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1 Revisit the performance of Doppler positioning with LEO satellites

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9 Abstract

10 Recently, the Doppler shifts from Low Earth Orbit (LEO) satellites have been used to augment GNSS
11 and provide standalone positioning, navigation and timing services in opportunistic navigation
12 frameworks or GNSS-like navigation systems. In this paper, we proposed a Doppler-only point-solution
13 algorithm for GNSS-like navigation systems operated in LEO. In these systems, LEO satellites are
14 assumed to be modified to support navigation capabilities. Their positions, velocities and clock drifts
15 could be obtained from ephemeris. The proposed point-solution algorithm can estimate the receiver clock
16 drift, position and velocity simultaneously. Then, the main error sources in Doppler positioning are
17 analyzed. As one of the main error sources, satellites' position error of several meters and velocity error
18 of several centimeters per second would reduce the positioning performance significantly. The LEO
19 doppler positioning accuracy will deteriorate if there are no error corrections for the ionospheric and
20 tropospheric delay rates. Subsequently, the performance of Doppler positioning algorithms is analyzed.
21 The Zero-Difference Doppler positioning accuracy is in meter-level, which is comparable with
22 pseudorange-based positioning in GNSS. By doing difference between satellites, the receiver clock drift
23 is eliminated and the Between-Satellite-Difference Doppler positioning accuracy is improved by about
24 10% as Doppler Dilution of Precision is reduced by 12.3%. Moreover, we analyzed the sensitivity of
25 initial position error in the LEO-based Doppler positioning and found that the initial position error should
26 be less than 300 km to ensure convergence.

Keywords

Doppler positioning; Low Earth Orbit (LEO); range rate; Doppler Dilution of Precision (DDOP)

Introduction

Recently, Low Earth Orbit (LEO) communication constellations such as Starlink, OneWeb and Kuiper have been in rapid development. These broadband internet providers plan to deploy thousands of LEO satellites into their constellations (Reid et al. 2018). As to Starlink, the SpaceX Company will deploy about 12000 satellites in phase I, and up to 30000 satellites in phase II. Compared with Global Navigation Satellite System (GNSS) satellites in medium Earth orbit (MEO), the LEO satellites have advantages in received signal strength. If the equivalent isotropically radiated power and signal frequency are the same, the received signal from LEO satellites is about 30 dB stronger than that from GNSS (Khalife and Kassas 2019a; Reid et al. 2018). Besides, LEO satellites travel at speed of about 7.5 km/s, resulting in rapid change of geometry in ranging, and large Doppler shift in received frequency. These desirable attributes would be of great use in GNSS-challengeable places like urban canyons and indoors. Therefore, LEO satellites have been considered as promising alternative positioning, navigation and timing (PNT) sources.

Existing applications of LEO satellites in navigation mainly include two aspects, i.e. augmenting GNSS or providing PNT services independently. In the aspect of GNSS augmentation, LEO satellites can serve as space-based monitoring stations to jointly determine the precise orbits of GNSS and LEO satellites with their onboard GNSS receivers (Yang et al. 2020; Zhao et al. 2017; Zeng et al. 2018), and can also be used to broadcast navigation augmentation information (Meng et al. 2018). Besides, some combined GNSS/LEO observation models and data fusion methods have been proposed to improve convergence and ambiguity-fixed performance of precise point positioning (PPP) and real-time kinematic (RTK) (Joerger et al. 2010; Ke et al. 2015; Li et al. 2019a, b).

When it comes to providing standalone PNT services with LEO satellites, the studies are mainly focused on two methods. One is to design the signal and satellite navigation payload to support navigation capabilities. For example, the Iridium NEXT has designed and broadcasted a Satellite Time and Location (STL) signal to provide alternative PNT services to backup or augment GNSS (Lawrence et al. 2016). Reid et al. (2016) explored how to leverage LEO constellations for navigation from the aspects of

1 constellation configuration, error modeling, onboard Chip-scale Atomic Clocks (CSAC) as well as orbit
2 determination methods. Psiaki (2021) proposed a LEO-based Doppler navigation system based on the
3 assumption that LEO satellites can be modified to carry special navigation beacon antennas and broadcast
4 beacon signals. Their results are optimistic and indicate that the navigation based on LEO constellation
5 is feasible and could provide alternative PNT services to GNSS.

