

Metrics for Object and Pixel Level Comparison

$$IoU = \frac{TP}{TP + FP + FN}$$

$$Sensitivity \text{ or True Positive Rate (TPR)} = \frac{TP}{TP + FN}$$

$$Precision = \frac{TP}{TP + FP}$$

$$Specificity \text{ or True Negative Rate (TNR)} = \frac{TN}{TN + FP}$$

$$False Negative Rate = \frac{FN}{TP + FN}$$

$$False Postive Rate = \frac{FP}{TN + FP}$$

$$False Discovery Rate = \frac{FP}{TP + FP}$$

$$False Omission Rate = \frac{FN}{TN + FN}$$

$$F0.5 \text{ Score} = \frac{(1 + 0.5^2) * TP}{(1 + 0.5^2) * TP + (0.5^2) * FN + FP}$$

$$F1 \text{ Score or Dice Index} = \frac{(2 * TP)}{2 * TP + FN + FP}$$

$$Fowlkes - Mallows Index = \frac{TP}{\sqrt{(TP + FP) * (TP + FN) * (TN + FN)}}$$

$$Negative Predictive Value = \frac{TN}{TN + FN}$$

$$Prevalence = \frac{TP + FN}{TOTAL}$$

$$Accuracy \text{ or Rand Index} = \frac{TP + TN}{TOTAL}$$

$$Balanced Accuracy = \frac{0.5 * TP}{TP + FN} + \frac{0.5 * TN}{TN + FP}$$

$$Prevalence Threshold = \frac{TNR - 1 + \sqrt{TPR * (1 - TNR)}}{TPR + TNR - 1}$$

$$Matthews Correlation Coefficient = \frac{(TP * TN) - (FP * FN)}{\sqrt{(TP + FP) * (TP + FN) * (TN + FN) * (TN + FP)}}$$

$$Bookmaker Informedness = \frac{TP}{TP + FN} + \frac{TN}{TN + FP} - 1$$

$$Markedness = \frac{TP}{TP + FP} + \frac{TN}{TN + FN} - 1$$

$$Expected Accuracy = \frac{(TP + FP) * (TP + FN) + (TN + FN) * (FP + TN)}{(TOTAL) * (TOTAL)}$$

$$Cohen's Kappa Index = \frac{Accuracy - Expected Accuracy}{1 - Expected Accuracy}$$

$$Mirkin Metric = (TOTAL) * (TOTAL) * (1 - Accuracy)$$

$$Adjusted Mirkin Metric = \frac{Mirkin Metric}{(TOTAL) * (TOTAL)}$$

$$Adjusted Rand Index = \frac{0.5 * \left(\binom{TP+FP}{2} + \binom{FN+TN}{2} + \binom{TP+FN}{2} + \binom{FP+TN}{2} \right) - \frac{\left(\binom{TP+FP}{2} + \binom{FN+TN}{2} \right) * \left(\binom{TP+FN}{2} + \binom{FP+TN}{2} \right)}{\binom{TOTAL}{2}}}{\left(\binom{TP+FP}{2} + \binom{FN+TN}{2} + \binom{TP+FN}{2} + \binom{FP+TN}{2} \right) - \frac{\left(\binom{TP+FP}{2} + \binom{FN+TN}{2} \right) * \left(\binom{TP+FN}{2} + \binom{FP+TN}{2} \right)}{\binom{TOTAL}{2}}}$$

Where,

TP = True Positive, TN = True Negative, FP = False Positive, FN = False Negative and TOTAL = TP + TN + FP + FN

Distribution Metrics for feature comparison

$$L1 \text{ or Manhattan Distance} = \sum_I |H_1(I) - H_2(I)|$$

$$L2 \text{ or Euclidean Distance} = \sum_I (H_1(I) - H_2(I))^2$$

$$L\infty \text{ or Chybyshchev Distance} = \max_I (|H_1(I) - H_2(I)|)$$

$$\text{Kolmogorov - Smirnov Divergence} = \max_I (|C_1(I) - C_2(I)|)$$

$$\text{Match Distance} = \sum_I |C_1(I) - C_2(I)|$$

$$\text{Cramer - von Mises Distance} = \sum_I (C_1(I) - C_2(I))^2$$

$$\text{PSI Value} = \sum_I \left((H_1(I) - H_2(I)) * \log \frac{H_1(I)}{H_2(I)} \right)$$

$$\text{Kullback-Leibler Divergence (KLD)} = \sum_I \left((H_1(I)) * \log \frac{H_1(I)}{H_2(I)} \right)$$

$$\text{Jensen Shannon Distance} = 0.5 * \text{KLD}(H_1, (H_1 + H_2)/2) + 0.5 * \text{KLD}(H_2, (H_1 + H_2)/2)$$

