Supplementary Information: Geometrical control of topological charge transfer in Shakti-Cairo colloidal ice

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SUPPORTING VIDEOS

With the article are 3 videoclips as supplements of the Figures and Main text.

- **MovieS1(.mp4)**: Simulation of the Shakti lattice (bond angle $\theta = 0^\circ$) when increasing the magnetic field strength from 0 mT to 25 mT. The first frames illustrate a schematic of the different colloidal configurations for the $z = 4$ (left) and $z = 3$ (right) vertices with the corresponding topological charges q. The ice rule vertices are highlighted in the green box. In the video, the colloidal particles are shown as open disks, while topological charges are indicated by filled disks with blue ($q < 0$) and red ($q > 0$) colours.

- **MovieS2(.mp4)**: Numerical simulation of the Cairo lattice (bond angle $\theta = 30^\circ$) when increasing the magnetic field from 0 mT to 25 mT. Also in this video the first frames illustrate the colloidal configurations for the $z = 4$ (left) and $z = 3$ (right) vertices and the values of the topological charges q. The ice rule vertices are highlighted in the green box. In the video, particles are shown as open disks, while topological charges are indicated by filled disks with blue ($q < 0$) and red ($q > 0$) colours.

- **MovieS3(.mp4)** Experimental realization of the Cairo lattice. The first frames of the video indicate the colloidal configurations in the $z = 4$ (left) and $z = 3$ (right) vertices and the values of the topological charges q. The ice rule vertices are highlighted in the green box. In the video, the pseudo-spins associated to each double well trap is superimposed as a green arrow, with the corresponding topological charge on each vertex. Also here blue (red) disks corresponds to negative (positive) monopoles. The external field is ramped up to a maximum value of 11.5mT, beyond which the paramagnetic colloids start jumping out of the topographic double wells. This video corresponds to Figure 3(c) of the main text.