6 The other is to exploit LEO satellites in opportunistic navigation frameworks. The first issue to be
7 addressed is how to estimate the satellites' and receivers' states (position, velocity, clock bias and drift)
8 simultaneously since the orbital elements of LEO satellites are not known precisely. Some researchers
9 have solved this either in a simultaneous tracking and navigation (STAN) framework (Ardito et al. 2019;
10 Kassas et al. 2019) or in a differential framework (Khalife et al. 2020). The other challenge is to design
11 specialized receivers (Farhangian and Landry 2020; Khalife and Kassas 2019a) to extract navigation
12 observables from LEO satellites communication signals. The measurements extracted from these signals
13 include pseudorange (Ardito et al. 2019), carrier phase (Khalife and Kassas 2019b; Khalife et al. 2020),
14 and Doppler (Morales et al. 2019b; Orabi et al. 2021). Among them, the Doppler shift is generally easy
15 to measure from these modulated signals, e.g., BPSK, QPSK, OFDM signals (Khalife et al. 2021;
16 Neinavaie et al. 2021; Psiaki and Slosman 2019). Besides, the Doppler measurement is less affected by
17 multipath than pseudorange and unaffected by cycle slips compared with carrier phase. Moreover, the
18 Doppler measurements can also be used for velocity estimation (Chen et al. 2013; Petovello 2015), orbit
19 and attitude determination (Capdeville et al. 2016; Jayles et al. 2010; Park et al. 2011). Therefore, the
20 Doppler shifts have attracted widespread attention in LEO-based navigation.

21 In LEO-based Doppler positioning, the pioneering early Transit navigation system (Parkinson 1996)
22 can solve the stationary receiver position with an accuracy of 200~300 meters by iteratively fitting the
23 Doppler curve of a satellite pass (Lehtinen 2002). However, a single positioning requires a complete
24 observation of 10~16 minutes, which limits its applications in real-time and high-dynamic services.
25 Levanon (1998) proposed an instantaneous positioning approach using one or two LEO satellites based
26 on the round-trip-delay and round-trip-Doppler measurements, while the horizontal position accuracy is
27 in the order of 10 km. In recent years, some scholars have studied the performance of Doppler positioning
28 using LEO constellations, such as Iridium NEXT (Benzerrouk et al. 2019; Farhangian and Landry 2020;
29 Orabi et al. 2021; Tan et al. 2019), ORBCOMM (Farhangian and Landry 2020; Khalife and Kassas 2019a;

Orabi et al. 2021), and Starlink (Neinavaie et al., 2021). In their studies, the positions and velocities of LEO satellites are mostly propagated with Two-line Element (TLE) files and SGP4 model. However, their position and velocity errors can be up to several kilometers and a few meters per second respectively, if the TLE file is not updated for a long time (Jiang et al. 2022; Khalife and Kassas 2019a; Khalife et al. 2020). Undoubtedly, this will degrade the positioning performance significantly. Besides, these literatures assume that the satellite and receiver clock drifts do not change with time (Khalife and Kassas, 2019a; Farhangian and Landry, 2020; Neinavaie et al., 2021; Orabi et al., 2021). However, this is not necessarily the case. Under this assumption, they did not estimate the receiver's and each satellite's clock drift individually, but lumped them into one term, i.e. the difference between the receiver's and each LEO satellite's clock drifts (Khalife and Kassas, 2019a; Neinavaie et al., 2021; Orabi et al., 2021). Moreover, most of them ignored the ionospheric and tropospheric delay rates in the measurement due to their negligible amounts compared with the errors in satellite's velocities (Khalife and Kassas, 2019a; Farhangian and Landry, 2020; Orabi et al., 2021). Nevertheless, these errors should be carefully considered in Doppler-based system, or they will reduce the position precision (Psiaki 2021).

Therefore, to analyze the main error sources of LEO-based Doppler positioning thoroughly, we propose a Doppler-only point-solution algorithm for a LEO-based navigation system. In the LEO-based navigation system, the receiver can obtain signals from a large number of satellites simultaneously. However, the case is not necessarily in opportunistic navigation frameworks with the existing LEO communication constellations. It is unlikely to listen to eight or more satellites simultaneously from one given LEO communication constellation due to their specific beam patterns and overlap possibilities (Psiaki 2021). Thus, we propose a LEO-based navigation system based on the assumption that LEO satellites can be modified to be equipped with navigation payloads and broadcast signals to support navigation capabilities. Besides, the satellites positions, velocities and clock drifts can be obtained from ephemeris. The receiver clock drift, position and velocity can be estimated simultaneously solely from Doppler shifts using the point-solution algorithm.

In the following sections, the principle of Doppler-only point-solution algorithm, and the Zero-Difference (ZD) and Between-Satellite-Difference (BSD) Doppler Positioning methods are presented. Then, the data simulation and positioning processing strategies are described. Subsequently, the main error sources and the performance of Doppler positioning with LEO satellites are analyzed. Afterwards,

the point-solution method is tested for its sensitivity to initial position error. Finally, the main conclusions of this paper are drawn.

