Volterra Operator-Based Fixed-Time Adaptive Parameter Estimation for DC-DC Buck Converters without Current Sensors

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Abstract This paper investigates the problem of parameter estimation for DC-DC buck converter without current sensors. For the circuit, all the parameters of capacitance, inductance, resistance and input voltage are unknown. A novel Volterra operator-based fixed-time adaptive algorithm is proposed by using only the output voltage and control input signals. By selecting proper kernel function for the Volterra integral operator, the influence of the system initial values can be eliminated and the calculation of the derivative of the system output can also be avoided. Strict analysis shows that the proposed estimation algorithm can ensure the estimation errors converge to zero in a fixed time independent of the initial error values. Finally, simulation results with different initial values verify the advantages of the proposed algorithm.

Keywords Parameter estimation · Buck converter · Fixed time · Adaptive algorithm · Volterra operator

1 Introduction

For the advantages of high efficiency, small size, high stability and so on, DC-DC converter has been widely used in DC motor drive, computer systems, communication equipment and other industrial systems [1]-[4]. DC-DC converter is the basic circuit unit for constructing various power electronic equip-
ment, and its stability plays a vital role in the application of power electronic equipment in some high-tech industries. With the continuous advancement of new applications, the requirements of DC-DC converter for dynamic response speed and stability accuracy are getting higher and higher\[5\]-\[6\].

For many applications of DC-DC converter\[7\]-\[10\], it is usually required that the parameters of the circuit are exactly known, which is very strict and can not be satisfied in practice. For example, because of the aging of components, the parameters of the circuit such as resistance, capacitance and inductance may also change over time. In addition, for the consideration of cost reduction, the accuracy of circuit components is often not high. In this case, the used values of the parameters may differ greatly from the true values, which will seriously affect the its dynamic performance. Accurate parameter identification will not increase the difficulty of the original system design, but also reduce the influence of parameter error on system and improve dynamic performance. Recent years, the problem of parameter identification has been fundamental in many recent state-of-the-art developments in DC-DC power circuits such as modeling, estimation, control and so on. In\[11\], an observer-based sliding mode control algorithm was proposed for DC-DC buck converter with mismatched disturbances. But only unknown load resistance was considered in\[11\]. In\[12\], a robust stabilization control algorithm was developed for DC-DC power converter by considering the simultaneous lack of the measurement on load resistance as well as input voltage. In the recent paper\[13\], the problem of fault detection and identification was considered for DC-DC converter with unknown inductance and capacitance. To learn more about this topic, one can see the review\[14\] for more details.

It can be observed that most of the existing algorithms for parameter identification of DC-DC converter can only achieve asymptotic estimation, whose convergence speed and precision are all not enough. Recently, finite-time parameter estimation has attracted much attention for its faster convergence speed, higher precision, and better robustness. In\[15\], a finite-time parameter observer-based sliding mode controller was proposed for a converter with constant power loads. However, only the parameter of input voltage considered is to be unknown. In\[16\], for DC-DC buck converter with unknown input voltage and load resistance, two observers were designed to estimate unknown parameters in finite time. It is noted that for both the asymptotic and finite-time parameter estimation algorithms, the convergence time depends on the initial estimation error and will increase with the increase of the initial error. When the range of the initial error is unknown, it is difficult to obtain an accurate estimation of the convergence time. To overcome this problem, the concept of fixed-time stability was developed and many positive results have been reported for different systems\[17\]-\[20\]. However, to the best of the authors’ knowledge, few results about fixed-time parameter estimation for DC-DC converter have been reported in the existing literature. Therefore, how to develop a new algorithm to estimate the parameters of DC-DC buck converter in fixed-time is one of the main motivations of the paper.
In addition, most of the existing parameter estimation algorithms are only applicable to circuit systems with partial parameters of the components unknown. For example, the reference [11] only considered unknown resistance, the reference [15] only considered unknown input voltage, the references [12] and [16] considered unknown resistance and input voltage, the reference [13] considered unknown inductance and capacitance, to just name a few. But, in the actual circuit systems, the parameters of the components including capacitance, inductance, resistance and input voltage are all not easy to be obtained accurately. When the circuit is considered without current sensors, the problem will become more challenging. As far as the authors’ knowledge, few results have been reported for DC-DC buck converter with all the parameters of the components unknown, not to mention that the current is also unmeasurable. Therefore, how to develop a new estimation algorithm for DC-DC buck converter without current sensors and with all the parameters of the components unknown is another important motivation of the paper.

