Validation of the mean systemic filling pressure assessment with preserved arterial blood flow by comparing two methods of calculation

Supplementary Information: Derivation of the relationship between cuff pressure and venular volume

Roberto Alberto De Blasi[1] and Stefano Finazzi[2]

[1]Dipartimento di Scienze Medico-Chirurgiche e Medicina Traslazionale, Universit degli studi di Roma Sapienza via di Grottarossa 1035, 00189 Roma - Italy - roberto.deblasi@uniroma1.it
[2]Laboratorio di Clinical Data Science, Dipartimento di Salute Pubblica, Istituto di Ricerche Farmacologiche Mario Negri IRCCS, Ranica (BG), Italy - stefano.finazzi@marionegri.it

We define R_a as the total vascular resistance upstream the venules, P_a and P_v as the arterial and venular pressures and C as the venular compliance. By applying to the arm a pressure P_{cuff} with a cuff, an extra resistance $R(P_{\text{cuff}})$ is generated downstream when the cuff pressure overcomes the baseline venule pressure P_0 (mean circulatory filling pressure). A resistor-capacitance model of this system is pictured in Fig. 1.

Venular pressure – cuff pressure. By considering either the full circuit or the downstream section, the blood flow Q can be expressed as

$$Q = \frac{P_a - P_{\text{cuff}}}{R_a + R(P_{\text{cuff}})} = \frac{P_v - P_{\text{cuff}}}{R(P_{\text{cuff}})}. \tag{1}$$

The venular pressure is derived as a function of the cuff pressure P_{cuff} from Eq. (1),

$$P_v = P_{\text{cuff}} + \frac{R(P_{\text{cuff}})}{R_a + R(P_{\text{cuff}})}(P_a - P_{\text{cuff}}). \tag{2}$$

Since the resistance R appears when the cuff pressure overcomes the baseline venular pressure P_0, we can approximate $R(P_{\text{cuff}})$ as

$$R(P_{\text{cuff}}) = R_0 \left(\frac{P_{\text{cuff}}}{P_0} - 1 \right), \tag{3}$$

for $P_{\text{cuff}} \geq P_0$ and $R(P_{\text{cuff}}) = 0$ for $P_{\text{cuff}} < P_0$. Thus $P_v = P_0$ for $P_{\text{cuff}} < P_0$, whereas Eq. (4) becomes

$$P_v = P_0 + \left[P_0 + \frac{R_0}{R_a} (P_a - P_0) \right] \frac{P_{\text{cuff}}/P_0 - 1}{1 + (R_0/R_a)(P_{\text{cuff}}/P_0 - 1)}, \tag{4}$$

for $P_{\text{cuff}} \geq P_0$ (see Fig 2, left panel). For the sake of simplicity, we define

$$\beta = \frac{R_0}{R_a} \tag{5}$$

and the two functions f and r as

$$f(x) = \begin{cases} 1, & \text{if } x \leq P_0, \\ \dfrac{x}{P_0} - 1, & \text{if } x > P_0, \end{cases} \tag{6}$$

$$r(x) = \frac{f(x)}{1 + \beta f(x)}. \tag{7}$$

With this notation, Eq. (4) reads

$$P_v = P_0 \left\{ 1 + [1 + \beta f(P_a)] \, r(P_{\text{cuff}}) \right\} \tag{8}$$

Venular volume – venular pressure. The volume of blood V in the venular compartment, represented in Fig. 2 by the capacitor C is the sum of the unstressed volume V_u and the stressed volume V_s

$$V = V_u + V_s(P_v). \tag{9}$$

The compliance varies with venular pressure P_v because it is generated by two different physiological mechanisms. At low pressure, compliance is associated to venule recruitments. For high pressure, all vessels are recruited and compliance is associated with venule wall elasticity. We can model this dependence with a step function

$$C(P_v) = \begin{cases} C_1, & \text{if } P_v \leq \bar{P}_v, \\ C_2, & \text{if } P_v > \bar{P}_v, \end{cases} \tag{10}$$

