

A research on age estimation methods of monumental Eastern Plane (*Platanus orientalis*) trees

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Research Article

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Abstract

The subject of this study is the investigation of age estimation methods in monumental old plane trees. Some trees are very old and have extraordinarily large diameters. These trees are preserved as monumental trees due to their unusual characteristics. Since they are mostly located in rural areas, they contribute to the development of the rural environment in terms of tourism. The ages of these trees are important for determining their monumental value. It is difficult to determine their age using standard methods. Also, some trees may require estimation of age without injury. These predictions methods can have big errors. In this study, an original approach has been used to reduce age prediction errors. The study presents some sample models so that different models can be developed in different regions. Some regression models are used to improve the age estimation in this study. A total of 54-sample trees were used for age estimation. Of this total, 44 sample trees were used for model development and 10 sample trees were used for suitability testing. The models were ranked according to the success criteria. The regression models showed that for plane trees, the best model based on diameter is M10 (R_{adj} : 0.71, score: 94). The best model based on diameter and annual ring variables was found as M22 (R_{adj} : 0.70, score: 157). It was determined that, the M22 regression model can be used in age estimation with 14.79% average absolute error.

1. Introduction

This study was funded by the Aegean Forestry Research Institute. Essentially, the study includes both age estimation methods some inventory and conservation methods. In this article, only information about age estimation methods are given.

Some trees have lifespans of thousands of years and have extraordinary physical features. These trees have a great effect on community life and shape the history of many societies. Because of their long life spans, they have the distinction of being the ancestors of their own kind. Some of them impact the human memory greatly with their striking physical features.

Monumental trees in Turkey have been attracting more attention since the 1950s. Scholars have studied old trees in Turkey (Eliçin, 1967; Asan, 2010) and published a series of articles. The monumental trees in Turkey have been identified and registered with the Law of Protection of Cultural and Natural Assets Number 2863, enacted in 1983. Some monumental trees are shown in Fig. 1.

The following traits are some of the features that make trees monumental: a) their physical dimensions, b) visual characteristics, and c) effects on moral and cultural values (Asan, 2016). Trees that have all or some of the aforementioned characteristics have recreational, touristic or, scientific importance. If the monumental trees are found in forest stands, they become the historical and genetic banks of these forest ecosystems. Monumental trees, with their important legacies should be protected for future generations, as they are not only of the common value of the countries they are found in but belong to the entire world.

Old trees with large diameters are generally located in untouched, remote parts of forests. It is imperative to develop official government strategies to protect these monumental trees which are in danger of extinction (Batur and Özçankaya, 2017).

Age, physical features, and cultural impact on society should be considered when defining a tree as a monumental. However, some deformations and their large diameters make it difficult to determine a tree age by counting all the annual rings (Moir 2004; Okdan and Yıldız 2018). In addition, the trees should not be damaged during age determination. This study was carried out to determine the least damaging method for age estimation.

In some studies, age estimation was made by adapting the annual number of rings in 15 cm long increment cores taken from the breast height to the radius (Genç and Güner, 2003). In northern Spain, some age estimates of *Fagus sylvatica* and *Quercus robur* have been made by adapting the annual ring thickness to the annual ring numbers (Rozas, 2003). Another study in England, grouped the annual rings according to their frequencies and their numbers per unit basal area. The age estimation was made by using the relationship between the unit surface area and the basal area (White, 1998).

In some studies with *Dacrycarpus dacrydioides*, the rate of age prediction errors were determined by studying the distance of the increment core from the stem center (Duncan, 1989). Some researchers have pointed out that age estimation error could depend on the increment core location in *Pinus ponderosa* and Douglas-fir, *Pseudotsuga rangei* species (Wong and Lertzman, 2001). The age estimation using the diameter or the crown-age relationship gave higher than normal results in some studies (Baker, 2003). Age estimation of old spruce and fir trees in the New England region of the US was conducted using regression equations derived from the diameter-age relationship (Leak, 1985). Some age estimates for the California black oak were made by utilizing a regression model (Garrison et al., 2002). Regression models were used to estimate the age of young trees along the streets (Lukaszkiwicz and Kosmala, 2008). Tree ages were estimated by using tree height, crown width and some stand variables in Finland (Koliavirta and Tokola, 2005). Asan (2018) stated that some different methods can be used to determine the age of monumental trees according to their physical conditions. It was suggested that some age estimates had very significant errors, sometimes as high as one hundred percent (Akkemik et al. 2018).