In view of the previous observations, in this article, we are interested in developing a novel fixed-time estimation algorithm for uncertain DC-DC buck converter without current sensors. The contributions are as follows:

- Firstly, a novel fixed-time adaptive estimation algorithm is proposed for DC-DC buck converter for the first time. Compared with [11]-[16] which can only achieve asymptotic or finite-time estimation, the proposed algorithm can guarantee that the estimation error converges to zero in a fixed time independent of the system initial values.
- Secondly, compared with [11]-[16] where only partial parameters of the circuit are unknown, this paper considers a DC-DC buck converter with more general condition for the first time, i.e., there is no current sensors and all the parameters of the circuit including capacitance, inductance, resistance and input voltage are unknown.
- Finally, the Volterra integral operator is used to estimate parameters of DC-DC buck converter for the first time. By properly selecting the kernel function for the Volterra integral operator, the influence of the system initial value can be eliminated and the calculation of the derivative of the system output can also be avoided.

2 Problem Formation and Preliminaries

2.1 Model Description of DC-DC Buck Converter

A DC-DC buck converter shown in Fig. 1 consists of a input voltage $V_{in}$, a diode $D$, an inductor $L$, a capacitor $C$, a controllable switch $S$, and a load resistor $R$. The average system model of the DC-DC buck converter is described as [11]

$$\left\{ \begin{array}{l}
\frac{dL}{dt} = \frac{1}{L}(\mu V_{in} - V_0), \\
\frac{dC}{dt} = \frac{1}{C}(i_L - \frac{V_0}{R}) \end{array} \right.,$$

(1)
where $i_L$ is the inductance current, and $V_0$ is the reference output voltage, and $V_{in}$ is the input voltage, and $\mu$ is the control signal of PWM.

**Assumption 1** The parameters of resistor $R$, inductor $L$, capacitor $C$ and input voltage $V_{in}$ are constant and unknown. The current $i_L$ is unmeasurable.

Define $V_{ref}$ as the reference output voltage and let $x_1 = V_{ref} - V_0, x_2 = -\frac{1}{L}(i_L - \frac{V_0}{R})$. Then, the model can be rewritten as follows

$$\begin{align*}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= \theta_1(x_1 - V_{ref}) + \theta_2 x_2 + \theta_3 \mu,
\end{align*}$$  
where $\theta_1 = -\frac{1}{LC}, \theta_2 = -\frac{1}{RC}, \theta_3 = -\frac{V_{in}}{LC}$.

**Remark 1** Note that how to design the control signal $\mu$ such that $x_2 = V_{ref} - V_0$ can be driven to zero (i.e., the output voltage $V_0$ can track the reference output voltage $V_{ref}$) has been widely investigated in the existing literature (see, e.g., [21]-[23]). However, these results usually require that all or at least some of the parameters $\theta_1, \theta_2, \theta_3$ are known. It can be clearly seen that for the DC-DC buck converter under Assumption 1, all the parameters $\theta_1, \theta_2, \theta_3$ are unknown, which implies that all the existing results cannot be directly applied here. Therefore, how to accurately estimate the parameters $\theta_1, \theta_2, \theta_3$ of system (2) has important theoretical and practical significance.

The goal of this paper is to develop a new fixed-time estimation algorithm for system (2) under Assumption 1 such that $\theta_1, \theta_2, \theta_3$ can be identified in a fixed time independent of the initial estimation errors.

