio$_{(P)}$=4.9441				
Analysis based on period ratio of Planet d to c as leading ratio [7]						Analysis based on the same divisor as leading ratio [8]				
Planet	P (day)	Ratio of $P_n:P_{n-1}$	ω_P	θ_P (%)		P (day)	Ratio of $P_n:P_{n-1}$	ω_P	θ_P (%)	The second result of calculation
c	1.38729				65.7%	0.91162				Equal to 100%
b	4.50717	3.2489	1	0	Reference	3.65366	4.0079	1	0	Equal to 123.36%
d	14.6435	3.2489	1	0	100%	14.6435	4.0079	1	0	Reference planet
	leading ratio$_{(P)}$=3.2489					leading ratio$_{(P)}$=4.0079				

Note 1.Symbol ω_P is period ratio parameter and θ_P is deviation rate of ω_P. They are calculated through equation (1) and (2).

Note 2. Analysis of the simulated Kepler-289 system:

If one doesn't know the observed planetary configuration, he will not confirm whether Calculation [3] or Calculation [4] is closer to the real planetary configuration of Kepler-289.

Note 3. Analysis of the observed EPIC 206024342 system:

Based on the calculation with equation (9), there will be three possibilities (See [6], [7] and [8]). It is impossible to confirm which calculation is closer to the previous planetary configuration of EPIC 206024342.

Table S71. A missing planet changes the property of the K2-136 system

Orbital relations among current planets						Possible planetary configuration in the past					
Planet	Observed P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Radius (R_{Jup})	Planet	Calculated P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Compare observed P to calculated P
b	7.975292				0.0883	b	7.92598				Equal to100.62%
c	17.307137	2.1701	1.1898	18.98	0.260	x	11.71221	1.4686	1	0	Missing planet
d	25.575065	1.4777	0.8102	-18.98	0.129	c	17.307137	1.4777	1	0	Reference planet
	average ratio$_{(P)}$ =1.7776					d	25.575065	1.4777	1	0	Equal to100%
							benchmark ratio$_{(P)}$=1.4777				

Note 1. The symbol ω_P is ratio parameter of periods and θ_P is deviation rate of ω_P.They are calculated with equation (1) and (3).

Note 2. The perfect coincidence between the calculated period and observed period for Planet b states that Planet b, c and d well follow the shared rules. Planet x is calculated on the basis of the planets with the shared rules. Because the current planets have relatively big eccentricities, showing $0.10^{+0.19}_{-0.07}$, $0.13^{+0.27}_{-0.11}$ and $0.14^{+0.13}_{-0.09}$, they may be influenced by some factor. It is possible that an alien body rushed into the system and impacted Planet x. Otherwise, it orbits in a different inclination.

Table S72. A missing planet changes the property of the K2-80 system

Orbital relations among current planets						Possible planetary configuration in the past					
Planet	Observed P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Mass (M_{Earth})	Planet	Calculated P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Compare observed P to calculated P
b	5.6050				4.70	b	5.5232				Equal to101.48%
c	19.0918	3.4062	1.3851	38.51	2.76	x	8.3511	1.5120	1	0	Undetected planet
d	28.8672	1.5120	0.6149	-38.51	7.04	y	12.629	1.5120	1	0	Undetected planet
average ratio$_{(P)}$ =2.4591						c	19.0918	1.5120	1	0	Equal to100%
						d	28.8672	1.5120	1	0	Reference planet
						benchmark ratio$_{(P)}$=1.5120					

Note 1. The symbol ω_P is ratio parameter of periods and θ_P is deviation rate of ω_P.They are calculated with equation (1) and (3).

Note 2. The coincidence between the calculated period and observed period for Planet b states that Planet b, c and d well follow the shared rules. Since Planet x and y are calculated on the basis of the planets showing the shared rules, they should exist now or once existed in the past. Because the adjacent planets are super-Earth planets and all current planets are at the regions of the shared rules, It is possible that some reason broke the stable motion of the two undetected planets, which caused them have collided with each other or been ejected off the system.

Table S73. A missing planet changes the property of the Kepler-267 system

Orbital relations among current planets						Possible planetary configuration in the past					
Planet	Observed P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Mass (M_{Earth})	Planet	Calculated P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Compare observed P to calculated P
b	3.35373				1.98	b	3.35373				Equal to100%
c	6.87745	2.0507	0.6627	-33.73	2.13	c	6.87745	2.0507	1	0	Reference planet
d	28.4645	4.1388	1.3373	33.73	2.27	x	14.1036	2.0507	1	0	Undetected planet
	average ratio$_{(P)}$ =3.0948					d	28.9222	2.0507	1	0	Equal to 98.42%
							benchmark ratio$_{(P)}$=2.0507				

Note 1. The symbol ω_P is ratio parameter of periods and θ_P is deviation rate of ω_P.They are calculated with equation (1) and (3).