It is important that, in the development of models, tree growth features and physical characteristics should be taken into account. Therefore, the models used in this study are valid for the plane species in the study region. We believe that this study will contribute to new research. In the future especially due to the method used in the evaluation of regression models.

2. Materials And Methods

2.1. Materials

For the development of age estimation models, 54 random sample trees were used in the diameter range of 40 cm and 108 cm. The annual rings of the tree samples used had very clear ring boundaries so they could be counted without errors. Trees with very large diameters were not sampled as their annual rings could not be counted because they were not clear. A sample tree with a diameter of 250 cm was used as an extrapolation control.

The sample trees were selected within the boundaries of the İzmir Regional Directorate of Forestry. Ten of the sample trees (18.5% of sample trees) were separated as a control group to be used in the confidence test of the models. The tree samples were chosen from each control group based on their diameter intervals. The diameter intervals were taken as 4 cm increments. The measurements and analysis of the samples required data collection from old trees with large diameters ($dbh > 40$ cm) since our study is based on monumental trees.

The sample trees were randomly selected from a pool of trees that were not very near to the other members of the forest stand within the boundaries of the study area.

This study is based on the assumption that the estimation errors would not differ between the young and old tree samples. To check the extrapolation estimates, a 485 years old tree (exact age known) with a 250 cm diameter was chosen as measured above.

2.2. Method

In the study, hypotheses (H_0) were established and then the hypotheses were tested with the statistical methods.

Hypotheses:

H_{0-1} : There is no regressional relationship between the tree age and the annual number of rings of the increment cores.

H_{0-2} : Models produced in this study cannot be used with 95% statistical confidence.

2.3. Collecting data

Increment cores were taken from all sample trees at breast height (130 cm). The samples were taken specifically from trees with regular and circular stems. Thus, it was aimed to reduce errors caused by eccentric growth. The annual rings in the increment cores were counted in the periods ($h_{5-10-15}$) with 5 cm intervals from the tree bark to the core (Fig. 2).

U_k : bark thickness, h_{5-25} : The number of rings in the 5-10-15 cm length increment intervals on the increment core, u_a : increment core length, $d_{1.30}/2$: Radius without bark thickness.

Increment cores were cleaned with sandpaper (Fig. 2). In order to see the rings clearly, they were treated with hematoxylin and some liquid solutions. A binocular microscope was used to count the annual rings.

We developed twenty five single and double entry regression equations were developed to carry out the model tests.

Tested model for age estimation:

$$1 - Y_t = b_0 + b_1 d_{12} \text{ (M1)}$$

$$2 - Y_t = b_0 + b_1 d_{12} + b_2 d_{12} \text{ (M2)}$$

$$3 - Y_t = b_0 + b_1 d_{12} \text{ (M3)}$$

$$4 - Y_t = b_0 + b_1 d_{12} + b_2 \frac{1}{d} \text{ (M4)}$$

$$5 - Y_t = b_0 + b_1 \frac{1}{d} \text{ (M5)}$$

$$6 - Y_t = b_0 + b_1 \ln d_{12} \text{ (M6)}$$

$$7 - Y_t = b_0 + b_1 d_{12} + b_2 \ln d_{12} \text{ (M7)}$$

$$8 - Y_t = b_0 + b_1 \log d_{12} \text{ (M8)}$$

$$9 - Y_t = b_0 + b_1 d_{12} + b_2 \log d_{12} \text{ (M9)}$$

$$10 - \log Y_t = b_0 + b_1 \log d_{12} \text{ (M10)}$$

$$11 - \log Y_t = b_0 + b_1 d_{12} + b_2 (\log d_{12}) \text{ (M11)}$$

$$12 - \log Y_t = b_0 + b_2 \log d_{12} + b_2 \frac{1}{d} \text{ (M12)}$$

$$13 - \log Y_t = b_0 + b_2 \log d_{12} + b_2 \left(\frac{1}{d}\right)^2 \text{ (M13)}$$

$$14 - Y_t = b_0 + b_1 d_{12} + b_2 h_5 \text{ (M14)}$$

$$15 - Y_t = b_0 + b_1 d_{12} + b_2 h_5 + b_3 h_{10} \text{ (M15)}$$

$$16 - Y_t = b_0 + b_1 d_{12} + b_2 h_5 + b_3 h_{10} + b_4 h_{15} \text{ (M16)}$$