### 2.2 Fixed-Time Stability

Consider the following systems:

$$\dot{x}(t) = f(x(t)), \tag{3}$$

where $f(0) = 0, x(t) \in \mathbb{R}^n, f(x(t)) : \mathbb{R}^n \to \mathbb{R}^n$ is a nonlinear function that can be discontinuous, and $x_0 = x(0)$ represents the initial value of the system.
Definition 1 [24] For the above system (3), the system is said to be globally finite-time stable, if for \( \forall x_0 \in \mathbb{R}^n \), the solution \( x(t, x_0) \) of (3) can reach zero in a finite time, i.e., \( x(t, x_0) = 0, \forall x_0 \geq T(x_0) \) with \( T : \mathbb{R}^n \rightarrow \mathbb{R}_+ \cup \{0\} \) being the setting-time function.

Definition 2 [17] For the above system (3), the system is said to be globally fixed-time stable, if it is globally finite-time stable and the setting time function \( T(x_0) \) is globally bounded, i.e., \( \exists T_{\text{max}} : T(x_0) \leq T_{\text{max}} \forall x_0 \in \mathbb{R}^n \).

Lemma 1 [17] If there exists a continuous radially unbounded and positive definite function \( V(x) \) such that \( \dot{V}(x) \leq -\alpha V^p(x) - \beta V^q(x) \) for some \( \alpha > 0, \beta > 0, 0 < p < 1, q > 1 \), then the origin of system (3) is globally fixed-time stable and the settling time can be estimated as \( T(x_0) \leq T_{\text{max}} = \frac{1}{\alpha(1-p)} + \frac{1}{\beta(q-1)} \).

2.3 Volterra Integral Operators Algebra

Given a function \( f(t) \in \ell^2_{\text{loc}}(\mathbb{R}_\geq 0) \), it is defined as a locally square-integrable function with domain \( \mathbb{R}_\geq 0 \) and range \( \mathbb{R} \) on the Hilbert space \( \ell^2_{\text{loc}}(\mathbb{R}_\geq 0) \). Its image through the Volterra operator \( V_K \) is denoted by \( [V_K f](\cdot) \) and defined as [25]:

\[
[V_K f](t) = \int_0^t K(t, \tau) f(\tau) d\tau, \tag{4}
\]

where the function \( K(\cdot, \cdot) : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R} \) is a Hilbert-Schmidt Kernel Function.

To facilitate the practical implementation of the operators \( [V_K f](t) \), a differential form for the operators is given in following equation:

\[
\begin{cases}
\dot{\xi}(t) = K(t, t) f(t) + \int_0^t \frac{\partial K(t, \tau)}{\partial t} f(\tau) d\tau, \\
\xi(0) = \xi_0 = 0,
\end{cases} \tag{5}
\]

where

\( \xi(t) = [V_K f](t), \forall t \in \mathbb{R}_\geq 0. \)

Lemma 2 [25] Define \( f^{(i)}(t), i \in \mathbb{Z}_+ \) is the i-order derivative of \( f(t) \), then \( [V_K f^{(i)}](t) \) can be expanded into:

\[
[V_K f^{(i)}](t) = \sum_{j=0}^{i-1} (-1)^{i-j-1} f^{(j)} K^{(i-j-1)}(t, t) \\
+ \sum_{j=0}^{i-1} (-1)^{i-j} f^{(j)}(0) K^{(i-j-1)}(t, 0) \\
+ (-1)^i [V_K f](t), \tag{6}
\]

where \( K^{(i)}(\cdot, \cdot) \) is the i-th derivative of \( K(\cdot, \cdot) \) with respect to the second argument.
3 Fixed-Time Parameter Estimation

For the convenience of parameter estimation, we define $y = x_1$ as measurable output. Then, the system (2) can be rewritten as the following input-output form:

$$\ddot{y} = \theta_1(y - V_{ref}) + \theta_2\dot{y} + \theta_3\mu. \quad (7)$$

Denote $y_1 = y$, $y_2 = y - V_{ref}$, $y_3 = \mu$. Then, system (7) can be further rewritten as

$$\ddot{y}_1 = \theta_1 y_2 + \theta_2\dot{y}_1 + \theta_3 y_3. \quad (8)$$

In the following, the parameters $\theta_1, \theta_2, \theta_3$ will be estimated from system (8).