Method and algorithm

Principle of positioning using Doppler observations

Doppler effect is caused by the relative movement between the transmitter and receiver, making the received signal frequency different from the transmitted signal frequency. This difference can be described as (Braasch and Dierendonck 1999):

$$D = f_R - f_T = \pm \frac{v_{los}}{c} \cdot f_T = \pm \frac{v_{los}}{\lambda_{f_T}} \quad (1)$$

where D is the Doppler shift; f_R and f_T are the received and transmitted frequency, respectively; c is the speed of light; λ_{f_T} is the wavelength of transmitted signal; v_{los} is the relative velocity magnitude between the transmitter (satellite) and receiver in the line-of-sight (LOS) direction. The Doppler shift is positive if transmitter and receiver are moving toward, while negative if they are moving away. The velocity magnitude is also referred to range rate in the context of Doppler positioning (Braasch and Dierendonck 1999; Lehtinen 2002):

$$v_{los} = (\vec{v}^s - \vec{v}_r) \cdot \frac{\vec{p}^s - \vec{p}_r}{\|\vec{p}^s - \vec{p}_r\|} = \frac{d\rho}{dt} = \dot{\rho} \quad (2)$$

where $\vec{v}^s$ and $\vec{v}_r$ are the satellite and receiver velocity vector; $\vec{p}^s$ and $\vec{p}_r$ are the satellite and receiver position vector; ρ and $\dot{\rho}$ are the pseudorange and the first derivative of pseudorange with respect to time, respectively. In the context of range-based navigation systems, the pseudorange measurement equation is modeled as:

$$P_r^s = \|\vec{p}^s - \vec{p}_r\| + c \cdot (\delta t_r - \delta t^s) + T_r^s + I_{r,f}^s + d_r^s + M_p^s + \varepsilon_p \quad (3)$$

where $\|\vec{p}^s - \vec{p}_r\|$ is the geometric distance between the satellite and the receiver; δt^s and δt_r are the satellite and receiver clock offset, respectively; T_r^s and $I_{r,f}^s$ are the signal propagation delay through the troposphere and ionosphere, respectively; d_r^s is other frequency independent errors (such as antenna phase center offset); M_p^s is the multipath effect; ε_p is the noise in pseudorange observation. Among

them, the multipath effect can be eliminated by selecting a suitable receiver location or assuming a perfect condition. Moreover, the rapid changes in LEO satellites' geometric distribution can effectively mitigate multipath effects (Li et.al. 2019b). Thus, the first derivative of pseudorange with respect to time is:

$$-\lambda_{f_r} \cdot D_r^s = \dot{P}_r^s = (\vec{v}^s - \vec{v}_r) \cdot \frac{\vec{p}^s - \vec{p}_r}{\|\vec{p}^s - \vec{p}_r\|} + c \cdot (\delta \dot{t}_r - \delta \dot{t}^s) + \dot{T}_r^s + \dot{I}_{r,f}^s + \varepsilon_{\dot{p}} \quad (4)$$

where D_r^s is the Doppler measurement; $\dot{P}_r^s$ is the pseudorange rate; $\vec{v}^s$, $\vec{p}^s$ and $\delta \dot{t}^s$ are the satellite velocity, position and clock drift, which can be obtained from ephemeris. $\vec{v}_r$, $\vec{p}_r$ and $\delta \dot{t}_r$ are the receiver velocity, position and clock drift, which need to be estimated. $\dot{T}_r^s$ and $\dot{I}_{r,f}^s$ are the tropospheric and ionospheric delay rates, which can be corrected by models or estimated as unknowns. Moreover, the ionospheric delay rates can be eliminated through combining observations of different frequencies. $\varepsilon_{\dot{p}}$ is the noise in Doppler observation.

Given simultaneous Doppler measurements from M LEO satellites, the pseudorange rate equation of the i^{th} LEO satellite is:

$$-\lambda_{f_r} \cdot D_r^{s,i} = \dot{P}_r^{s,i} = (\vec{v}^{s,i} - \vec{v}_r) \cdot \frac{\vec{p}^{s,i} - \vec{p}_r}{\|\vec{p}^{s,i} - \vec{p}_r\|} + c \cdot (\delta \dot{t}_r - \delta \dot{t}^{s,i}) + \dot{T}_r^{s,i} + \dot{I}_r^{s,i} + \varepsilon_{\dot{p}}^i \quad (5)$$

where $D_r^{s,i}$ is the Doppler shift of the i^{th} satellite; $\vec{v}^{s,i} = [v_x^{s,i}, v_y^{s,i}, v_z^{s,i}]^T$, $\vec{p}_i^s = [x^{s,i}, y^{s,i}, z^{s,i}]^T$ and $\delta \dot{t}^{s,i}$ are the i^{th} satellite's velocity, position and clock drift, respectively; $\vec{v}_r = [v_{r_x}, v_{r_y}, v_{r_z}]^T$, $\vec{p}_r = [x_r, y_r, z_r]^T$ and $\delta \dot{t}_r$ are the receiver's velocity, position and clock drift, respectively. When there are only 7 unknowns (receiver's velocity, position and clock drift) to be solved for, the solution will be obtained using at least 7 different satellites at the same time. Ignoring the higher-order term, linearize the equation (5) at the initial value $X_0 = [x_r^0, y_r^0, z_r^0, v_{r_x}^0, v_{r_y}^0, v_{r_z}^0, \delta \dot{t}_{r,0}]$, the linearized observation equation can be expressed as:

