

1 Supplemental Information for *Anisotropic resistance with a 90°*
2 *twist in a ferromagnetic Weyl semimetal, Co₂MnGa*

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A. Methods

1. Crystal growth

Single crystals of Co_2MnGa were grown using the Bridgman-Stockbarger crystal growth technique. First, we prepared a polycrystalline ingot using the induction melt technique with the stoichiometric mixture of Co, Mn and Ga metal pieces of 99.99% purity. Then, we poured the powdered material into an alumina crucible and sealed it in a tantalum tube. The growth temperature was controlled with a thermocouple attached to the bottom of the crucible. For the heating cycle, the entire material was melted above 1200 °C and then slowly cooled below 900 °C. The L2_1 crystal structure was confirmed by X-ray diffraction and transmission electron microscopy.

2. Thin lamella fabrication

Thin Co_2MnGa lamellae were prepared by focus ion beam milling using a ThermoFisher Scientific Helios NanoLabTM 600 dual-beam system (FIB/SEM) and standard TEM sample preparation techniques. The lamellae were gently polished at a glancing angle using a 2 kV Ga⁺ ion beam in order to minimize surface damage. The very last polishing steps used beam currents of < 100 pA. In order to make electrical contacts to the corners, we first apply a thick layer of amorphous (insulating) carbon to the side faces using the FIB. Then we use the FIB to deposit a thick platinum “nib” at the corners on the backside of the sample. We then flip the lamella over (mounted on a lift-out grid) and use an *in situ* micro-manipulator needle to maneuver it into place on the substrate. We fix it to the substrate using Pt and cut it away from the needle with the ion beam. We then use thick Pt strips to connect the nibs to nearby gold electrodes pre-patterned by photolithography. To make the top contacts, we first build up a stack of Pt lines in a staircase geometry and then connect it across to the corners of the upper face. In order to minimize the resistances of the Pt connections, we build thick ($\sim 1\text{ }\mu\text{m}$) lines with the scanning direction parallel to the long axis and the shortest lengths possible.

3. *Magnetic force microscopy*

Magnetic domains were imaged using a Bruker ICON3 atomic force microscope equipped with Bruker’s MFM probes (MESP-V2). The MFM probe was coated with Co-Cr for high sensitivity and magnetic contrast. The probe had a nominal tip radius of 35 nm, a nominal resonant frequency of 75 kHz, a nominal coercivity of 400 Oe (medium) and a magnetic moment of $1\text{e-}13$ emu (medium). Before the experiment, the MFM probe was magnetized under an external magnetic field (top-down) for 15 minutes.

4. *Electrical transport*

Resistance measurements were carried out in a Quantum Design (QD) Physical Property Measurement System (PPMS) equipped with a 9 T magnet. A Keithley 6221 current source (100-500 μA) and 2182a nanovoltmeter in Delta mode were used to record each four-point resistance. There was no observed difference between single-direction (DC) and alternating (AC) current pulses. The contact resistances were $\sim 200\ \Omega$ at $T = 290\ \text{K}$. Measurements with in-plane field were accomplished using the QD Horizontal Rotator option in the PPMS.

B. Complete in-plane anisotropy in Sample O1

Table S1 contains the full set of in-plane $R_{ij,kl}$ ($T = 290\ \text{K}$, $H = 0\ \text{T}$) for Sample O1 (Fig. 3a of the main text) and compares each measurement to the expected value for the isotropic equivalent sample. The first 7 rows of each sub-table provide the resistances shown in Fig. 3a. The lower rows give the resistances for current directed along the opposite parallel edge on that face, i.e., $ij = 14 \rightarrow ij = 23$ ($\parallel \hat{\mathbf{y}}$), which were omitted from Fig. 3a. They are nearly identical, as expected (Onsager relation at $H = 0\ \text{T}$). Values in bold font indicate that the voltage drop deviates significantly from the isotropic equivalent value (depicted with red arrows in Fig. 3a.) The 90° -twist anisotropy is apparent in the fact that bold values only appear in the second and third sub-tables, corresponding to current directed in the upper (lower) face along $\hat{\mathbf{x}}$ ($\hat{\mathbf{y}}$).

C. Sample N2, $d = 400$ nm

A large 90° -twist resistance anisotropy invariably arises in every thin-plate sample we have studied. Here, we discuss in detail the anisotropy observed in a second sample (N2) with a thickness of 400 nm. Figure S1 shows the temperature dependences of the four in-plane resistances ($H = 0$ T). All four are qualitatively similar to those of Sample O1 (Fig. 1b in the main text), including the metallic profiles of the high-conductance-axis resistances and the anomalous profiles of the low-conductance-axis resistances. The abrupt jump in $R_{1'4',2'3'}$ at $T \simeq 120$ K disappeared after thermal cycling (this was the first cool-down). The magnitude of the anisotropy is somewhat lower (at most about $5\times$) and not as symmetric between x and y as in Sample O1. Additionally, the high-conductance-axis resistances ($R_{1'2',4'3'}$ and $R_{14,23}$) do not perfectly match. Furthermore, we note that the isotropic equivalent value quoted (0.478 Ω , open star in Fig. S1) is actually the simulated value for a 600 nm thick slab. The closed star gives the isotropic equivalent value for a 400 nm slab: 0.716 Ω . This value increases exponentially with decreasing sample thickness so the deviation may be indicative of some uncertainty in the thickness measurement of the device (SEM imaging).