Remark 2 Note that for the unknown parameters and current sensorless in Assumption 1, only the variables $y_1, y_2, y_3$ of system (8) are known. The derivatives $\dot{y}_1, \ddot{y}_1$ are unknown.

3.1 Selection of Kernel Function

Note that the selection of kernel function $K(t, \tau)$ affects the implementation of Volterra integral operator $[V_K f](t)$. Therefore, in this subsection, we will first select a suitable kernel function and discuss some of its good properties.

In the paper, the kernel function is selected as follows

$$K(t, \tau) = e^{-(\omega_h(t-\tau)(1-e^{-\omega\tau})^2[1-e^{-\omega(t-\tau)}])}, \quad (9)$$

where $\omega_h, \omega > 0$ are the tuning parameters to be designed. Then, it can be represented as:

$$K(t, \tau) = e^{-\omega_h t}(e^{\omega_h \tau} - 2e^{(\omega_h - \omega)\tau} + e^{(\omega_h - 2\omega)\tau})$$

$$-2e^{-(\omega_h - \omega)t}e^{\omega\tau}(e^{\omega_h \tau} - 2e^{(\omega_h - \omega)\tau} + e^{(\omega_h - 2\omega)\tau})$$

$$+e^{-(\omega_h - 2\omega)t}e^{2\omega\tau}(e^{\omega_h \tau} - 2e^{(\omega_h - \omega)\tau} + e^{(\omega_h - 2\omega)\tau})$$

$$\triangleq \sum_{p=0}^{2} e^{-(\omega_h + \omega p)\tau} f_p(\tau), \quad (10)$$

where $f_p(\tau), p = 0, 1, 2$ satisfies

$$f_p(\tau) = \sum_{q=0}^{2} \tilde{k}_{p,q} e^{(\omega_h - q\omega + p\omega)\tau},$$

$$k_{0,0} = 1, k_{0,1} = -2, k_{0,2} = 1,$$

$$k_{1,p} = -2k_{0,p}, k_{2,p} = k_{0,p}, \forall p = 0, 1, 2. \quad (11)$$

Then, by using (11), it yields that $f_p^{(i)}(\tau), p = 0, 1, 2$ satisfies

$$f_p^{(i)}(\tau) = \sum_{q=0}^{2} \tilde{k}_{p,q} e^{(\omega_h - q\omega + p\omega)\tau},$$

$$\tilde{k}_{p,q} = k_{p,q}(\omega_h - q\omega + p\omega)^i. \quad (12)$$
From (10) and (12), the derivative of function with respect to $\tau$ satisfies:

$$K^{(i)}(t, \tau) \triangleq \sum_{p=0}^{2} F_{i,p}(t, \tau), \quad i = 0, 1, 2,$$

$$F_{i,p}(t, \tau) = e^{-{(\omega_h + p\omega)t}}f_{p}^{(i)}(\tau).$$  \hfill (13)

Substituting (12) into (13), it follows that

$$K^{(i)}(t, t) = 0, \quad K^{(i)}(t, 0) = 0, \forall i = 0, 1.$$  \hfill (14)

### 3.2 Parameter Estimation

Apply the integral operator $V_K$ to both sides of (8), we have

$$[V_K\ddot{y}_1](t) = \theta_1[V_Ky_2](t) + \theta_2[V_K\dot{y}_1](t) + \theta_3[V_Ky_3](t).$$  \hfill (15)

Note that $y_1, y_2, y_3$ are known, but its derivative $\dot{y}_1, \ddot{y}_1$ are unknown. Therefore, $[V_K\dot{y}_1](t), [V_K\ddot{y}_1](t)$ are unknown. Substituting (14) into (6) will produce

$$[V_K\dot{y}_1](t) = -[V_K^{(1)}y_1](t), [V_K\ddot{y}_1](t) = [V_K^{(2)}y_1](t),$$  \hfill (16)