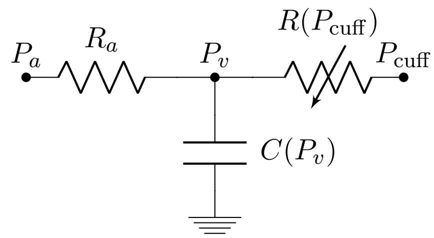

Figure 1: Resistor-capacitance model. P_a, P_v, and $P_{\rm cuff}$ are the arterial, venular, and cuff pressure, $C(P_v)$ is the pressure-dependent venular compliance, R_a the total resistance upstream of venules, $R(P_{\rm cuff})$ is the variable resistance generated by the cuff pressure.

for some threshold value $\bar{P}_v$ of the venular pressure. With this assumption the total volume of the venule compartment is obtained by integrating

$$V = V_u + \int_0^{P_v} C(p)\,\mathrm{d}p. \tag{11}$$

We obtain

$$V = \begin{cases} V_u + C_1\,P_v, & \text{if } P_v \le \bar{P}_v, \\ V_u + (C_1 - C_2)\,\bar{P}_v + C_2\,P_v, & \text{if } P_v > \bar{P}_v. \end{cases} \tag{12}$$

Inserting Eq. (8) in the above expression, we obtain an explicit relation for the venular volume V as a function of the cuff pressure $P_{\rm cuff}$:

$$V(P_{\rm cuff}) = \begin{cases} V_0 + V_1\,r(P_{\rm cuff}), & \text{if } P_{\rm cuff} \le \bar{P}_{\rm cuff}, \\ V_2 + V_3\,r(P_{\rm cuff}) & \text{if } P_{\rm cuff} > \bar{P}_{\rm cuff}, \end{cases} \tag{13}$$

where

$$V_0 = V_u + C_1 P_0, \tag{14}$$
$$V_1 = C_1\,P_0\left[1 + \beta f(P_a)\right], \tag{15}$$
$$V_2 = V_u + (C_1 - C_2)\bar{P}_v + C_2 P_0, \tag{16}$$
$$V_3 = C_2 P_0\left[1 + \beta f(P_a)\right], \tag{17}$$

for a certain threshold value $\bar{P}_{\rm cuff}$ of the cuff pressure $P_{\rm cuff}$, which is determined by solving Eq. (8) for $P_v = \bar{P}_v$,

$$\bar{P}_v = P_0\left\{1 + \left[1 + \beta f(P_a)\right] r(\bar{P}_{\rm cuff})\right\}. \tag{18}$$

Fitting the system parameters. The venular volume depends on the cuff pressure through a complex expression involving six free parameters V_u, C_1, C_2, P_0, $\bar{P}_v$, β that must be estimated by fitting the function $V(P_{\rm cuff})$ to measured data. Computationally, it is simpler to fit $V(P_{\rm cuff})$ as in Eq. (13) expressed with the parameters V_i, $i = 0, \ldots, 3$, P_0, and β, and then invert Eqs. (15)–(17) to determine the relevant physiological parameters.

By measuring the arterial pressure P_a:

$$C_1 = \frac{V_1}{P_0\left[1 + \beta f(P_a)\right]}, \tag{19}$$
$$C_2 = \frac{V_3}{V_1}\,C_1 \tag{20}$$
$$V_U = V_0 - C_1 P_0. \tag{21}$$

