SUPPLEMENTARY INFORMATION - I
Competition and cooperation in sociophysical settings: Simulation of a strategic model

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Using the model parameters as given in the model & method section of the main text, the dynamics of deceptive invasion under two different defensive manoeuvres of the empires are visually realized in the Videos: Video I for idle PPs and Video II for active PPs (see supplementary information-II). One would clearly see in Video I that, as soon as the simulation begins, the invading domain (red in color) pierces the circular territory of the empire at any arbitrary point and starts occupying the weak (empty) region of the empire, expands its size, enhances its military strength, and proceeds towards the emperor. During this process, any two neighbouring PPs (cyan in color) that lie within a radial distance ($r \leqslant r_d$) from the invaders, either get alerted for external threat or become the victim of the invaders's deception. In any case, they start expanding at a predefined constant rate, f_{gP}. If such PPs are expanding out of a state of alert, their intention will be to coalesce among themselves and to resist the invaders. Prior to such a coalescence takes place, the invaders may trigger an internal conflict among the expanding PPs through some art of deceit. While such deceived PPs expand and grow larger and larger, invaders retreat at a rate f_{R} until they engage in a devastating war among themselves. The internal conflict is considered to be devastating enough that can destroy the other smaller PPs (green in color) that come into contact with the deceived PPs (cyan in color). The deception is assumed to be so strong that the expanding PPs are unable to identify the invaders, and the invading domain can remain alive within the empire although it is occupied by any one of the expanding PPs. The moment the expanding PPs collide, the smaller PP (radius, r_i) gets annihilated and the bigger one (radius, r_j) suffers the casualty as a consequence of a

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vicious war. The surviving PP suffers the after-war effect and shrinks in size at a constant rate f_{gP} to an extent $r_j - \varepsilon r_i$, where ε is the fairness parameter of the war whose value can be chosen to lie between 0 and 1.[1–3] For the entire simulation, we set $\varepsilon = 1$ that corresponds to a fair war. Being dependent on the decay rate and the magnitude of casualties, the decay may last longer and it bears a resemblance to the historical period during which acute economic depression, declining population, declining standards of living, and other after-war effects prevail. As soon as the survivor PP starts suffering the after-war effect, the invading domain stops retreating and proceeds again towards the emperor by expanding at a predefined rate f_{gn}. The process of deception repeats with other neighbouring PPs. Meanwhile, a surviving deceived PP who continues its expansion after being suffered from the casualty, could come in direct contact with the invading domain, resulting in a vicious war. In such a war, if the size of the invading domain is smaller than that of the PP, the invaders get destroyed and the empire wins over the invader. By contrast, if the invading domain is bigger than that of the PP, the PP gets annihilated and the invading domain suffers the casualty. Once the casualty is over, the invaders again grow and proceed toward the emperor at the same rate as before (f_{gn}). As soon as the invader attacks the emperor (the grey circular domain at the core), the central administrator puts its remaining PPs on high alert to defend aggressively against the attack. With an aim to enhance the military strength and intensify the coordination, the remaining green PPs start expanding at a predefined rate f_{gP} and coalesce among themselves at a rate f_{C}. The invaders can still deploy their stratagem and strive to destroy all the remaining PPs through internal conflicts. As the deception process continues, there will be only one survivor PP in the long run. Depending on the sizes of the surviving PP and the invading domain at the moment of their interaction, the final outcome would be a victory of either the invaders or the empire.

For the case of active PPs (see the Video II), a surviving deceived PP that comes in a direct contact with the invaders, sends a message to its nearby PPs who lies within a radial distance ($r \leqslant r_m$). The message could be any means of communication for a vital piece of information regarding the invader's intentions and techniques. Thus, the empires with active PPs are assumed to have some mechanisms to acquire a knowledge of the deception plan and to use it against the deceiver. Those PPs who receive the message (yellow in color), start expanding at a predefined constant rate f_{gP} and coalesce with the neighbouring PPs (green in color) to enhance their coordination. Meanwhile, if the growing invading domain collides with the yellow PP, a vicious war will take place. If the invading domain is able to defeat the yellow PP, it advances forward and tries to capture the emperor.

