

TABLE I. Detailed comparison with SOTA algorithms for each of the five index data sets and on seven different performance indicators described in Appendix A2. Entries in **red** correspond to cases where TN-GEO performed better or tied compared to the other algorithm. Entries in **bold**, corresponding to the best (lowest) value, for each specific indicator.

Data Set	Performance Indicator	GTS	IPSO	IPSO-SA	PBILD	GRASP	ABCFEIT	HAAG	VNSQP	RCABC	TN-GEO
Hang Seng	Mean	1.0957	1.0953	-	1.1431	1.0965	1.0953	1.0965	1.0964	1.0873	1.0958
	Median	1.2181	-	-	1.2390	1.2155	1.2181	1.2181	1.2155	1.2154	1.2181
	Min	-	-	-	-	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	Max	-	-	-	-	1.5538	1.5538	1.5538	1.5538	1.5538	1.5538
	MEUCD	-	-	0.0001	-	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
	VRE	-	-	1.6368	-	1.6400	1.6432	1.6395	1.6397	1.6342	1.6392
	MRE	-	-	0.6059	-	0.6060	0.6047	0.6085	0.6058	0.5964	0.6082
DAX100	Mean	2.5424	2.5417	-	2.4251	2.3126	2.3258	2.3130	2.3125	2.2898	2.3142
	Median	2.5466	-	-	2.5866	2.5630	2.5678	2.5587	2.5630	2.5629	2.5660
	Minimum	-	-	-	-	0.0059	0.0023	0.0023	0.0059	0.0059	0.0023
	Maximum	-	-	-	-	4.0275	4.0275	4.0275	4.0275	4.0275	4.0275
	MEUCD	-	-	0.0001	-	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
	VRE	-	-	6.7806	-	6.7593	6.7925	6.7806	6.7583	6.8326	6.7540
	MRE	-	-	1.2770	-	1.2769	1.2761	1.2780	1.2767	1.2357	1.2763
FTSE100	Mean	1.1076	1.0628	-	0.9706	0.8451	0.8481	0.8451	0.8453	0.8406	0.8445
	Median	1.0841	-	-	1.0841	1.0841	1.0841	1.0841	1.0841	1.0841	1.0841
	Minimum	-	-	-	-	0.0016	0.0047	0.0006	0.0045	0.0016	0.0047
	Maximum	-	-	-	-	2.0576	2.0638	2.0605	2.0669	2.0670	2.0775
	MEUCD	-	-	0.0000	-	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	VRE	-	-	2.4701	-	2.4350	2.4397	2.4350	2.4349	2.4149	2.4342
	MRE	-	-	0.3247	-	0.3245	0.3255	0.3186	0.3252	0.3207	0.3254
S&P100	Mean	1.9328	1.6890	-	1.6386	1.2937	1.2930	1.2930	1.2649	1.3464	1.2918
	Median	1.1823	-	-	1.1692	1.1420	1.1369	1.1323	1.1323	1.1515	1.1452
	Minimum	-	-	-	-	0.0009	0.0000	0.0000	0.0000	0.0009	0.0000
	Maximum	-	-	-	-	5.4551	5.4422	5.4642	5.4551	5.4520	5.4422
	MEUCD	-	-	0.0001	-	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
	VRE	-	-	2.6281	-	2.5211	2.5260	2.5255	2.5105	2.5364	2.5269
	MRE	-	-	0.7846	-	0.9063	0.8885	0.7044	0.9072	0.8858	0.9117
Nikkei	Mean	0.6066	0.6870	-	0.5972	0.5782	0.5781	0.5781	0.5904	0.5665	0.5793
	Median	0.6093	-	-	0.5896	0.5857	0.5856	0.5854	0.5857	0.5858	0.5855
	Minimum	-	-	-	-	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	Maximum	-	-	-	-	1.1606	1.1606	1.1607	1.1606	1.1606	1.1606
	MEUCD	-	-	0.0000	-	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	VRE	-	-	0.9583	-	0.8359	0.8396	0.8191	0.8561	0.8314	0.8353
	MRE	-	-	1.7090	-	0.4184	0.4147	0.4233	0.4217	0.4042	0.4229

TABLE II. Pairwise comparison of TN-GEO against each of the SOTA optimizers. The asymptotic significance is part of the Wilcoxon signed-rank test results. The null hypothesis that the performance of the two algorithms is the same is tested at the 95% confidence level (significance level: $\alpha = .05$). Results show that TN-GEO is on par with all the SOTA algorithms, and in two cases, GTS and PBILD, it significantly outperforms them. We also report the count for TN-GEO wins, losses, and ties, compared to each of the other algorithms.

TN-GEO vs Other:	GTS	IPSO	IPSO-SA	PBILD	GRASP	ABCFEIT	HAAG	VNSQP	RCABC
Wins(+)	6	4	6	9	12	10	11	11	8
Loss(-)	2	1	4	0	12	9	11	12	16
Ties	2	0	5	1	11	16	13	12	11
(Wins+Ties)/Total	80%	80%	67%	100%	66%	74%	69%	66%	54%
Asymptotic significance (p)	.036	.080	.308	.008	.247	.888	.363	.594	.110
Decision	Reject	Retain	Retain	Reject	Retain	Retain	Retain	Retain	Retain

TABLE III. State-of-the-art (SOTA) algorithms used in the comparison with our proposal, TN-GEO.

Acronym	Algorithm	Paper
GTS	Genetic algorithm, tabu search and simulated annealing algorithms.	Chang, et al., 2000 [1]
IPSO	An Improved Particle Swarm Optimization.	Deng, et al., 2012 [2]
IPSO-SA	An Improved Particle Swarm Optimization with simulated annealing.	Mozafari, M., 2011 [3]
PBILD	A population-based incremental learning and differential evolution algorithms.	Lwin & Qu, 2013 [4]
GRASP	A greedy randomized adaptive search procedure.	Baykasoglu et al., 2015 [5]
ABCFEIT	An artificial bee colony algorithm with feasibility enforcement and infeasibility toleration.	Kalayci et al., 2017 [6]
HAAG	A hybridization of ant Colony, artificial Bee Colony and genetic algorithms.	Kalayci et al., 2020 [7]
VNSQP	A variable neighborhood search algorithm with quadratic programming.	Kalayci et al., 2020 [8]
RCABC	A rapidly converging Artificial Bee Colony Algorithm.	Cura, 2021 [9]
TN-GEO	Our proposed quantum-enhanced optimization	

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