Substituting (16) into (15) will produce

$$[V_K^{(2)}y_1](t) = \theta_1[V_Ky_2](t) - \theta_2[V_K^{(1)}y_1](t) + \theta_3[V_Ky_3](t).$$  \hfill (17)

**Remark 3** Note that in addition to $\theta_1, \theta_2$ and $\theta_3$, other variables in (17) are all known. However, to calculate the Volterra integral operator mapping $[V_K^{(i)}y_j](t)$ in (17) directly according to the definition of Volterra integral operator (4) is very complicated. It is more convenient to use differential equations to generate these variables required in (17).

In the following lemma, we will give the differential equation of $[V_K^{(i)}y_j](t), \forall i \in 0, 1, 2, j \in 1, 2, 3$.

**Lemma 3** For $\forall i \in \{0, 1, 2\}, \forall j \in \{1, 2, 3\}$, the differential equation is constructed

$$\begin{cases} \dot{\xi}_{i,p}^{y_j}(t) = F_{i,p}(t, t)y_j(t) - (\omega_h + p\omega)\xi_{i,p}^{y_j}(t), \\ \xi_{i,p}^{y_j}(0) = 0, p \in \{0, 1, 2\}, \end{cases}$$  \hfill (18)

where

$$\xi_{i,y_j}(t) = \sum_{p=0}^{2} \xi_{i,p}^{y_j}(t), \forall t \in \mathbb{R}_{\geq 0}$$

and $F_{i,p}(t, t) = e^{-{(\omega_h + p\omega)t}}f_{p}^{(i)}(t)$. If the kernel function $K(t, \tau)$ satisfies (9), then

$$\xi_{i,y_j}(t) = [V_K^{(i)}y_j](t), \forall i \in \{0, 1, 2\}, \forall j \in \{1, 2, 3\}.$$
Proof of Lemma 3: According to the definition of Volterra integral operator (4) and (13), we have

\[
\left[ V^{(i)} K \right] y_j (t) = \int_0^t K^{(i)} (t, \tau) y_j (\tau) \, d\tau
\]

\[
= \sum_{p=0}^{2} \int_0^t F_{i,p} (t, \tau) y_j (\tau) \, d\tau
\]

\[
= 2 \sum_{p=0}^{2} \left[ V_{F_{i,p}} y_j \right] (t).
\]

It follows from (5) that \( V_{F_{i,p}} y_j \) can be produced by the following system:

\[
\begin{cases}
\dot{\xi}_{i,y_j} (t) = F_{i,p} (t, t) y_j (t) + \int_0^t \frac{\partial F_{i,p}(t, \tau)}{\partial t} y_j (\tau) \, d\tau, \\
\xi_{i,y_j} (0) = 0,
\end{cases}
\]

\[
\xi_{i,y_j} (t) = \left[ V_{F_{i,p}} y_j \right] (t), \forall t \in \mathbb{R}_{\geq 0}.
\]

It can be calculated from (13) that

\[
\int_0^t \frac{\partial F_{i,p}(t, \tau)}{\partial t} y_j (\tau) \, d\tau = - (\omega_h + \omega_p) \int_0^t F_{i,p}(t, \tau) y_j (\tau) \, d\tau
\]

\[
= - (\omega_h + \omega_p) \left[ V_{F_{i,p}} y_j \right] (t)
\]

\[
= - (\omega_h + \omega_p) \xi_{i,p} (t).
\]

Substituting \( \xi_{i,y_j} (t) = \left[ V_{F_{i,p}} y_j \right] (t) \) into (19), and substituting (21) into (20), it follows that

\[
\left\{ \begin{array}{l}
\dot{\xi}_{i,y_j} (t) = F_{i,p} (t, t) y_j (t) - (\omega_h + \omega_p) \xi_{i,p} (t), \\
\xi_{i,y_j} (0) = 0,
\end{array} \right.
\]

\[
\left[ V^{(i)} K \right] y_j (t) = 2 \sum_{p=0}^{2} \xi_{i,p} (t), \forall t \in \mathbb{R}_{\geq 0}.
\]