I. Some additional simulation results

Following the history-inspired rules as mentioned in Sec. I and in the Fig. 1 of the main text, we construct the dynamics of deceptive invasion on empires of different internal configurations. The results that are obtained in addition to the reported results of the main text, are presented below.

A. For idle PPs

Fig. 1**a, b** indicate that the number of events in which the empire is captured by the invaders (small black circles on the blue curve), is more in the case of faster invasion. Further, the number of events in which the invaders are unable to trigger deception (correspond to nearly zero defence costs) is also more for faster invasion. This is more clearly visible from the distribution of defence costs against $N = 15000$ number of events (Fig. 1**c**). That the number of such events is greatest for faster invasion and least for slower invasion, is observed for the empires with $n = 5$ and $n = 10$ number of idle PPs (Fig. 1**c,d**). This indicates that although the probability of capturing the empire is higher for faster invaders, they fail in triggering deception in more number of events than the slower invaders. This trend is somewhat different for the case of $n = 10$ and $n = 20$ number of idle PPs (Fig. 1**e,f**) where deception fails in least number of event for normal invasion. By comparing the plots of Fig. 1**c-f**, one would clearly see that in the empires with increasing number of PPs, invaders' deception fails in increasingly large number of events. This is due to the fact that the invading domain in such configurations comes into direct contact with one of the stronger PPs and gets destroyed well before the initiation of deception. In addition to this, the empires with increasingly large number of PPs are subjected to gradually increasing defence costs. This is because, once deception starts, more number of PPs may become the victim of invader's deception in such an empire. Although the total defence cost, on average, increases with increasing n, the number of events where the empire wins victory increases gradually, as evident from the computation results of the invader's capture probability, P_c (Fig. 3 in the main text) on different configurations of the empire.

While performing the simulation on empires with a wider distribution of PPs, we obtain the results displayed in Fig. 2. Comparing these plots with those of Fig. 1, one can see that the deception fails in more number of events when PPs have a wider distribution.

