Supplementary information for Dynamics of multipartite quantum steering for different types of decoherence channels

Wei-Chen Li[1], *Ya Xiao*[1], *Xing-Hong Han*[1], *Xuan Fan*[1], *Xiao-Bing Hei*[1], *and Yong-Jian Gu*[1]

[1]*College of Physics and Optoelectronic Engineering, Ocean University of China, Qingdao* 266100, *People's Republic of China*

◇◇◇

I. DENSITY MATRIX ELEMENTS OF DIFFERENT DECOHERENCE CHANNELS

Decoherence caused by an unavoidable interaction between a system and environment can be expressed by the corresponding Kraus operations acting on the system state. As mentioned in the main text, given a generalized three-qubit W state ρ_{gW}, when only Charlie's qubit under a quantum channel with the decoherence strength d, the state ρ_{gW} evolves to $\varepsilon(\rho_{ABC}) = \sum_{m=1}^{n}(I^A \otimes I^B \otimes K_m^C)\rho_{gW}(I^A \otimes I^B \otimes K_m^C)^\dagger$. With the corresponding Kraus operators shown in Table I in the main text, the nonzero matrix elements ρ_{ij} of the evolved state in the ADC becomes

$$\begin{aligned}
&\rho_{11} = \alpha^2 d, \quad \rho_{22} = \alpha^2(1-d), \quad \rho_{23} = \rho_{32} = \alpha\beta\sqrt{1-d}, \\
&\rho_{33} = \beta^2, \quad \rho_{25} = \rho_{52} = \alpha\sqrt{1-d}\sqrt{1-\alpha^2-\beta^2}, \\
&\rho_{35} = \rho_{53} = \beta\sqrt{1-\alpha^2-\beta^2}, \quad \rho_{55} = 1-\alpha^2-\beta^2,
\end{aligned} \tag{1}$$

the nonzero matrix elements ρ_{ij} of the evolved state in the PDC becomes

$$\begin{aligned}
&\rho_{22} = \alpha^2, \quad \rho_{23} = \rho_{32} = \alpha\beta\sqrt{1-d}, \\
&\rho_{33} = \beta^2, \quad \rho_{25} = \rho_{52} = \alpha\sqrt{1-d}\sqrt{1-\alpha^2-\beta^2}, \\
&\rho_{35} = \rho_{53} = \beta\sqrt{1-\alpha^2-\beta^2}, \quad \rho_{55} = 1-\alpha^2-\beta^2,
\end{aligned} \tag{2}$$

and the nonzero matrix elements ρ_{ij} of the evolved state in the DC becomes

$$\begin{aligned}
&\rho_{11} = \alpha^2 d/2, \quad \rho_{22} = \alpha^2(2-d)/2, \quad \rho_{23} = \rho_{32} = \alpha\beta(1-d), \\
&\rho_{33} = \beta^2(2-d)/2, \quad \rho_{25} = \rho_{52} = \alpha(1-d)\sqrt{1-\alpha^2-\beta^2}, \\
&\rho_{35} = \rho_{53} = \beta(2-d)\sqrt{1-\alpha^2-\beta^2}/2, \quad \rho_{44} = \beta^2 d/2, \\
&\rho_{46} = \rho_{64} = \beta d\sqrt{1-\alpha^2-\beta^2}/2, \\
&\rho_{55} = (2-d)\left(1-\alpha^2-\beta^2\right)/2, \quad \rho_{66} = d\left(1-\alpha^2-\beta^2\right)/2.
\end{aligned} \tag{3}$$

In addition, the reduced state between Alice and Bob, Alice and Charlie, Bob and Charlie can be obtained by taking the partial trace of the density matrix $\varepsilon(\rho_{ABC})$, i.e., $\varepsilon(\rho_{AB}) = \mathrm{Tr}_C[\varepsilon(\rho_{ABC})]$, $\varepsilon(\rho_{AC}) = \mathrm{Tr}_B[\varepsilon(\rho_{ABC})]$, and $\varepsilon(\rho_{BC}) = \mathrm{Tr}_A[\varepsilon(\rho_{ABC})]$. The nonzero matrix elements ρ_{ij} of the state $\varepsilon(\rho_{AB})$ in the PDC becomes

$$\rho_{11} = \alpha^2, \quad \rho_{22} = \beta^2, \quad \rho_{33} = 1-\alpha^2-\beta^2, \quad \rho_{23} = \rho_{32} = \beta\sqrt{1-\alpha^2-\beta^2}. \tag{4}$$

The nonzero matrix elements ρ_{ij} of the state $\varepsilon(\rho_{AC})$ in the PDC becomes

$$\rho_{11} = \beta^2, \quad \rho_{22} = \alpha^2, \quad \rho_{33} = 1 - \alpha^2 - \beta^2, \quad \rho_{23} = \rho_{32} = \alpha\sqrt{1-d}\sqrt{1-\alpha^2-\beta^2}. \tag{5}$$

The nonzero matrix elements ρ_{ij} of the state $\varepsilon(\rho_{BC})$ in the PDC becomes

$$\rho_{11} = 1 - \alpha^2 - \beta^2, \quad \rho_{22} = \alpha^2, \quad \rho_{33} = \beta^2, \quad \rho_{23} = \rho_{32} = \alpha\beta\sqrt{1-d}. \tag{6}$$