$$\begin{aligned} \Delta \dot{p}_i &= \dot{p}_i - \dot{p}_i^0 \\ &= \frac{\Delta v_{x_i}}{\rho_i^0} \cdot (\bar{e}_{x_i}^2 - 1) \cdot \Delta x_i + \frac{\Delta v_{y_i}}{\rho_i^0} \cdot (\bar{e}_{y_i}^2 - 1) \cdot \Delta y_i + \frac{\Delta v_{z_i}}{\rho_i^0} \cdot (\bar{e}_{z_i}^2 - 1) \cdot \Delta z_i \\ &\quad - \bar{e}_{x_i} \cdot \Delta V_{r_{x_i}} - \bar{e}_{y_i} \cdot \Delta V_{r_{y_i}} - \bar{e}_{z_i} \cdot \Delta V_{r_{z_i}} + c \cdot \Delta \delta \dot{t}_{r,i} + \varepsilon_{\dot{p}_i} \end{aligned} \quad (6)$$

where $\dot{p}_i$ is the pseudorange rate of the i^{th} satellite; $\dot{p}_i$ and $\dot{p}_i^0$ are the initial pseudorange and

1 pseudorange rate; $\Delta v_{x_i} = v_x^{s,i} - v_{r_x}^0$, $\Delta v_{y_i} = v_y^{s,i} - v_{r_y}^0$ and $\Delta v_{z_i} = v_z^{s,i} - v_{r_z}^0$ are the relative velocities
2 between the satellite and the receiver; $\vec{e}_i = [\vec{e}_{x_i}, \vec{e}_{y_i}, \vec{e}_{z_i}]$ is the unit LOS vector from the satellite to
3 the user; Δx_i , Δy_i , Δz_i , $\Delta V_{r_{x_i}}$, $\Delta V_{r_{y_i}}$, $\Delta V_{r_{z_i}}$ and $\Delta \delta t_{r,i}$ are the corrections to position, velocity,
4 and clock drift of the receiver, respectively. Combining the M satellites' observations, the least-square
5 iteration equation and the design matrix G can be expressed as:

$$6 \quad \Delta \dot{\rho} = \begin{bmatrix} \Delta \dot{\rho}_i \\ \Delta \dot{\rho}_j \\ \vdots \\ \Delta \dot{\rho}_k \end{bmatrix} = G \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta V_{r_x} \\ \Delta V_{r_y} \\ \Delta V_{r_z} \\ \Delta \delta t_r \end{bmatrix} + \begin{bmatrix} \varepsilon_{\dot{\rho}_i} \\ \varepsilon_{\dot{\rho}_j} \\ \vdots \\ \varepsilon_{\dot{\rho}_k} \end{bmatrix} = G \cdot \Delta X_r + \varepsilon_{\dot{\rho}} \quad (7)$$

$$7 \quad G = \begin{bmatrix} \frac{\Delta v_{x_1}}{\rho_1} (\vec{e}_{x_1}^2 - 1) & \frac{\Delta v_{y_1}}{\rho_1} (\vec{e}_{y_1}^2 - 1) & \frac{\Delta v_{z_1}}{\rho_1} (\vec{e}_{z_1}^2 - 1) & -\vec{e}_{x_1} & -\vec{e}_{y_1} & -\vec{e}_{z_1} & c \\ \frac{\Delta v_{x_2}}{\rho_2} (\vec{e}_{x_2}^2 - 1) & \frac{\Delta v_{y_2}}{\rho_2} (\vec{e}_{y_2}^2 - 1) & \frac{\Delta v_{z_2}}{\rho_2} (\vec{e}_{z_2}^2 - 1) & -\vec{e}_{x_2} & -\vec{e}_{y_2} & -\vec{e}_{z_2} & c \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\Delta v_{x_m}}{\rho_m} (\vec{e}_{x_m}^2 - 1) & \frac{\Delta v_{y_m}}{\rho_m} (\vec{e}_{y_m}^2 - 1) & \frac{\Delta v_{z_m}}{\rho_m} (\vec{e}_{z_m}^2 - 1) & -\vec{e}_{x_m} & -\vec{e}_{y_m} & -\vec{e}_{z_m} & c \end{bmatrix}_{m \times 7} \quad (8)$$

8 Using the matrix methods, a correction to the initial value is computed:

$$9 \quad \Delta X_r = (G^T G)^{-1} G^T (\Delta \dot{\rho} - \varepsilon_{\dot{\rho}}) \quad (9)$$

10 Then, the solution is obtained by adding the calculated correction to the initial value $X_0 + \Delta X_r$,
11 and the algorithm iterates until it converges.

12 **Doppler positioning DOP (DDOP)**

13 The 3D positioning error of standard point positioning can be expressed as (Guan et al. 2020):

$$14 \quad \sigma_{3D} = GDOP \cdot \sigma_{URE} \quad (10)$$

15 where σ_{URE} is the user ranging error including clock error, atmospheric effect, multipath error, etc.

16 GDOP is the geometric dilution of precision and can reflect the positioning error caused by the geometric
17 distribution of visible satellites. Similarly, the Doppler positioning error and DDOP can be (Morales-
18 Ferre et al. 2020):

$$19 \quad \sigma_{3D, Doppler} = DDOP \cdot \sigma_{URE, Doppler} \quad (11)$$

$$DDOP = \sqrt{\text{trace}[(G^T G)^{-1}]} \quad (12)$$

where DDOP is Doppler dilution of precision, and can be obtained by the design matrix G .