$$17 - Y_t = b_0 + b_2 h_{15} + b_1 d_{12} \text{ (M17)}$$

$$18 - Y_t = b_0 + b_1 d_{12} + b_2 h_{15} \text{ (M17)}$$

$$19 - Y_t = b_0 + b_1 \left(\frac{d_{12}}{h_{15}}\right) \text{ (M19)}$$

$$20 - Y_t = b_0 + b_1 d_{12} + b_2 \left(\frac{d_{12}}{h_{15}}\right) \text{ (M20)}$$

$$21 - Y_t = b_0 + b_1 d_{12} + b_2 \left(\frac{d_{12}}{h_{15}}\right)^2 \text{ (M21)}$$

$$22 - Y_t = b_0 + b_1 d_{12} + b_2 \left(\frac{h_{15}}{d_{12}}\right) \text{ (M22)}$$

$$23 - Y_t = b_0 + b_1 d_{12} + b_2 \left(\frac{h_{15}}{d_{12}}\right)^2 \text{ (M23)}$$

$$24 - Y_t = b_0 + b_1 h_{15} \text{ (M24)}$$

$$25 - Y_t = b_0 + b_1 d_{12} + b_2 h_{15} \text{ (M25)}$$

In models; Y_t : estimated age, Y_g : real age, b_0, b_1, b_2, b_3, b_4 : coefficients, d_{12} : diameter without bark at breast height, h_{5-15} : the annual rings number in the increment period (5,10,15 cm).

In regression models, the actual ages (dependent variable) the number of rings and increment period (independent variables) were related to each other. The coefficients of the regression equations were produced by using the data collected from the sample trees. In recent studies, it is recommended that some success criteria should also be evaluated using the coefficient of expression in the selection of the best model (Rawling et al. 1998; Yavuz, 1999; Aydın, 2014; Alpar, 2017). In these studies, it is recommended to evaluate some success criteria in addition to the coefficient of expression in the selection of the best model. Therefore, in choosing the regression models, the mean error (ME), mean absolute error (MAE), mean absolute percentage error (MAPE), standard deviation (S), standard error ($S_{x,y}$), adjusted coefficient of determination (R^2_{adj}), Baesian Knowledge Criteria (SBC), and success criteria were used. In addition, the criteria absolute total score (ATS) and, extrapolation control error (EXT) were evaluated. The values of the success criteria of the models were calculated by using the actual ages (Y_g) and estimated ages (Y_t) of the sample trees.

General formulas of evaluation criteria that are used in the models and formulas are shown below.

$$R^2_{adj} = 1 - \frac{\sum (Y_g - Y_t)^2 / (n - k - 1)}{\sum (Y_g - Y_{g-m})^2 / (n - 1)}$$

$$S = \sqrt{\frac{(Y_t - Y_g)^2}{n - k}}$$

$$S_{x,y} = \frac{S}{\sqrt{n}}$$

$$ME = \frac{1}{n} \sum (Y_t - Y_g)$$

$$MAE = \frac{1}{n} \sum |Y_t - Y_g|$$

$$\text{MAPE} = \frac{1}{n} \sum \frac{|Y_t - Y_g|}{Y_g} 100$$

$$\text{SBC} = n \bullet \ln \left(\frac{\sum (Y_t - Y_g)^2}{n} \right) + k \bullet \ln (n)$$

$$\text{ATS} = \text{ME} + \text{MAE} + \text{TPE} + \text{MAPE} + \text{S} + \text{S}_{x,y}$$

$$\text{EXT} = \frac{Y_{te} - Y_g}{Y_g} 100$$

In formulas; n: number of sample, k: number of variable, Y_{g-m} : mean real age, Y_{te} : extrapolation estimate

The best univariate and bivariate models with the best scores were determined by evaluating the model results with the success criteria. Thus, H0-1 hypothesis was tested. The Wilcoxon suitability test was performed using the control group for testing H0-2 hypothesis.

3. Results

Before the regression analysis, the structure of the data in the models was examined. The scatter diagram shows that the sample structure was homogeneous, the samples were independent from each other and they were distributed normally. Autocorrelation and multicollinearity were not detected among the variables.

Predictive models can be established in many disciplines with regression analysis. Regression models are very useful in making volume tables or other estimates used in forestry. In regression analysis, the actual linear models are used. The significance of the model coefficients was tested. Statistics for the selected models are shown in Table 1.