Note 2. The coincidence between the calculated period and observed period for Planet b states that Planet b, c and d well follow the shared rules. Planet x is calculated on the basis of the planets showing the shared rules. Because the adjacent planets are only super-Earth planets and they are at the regions of the shared rules, it is possible that an alien body rushed into the system and impacted Planet x. Otherwise, it orbits the star in a different inclination.

Table S74. The calculation according to the shared rules suggest the existence of the unconfirmed planet of GJ 887

Analysis of the periods of possible planets					The orbit of unconfirmed planet calculated by confirmed planets					
Planet	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	P (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	M ($M_{Earth.}$)	Mass of M_n:M_{n-1}
b	9.262				9.262				4.2	
c	21.789	2.3525	1.0124	1.24	21.789	2.3525	1	0	7.6	1.81
d (?)	~50	~2.2947	0.9876	-1.24	51.259	2.3525	1	0		
	average ratio$_{(P)}$=2.3236				leading ratio$_{(\alpha)}$=1.5013					

Note 1. "ω_P" is the ratio parameter of periods and "θ_P" is the deviation rate of period ratio. They are calculated by equation (1) and (3).

Note 2. Possibility of the unconfirmed planet.

In 2020, Jeffers et al. (2020) found two planets in the habitable zones of GJ 887. Meanwhile, they also detected an unconfirmed signal with the period of ~50 days, which could correspond to a third planet. If considering the mass relations between planets, there should be planet outside the orbit of Planet c. If the shared rules still exist in the system between the two confirmed planets, the calculation by referring to the orbits of the two planets may predict the planet by $P_x=P_c\div(P_c:P_b)^{(2-3)}=51.259$ (day). If the unconfirmed planet is confirmed someday, the confirmation will further support the shared rules.

Table S75. A pairs of four-planet systems with similar stellar physical parameters show the similar planetary physical parameters.

The Kepler-221 System (Stellar radius=0.821 R_{Sun}, stellar age=$4.57^{3.67}_{-2.65}$ Gyr)										
		Analysis of current planets				Calculation according to the planets with shared rules				
Planet	Radius (R_{Jup})	Observed Period (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Previous Period (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Compare observed P to previous P
b	0.153	2.795906				3.00043				Equal to 93.2%
c	0.261	5.690586	2.0353	1.1126	11.26	5.48899	1.8294	1	0	Equal to 103.67%
d	0.244	10.04156	1.7646	0.9646	-3.54	10.04156	1.8294	1	0	Reference planet
e	0.235	18.36992	1.8294	1	0	18.36992	1.8294	1	0	Equal to 100%
		benchmark $ratio_{(P)}$=1.8294				leading $ratio_{(P)}$=1.8294				

The Kepler-305 System (Stellar radius=0.827 R_{Sun}, stellar age=$4.37^{3.26}_{-2.35}$ Gyr)										
		Analysis of current planets				Calculation according to the planets with shared rules				
Planet	Radius (R_{Jup})	Observed Period (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Previous Period (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Compare observed P to previous P
e	0.160	3.20538				3.20538				Equal to 100%
b	0.321	5.487	1.7118	1	0	5.487	1.7118	1	0	Reference planet
c	0.294	8.291	1.5110	0.8827	-11.73	9.3927	1.7118	1	0	Equal to 88.27%
d	0.242	16.73866	2.0189	1.1794	17.94	16.0783	1.7118	1	0	Equal to 104.1%
		benchmark $ratio_{(P)}$=1.7118				leading $ratio_{(P)}$=1.7118				

Note 1.The symbol ω_P is period ratio parameter and θ_P is the deviation rate of ω_P. They are calculated through equation (1) and (3).

Note 2. If calculated according to the planets following the shared rules, the planetary configuration of each system shows a little different from its previous configuration. Even so, the possibly previous planetary configurations of the two systems were still similar.

Note 3. Although the two systems show the deviation rates bigger than 10%, the relatively big deviation rates can be explained. For the Kepler-305 system, the dragging force should cause the opposite migration of Planet c and d. For the Kepler-221 system, it is the mutual dragging force that caused the opposite migration of Planet b and c. The exchange of kinetic energy is in line with the law of kinetic energy conversion.

Note 4. The observation data come from NASA Exoplanet Archive by https://exoplanetarchive.ipac.caltech.edu/.

Table S76. A pairs of four-planet systems with similar stellar physical parameters show the similar planetary physical parameters.