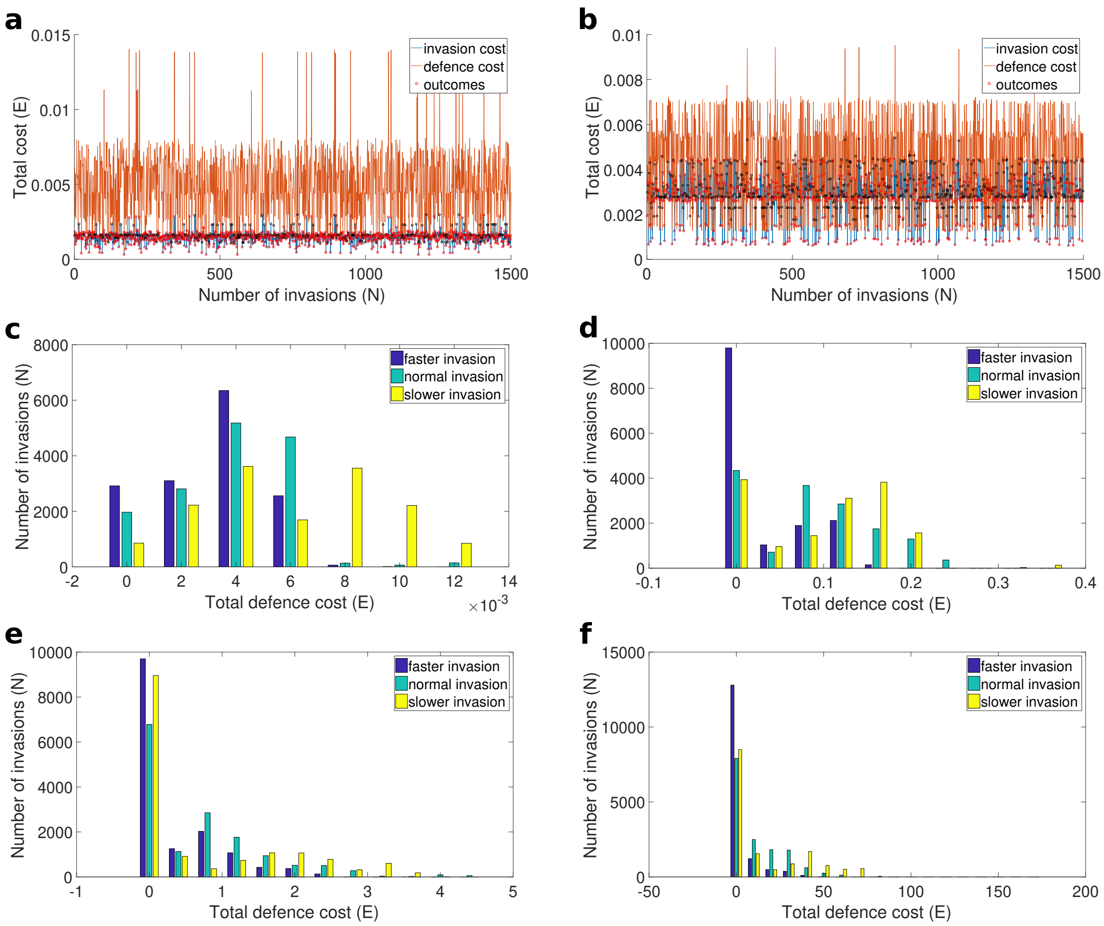

Figure 1: **Variation of costs in empires comprising idle PPs**. **a**, Variation of the total defence and invasion costs against $N = 1500$ number of normal invasion in an empire comprising $n = 5$ idle PPs distributed over a constant map. **b**, The corresponding variation for the case of faster invasion in the same empire. In both the figures, the outcome of each invasion is shown as the victory of either the empire (small red circle) or the invaders (small black circle). **c-f**, The distributions of defence cost against $N = 15000$ number of events for three different strength of invasion in empires comprising $n = 5, 10, 15$ and 20 number of idle PPs, respectively. A gradually increasing trend of defence costs is observed for increasing number of PPs. Further, the number of events with nearly zero defence cost is also increases with increasing n. That the empires incur a relatively more defence costs for slower invasion, is evident from the plots.