Table 1: Statistics of models

No	b0		b1		b2		b3	b4	cf	sign
10	0.072	NS	1.052	***	-		-	-	1,05	***
22	-81.338	***	2.090	***	47.990	***	-	-	-	***
P > 0.05 => NS, p < 0.05 => *, p < 0.01 => **, p < 0.001 => ***										
b ₀₋₄ :model coefficients, cf: correction factor, sign: significance										

The correction factor ($df = 10^{1,1513 \bullet Se^2}$) recommended in the literature was used to eliminate systematic errors in estimations with logarithmic equations (Alemdağ, 1962; Asan, 1984; Birler, 1986).

A total of seven criteria were used in the selection of regression models. Schwarz's Bayes (SBC) and Akaike (AIC) information criteria were calculated, but a stronger criterion SBC was preferred to prevent any increase in the weight of similar criteria in scoring. In scoring, for each criterion, the best indicator was given 25 points, and the others were given decreasing points between 25 and one according to the rankings. Models that did not fit the natural growth models were not selected.

The models between 1-13 are single variable models depending on diameter only, and models between 13-25 are bivariate models. The univariate and the bivariate models were scored together and a ranking was done. A selection was made among the models with the highest scores from the univariate and the bivariate groups, with the non-NS coefficients except b_0 . As a result of this evaluation, the points and ranks of the selected models with one and two variables that got the highest score are shown in Table 2.

Table 2: Statistics of the success criteria of the models subject to scoring

Univariable model summary									
Rank/Model	ME	MAE	$S_{x,y}$	MAPE	R^2_{adj}	SBC	EXT.	Total Point	General Ranks
7/10	0.16	16.75	22.45	26.27	0.71	549.27	-13.83		14
Point	24	12	13	8	21	9	18	105	
Bivariable model summary									
Rank/Model	ME	MAE	$S_{x,y}$	MAPE	R^2_{adj}	SBC	EXT.	Total Point	General Ranks
2/22	0.00	13.78	19.34	14.79	0.70	267.23	-7.80		2
Point	25	23	22	22	21	23		157	

Although models 9, 12, 4, 7, 2, 13 were ahead among the univariate models, they were not chosen because their coefficients of the two terms are not significant. Although the first term of Model 10 (M10) is not significant, it was chosen as the best single-entry model. This constant value does not impact the model because it is near zero, considering the literature recommendations (Ercan 1995; Aydın, 2014). The graph of Model 10 is shown in Figure 4.

Our results suggest that Model 22 should be used primarily for age estimation because of its bivariate. The summary of the models is shown in Table 3.

Table 3: Summary of the models suggested.

Priority	MODELS SUMMARY		
	Models	Formulas	Coefficients
1	M22		b_0 : -81.3382042642323
			b_1 : 2.09028353942026
			b_2 : 47.9904525868996
2	M10		b_0 : 0.07282174863217
			b_1 : 1.05220303428384
			df: 1.05963218479104

One univariate and one bivariate regression model were chosen by correlating the variables of age, diameter and annual number of rings. In this case, the H0-1 hypothesis was rejected.

The suitability of the selected models was tested with the Wilcoxon test using 10 control data outside the sample pool. As a result of examining the p values of the tests, it was determined that there was no significant difference between the data produced from the models and the real age values of the control group with 95% confidence ($p > 0.05$). The fitness of the models was accepted. In this case, the H0-2 hypothesis was rejected. Test results are summarized in Table 4.

Table 4: Wilcoxon tests statistics

Models	p	z
22	0.202622	-1.274118
10	0.114128	-1.579906
P > 0.05: fitted, p < 0.05: non fitted		

4. Conclusion And Recommendations

According to the research results, both H0-1 and H0-2 hypotheses were rejected. The annual rings counted from the increment core and the diameter can be used in tree age estimation. The estimation can be made with the help of the models produced by the study data, with an average absolute error percentage (MAPE) of 14.79% for the double-input model (M22) and 26.27% for the single-input model (M10).

The exact age of a tree can be calculated by counting the annual rings. However, not all annual rings are visible in very old trees for various reasons. The formulas and models suggested in this study are estimators, and the proposed models should be used according to the conditions of the variables. For

example; M22 should be used if both the diameter measurement and 15 cm increment core can be taken, and M10 model should be used, if only the diameter can be measured. If the tree is in extreme condition or better data can be obtained for age estimation, other methods can be used, including different dendrological variables. However, it is possible to use combinations of different methods.

The simplest model that meets the needs in regression analysis is the best model. The more complex the models, the more difficult it is to apply it and the possibility of error may increase.