Comparing (18) and (22), we can prove that \( \xi_{i,y_j} (t) = \left[ V_{K^{(i)}} y_j \right] (t), \forall i \in \{0, 1, 2\}, \forall j \in \{1, 2, 3\} \) holds, which completes the proof of Lemma 3. ■

From lemma 3, \( \xi_{i,y_j} (l) = \left[ V_{K^{(i)}} y_j \right] (l) \) can be obtained, so (17) can be rewritten as :

\[
\xi_{2,y_1} (t) = \theta_1 \xi_{0,y_2} (t) - \theta_2 \xi_{1,y_1} (t) + \theta_3 \xi_{0,y_3} (t).
\]

To facilitate the estimator design, (23) can be rewritten as the following compact form:

\[
s(t) = v(t) \theta,
\]

\[
(24)
\]
where
\[ s(t) = \xi_2 y_1(t), \theta = [\theta_1, \theta_2, \theta_3]^T, \]
\[ v(t) = [\xi_0 y_2(t), \xi_1 y_1(t), \xi_0 y_1(t)]. \]

Applying the integral operator following persistency of excitation assumption:

**Assumption 2** There exists constants \( r > 0, T_0 > 0 \) such that:
\[ \int_{t-T_0}^{t} v^T(\tau)v(\tau)d\tau \geq r I_{3 \times 3}. \]

Multiplying both sides of (24) by \( v^T(t) \), one has:
\[ s(t) = \bar{v}(t)\dot{\theta}, \]
where
\[ s(t) = v^T(t)s(t), \bar{v}(t) = v^T(t)v(t). \]

Applying the integral operator \( V_{K_y} \) with \( K_y = e^{-g(t-\tau)}, g > 0 \) on both sides of (26), we have
\[ s_{1,f}(t) = v_{1,f}(t)\dot{\theta}, \]
where \( s_{1,f}(t) = [V_{K_y}s](t), v_{1,f}(t) = [V_{K_y}\bar{v}](t) \). By applying Leibnitz rule in deriving the integral, \( s_{1,f}(t), v_{1,f}(t) \) can be obtained as the output of following system:
\[ \dot{s}_{1,f}(t) = -gs_{1,f}(t) + \bar{s}(t), s_{1,f}(0) = 0 \in \mathbb{R}^3, \]
\[ \dot{v}_{1,f}(t) = -gv_{1,f}(t) + \bar{v}(t), v_{1,f}(0) = 0 \in \mathbb{R}^{3 \times 3}. \]

Define the following auxiliary variables
\[ R_{1,f}(t) = v_{1,f}(t)\hat{\theta}(t) - s_{1,f}(t), \]
where \( \hat{\theta}(t) \) is an adaptive parameter estimation whose update law is given by the following theorem.

**Theorem 1** The adaptive parameter estimator is designed as follows:
\[ \hat{\theta}(t) = \begin{cases} v_{1,f}^{-1}(t)[-\alpha_1 |R_{1,f}(t)|^{\kappa_1} - \alpha_2 |R_{1,f}(t)|^{\kappa_2}] \\
+ (gv_{1,f}(t) - \bar{v}(t))\bar{\theta}(t) + \dot{s}_{1,f}(t), \\
\text{if } \min\{eig(v_{1,f}(t))\} \geq \varepsilon \\
0, \text{ if } \min\{eig(v_{1,f}(t))\} < \varepsilon \end{cases} \]