Usually, to achieve a meter-level positioning accuracy in standard point positioning, the GDOP is considered usable when it is less than 3 (Han et al. 2014). Similarly, in Doppler positioning, one way to improve the positioning accuracy is to reduce the DDOP.

In the context of GNSS Doppler positioning, the velocity and altitude of GNSS satellite are about 3.9 km/s and 20200 km. Thus, the item $\Delta v_{x_1} \cdot (\bar{e}_{x_1}^2 - 1) / \rho_1$ of G matrix is in the order of 10^{-4} and the GNSS-DDOP can be up to the order of 10^4 . In LEO Doppler positioning, the item $\Delta v_{x_1} \cdot (\bar{e}_{x_1}^2 - 1) / \rho_1$ is in the order of 10^{-2} and the LEO-DDOP is in the order of 10^2 . The LEO-DDOP is less than the GNSS-DDOP by two orders, and the accuracy of LEO Doppler positioning could be better than GNSS Doppler.

There are several ways to reduce DDOP as many satellites are visible. One way is to select a group of satellites from a large number of visible satellites, and the goal is to improve the structure of G matrix and minimize DDOP. The other way is doing the BSD, which will eliminate some errors related to the receiver and reduce DDOP simultaneously. Both methods have been widely used in GNSS positioning. However, the purpose of using BSD in GNSS is to eliminate the receiver-related errors, not for improving the DOP.

Between-Satellite-Difference Doppler Positioning

The BSD Doppler positioning algorithm is described in this section. Firstly, this method needs to choose a satellite from all the visible satellites as the reference. There are many ways to choose the reference satellite. One way is to select the satellite with the maximum elevation angle, the other way is to choose the one with the largest signal-to-noise ratio. Then, doing the single difference between every other satellite and the reference satellite. The BSD pseudorange rate observation equation is:

$$\begin{aligned} -\lambda_{f_r} \cdot (D_r^i - D_r^j) &= \dot{P}_r^{s,i} - \dot{P}_r^{s,j} \\ &= (\bar{v}^i - \bar{v}_r) \cdot \frac{\bar{p}^i - \bar{p}_r}{\|\bar{p}^i - \bar{p}_r\|} - (\bar{v}^j - \bar{v}_r) \cdot \frac{\bar{p}^j - \bar{p}_r}{\|\bar{p}^j - \bar{p}_r\|} + c \cdot (\delta \dot{t}^j - \delta \dot{t}^i) + (\dot{T}_r^i - \dot{T}_r^j) + (\dot{I}_{r,f}^i - \dot{I}_{r,f}^j) + \varepsilon_{\dot{\rho}}^i - \varepsilon_{\dot{\rho}}^j \end{aligned} \quad (13)$$

where the superscript j represents the reference satellite; the superscript i represents the i^{th} visible satellite except for the reference satellite. $\delta \dot{t}^j - \delta \dot{t}^i$ is the clock drift difference between the reference satellite and every other satellite; $\dot{T}_r^i - \dot{T}_r^j$ and $\dot{I}_{r,f}^i - \dot{I}_{r,f}^j$ are the difference in tropospheric and

ionospheric delay rates between every other satellite and the reference satellite, which can be corrected by models. Ignoring the higher-order term, the linearized observation equation is:

$$\begin{aligned}\Delta\dot{\rho}_i - \Delta\dot{\rho}_j &= (\dot{\rho}_i - \dot{\rho}_i^0) - (\dot{\rho}_j - \dot{\rho}_j^0) \\ &= \left(\frac{\Delta v_{x_i}}{\rho_i} (\bar{e}_{x_i}^2 - 1) - \frac{\Delta v_{x_j}}{\rho_j} (\bar{e}_{x_j}^2 - 1) \right) \cdot \Delta x_i + (\bar{e}_{x_j} - \bar{e}_{x_i}) \cdot \Delta V r_{x_i} \\ &\quad + \left(\frac{\Delta v_{y_i}}{\rho_i} (\bar{e}_{y_i}^2 - 1) - \frac{\Delta v_{y_j}}{\rho_j} (\bar{e}_{y_j}^2 - 1) \right) \cdot \Delta y_i + (\bar{e}_{y_j} - \bar{e}_{y_i}) \cdot \Delta V r_{y_i} \\ &\quad + \left(\frac{\Delta v_{z_i}}{\rho_i} (\bar{e}_{z_i}^2 - 1) - \frac{\Delta v_{z_j}}{\rho_j} (\bar{e}_{z_j}^2 - 1) \right) \cdot \Delta z_i + (\bar{e}_{z_j} - \bar{e}_{z_i}) \cdot \Delta V r_{z_i} + \varepsilon_{\dot{\rho}}^i - \varepsilon_{\dot{\rho}}^j\end{aligned}\tag{14}$$