When all models are examined, the scores of the bivariate models are found to be higher. For this reason, a bivariate model should be used at first, otherwise if data for a multivariate model is not available, an estimation should be made by using the univariate model.

The annual number of rings of plane trees in a period show a wide variation, depending on the conditions of the habitat. Therefore, it is difficult to create a strong age prediction model based on diameter alone. Therefore, it is recommended to use multivariate models with annual ring widths as the variables for a better estimate.

Each tree species displays different growth characteristics. Since the models in this study are developed for the region and for the particular species, it is not recommended to be used without compatibility tests for other regions or other species.

Some estimation errors (nearly 100%) were made in the recent age estimates of monumental trees (Akkemik et al. 2018). Examining the absolute error percentages of the models produced in this study, the results vary between 10–26%. Considering the relationship of the independent variables with age, it can be considered that the model error rates produced with the 0.70–0.90 R^2 values are reasonable.

Rays are very frequent and evident in plane trees. The presence of rays might cause errors in age estimation because the rays can be confused with annual rings. Due to the mixing of the rays with the annual ring, significant age estimation errors are made. The annual ring traces on the increment core should be determined correctly. For better estimation, it is recommended that predictions should be checked by other dendrological methods.

Stem and branch mergers can be seen on some trees with high cambium activity such as plane trees. Samples should be taken to estimate the age of such trees on a single stem.

If an estimation of the tree without injury is desired, the single-entry M10 model can be used. However, the prediction error will be higher in this model.

The models produced in this study can be used in a particular region of Turkey. However, it can also be used for other regions and countries, provided that it is checked with conformity tests. The model coefficients can be redeveloped by type and region with new samples for better estimation.

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Figures

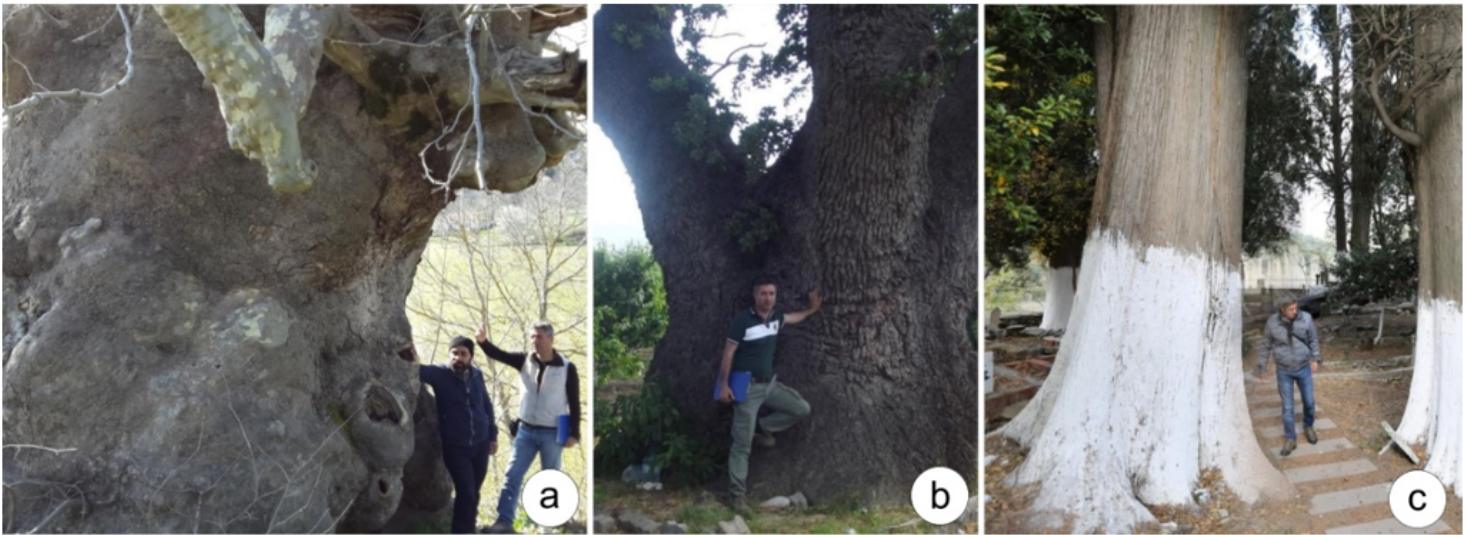


Figure 1

Some of monumental trees which are in Izmir Province (a: Subatan Plane, b: King Oak, c: Hamzababa Cypress)