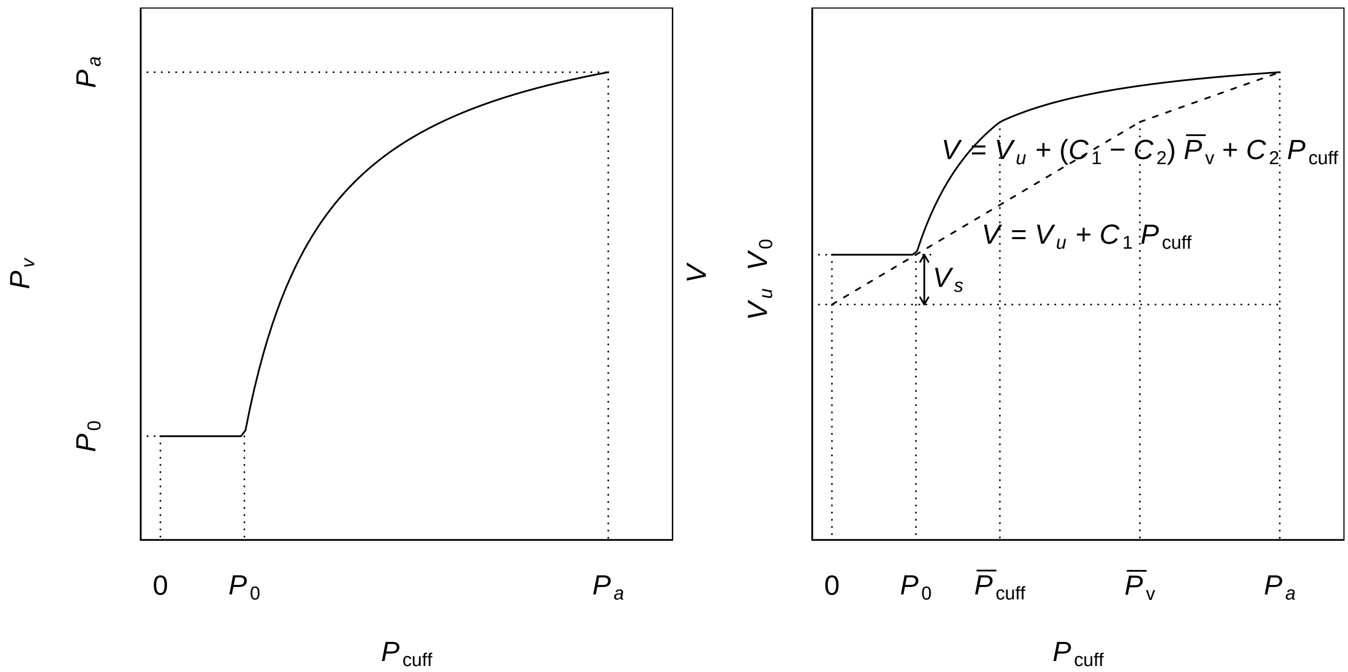

Figure 2: Venular pressure P_v (left panel) and venular volume V (right panel) as functions of the cuff pressure P_{cuff}. P_a is the arterial pressure, P_0 the baseline venular pressure, V_u and V_s the unstressed and stressed volume, respectively, and C the venular compliance.

The calculation of $\bar{P}_{\text{cuff}}$ and $\bar{P}_v$ is slightly more involved. Subtracting Eq. (14) from Eq. (16) and Eq. (17) from Eq. (15)

$$V_2 - V_0 = (C_1 - C_2)P_0 f(\bar{P}_v). \tag{22}$$
$$V_1 - V_3 = (C_1 - C_2)P_0 \left[1 + \beta f(P_a)\right]. \tag{23}$$
$$\tag{24}$$

We obtain

$$f(\bar{P}_v) = \frac{V_2 - V_0}{V_1 - V3}\left[1 + \beta f(P_a)\right], \tag{25}$$

from which

$$\bar{P}_v = P_0 \left\{1 + \frac{V_2 - V_0}{V_1 - V_3}\left[1 + \beta f(P_a)\right]\right\} \tag{26}$$

Comparing with Eq. (18), we obtain

$$r(\bar{P}_{\text{cuff}}) = \frac{V_2 - V_0}{V_1 - V_3} \tag{27}$$

and, finally,

$$\bar{P}_{\text{cuff}} = P_0 \left[1 + \left(\frac{V_1 - V_3}{V_2 - V_0} - \beta\right)^{-1}\right]. \tag{28}$$