Tables S2 and S3 give the full $R_{ij,kl}$ for Sample N2 at $T = 290$ K and $H = 0$ T. The anomalous resistances are again presented in bold font. In this sample, the low-conductance-axis resistances vary by about 3 times. On the upper face, $R_{xx} \simeq 6.5 \times R_{yy}$, but on the lower face, $R_{yy} \simeq 1.6 \times R_{xx}$. There appears to be an issue with one of the electrical contacts in this sample. For example, $R_{44',12} = 1.11 \Omega$, $R_{44',2'3'} = 0.05 \Omega$, and $R_{33',1'4'} = 0.05 \Omega$ should all be $\sim 1 \Omega$ by C_4I symmetry. Because $R_{14',33'}$ and $R_{2'3',44'}$ are also too small, it is likely that these inconsistencies simply arise from an issue with the electrical contact at 3'. Some of our other samples exhibited similar asymmetric $R_{ij,kl}$, which may be attributed to contact issues (see Table S4).

D. Thickness, aspect ratio, and lattice orientation studies

In addition to Samples O1 and N2, we fabricated four other thin-plate crystals with contacts on both the upper and lower faces (Fig. S2). Their geometrical details and resistance anisotropies are listed in Table S4.

1. Thickness dependence

The first four devices listed in Table S4 have the same lateral geometry but different thicknesses, d . In the Van der Pauw corner-contact geometry, as d is increased the expected value of each in-plane resistance in the isotropic equivalent sharply decreases. The expected isotropic values are 0.478, 0.287, 0.095, and 0.058 Ω for samples with $d = 0.6$ (N2), 1 (O1), 3 (D1), and 5 μm (M1), respectively. In each sample, the high-conductance-axis resistances ($R_{14,23}$ and $R_{1'2',4'3'}$) are close to these expected isotropic values whereas the low-conductance-axis resistances are $2\times$ (N2) - $58\times$ (M1) larger. There is a large amount of variation in the low-conductance-axis resistances but they are consistently $\gtrsim 1 \Omega$, regardless of the fact that they should decrease sharply with increasing d . This observation agrees with the “thin-surface” model described in the main text. If the anisotropic conductivity is limited to states in the upper and lower surfaces, then the low-conductance-axis resistances should not change significantly with sample thickness.

2. Lattice orientation dependence

In order to determine if the twisted anisotropy depends strongly on the lattice orientation, we made a sample with edges oriented to $\hat{\mathbf{x}} \parallel \langle 100 \rangle$, $\hat{\mathbf{y}} \parallel \langle 010 \rangle$ (Sample F1). Regardless of this difference, this device still exhibited a robust 90° -twist resistance anisotropy. The high-conductance-axis resistances agree with the isotropic equivalent expectation (0.239 Ω): $R_{14,23} = 0.211 \Omega$ and $R_{1'2',4'3'} = 0.250 \Omega$, whereas the low-conductance-axis resistances are about 2-5 times greater: $R_{12,43} = 0.77 \Omega$ and $R_{1'4',2'3'} = 0.473 \Omega$.

3. Aspect ratio dependence

In order to determine if the square geometry of the devices was essential to realizing the 90° -twist anisotropy in transport, we fabricated a sample with a 2:1 aspect ratio (J1) ($14 \times 7 \times 1 \mu\text{m}$). Due to the Van der Pauw geometry, the rectangular shape generates a large difference between the resistance measured along the long side $\vec{ij} \parallel \hat{\mathbf{x}}$ and the short side $\vec{ij} \parallel \hat{\mathbf{y}}$. The expected isotropic equivalent resistances in this geometry are 1.457 and 0.012 Ω for $\vec{ij} \parallel \hat{\mathbf{x}}$ and $\vec{ij} \parallel \hat{\mathbf{y}}$, respectively. On the upper face this geometry-induced anisotropy is parallel to the high-conductance axis ($\hat{\mathbf{y}}$), but on the lower face it counteracts the intrinsic

anisotropy ($\parallel \hat{\mathbf{x}}$). On the upper surface $R_{14,23} = 0.016 \Omega$ matches the isotropic equivalent, but $R_{12,43} = 4.442 \Omega$ is anomalously large. On the lower face the two resistances are roughly equal in magnitude, but actually $R_{1'4',2'3'} = 1.58 \Omega$ is anomalously large and $R_{1'2',4'3'} = 1.269 \Omega$ agrees with the isotropic equivalent. Thus, the 90° -twist anisotropy arises in much the same way in this 2:1 rectangular sample as in a square one.

E. Weak antisymmetric magnetoresistance

At low-field ($H < 1$ T), the $R_{ij,kl}$ exhibit weak anisotropic and field-antisymmetric magnetoresistance (MR). At high-field (measured up to 9 T), the MR of each configuration is very weak and featureless. It is ~ 20 m Ω at $H = 9$ T for each configuration. Figure S3a shows the MR of the four in-plane $R_{ij,kl}$ in Sample O1 at $T = 50$ K ($\mathbf{H} \parallel \hat{\mathbf{z}}$). This is qualitatively identical to Fig. 3a of the main text (Sample N2). Again, there is a clear saturation field $H_S \simeq 1.3$ T above which the MR of each channel is very weak and slightly negative. As in Sample N2, below H_S the MR measured with current $\parallel \hat{\mathbf{y}}$, $R_{14,23}$ and $R_{1'4',2'3'}$, feature local maxima at $|H| \simeq 0.75$ T whereas the MR $\parallel \hat{\mathbf{x}}$, $R_{12,43}$ and $R_{1'2',4'3'}$, have minima. More interesting is the fact that the MR $\parallel \hat{\mathbf{y}}$ has a significant H -antisymmetric component (and the MR $\parallel \hat{\mathbf{x}}$ does not). Figure S3b shows the H -antisymmetrized MR $\parallel \hat{\mathbf{y}}$ on the same plot as the Hall resistance. They have a similar profile to the anomalous Hall curves and share the same knee at $|H| \simeq 1$ T. However, they are less than the Hall resistance by a factor of 10.