where \( \bar{v}(t) \) is defined in (27), and \( v_{1,f}, s_{1,f} \) are defined in (29), and \( R_{1,f}(t) \) is defined in (30). \( \min\{eig(v_{1,f}(t))\} \) represents the minimum eigenvalue of \( v_{1,f}(t) \). If the parameters in adaptive law satisfy:
\[ 0 < \kappa_1 < 1, \kappa_2 > 1, 0 < \varepsilon < e^{-gT_0 r}, \]
\[ g > 0, \alpha_1 > 0, \alpha_2 > 0, \]
\[ 0 < \kappa_1 < 1, \kappa_2 > 1, 0 < \varepsilon < e^{-gT_0 r}, \]
\[ g > 0, \alpha_1 > 0, \alpha_2 > 0, \]
\[ 0 < \kappa_1 < 1, \kappa_2 > 1, 0 < \varepsilon < e^{-gT_0 r}, \]
\[ g > 0, \alpha_1 > 0, \alpha_2 > 0, \]
then there is a fixed time $T_{\text{max}}$ that does not depend on the initial estimation error such that $\hat{\theta} = \theta, \forall t \geq T_{\text{max}}$ holds, where $T_{\text{max}}$ satisfies:

$$T_{\text{max}} = \frac{2^{1-\kappa_1}}{\alpha_1(1 - \kappa_1)} + \frac{2^{1-\kappa_2}}{\alpha_2(\kappa_2 - 1)} + T_0. \quad (33)$$

**Proof:** The proof is divided into two steps.

**Step 1:** In the first step, we will prove that the auxiliary variable $R_{1,f}(t)$ can converge to zero within a fixed time.

The derivative of $R_{1,f}(t)$ can be calculated as:

$$\dot{R}_{1,f}(t) = \dot{v}_{1,f}(t)\hat{\theta}(t) + v_{1,f}(t)\dot{\hat{\theta}}(t) - \dot{s}_{1,f}(t). \quad (34)$$

According to the definition of $v_{1,f}(t)$ and Assumption 1, we can calculate that for $t \geq T_0$:

$$v_{1,f}(t) = [V_{K_c} \bar{v}](t) = \int_0^t e^{-g(t-\tau)}v^T(\tau)v(\tau)d\tau \geq e^{-gT_0} \int_{t-T_0}^t v^T(\tau)v(\tau)d\tau \geq e^{-gT_0}r_{I_3}^{3 	imes 3}. \quad (35)$$

which implies that

$$\min\{\text{eig}(v_{1,f}(t))\} \geq \varepsilon, \quad (36)$$

holds for $\forall t \geq T_0$. Therefore, when $t \geq T_0$, the adaptive law takes the form:

$$\dot{\hat{\theta}}(t) = v_{1,f}^{-1}(t)[-\alpha_1[R_{1,f}(t)]^{\kappa_1} - \alpha_2[R_{1,f}(t)]^{\kappa_2} + (gv_{1,f}(t) - \bar{v}_1(t))\dot{\hat{\theta}}(t) + \dot{s}_{1,f}(t)]. \quad (37)$$

Substituting (29) and (37) into (34), we have

$$\dot{R}_{1,f}(t) = -\alpha_1[R_{1,f}(t)]^{\kappa_1} - \alpha_2[R_{1,f}(t)]^{\kappa_2}, t \geq T_0, \quad (38)$$

where $R_{1,f}(t) \in \mathbb{R}^3$. Define

$$R_{1,f}(t) = [r_1(t), r_2(t), r_3(t)]^T. \quad (39)$$

Then, (38) can be written as

$$\dot{r}_i(t) = -\alpha_1 r_i^{\kappa_1}(t) - \alpha_2 r_i^{\kappa_2}(t), \forall t \geq T_0, i = 1, 2, 3. \quad (40)$$

Define the following Lyapunov function

$$V_i = \frac{1}{2} r_i^2(t), i = 1, 2, 3. \quad (41)$$
Then, the derivative of $V_i$ satisfies

$$
\dot{V}_i = -\alpha_1 |r_i(t)|^{1+\kappa_1} - \alpha_2 |r_i(t)|^{1+\kappa_2}
= -2^{1+\kappa_1} \alpha_1 V_i^{1+\kappa_1} - 2^{1+\kappa_2} \alpha_2 V_i^{1+\kappa_2}
$$

(42)

for $\forall t \geq T_0, i = 1, 2, 3$. It can be concluded from Lemma 1 that $r_i(t) = 0$ holds for $\forall t \geq T_{max}, i = 1, 2, 3$ and thus

$$
R_{1,f}(t) = 0 \in \mathbb{R}^3, \forall t \geq T_{max},
$$

(43)

where $T_{max}$ satisfies (33).