Implemented using OpenCV functions

$$\text{Histogram Intersection} = \sum_I \min(H_1(I), H_2(I))$$

$$\text{Correlation} = \frac{\sum_I (H_1(I) - \bar{H}_1)(H_2(I) - \bar{H}_2)}{\sqrt{\sum_I (H_1(I) - \bar{H}_1)^2 \sum_I (H_2(I) - \bar{H}_2)^2}}$$

$$\text{Chi Square} = \sum_I \frac{(H_1(I) - H_2(I))^2}{H_1(I)}$$

$$\text{Bhattacharya Distance} = \sqrt{1 - \frac{\sum_I (H_1(I) \cdot H_2(I))}{\sqrt{\bar{H}_1 \cdot \bar{H}_2 \cdot N^2}}}$$

Implemented using SciPy

$$\text{Cosine Distance} = 1 - \frac{H_1 \cdot H_2}{||H_1||_2 ||H_2||_2}$$

$$\text{Canberra Distance} = \sum_I \frac{|H_1(I) - H_2(I)|}{|H_1(I)| + |H_2(I)|}$$

$$\text{Wasserstein Distance} = \int_{-\infty}^{\infty} |C_1 - C_2|$$

Where,

H_1 and H_2 are probability distribution functions for ground truth and predicted features,
 C_1 and C_2 are cumulative distribution functions for ground truth and predicted features, $\bar{H}_k = \frac{1}{N} \sum_J H_k(J)$ and $N = \text{Total Number of Histogram Bins}$

Error Metrics

$$\text{Mean Squared Error} = \frac{\sum_l (H_1(I) - H_2(I))^2}{TOTAL}$$

$$\text{Root Mean Squared Error} = \sqrt{\frac{\sum_l (H_1(I) - H_2(I))^2}{TOTAL}}$$

$$\text{Normalized Root Mean Squared Error} = \frac{\sqrt{\frac{\sum_l (H_1(I) - H_2(I))^2}{TOTAL}}}{\max(H_1(I)) - \min(H_1(I))}$$

$$\text{Mean Error} = \frac{\sum_l H_1(I) - H_2(I)}{TOTAL}$$

$$\text{Mean Absolute Error (MAE) or Mean Absolute Deviation} = \frac{\sum_l |H_1(I) - H_2(I)|}{TOTAL}$$

$$\text{Geometric Mean Absolute Error} = e^{\frac{\sum_l \log(|H_1(I) - H_2(I)|)}{TOTAL}}$$

$$\text{Median Absolute Error} = \text{median}(H_1(I) - H_2(I))$$

$$\text{Percentage Error} = \frac{H_1(I) - H_2(I)}{H_1(I)}$$

$$\text{Mean Percentage Error} = \frac{\sum_l \text{Percentage Error}}{TOTAL}$$

$$\text{Mean Absolute Percentage Error} = \frac{\sum_l |\text{Percentage Error}|}{TOTAL}$$

$$\text{Median Absolute Percentage Error} = \text{median}(|\text{Percentage Error}|)$$

$$\text{Symmetric Mean Absolute Percentage Error} = \frac{\sum_l 2.0 * \frac{|H_1(I) - H_2(I)|}{|H_1(I)| + |H_2(I)|}}{TOTAL}$$

$$\text{Symmetric Median Absolute Percentage Error} = \text{median}(2.0 * \frac{|H_1(I) - H_2(I)|}{|H_1(I)| + |H_2(I)|})$$

$$\text{Mean Arctangent Absolute Percentage Error} = \frac{\sum_l \arctan(|\text{Percentage Error}|)}{TOTAL}$$

$$\text{Normalized Absolute Error} = \sqrt{\frac{\sum_l (H_1(I) - H_2(I) - MAE)^2}{TOTAL_{H_1(I)} - 1}}$$

$$\text{Normalized Absolute Percentage Error} = \sqrt{\frac{\sum_l (\text{Percentage Error} - MAE)^2}{TOTAL_{H_1(I)} - 1}}$$

$$\text{Root Mean Squared Percentage Error} = \sqrt{\frac{\sum_l (\text{Percentage Error})^2}{TOTAL}}$$

$$\text{Root Median Squared Percentage Error} = \sqrt{\text{median}((\text{Percentage Error})^2)}$$

$$\text{Integral Normalized Root Squared Error} = \sqrt{\frac{\sum_l (H_1(I) - H_2(I))^2}{\sum_l (H_1(I) - \text{mean}(H_1(I)))^2}}$$

$$\text{Root Relative Squared Error} = \sqrt{\frac{\sum_l (H_1(I) - H_2(I))^2}{\sum_l (H_1(I) - \text{mean}(H_1(I)))^2}}$$

$$\text{Relative Absolute Error} = \frac{\sum_l |H_1(I) - H_2(I)|}{\sum_l |H_1(I) - \text{mean}(H_1(I))|}$$

$$\text{Mean Directional Accuracy} = \frac{\sum_l \sin(H_1(I)[1:] - H_1(I)[: -1])}{TOTAL} = \frac{\sum_l \sin(H_2(I)[1:] - H_2(I)[: -1])}{TOTAL}$$

Where,

H_1 and H_2 are probability distribution functions for ground truth and predicted feature, $TOTAL$ is the number of samples and $TOTAL_{H_1(I)}$ is the length of distribution H_1 .