We find additional H -antisymmetric MR when the field is directed in the x - y plane (Fig. S4). These present a puzzling pattern that we have yet to solve. Starting with the in-plane MR measured on the left face of the thin crystal, $R_{14,1'4'}(H)$ (Fig. S4a), there is a large H -antisymmetric MR with a negative slope. When $\mathbf{H}$ is normal to this face ($\parallel \hat{\mathbf{x}}$), $\Delta R(H) = R(H = 3 \text{ T}) - R(H = 0 \text{ T}) \simeq -6$ m Ω . When $\mathbf{H}$ is rotated to be *parallel* to the applied-current direction ($\hat{\mathbf{y}}$), $\Delta R(H)$ *increases* in magnitude to -14 m Ω . Then, when the MR of the right face $R_{23,2'3'}$ is measured, the antisymmetrized $\Delta R(H)$ is exactly opposite to that of the left face: 14 m Ω for $\mathbf{H} \parallel \hat{\mathbf{y}}$ and 6 m Ω for $\mathbf{H} \parallel \hat{\mathbf{x}}$. Thus, this in-plane H -antisymmetric MR is actually largest when the current is directed parallel to $\mathbf{H}$, and its slope changes sign when the current is directed along opposite parallel edges of the crystal.

When the current is directed along the x -axis edges, $R_{12,1'2'}$ and $R_{43,4'3'}$, the pattern is

somewhat different. In fact, these configurations only feature antisymmetric MR for $\mathbf{H} \parallel \hat{\mathbf{x}}$ —the same direction as the applied current. $R_{12,1'2'}$ ($\mathbf{H} \parallel \hat{\mathbf{x}}$) closely matches $R_{14,1'4'}$ ($\mathbf{H} \parallel \hat{\mathbf{y}}$). $R_{43,4'3'}$ ($\mathbf{H} \parallel \hat{\mathbf{x}}$) is equal in magnitude but opposite in sign and matches $R_{23,2'3'}$ ($\mathbf{H} \parallel \hat{\mathbf{y}}$). Perhaps this suggests that the same mechanism generates the antisymmetric MR for the left and back faces (sharing the 11' edge) and likewise for the front and right sides (sharing the 33' edge). Finally, we note that although these patterns of H -antisymmetric MR are very interesting, they are very small in magnitude ($\sim m\Omega$) compared with the intrinsic 90° -twist anisotropy (Ω).

F. Extended symmetry analysis

We consider the configuration $R_{11',aa'}$ (see diagram in Fig. S5). For simplicity, we take the crystal symmetry to be $C_{4z}M_z$ (as $M_z = IC_{2z}$, this is equivalent to $C_{4z}I$ plus the 2-fold rotation C_{2z}). The applied current density leads to a voltage contour pattern that transforms as a subgroup G_c of the full point group $\mathcal{G}$ of the twisted lattice ($G_c \subset \mathcal{G}$). The observed potential contours are invariant (up to an overall sign) under any operation g belonging to the subgroup ($g \in G_c$). For e.g., a reflection M_z across the x - y midplane followed by a dihedral reflection M_d across the diagonal plane normal to $\hat{\mathbf{n}} = (\hat{\mathbf{x}} + \hat{\mathbf{y}})/\sqrt{2}$ leaves the contours invariant apart from a sign change. Hence $M_d M_z \in G_c$. Likewise, the rotation C_{2d} by π about a diagonal in the x - y mid-plane also belongs to G_c .

We focus on the two side faces 233'2' and 344'3' and adopt the “flattened” representation with planar coordinates (u, z) and origin at the midpoint of 33' (see Fig. S5). The potential is defined to be zero at the origin. Both the symmetry operations $M_d M_z$ and C_{2d} correspond to rotation by π about the axis normal to the (u, z) plane, which leads to $u \rightarrow -u$ and $z \rightarrow -z$. Acting alone, M_d inverts the sign of u while M_z inverts the sign of z .

Under any operation $g \in G_c$, $V(u, z)$ transforms as

$$\mathcal{P}_g V(\mathbf{u}) = V(g^{-1}\mathbf{u}), \quad (1)$$

where $\mathbf{u} = (u, z)$. For $g = C_{2d}$ ($(u, z) \rightarrow (-u, -z)$), the contours remain invariant but

173 acquire a sign change. Hence $V(u, z)$ satisfies

$$V(u, z) = -V(-u, -z). \quad (2)$$

174 This is our first constraint. A second constraint is that $V(u, z)$ must not remain invariant
175 under the reflection $u \rightarrow -u$ (effected by M_d alone),

$$\mathcal{P}_{M_d} V(u, z) = V(-u, z) \neq V(u, z). \quad (3)$$

176 This is because the edges 32 and 3'4' are identical twins as are the edges 3'2' and 34, but
177 the two pairs are distinct.

