Supplementary Information for Near-unity fidelity four-dimensional speckle imaging through scattering media

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Supplementary Section 1. Image reconstruction evaluating indicators

The Pearson correlation coefficient (PCC) is defined as:

$$PCC(G,R) = \frac{\sum_i (g_i - \overline{G})(r_i - \overline{R})}{\sqrt{\sum_i (g_i - \overline{G})^2 \sum_i (r_i - \overline{R})^2}} \tag{1}$$

where g_i and r_i represent the pixelate value with index i of the encoded and retrieved images, respectively. $\overline{G}$ and $\overline{R}$ are the mean values of g and r, respectively.

While the structural similarity index (SSIM) that is more in line with human perception is defined as follows:

$$SSIM(G,R) = \frac{(2\mu_g\mu_r + C_1)(2w_{gr} + C_2)}{(\mu_g^2 + \mu_r^2 + C_1)(w_g^2 + w_r^2 + C_2)} \tag{2}$$

where μ_g and μ_r are the average of the encoded and retrieved images, respectively. w_g and w_r are the standard deviations of the encoded and retrieved images, respectively. w_{gr} is the cross-covariance between the encoded and retrieved images. C_1 and C_2 are regularization parameters.

Supplementary Section 2. Normalized singular values

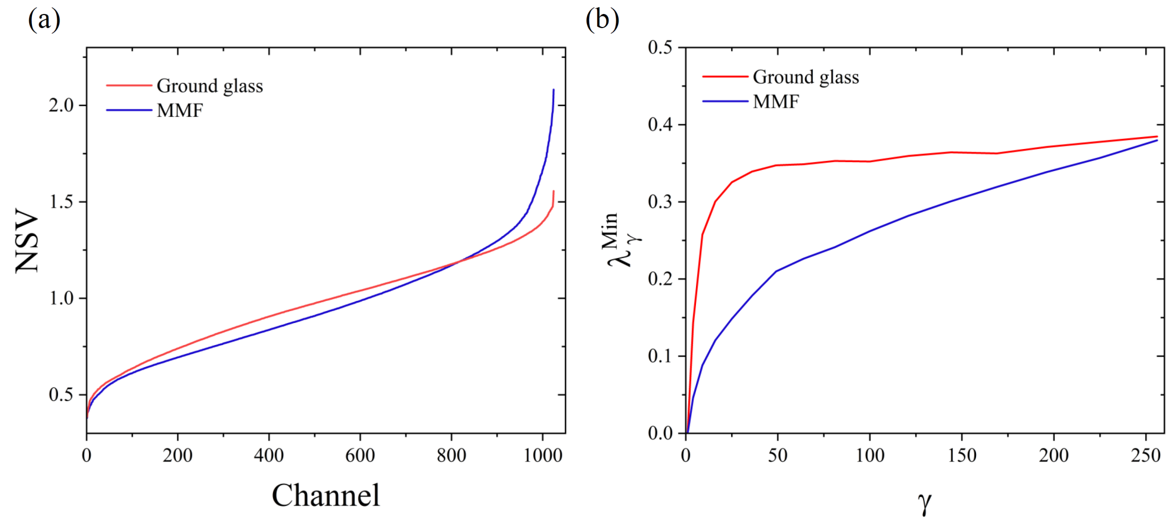
According to the random matrix theory[1], the singular value of the measured TM is proportional to the energy of the scattered light transmitted in the corresponding channel. Furthermore, the relationship between the normalized minimum singular value and the experimental noise deterministically affects the quality of speckle imaging[2].

In order to quantify the energy distribution of each channel in the multiple scattering system, we perform singular value decomposition (SVD) of the TM. The SVD of the calibrated TM can be defined as

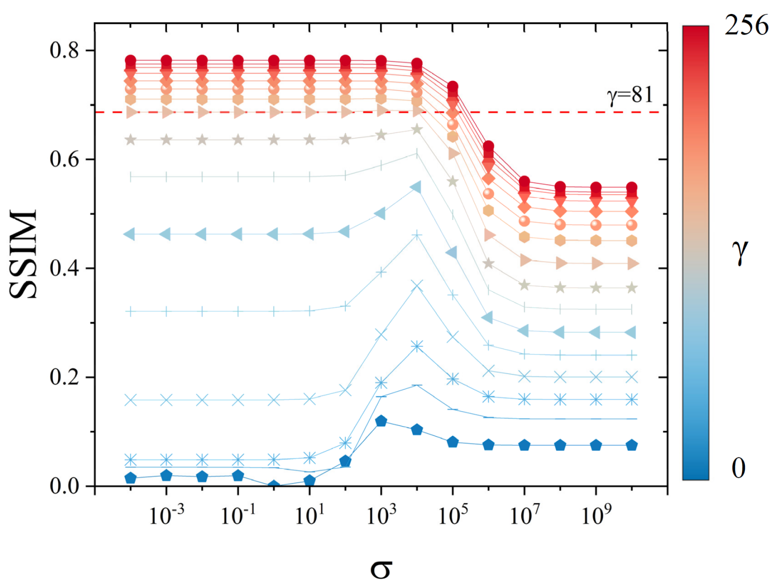
$$T = USV^* \tag{3}$$

Where U and V are unitary matrices. V^* is the transpose conjugate matrix of V. S is a diagonal matrix containing positive real values called singular values of T and denoted by λ_i. The normalized singular value (NSV)[3] can be expressed as

$$\lambda_i^n = \frac{\lambda_i}{\sqrt{\sum \lambda_i^2 / N}} \tag{4}$$



Supplementary Fig. 1. (a) The corresponding NSV for the energy transfer channels of the TM(γ=256). (b)The corresponding λ_γ^{Min} as a function of output-input ratio γ.