The Kepler-172 System (Stellar radius=1.085 R_{Sun}, stellar age=$6.46^{3.14}_{-3.69}$ Gyr)										
Analysis of current planets [3]						Calculation according to the planets with shared rules [4]				
Planet	Radius (R_{Jup})	Observed Period (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Previous Period (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Compare observed P to previous P
b	0.21	2.940309				2.790686				Equal to 105.36%
c	0.255	6.388996	2.1729	0.9491	-5.09	6.388996	2.2894	1	0	Reference planet
d	0.201	14.62712	2.2894	1	0	14.62712	2.2894	1	0	Equal to 100%
e	0.246	35.11874	2.4009	1.0487	4.87	33.48733	2.2894	1	0	Equal to104.87%
benchmark ratio$_{(P)}$=2.2894						leading ratio$_{(P)}$=2.2894				

The Kepler-299 System (Stellar radius=1.032 R_{Sun}, stellar age=$6.03^{3.05}_{-3.59}$ Gyr)										
Analysis of current planets [1]						Calculation according to the planets with shared rules [2]				
Planet	Radius (R_{Jup})	Observed Period (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Previous Period (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Compare observed P to previous P
b	0.118	2.927128				2.927128				Equal to 100%
c	0.236	6.885875	2.3524	1	0	6.885875	2.3524	1	0	Reference planet
d	0.166	15.054786	2.1863	0.9294	-7.06	16.1983	2.3524	1	0	Equal to 92.94%
e	0.167	38.285489	2.5431	1.0810	8.10	38.1050	2.3524	1	0	Equal to 100.47%
benchmark ratio$_{(P)}$=2.3524						leading ratio$_{(P)}$=2.3524				

Note 1.The symbol ω_P is period ratio parameter and θ_P is the deviation rate of ω_P. .

Note 2. Calculated through equation (1) and (3), the two systems show the orbital relations of the observed planets. It can be seen that they have the similar benchmark ratios.

Note 3. The calculations reflect that the two systems are normal systems. If calculated according to the planets following the shared rules, the planetary configuration of each system shows a little different from its previous configuration. Even so, the possibly previous planetary configurations of the two systems were still similar. Because the stellar size and age of Kepler-172 are a little bigger than that of Kepler-299, the planetary physical parameters of Kepler-172 are a little bigger than that of Kepler-299.

Note 2. The observation data come from NASA Exoplanet Archive by https://exoplanetarchive.ipac.caltech.edu/.

Table S77. Two pairs of three-planet systems with similar stellar physical parameters show the similar planetary physical parameters.

The Kepler-244 System (Stellar radius=0.803 R_{Sun}, stellar age=$4.79^{+6.15}_{-3.08}$ Gyr)						The TOI-125 System (Stellar radius=0.848 R_{Sun}, stellar age=$6.80^{+4.60}_{-4.20}$ Gyr)					
Planet	Radius (R_{Jup})	Period (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Planet	Radius (R_{Jup})	Period (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)
b	0.246	4.31179				b	0.243	4.65382			
c	0.183	9.76729	2.2653	1.0492	4.92	c	0.246	9.15059	1.9663	0.9477	-5.23
d	0.206	20.0504	2.0528	0.9508	-4.92	d	0.261	19.9800	2.1835	1.0523	5.23
		average ratio$_{(P)}$=2.1590						average ratio$_{(P)}$=2.0749			
The Kepler-60 System (Stellar radius=1.257 R_{Sun}, stellar age=$5.13^{+1.00}_{-1.69}$ Gyr)						The Kepler-431 System (Stellar radius=1.092 R_{Sun}, stellar age=$4.27^{+0.90}_{-0.962}$ Gyr)					
Planet	Radius (R_{Jup})	Period (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)	Planet	Radius (R_{Jup})	Period (day)	Ratio of P_n:P_{n-1}	ω_P	θ_P (%)
b	0.153	7.1334				b	0.068	6.803			
c	0.170	8.9187	1.2503	0.9676	-3.24	c	0.060	8.703	1.2793	0.9658	-3.42
d	0.178	11.8981	1.3341	1.0324	3.24	d	0.10	11.922	1.3699	1.0342	3.42
		average ratio$_{(P)}$=1.2922						average ratio$_{(P)}$=1.3246			

Note 1.The symbol ω_P is period ratio parameter and θ_P is the deviation rate of ω_P. They are calculated with equation (1) and (3).

Note 2. Here we list two pairs of three-planet systems. Each pair of systems have similar physical parameters. They are the systems with the shared rules according to trait (1). Although some planets of a system may migrate for the mutual perturbations among them, it is difficult to convincingly judge which planet migrated because the migratory planet can not be proved. However, if considering the migrations of some planets, the average ratios for the corresponding pair of systems will become closer.

Note 3. Because the physical conditions of each protoplanetary disk is different from others, it is almost impossible to find a pair of system showing completely identical stellar physical parameters and planetary physical parameters. The difference of one condition between the two disks may cause some physical parameter to be different, such as the pair of the systems of Kepler-60 and Kepler-431, which show the difference of planetary sizes between the two corresponding planets. Therefore, two systems showing the same physical parameters are accidental, and it is necessary for most systems to show different physical parameters. However, if a pair of systems with similar physical parameters show the shared rules, they should demonstrate that the shared rules appear along with the formation of the systems.

The end