178 Together, Eqs. 2 and 3 suggest that, in the flattened representation, z appears in $V(u, z)$
179 in the combination $z.\text{sign}(u)$. The potential contours that satisfy Eqs. 2 and 3 then have
180 the form

$$V(u, z) = U(u)F(z.\text{sign}(u) - c), \quad (c \neq 0) \quad (4)$$

181 where U and F are arbitrary functions that are odd in their arguments, i.e. $U(-u) = -U(u)$
182 and $F(-x) = F(x)$, and c is a non-zero constant. Note that if $c = 0$, $V(u, z)$ violates
183 constraint Eq. 3. Similarly, a surface E -field that is uniform ($V \sim \mathbf{a} \cdot \mathbf{u}$ with any constant
184 $\mathbf{a}$) trivially satisfies Eq. 2 but is excluded by Eq. 3.

185 The simplest example of Eq. 4 is

$$V(u, z) = u(z.\text{sign}(u) - c), \quad (c \neq 0). \quad (5)$$

186 Fixing $c = d/2$, Eq. 5 displays the symmetry observed for $V(u, z)$ in our experiment.
187 Specifically, both Eqs. 4 and 5 predict that $V = 0$ all along the edge 33' ($u = 0$). Moreover,
188 $V = 0$ along the edges 32 and 3'4' with the choice $c = d/2$. Consequently, the zero-potential
189 edges are strictly enforced by the 90°-twist geometry.

190 *Negative non-local resistance*

191 Although the above symmetry arguments enforce $V = 0$ along edges 32, 3'4', and 33'
192 when current is directed along $ij = 11'$, in the experiment these edges have very small finite

values. In particular, the non-local resistance along the diametrically opposed edge, e.g.,
 $R_{11',33'}$, always has a tiny negative (anti-parallel) value. Although not explicitly mentioned
in the main text, the column charts in Figs. 4b and 5b show that the simulations also
capture this negativity. Thus, this finite negative value seems to arise from an isotropic
contribution of bulk states that is not directly considered in our symmetry analysis.

i,j	k,l	Iso. Equiv. R (Ω)	Sample O1 R (Ω)	i,j	k,l	Iso. Equiv. R (Ω)	Sample O1 R (Ω)	i,j	k,l	Iso. Equiv. R (Ω)	Sample O1 R (Ω)	i,j	k,l	Iso. Equiv. R (Ω)	Sample O1 R (Ω)
1,4	2,3	0.29	0.2	1,2	4,3	0.29	1.9	1',4'	2',3'	0.29	2.5	1',2'	4'3'	0.29	0.18
	2,2'	-10^{-8}	-0.03		4,4'	10^{-8}	1.0		2,2'	-10^{-8}	-1.5		4,4'	10^{-8}	0.04
	3,3'	10^{-8}	0.04		3,3'	-10^{-8}	-0.7		3,3'	10^{-8}	1.0		3,3'	-10^{-8}	-0.05
	1',2'	1.87	2.1		1',2'	4.0	4.4		1,2	1.87	2.1		1,2	4.0	4.6
	2',3'	0.29	0.3		2',3'	-1.87	-2.0		2,3	0.29	0.3		2,3	-1.87	-2.1
	4',3'	-1.87	-1.3		4',3'	0.29	0.2		4,3	-1.87	-1.2		4,3	0.29	0.25
	1',4'	4.0	3.7		1',4'	1.87	2.0		1,4	4.0	3.8		1,4	1.87	2.0
2,3	1,4	0.29	0.2	4,3	1,2	0.29	1.9	2',3'	1',4'	0.29	2.7	4',3'	1',2'	0.29	0.15
	1,1'	-10^{-8}	-0.06		1,1'	10^{-8}	1.0		1,1'	-10^{-8}	-1.5		1,1'	10^{-8}	0.02
	4,4'	10^{-8}	0.07		2,2'	-10^{-8}	-0.7		4,4'	10^{-8}	1.0		2,2'	-10^{-8}	-0.05
	1',2'	-1.87	-2.1		1',2'	0.29	0.3		1',2'	-1.87	-2.0		1,2	0.29	0.2
	2',3'	4.0	4.1		2',3'	1.87	1.6		2',3'	4.0	4.1		2,3	1.87	1.6
	4',3'	1.87	1.6		4',3'	4.0	3.1		4',3'	1.87	1.7		4,3	4.0	3.1
	1',4'	0.29	0.4		1',4'	-1.87	-1.2		1',4'	0.29	0.3		1,4	-1.87	-1.3

TABLE S1. **Sample O1 in-plane resistances.** ($T = 290$ K, $H = 0$ T.) The first two columns identify $R_{ij,kl}$ according to the convention shown in the inset of Fig. S1. The third column shows the simulated resistances for the isotropic equivalent. The last column provides the experimental data. The first (second) sub-table is with current directed on the upper face $\parallel \hat{y}$ ($\parallel \hat{x}$). The third and fourth sub-tables correspond to the same current directions on the lower face. Only the values in bold font deviate significantly from the isotropic equivalent.