**Step 2:** In the second step, we will prove that once the auxiliary variable $R_{1,f}(t) = 0$ is established, one has $\hat{\theta}(t) = \theta$.

From (30), it can be known that when $R_{1,f} = 0$, it holds that

$$
s_{1,f}(t) = v_{1,f}(t)\hat{\theta}(t).
$$

(44)

Subtracting (28) from (44), one has

$$
v_{1,f}(t)(\theta - \hat{\theta}(t)) = 0, t \geq T_{max},
$$

(45)

where $v_{1,f}(t)$ is invertible. So it can be known that

$$
\theta = \hat{\theta}(t), \forall t \geq T_{max}.
$$

(46)

Therefore, the proof is completed.

\[\blacksquare\]

4 Simulation Results

<table>
<thead>
<tr>
<th>Table 1. SPECIFICATIONS OF THE BUCK CONVERTER.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Input voltage</strong> ($V_{in}$)</td>
</tr>
<tr>
<td><strong>Reference voltage</strong> ($V_{ref}$)</td>
</tr>
<tr>
<td><strong>Inductance</strong> ($L$)</td>
</tr>
<tr>
<td><strong>Capacitance</strong> ($C$)</td>
</tr>
<tr>
<td><strong>Resistance</strong> ($R$)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2. SPECIFICATIONS OF THE CONVERTER.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Adaptive parameter</strong></td>
</tr>
<tr>
<td>$\alpha_1, \alpha_2$</td>
</tr>
<tr>
<td>$\kappa_1$</td>
</tr>
<tr>
<td>$\kappa_2$</td>
</tr>
<tr>
<td>$\epsilon$</td>
</tr>
</tbody>
</table>

In order to verify the effectiveness of the algorithm, this section uses the estimation algorithm proposed in Theorem 1 for simulation verification. The parameters of DC-DC buck circuit system are selected as Table 1. The fixed time adaptive parameter estimation algorithm is designed according to Theorem 1, and its parameter selection is shown in Table 2. To verify the fixed time convergence of the proposed algorithm, two different adaptive initial values are
selected respectively:

\[ \hat{\theta}(0) = [0, 0, 0]^T, \]
\[ \hat{\theta}(0) = [10^3, 10^3, -10^3]^T. \]

The simulation verification of the algorithm will be carried out for the above two different adaptive initial values.

The simulation results are given in Figs. 2-4. The estimation of the convergence time can be calculated by Theorem 1 as \( T_{max} \approx 1.789s \). The time history of estimation errors under two different initial values are presented in Figs. 2 and 3, respectively. It can be clearly concluded from Figs. 2 and 3 that the proposed algorithm can achieve accurate estimation of the system parameters within a fixed-time less than \( T_{max} \approx 1.789s \) even under different initial values. This nature can also be seen from Fig. 4 where

\[ \| \theta - \hat{\theta}(0) \|_2 := \sqrt{(\theta_1 - \hat{\theta}_1(0))^2 + (\theta_2 - \hat{\theta}_2(0))^2 + (\theta_3 - \hat{\theta}_3(0))^2}. \]

It can be clearly seen from Fig. 4 that the convergence time of the proposed estimator is free of the initial estimation errors.

5 Conclusion

In this paper, a fixed-time adaptive parameter estimation algorithm has been proposed for DC-DC buck converter. Compared with the traditional asymptotic estimation and finite-time estimation algorithms, the proposed method can achieve accurate estimation of parameters in a fixed time independent of initial estimation error. Our future work will focus on design if parameter estimation based controller for DC-DC converter.
Acknowledgements

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Data Availability Statement

The data that support the findings of this study are not openly available since it also forms part of an ongoing study and are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest to report.

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