The least-square iteration equation and the design matrix G^{BSD} can be expressed as:

$$\Delta\dot{\rho}_i - \Delta\dot{\rho}_j = \begin{bmatrix} \Delta\dot{\rho}_1 - \Delta\dot{\rho}_j \\ \Delta\dot{\rho}_2 - \Delta\dot{\rho}_j \\ \vdots \\ \Delta\dot{\rho}_{m-1} - \Delta\dot{\rho}_j \end{bmatrix} = G^{BSD} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta V r_x \\ \Delta V r_y \\ \Delta V r_z \end{bmatrix} + \begin{bmatrix} \varepsilon_{\dot{\rho}}^1 - \varepsilon_{\dot{\rho}}^j \\ \varepsilon_{\dot{\rho}}^2 - \varepsilon_{\dot{\rho}}^j \\ \vdots \\ \varepsilon_{\dot{\rho}}^{m-1} - \varepsilon_{\dot{\rho}}^j \end{bmatrix} = G^{BSD} \Delta V_r + \varepsilon_{\dot{\rho}}\tag{15}$$

$$G^{BSD} = \begin{bmatrix} \frac{\Delta v_{x_1}}{\rho_1} (e_{x_1}^2 - 1) - \frac{\Delta v_{x_j}}{\rho_j} (e_{x_j}^2 - 1) & \frac{\Delta v_{x_{m-1}}}{\rho_{m-1}} (e_{x_{m-1}}^2 - 1) - \frac{\Delta v_{x_j}}{\rho_j} (e_{x_j}^2 - 1) & \cdots \\ \frac{\Delta v_{y_1}}{\rho_1} (e_{y_1}^2 - 1) - \frac{\Delta v_{y_j}}{\rho_j} (e_{y_j}^2 - 1) & \frac{\Delta v_{y_{m-1}}}{\rho_{m-1}} (e_{y_{m-1}}^2 - 1) - \frac{\Delta v_{y_j}}{\rho_j} (e_{y_j}^2 - 1) & \cdots \\ \frac{\Delta v_{z_1}}{\rho_1} (e_{z_1}^2 - 1) - \frac{\Delta v_{z_j}}{\rho_j} (e_{z_j}^2 - 1) & \frac{\Delta v_{z_{m-1}}}{\rho_{m-1}} (e_{z_{m-1}}^2 - 1) - \frac{\Delta v_{z_j}}{\rho_j} (e_{z_j}^2 - 1) & \cdots \\ -(\bar{e}_{x_1} - \bar{e}_{x_j}) & \cdots & -(\bar{e}_{x_{m-1}} - \bar{e}_{x_j}) \\ -(\bar{e}_{y_1} - \bar{e}_{y_j}) & \cdots & -(\bar{e}_{y_{m-1}} - \bar{e}_{y_j}) \\ -(\bar{e}_{z_1} - \bar{e}_{z_j}) & \cdots & -(\bar{e}_{z_{m-1}} - \bar{e}_{z_j}) \end{bmatrix}^T_{6 \times (m-1)}\tag{16}$$

By doing the single difference between satellites, the clock drift of the receiver is cancelled, and the errors related to satellites are also diminished. Therefore, the BSD approach can benefit Doppler positioning in both reduced DDOP and diminished errors in observations.

Simulation and experiment

Data simulation

As most LEO constellations are designed for communication and still under construction (e.g., Starlink, OneWeb and Kuiper), there are no public data released for navigation. Thus, we have to simulate the ephemeris and ground observations for LEO satellites.

Among all of the LEO constellations, Starlink is a typical heterogeneous mega-constellation, which consists of different orbital shells. Every orbital shell is a walker constellation (Walker 1984) and its geometric configuration can be described by the total number of satellites, the number of orbital planes, the orbital altitude and inclination. In order to investigate the influence of different constellations on positioning, two Starlink-like LEO constellations are used and their orbital shells parameters (Zhang et al. 2022) are shown in Table 1. The 1584 satellites of Constellation A are evenly distributed on the first orbital shell, which consists of 72 orbital planes with an altitude of 550 km. Given the orbital inclination of 53° , this constellation cannot guarantee the simultaneous visibility of seven or more satellites at locations within 30° of the North and South poles. With a cut-off elevation angle of 5° , the number of visible satellites at covered locations is 30~70. Constellation B has 4408 satellites and consists of 5 orbital shells at altitudes of 540~570 km. By adding satellites with inclinations of 70° and 97.6° , Constellation B can guarantee the simultaneous visibility of 60 or more satellites at every location in the globe.