i,j	k,l	Iso. Equiv. R (Ω)	Sample N2 R (Ω)	i,j	k,l	Iso. Equiv. R (Ω)	Sample N2 R (Ω)	i,j	k,l	Iso. Equiv. R (Ω)	Sample N2 R (Ω)	i,j	k,l	Iso. Equiv. R (Ω)	Sample N2 R (Ω)
1,4	2,3	0.48	0.44	1,2	4,3	0.48	2.88	1',4'	2',3'	0.48	1.07	1',2'	4',3'	0.48	0.69
	2,2'	-10^{-7}	-0.05		4,4'	10^{-7}	1.11		2,2'	-10^{-7}	-0.51		4,4'	10^{-7}	0.07
	3,3'	10^{-7}	0.02		3,3'	-10^{-7}	-0.89		3,3'	10^{-7}	0.05		3,3'	-10^{-7}	-0.07
	1',2'	3.83	2.94		1',2'	8.15	7.07		1,2	3.83	2.92		1,2	8.15	7.05
	2',3'	0.48	0.51		2',3'	-3.83	-3.29		2,3	0.48	0.51		2,3	-3.83	-3.28
	4',3'	-3.83	-3.69		4',3'	0.48	0.89		4,3	-3.83	-3.71		4,3	0.48	0.83
	1',4'	8.15	7.13		1',4'	3.83	2.90		1,4	8.15	7.16		1,4	3.83	2.95
2,3	1,4	0.48	0.44	4,3	1,2	0.48	2.88	2',3'	1',4'	0.48	1.07	4',3'	1,2	0.48	0.69
	1,1'	-10^{-7}	-0.05		1,1'	10^{-7}	1.14		1,1'	-10^{-7}	-0.51		1,1'	10^{-7}	0.11
	4,4'	10^{-7}	0.02		2,2'	-10^{-7}	-0.91		4,4'	10^{-7}	0.05		2,2'	-10^{-7}	-0.09
	1',2'	-3.83	-3.31		1',2'	0.48	0.83		1,2	-3.83	-3.27		1,2	0.48	0.89
	2',3'	8.15	7.66		2',3'	3.83	3.86		2,3	8.15	7.66		2,3	3.83	3.85
	4',3'	3.83	3.84		4',3'	8.15	8.41		4,3	3.83	3.88		4,3	8.15	8.41
	1',4'	0.48	0.51		1',4'	-3.83	-3.72		1,4	0.48	0.51		1,4	-3.83	-3.67

TABLE S2. **Sample N2 in-plane resistances.** ($T = 290$ K, $H = 0$ T.) These tables compare the measured resistances in Sample N2 to the isotropic equivalent expected values. The bold values are the anomalous $R_{ij,kl}$. Note that the third sub-table only has 4 anomalous resistances: $R_{1'4',33'}$ and $R_{2'3',44'}$ do not capture the full anisotropy because of bad electrical contact. The sample thickness used in the isotropic equivalent simulation is 600 nm.

ij,kl	Sample N2 R (Ω)	ij,kl	Sample N2 R (Ω)	ij,kl	Sample N2 R (Ω)	ij,kl	Sample N2 R (Ω)
11',22'	0.46	22',33'	0.82	33',44'	0.03	44',11'	1.03
11',33'	-0.001	22',44'	-0.001	33',11'	-0.001	44',22'	-0.001
11',44'	1.03	22',11'	0.46	33',22'	0.82	44',33'	0.03
11',23	-0.05	22',14	-0.05	33',14	0.02	44',23	0.02
11',43	1.14	22',43	-0.91	33',12	-0.89	44',12	1.11
11',2'3'	-0.51	22',1'4'	-0.51	33',1'4'	0.05	44',2'3'	0.05
11',4'3'	0.11	22',4'3'	-0.09	33',1'2'	-0.07	44',1'2'	0.07

TABLE S3. **Sample N2 out-of-plane resistances.** ($T = 290$ K, $H = 0$ T.) These tables show the resistances measured with current applied $\parallel \hat{z}$. In the isotropic equivalent slab each resistance is $< 10^{-6} \Omega$. As with Sample O1, these vertical-current configurations feature anomalous $\simeq 1\text{-}\Omega$ voltage drops and an alternating pattern of high and low resistances that is enforced by the C_4I symmetry of the 90° -twist anisotropy. The aforementioned contact issue affects these measurements as well, however: the third and fourth sub-tables each only have two anomalous resistances whereas the first and second each have four.