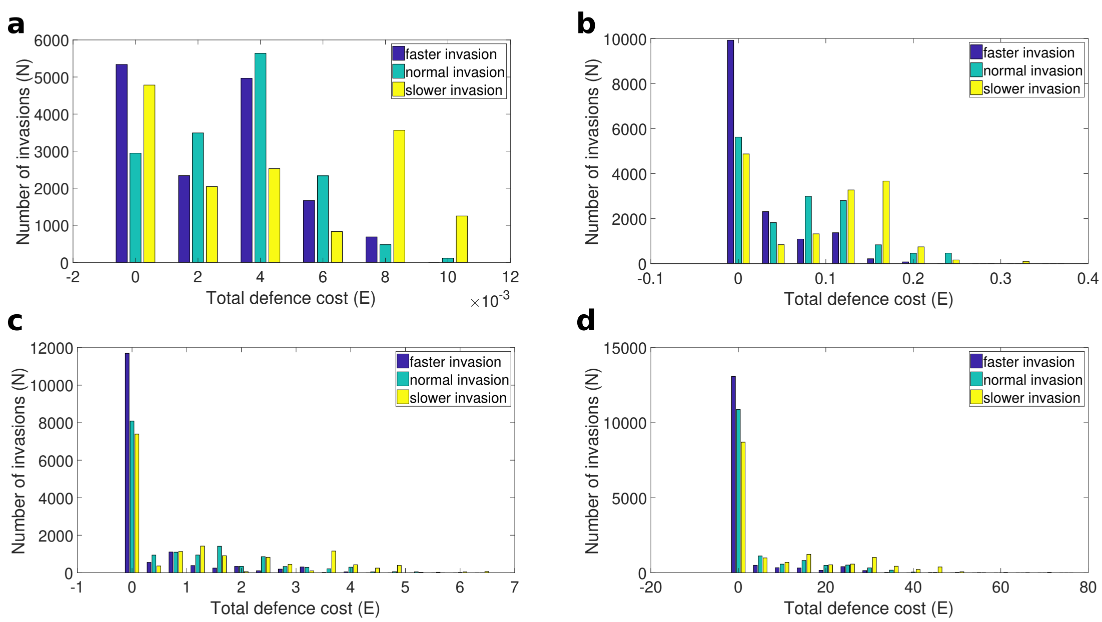

Figure 2: **Results for wider distribution of idle PPs. a-d**, The distribution of total defence costs against $N = 15000$ number of invasions in the empires comprising $n = 5, 10, 15$ and 20 number of idle PPs, respectively. The PPs are distributed in the wider belt $15 \leqslant r'_b \leqslant 40$ and the simulations are performed for three different strengths of invasion. As in Fig. 1, defence cost incurred by the empires against slower invaders is always higher. Further, the number of events against zero defence costs are more than the corresponding cases of Fig. 1.

This happens simply because some of the PPs are now distributed more or less close to the territory and, thus, possess a greater possibility of destructing the invaders quickly. This is, indeed, an important prediction of our model, hinting at the fact that an empire can greatly enhance the chance of fast destruction of the intruders when it deploys more and more military powers towards the territorial regions. One can also see from the plots of Fig. 2 that except for $n = 5$, the slower invaders fail in triggering deception in a less number of events than those of normal and faster invasions. Further, the defence costs incurred by the empires against the slower invasion are comparatively higher than the corresponding cases of normal and faster invasion.

B. For active PPs

For the case of active PPs, we obtain nearly similar trends in the variations of costs (Fig. 3). As in the case of idle PPs, here also the total defence costs and the number of events at nearly zero defence cost increase with increasing number of PPs. Fig. 3**a** shows that the empire with $n = 5$ incurs a relatively higher defence cost for slower invaders. However, the slower invaders are unable to capture the empire ($P_c = 0$), as evident from Fig. 3 of the main text. In contrast, if the active PPs are distributed in a wider belt, invader's deception fail in a more number of events, as can be visualized from Fig. 4. But still, invader's capture probability remains always higher (Fig. 3 of the main text). Thus the defensive manoeuvre of active PPs is more effective if PPs are confined in a narrow belt. Another interesting conclusion can be drawn by comparing the results of two different defensive manoeuvres. One can visualize from the comparison of Figs. 2a and 3a that invaders deception fail in more number of events if the empire with $n = 5$ follows the defensive manoeuvre of idle PPs and if the PPs are distributed in a wider belt. But still invader's capture probabilities are more in idle PPs than those of active PPs (Fig. 3 of the main text). Thus, the correlation between the execution of deception and the outcomes remains unclear and, perhaps, the trickiness of defensive manoeuvre plays the dominant role in determining the final outcomes. The strategy of active PPs is more effective as compared to that of the idle PPs because the empire's probability of winning victory is always higher irrespective of the strength of the invaders.

C. For optimal defensive structures

We consider two configurations comprising $n = 5$ and $n = 10$ PPs with a relatively wider distribution of both the size and position of the PPs: the sizes are set as $8 \leqslant r_p \leqslant 10$ while the centres are allowed to lie within a belt of radius $35 \leqslant r_b' \leqslant 45$. While these empires are