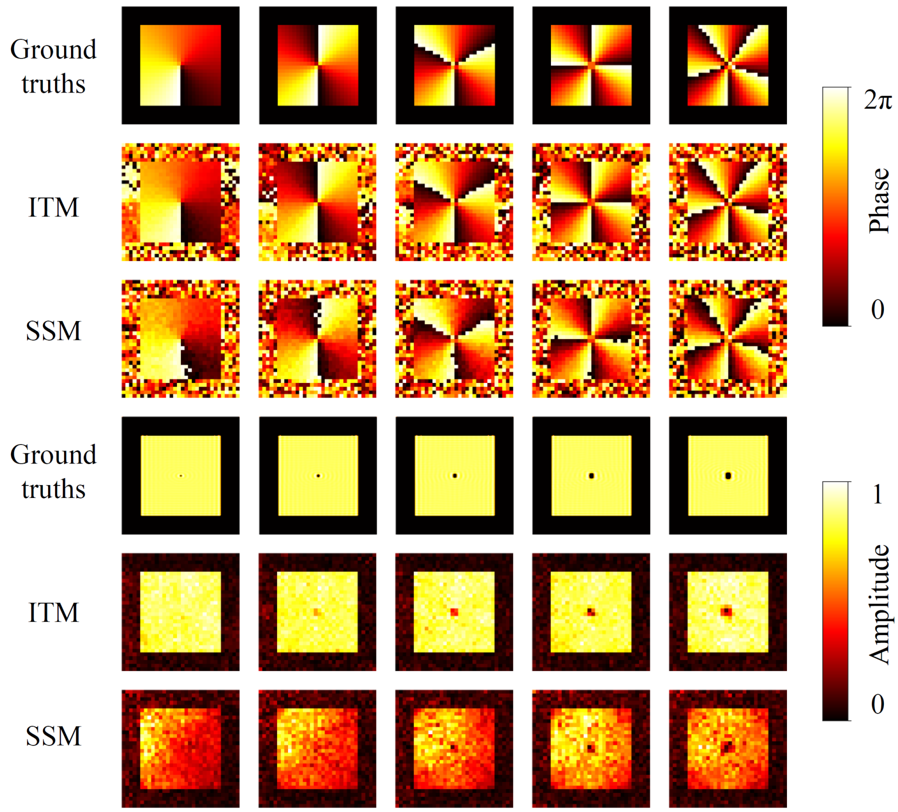


Supplementary Fig. 2. The SSIM of the retrieved OAM hologram (l = 4) utilizing the MSO based method (50 averages) as a function of experimental noise factor σ.

The distribution of NSVs indicates the transmission characteristics of the scattering medium by counting the energy distribution of each independent transmission channel, as shown in Supplementary Fig .1 (a). Furthermore, we calculate the minimum NSV (λ_γ^{Min}) of the TM as a function of γ, as shown in Supplementary Fig .1 (b), where λ_γ^{Min} increases together with the increasing of the output-input ratio γ.

In the MSO based method, the experimental noise factor σ is a very important parameter. It can be seen from Supplementary Fig .2 that the experimental noise factor σ can be used to optimize the fidelity of speckle imaging, which is corresponding to a regularization process[4,5]. The singular value distribution of the reconstruction operator is readjusted by the experimental noise factor to reduce the influence of experimental noise on image reconstruction. However, when the output-input ratio $\gamma(\gamma > 81)$ reaches a certain threshold value, optimizing the experimental noise factor σ does not yield a better result compared with the case of $\sigma = 0$ (i.e. inversion or pseudo-inversion of TM), because the minimum normalized singular value associated with the most inefficient energy transport channel is larger than the experimental noise level.

Supplementary Section 3. Light field imaging



Supplementary Fig. 3. Comparisons of the complex light field imaging between the ITM and state-of-the-art SSM methods through a ground glass. The first and fourth rows show the ground truths of input phase and amplitude information, respectively. The second and fifth rows provide the complex light field reconstruction results based on the ITM method. The third and sixth rows show the retrieved complex light field based on the SSM method. The averaged PCC/SSIM of the retrieval phase (amplitude) based on ITM method is 0.99/0.98 (0.99/0.42), while the averaged PCC/SSIM of the retrieval phase (amplitude) based on the SSM method is 0.96/0.91 (0.89/0.25). The fidelity of phase retrieval results are evaluated at the positions where its amplitude is not equal to 0.

Supplementary Fig .3 show the high-fidelity light field imaging results through a ground glass based on the inverse transmission matrix (ITM) and speckle correlation scattering matrix (SSM) methods, respectively. It can be seen from these results that the demonstrated ITM method is substantially better than the SSM method that represents the state-of-the-art of light field speckle imaging.

References

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