Table 1 Orbital shells parameters of Starlink-like Constellation A and Constellation B (Zhang et al. 2022)

Constellation	Shell	Total number of satellites	Number of orbital planes	Orbital inclination ($^\circ$)	Orbital altitude (km)
A	1	1584	72	53	550
	1	1584	72	53	550
	2	1584	72	53.2	540
B	3	720	36	70	570
	4	348	6	97.6	560
	5	172	4	97.6	560

In the ephemeris simulation, the initial orbital elements of each satellite are calculated according to the parameters in Table 1. Then, the satellite position and velocity at any time are propagated using orbital force models. The N-Body perturbation employs JPL DE405 (Earth, Sun, Moon, Mercury, Venus, Mars, Jupiter, Saturn, Uranus, Neptune, Pluto) (Standish 1998). The other orbital models include high order earth gravity (120×120 of the EIGEN 6C model), solar radiation pressure (box-wing model), atmospheric drag (DTM94 for atmospheric density), tides and relativistic effects (IERS 2010 convention) (Berger et al. 1998; Marshall et al. 1992; Petit and Luzum 2010; Reid et al. 2018; Reigber et al. 2005). Finally, some random errors are added to approximate the real orbit. Noted that the satellite's position

and velocity coordinates are simulated in the Earth Centred Interval (ECI) frame and then transformed into the Earth Centred Earth Fixed (ECEF) frame.

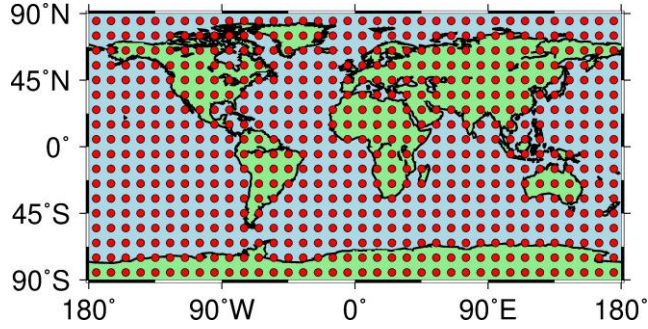


Fig. 1 Distribution of 648 stations with a spatial resolution of 10°

Table 2 Data simulation and Doppler Positioning processing strategies

Items	Simulation strategy	Processing strategy
Date	April 1 st , 2022	
Signal frequency	C-band: $f_1=5$ GHz, $f_2=7$ GHz	
Cut-off elevation angle	5°	
Satellite position and velocity	Initial orbital elements + orbital force models + random error	From ephemeris
Satellite clock	Using the CSACs and the atomic clock model	From ephemeris
Receiver position	Stationary	Estimated as constant
Receiver clock	No	Estimated as white noise process
Tropospheric delay (rate)	Saastamoninen + GPT3 + VMF3	
Ionospheric delay (rate)	NeQuick-G model	

Assuming that LEO satellites are equipped with the Clip Scale Atomic Clocks (CSACs), the satellites clocks are simulated based on a typical atomic clocks model (Tavella 2008):

$$\delta t^s = a_0 + a_1(t - t_0) + \frac{1}{2} a_2(t - t_0)^2 + \psi(t) \quad (17)$$

$$\delta \dot{t}^s = a_1 + a_2(t - t_0) + \varepsilon(t) \quad (18)$$

where δt^s and $\delta \dot{t}^s$ represent the time and frequency variations of clocks; a_0 , a_1 and a_2 are clock bias, clock drift and frequency drift at time t_0 ; $\psi(t)$ and $\varepsilon(t)$ are the stochastic noises, respectively. In simulation, a_0 and a_1 are randomly obtained with a magnitude of 1 us and 10^{-12} s/s,

respectively, while the higher-order term is ignored. The stochastic noise is obtained from a zero-mean Gaussian distribution with a standard deviation of 10^{-12} s. The reasons why we use the CSACs are: i) GNSS satellites are usually equipped with high-performance atomic clocks, whose power consumption, size and weight are not suitable for low-cost LEO satellites; ii) GPS receivers or Oven-controlled Crystal Oscillators (OCXOs) are more common for LEO satellites launched in early times. However, the clock bias and drift of ORBCOMM satellite with a typical OCXO may be up to 70 m and 0.2 m/s over a 10-minute period (Kassas et al. 2019); iii) as a compromise, the CSACs are designed to pursue the performance of atomic clock, while achieving low-cost, less power consumption, reduced weight and size (Reid et al. 2016). Their clock stability at an averaging time of 100 minutes is about 10^{-12} s/s, and they have been proposed to be used in the Cubesat Handling of Multisystem Precision Time Transfers (CHOMPTT) mission (Nydam et al. 2017), and for small satellite navigation systems (Batista et al. 2012; Conklin et al. 2014; Rybak et al 2021).

Then, 648 stations with a spatial resolution of 10° are chosen globally and their distribution is shown in Fig. 1. The data simulation and Doppler positioning processing strategies are shown in Table 2. Among them, the cut-off elevation angle is set to 5° and the receiver is assumed to measure a valid Doppler shift if the satellite lie above a 5° elevation at the truth receiver position. The existing LEO constellations transmit their signals at different bands, such as ORBCOMM VHF-band, Boeing V/C-band, Starlink Ku/Ka-band. Among them, the Ku/Ka/V-band are used mainly for high signal-to-noise-ratio (SNR) and large bandwidth. However, signals at these high frequencies will experience significant attenuation effects such as free space loss, rainfall attenuation and foliage attenuation (Irsigler et al. 2004; Ma et al. 2022). Thus, it would be better to have signals in lower frequencies if they are used for navigation (Psiaki 2021). However, signals at lower frequencies will suffer more significant ionospheric effects, which is inversely proportional to the squared frequency. Therefore, the C-band is chosen as the tradeoff because it suffers smaller ionospheric effects than VHF/L-band, and smaller attenuation than Ku/Ka/V-band.