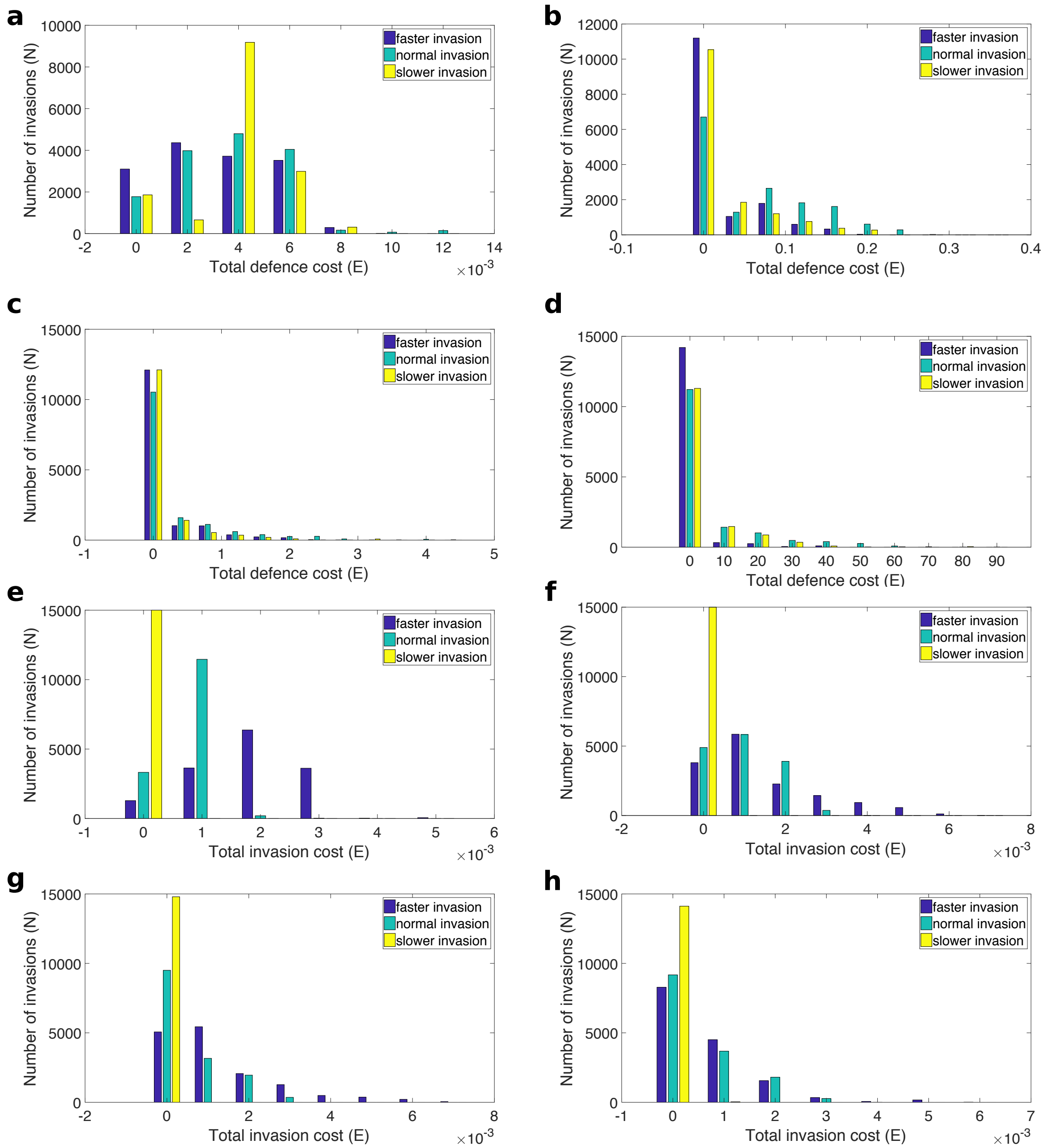

Figure 3: Distribution of total defence costs incurred by the empires comprising (a) $n = 5$, (b) $n = 10$, (c) $n = 15$, and (d) $n = 20$ number of active PPs that are distributed over a constant map. **e-h**, The corresponding distribution of total invasion costs, indicating that the invasion costs remain nearly zero for slower invaders. Further, the invasion costs are always remain smaller than the corresponding defence costs.