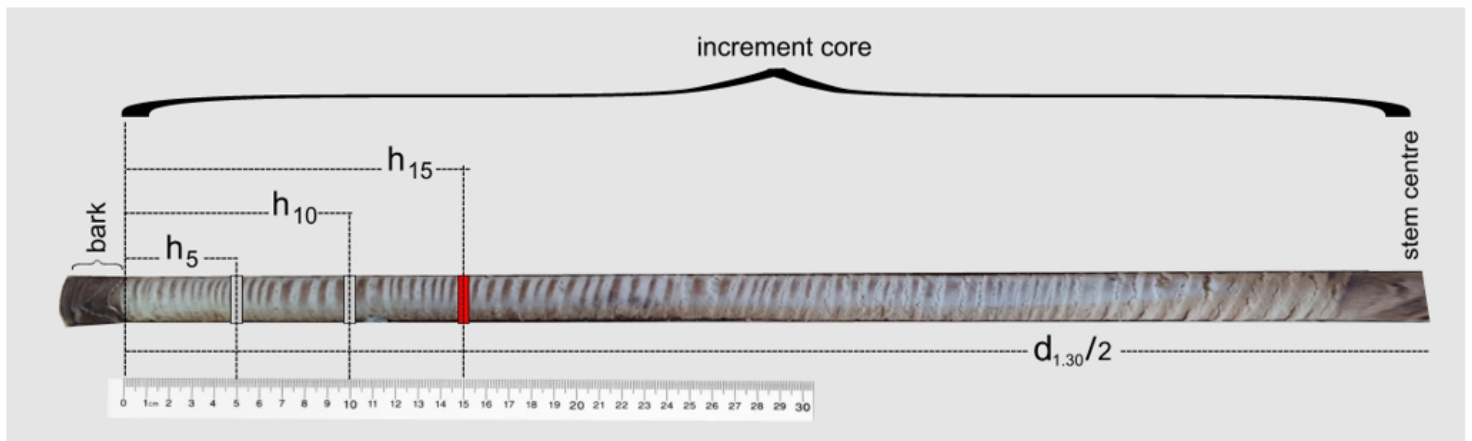


Figure 2

Representation of the annual ring variables on the increment core

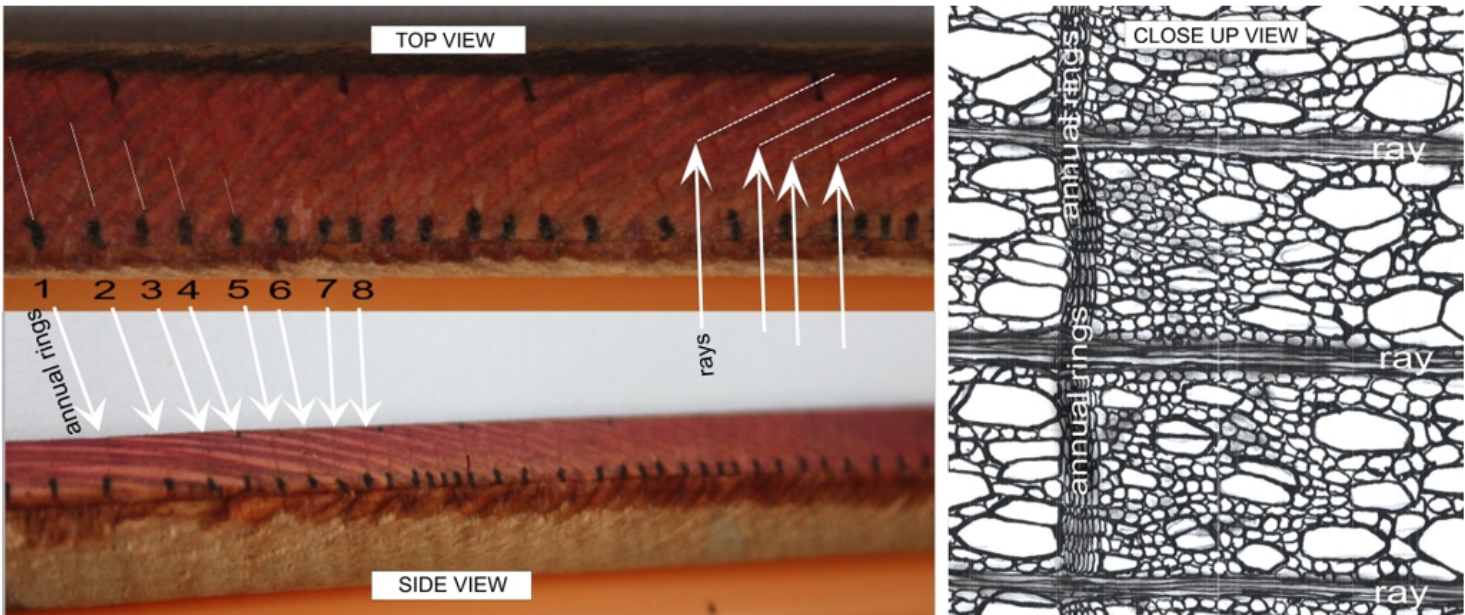


Figure 3

Identification of annual ring traces in plane trees