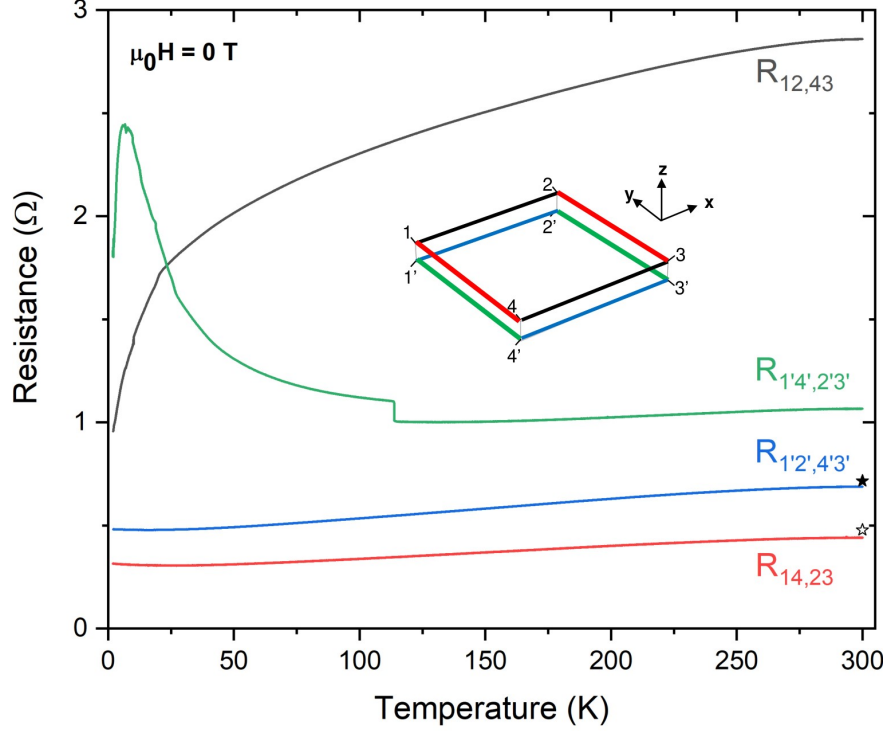


FIG. S1. **Sample N2 temperature dependences.** The low-conductance-axis resistances ($R_{14,23}$ and $R_{1'2',4'3'}$) have metallic profiles matching the isotropic equivalent whereas the high-conductance-axis resistances ($R_{12,43}$ and $R_{1'4',2'3'}$) have anomalous profiles, similar to Sample O1 of the main text. The abrupt jump at $T = 120$ K in $R_{1'4',2'3'}$ was not seen in repeated cool-downs. The white (black) star symbols at $T = 290$ K give the isotropic equivalent values for $d = 600$ (400) nm.

Samp.	Dimensions (μm)	Alignment $\hat{\mathbf{x}}, \hat{\mathbf{y}} \parallel$	Iso. Equiv. R (Ω)	$R_{14,23}$ (Ω)	$R_{12,43}$ (Ω)	$R_{1'4',2'3'}$ (Ω)	$R_{1'2',4'3'}$ (Ω)
N2	$12 \times 12 \times 0.4$	$\langle 110 \rangle, \langle \bar{1}, 1, 0 \rangle$	0.478	0.44	2.88	1.068	0.689
O1	$10 \times 10 \times 1$	$\langle 110 \rangle, \langle \bar{1}, 1, 0 \rangle$	0.287	0.2	1.9	2.6	0.17
D1	$12 \times 12 \times 3$	$\langle 110 \rangle, \langle \bar{1}, 1, 0 \rangle$	0.095	0.105	0.928	4.41	0.38
M1	$10 \times 10 \times 5$	$\langle 110 \rangle, \langle \bar{1}, 1, 0 \rangle$	0.058	0.061	3.355	3.243	0.042
F1	$14 \times 7 \times 1.2$	$\langle 100 \rangle, \langle 0, 1, 0 \rangle$	0.239	0.211	0.77	0.473	0.250
J1	$14 \times 7 \times 1$	$\langle 110 \rangle, \langle \bar{1}, 1, 0 \rangle$	1.457 ($\parallel \hat{\mathbf{x}}$) 0.012 ($\parallel \hat{\mathbf{y}}$)	0.016	4.442	1.58	1.269

TABLE S4. **Sample details.** The first four samples have roughly the same lateral dimensions but varying thicknesses. The fifth (F1) is oriented to the crystal lattice at angle of 45° with respect to the others. The sixth (J1) has a 2:1 rectangular aspect ratio.

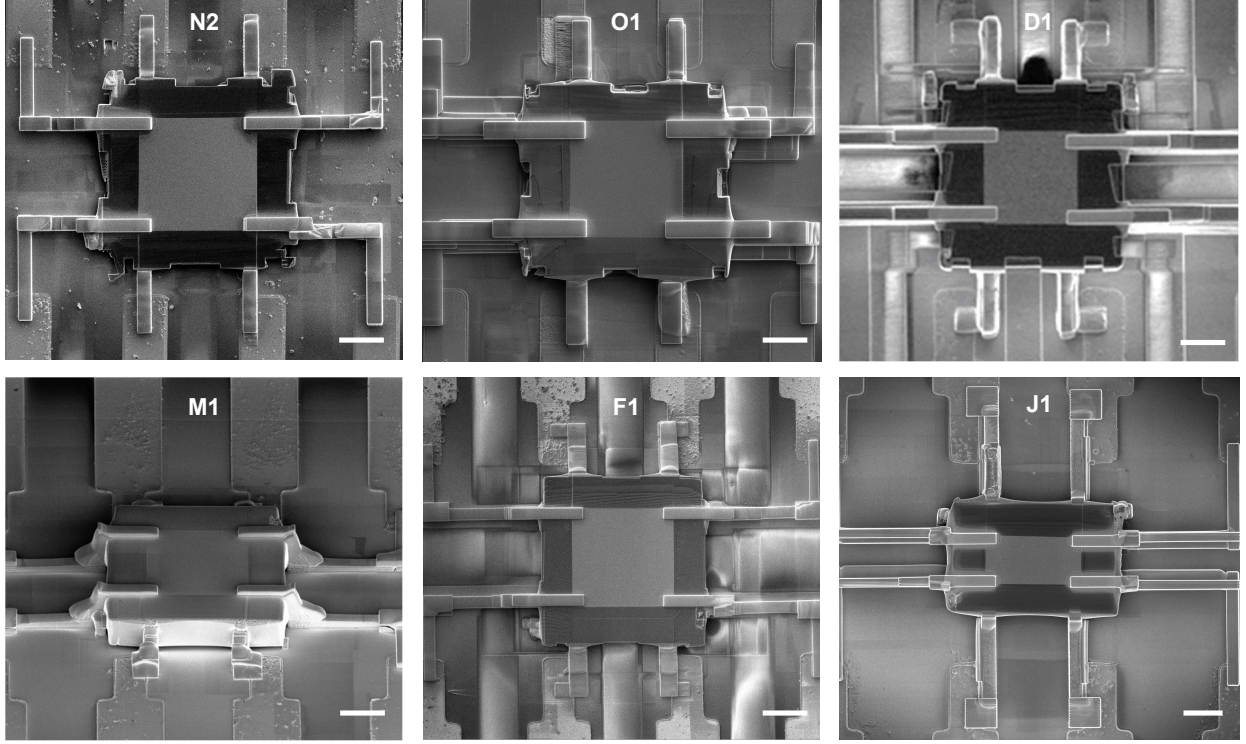


FIG. S2. SEM micrographs of thin-plate devices. Scale bars = 5 μm .