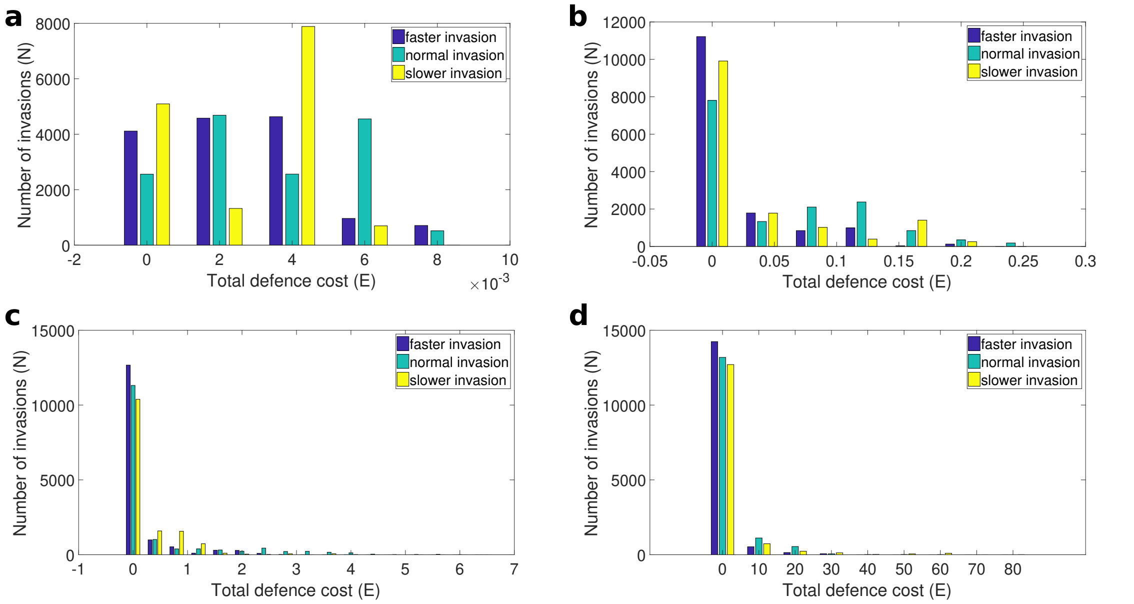

Figure 4: Distribution of total defence costs incurred by the empires comprising (a) $n = 5$, (b) $n = 10$, (c) $n = 15$, and (d) $n = 20$ number of active PPs that are distributed in wider belts.

set for the defensive manoeuvres of active PPs, the minimum radial distance to which the message can be transmitted is set as $r_m = 45$ units. The dynamics of deceptive invasion in the optimal defensive structure with $n = 5$ is visualized in two supplementary Videos: Video 3 for strong deceit and Video 4 for weak deceit. Keeping all the other parameters the same as given in Table **??**, we perform the simulation on five different configurations for a particular n. The configurations differ from each other because of the randomly assigned size of the PPs. For each of these empires and for each invasion strengths, we run the simulation for $N = 15000$ number of random invasion and, accordingly, we have a total $15000 \times 5 \times 3 = 225000$ number of simulation on an empire. We thus calculate the capture probability (P_c) and obtain the results as displayed in Fig. 4**f** of the main text.

As shown in Fig. 5, we find that the defence costs incurred by the empire with $n = 5$ are considerably smaller than those of $n = 10$ and, as such, we consider the empire with $n = 5$ as the optimal defensive structure. We see from Fig. 5 that when the invaders employ the strategy of strong deceit, there is a finite but very low probability of capturing the empire even for slower invasion rate (Fig. 5**a, c**). This is irrespective of the defensive manoeuvre of the empire. In contrast, this probability diminishes to zero for the case of weak deceit on the same empire with active PPs (Fig. 5**b**). As shown in Fig. 5**d**, although the empire incurs relatively high defence costs in some of the events against the low dispersal ability