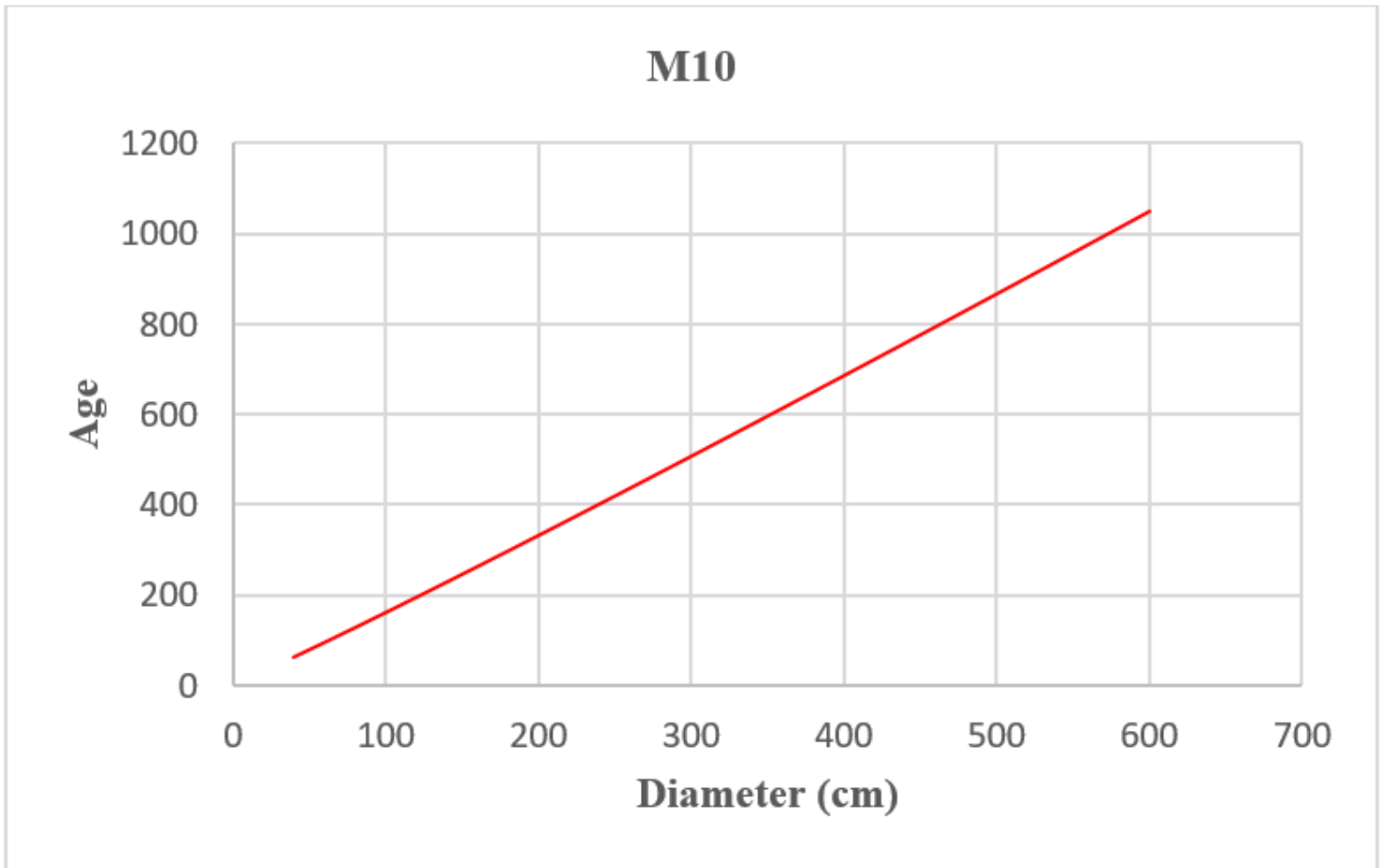


Figure 4

Model 10’s graph Model 22 (M22) was chosen because all the coefficients were significant among the bivariate models. The graph of Model 22 is shown in Figure 5.