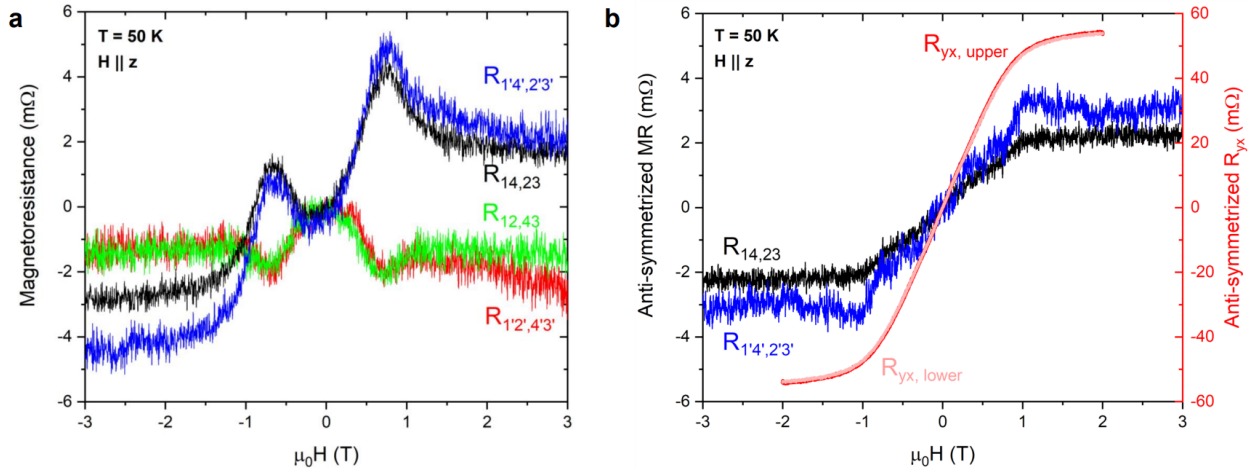


FIG. S3. **Sample O1 magnetotransport.** ($\mathbf{H} \parallel \hat{\mathbf{z}}$) **a.** Magnetoresistances $R(H) - R(H = 0)$ of the four in-plane $R_{ij,kl}$. The MR $\parallel \hat{\mathbf{y}}$ features a field-antisymmetric component resembling the anomalous Hall resistance, but the MR $\parallel \hat{\mathbf{x}}$ does not. The zero-field values which have been subtracted off reflect the intrinsic 90° -twist anisotropy. At $H = 0$ T ($T = 50$ K), $R_{14,23} = 0.141$ Ω , $R_{12,43} = 1.241$ Ω , $R_{1'4',2'3'} = 1.61$ Ω , and $R_{1'2',4'3'} = 0.114$ Ω . **b.** This panel shows the field-antisymmetrized MR $\parallel \hat{\mathbf{y}}$ alongside the anomalous Hall resistances (upper face = dark red, lower face = light red). The two sets of curves share a knee at $|H| \simeq 1$ T but the anomalous Hall resistances are an order of magnitude larger.