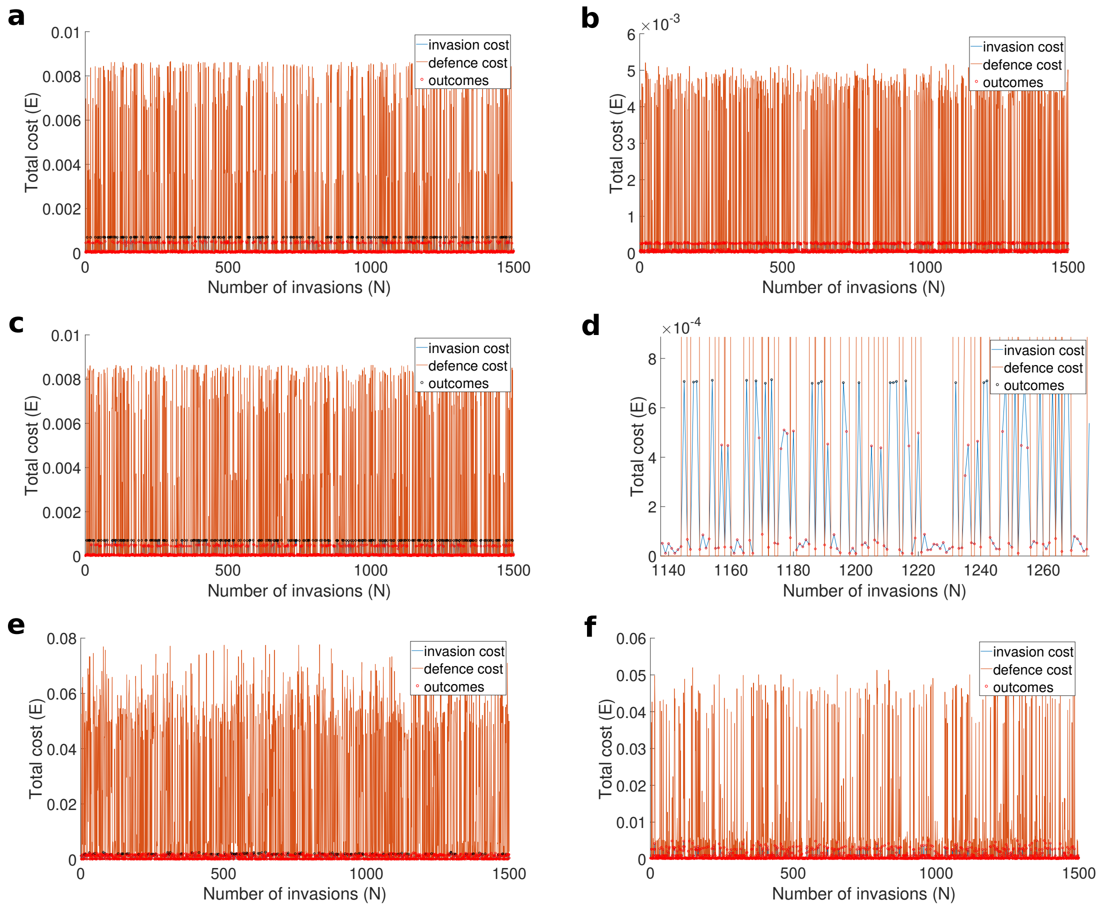

Figure 5: **Results for optimal defensive structures**. **a**, The variation of total costs in $N = 1500$ events in the empire with $n = 5$ active PPs and for the case of slower invasion and strong deceit, showing a finite number of events (small black circle) where invaders capture the empire. **b**, The corresponding variation in the same empire but for the case of slower invasion and weak deceit, indicating that the invaders never succeed in capturing the empire. **c**, The variation of total costs in the empire with $n = 5$ idle PPs and for the case of slower invasion. Again, as in **a**, the invader capture the empire in a finite number of events. **d**, The zoom-in view of Fig. **c**, showing some of the events where the empire incurs very high defence cost but defeated. **e** The variation of total costs in the empire with $n = 10$ idle PPs and for the case of normal invasion. **f**, The variation of total costs in the same empire as in **e** but for the case of faster invasion. **e** and **f** clearly show that the defence costs incurred by the empire with $n = 10$ PPs are considerably higher than those of $n = 5$ PPs.

of the invaders, the empire is defeated. This signifies that the slower invaders have a relatively higher chance of capturing the empire than that of faster ones.

References

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