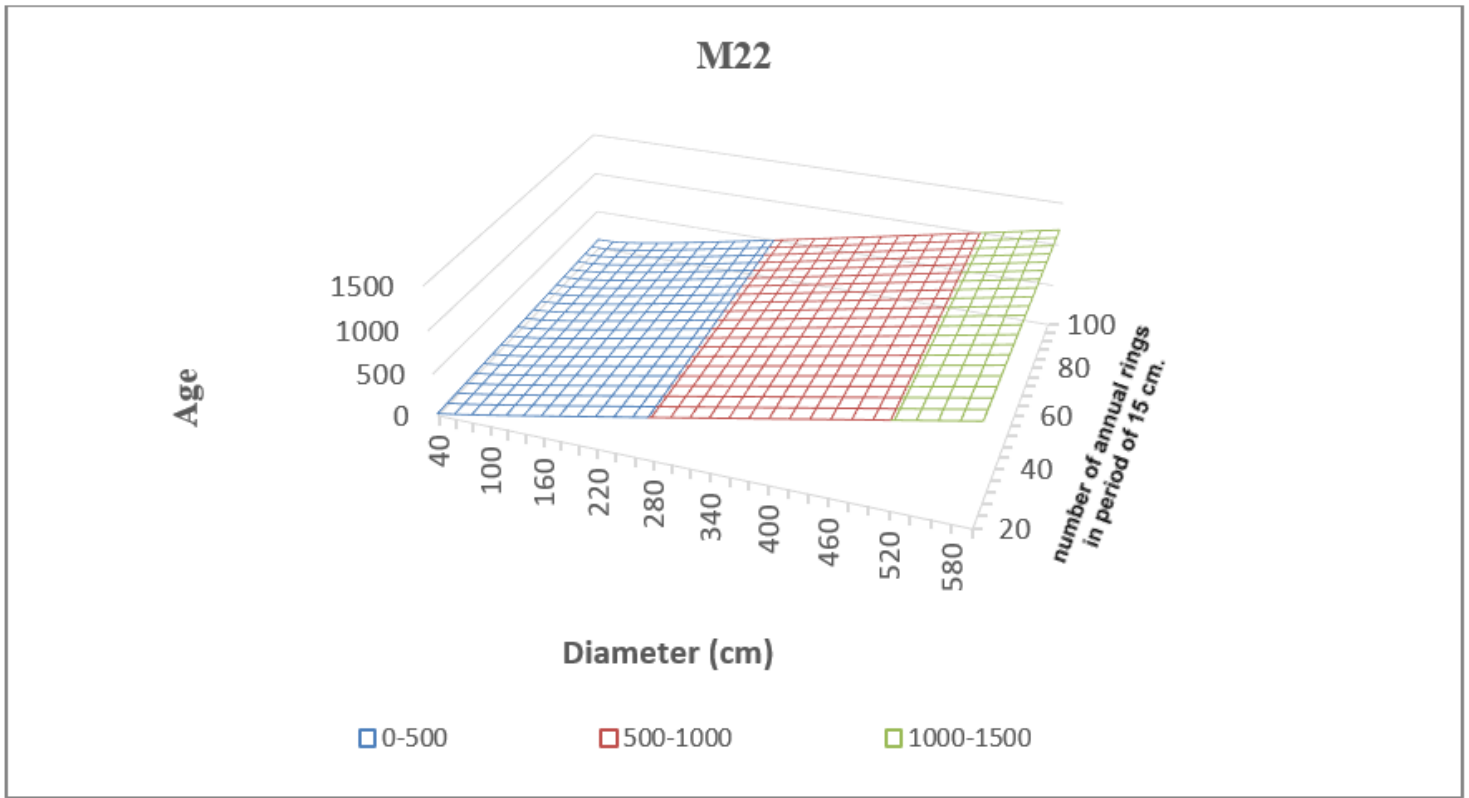


Figure 5

Model 22's graph