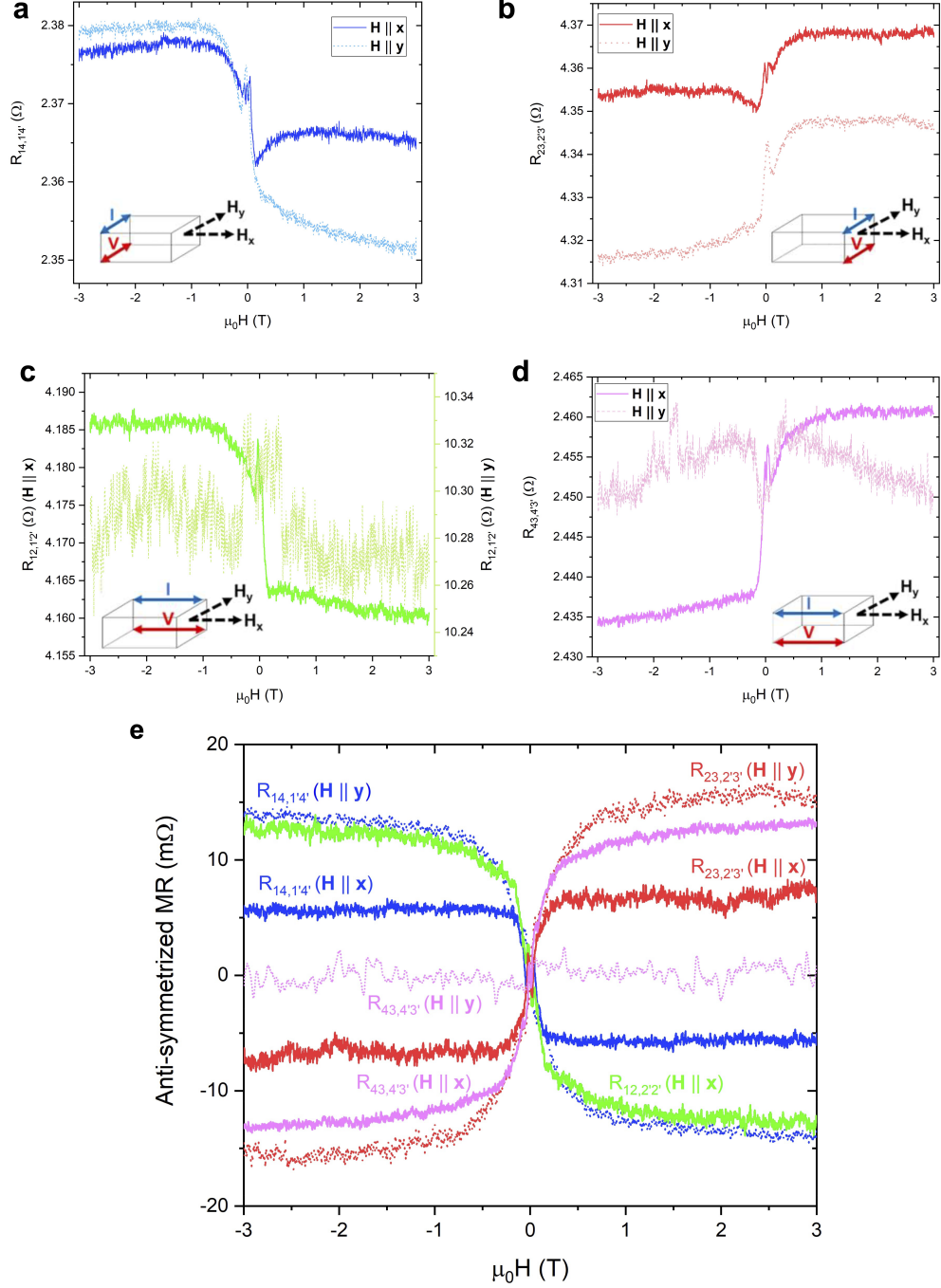


FIG. S4. **In-plane antisymmetric magnetoresistances.** Sample O1. **a-d.** These panels show the raw resistances of the side faces ($R_{ij,i'j'}$) with $\mathbf{H}$ directed in the x - y plane. The solid (dashed) curves represent the data for $\mathbf{H} \parallel \hat{x}$ ($\mathbf{H} \parallel \hat{y}$). A contact-related issue caused the measurement of $R_{12,1'2'}$ $\mathbf{H} \parallel \hat{y}$ (panel d) to be abnormally large. The inset diagrams depict the edges chosen for the applied current (blue) and measured voltage drop (red). **e.** Panels a-d after H antisymmetrization. With $\mathbf{H} \parallel \hat{y}$, the resistances measured on the left ($R_{14,1'4'}$) and back ($R_{12,1'2'}$) faces are mirror images of those measured on the right ($R_{23,2'3'}$) and front ($R_{43,4'3'}$) faces. The magnitudes are all largest when the current is directed $\parallel \mathbf{H}$.

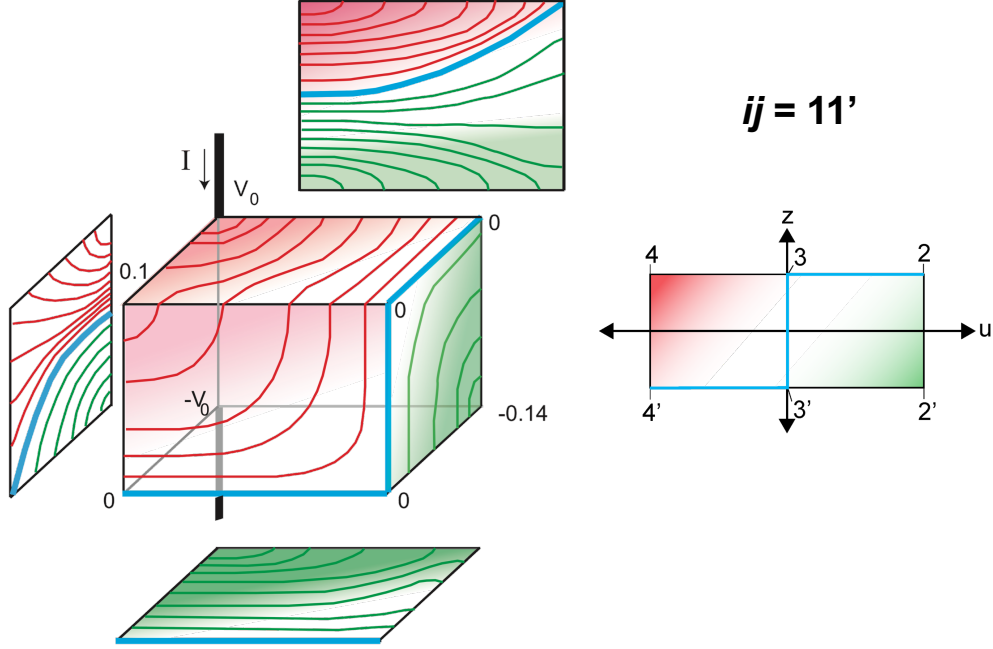


FIG. S5. **Surface potential distribution for vertically applied current.** This diagram shows how the C_4I symmetry gives rise to a surface potential that is zero along the $4'-3'-3'-2$ edge for current contacts $ij = 11'$. The “flattened representation” for the analysis in Section F is shown on the right.