As to the observations simulation, firstly, the geometric distance between the ground station (receiver) and the satellite is calculated. Then, the simulated pseudorange, Doppler, and carrier phase observations are obtained by adding various error terms. For example, the atmospheric error terms are calculated using the tropospheric and ionospheric models with the assumption that they can match the truth atmospheric errors. Finally, some random errors are added into the observations to approximate the real observation

noises. Previous studies show that the Doppler shifts can be measured precisely (Neinavaie et al. 2021; Psiaki and Slosman 2019) and they set the accuracy of measured Doppler shift to be 0.1 Hz (Jiang et al. 2022; Kanamori et al. 2022). Thus, in this paper, the Doppler measurement random errors are obtained from a zero-mean Gaussian distribution with a standard deviation of 0.1 Hz.

In the following part, the receiver position and velocity are solved in ECEF coordinate, and then converted into a North-East-Up frame centered at the true receiver position. The Root Mean Square (RMS) is used to evaluate the positioning accuracy.

Error analysis

This section discusses the main error sources in LEO-based Doppler positioning, including satellite position and velocity errors, the ionospheric and tropospheric delay rate errors.

Satellite position and velocity errors

One of the main error sources of accuracy degradation is the satellite position and velocity errors. The precision orbit determination (POD) of GNSS satellites has been widely studied and the precision of ephemeris can be in centimeter-level or better. However, this level of errors maybe challenging to achieve for LEO satellites without onboard GNSS receivers and atomic clocks. The position and velocity errors of LEO satellites extrapolated by the TLE-SGP4 can be up to 3 km and 3.47 m/s (Jiang et al. 2022; Kassas et al. 2019a), which will reduce the positioning accuracy significantly. The ORBCOMM satellite can obtain their more precise position and velocity from onboard GPS receiver (Morales et al. 2019a). Neinavaie et al. (2021) adjusted the TLE epoch time to account for ephemeris errors, and the 2-D position error can be down to 10 m. Thus, the impact of satellite orbital errors in Doppler positioning should be considered in detail. In simulation, the random errors with different orders are added into the satellite position or velocity of every axis. The positioning results are shown in Table 3, where N, E, and U denote north, east, and up directions, respectively.

Table 3 Doppler positioning accuracy with different orbital errors

Satellite position	Satellite velocity	RMS-N	RMS-E	RMS-U	RMS-3D
error (m)	error (cm/s)	(m)	(m)	(m)	(m)
0.0	0.0	1.379	2.920	4.070	5.195
0.03	0	1.378	2.921	4.070	5.196

0	0.3	1.384	2.950	4.091	5.230
3	0	1.602	3.516	4.364	5.829
0	3	1.749	3.934	5.109	6.681
30	0	8.196	18.776	18.261	27.444
0	30	10.953	25.828	29.017	40.361
300	0	80.279	178.167	183.805	268.277
0	300	108.827	255.685	286.723	399.285
3000	0	790.073	1865.394	1773.159	2692.212
0	3000	1085.340	2561.022	2842.690	3977.145

Results show that the satellites' position error of several meters and velocity error of several centimeters per second would reduce the positioning performance significantly. The 30 m/s position error and 30 cm/s velocity errors reduce the positioning accuracy by 4.28 and 6.77 times respectively. What's worse, the 300 m position error and 3 m/s velocity error increase the positioning error by about 50~77 times. Worse still, the errors of 3 km and 30 m/s will cause about 3 km positioning error. Therefore, it is necessary to carry out research on the POD of LEO constellations. Reid et al. (2018) have discussed this issue from the aspects of ground station density, inter-satellite links (ISL) observations, on-board GPS receivers and accelerometer measurements.

Ionospheric delay rates

Neglecting higher order terms, the ionospheric delay is:

$$I_{r,f}^s = \frac{40.3}{f^2} \cdot \int N \cdot dl = \frac{40.3}{f^2} \cdot STEC = \frac{40.3}{f^2} \cdot \alpha(E) \cdot VTEC \quad (19)$$

where $I_{r,f}^s$ is the ionospheric delay in meters unit; f is frequency; N is the electron density; $\alpha(E)$ is the factor related to the satellite elevation angle E . $STEC$ is the slant total electron content in the signal propagation path, while $VTEC$ is the vertical total electron content. Their unit is TECU. And 1TECU equals to 10^{16} electrons/m². The ionospheric delay rate $\dot{I}_{r,f}^s$ can be defined as:

$$\dot{I}_{r,f}^s = \frac{40.3}{f^2} \cdot \frac{dSTEC}{dt} \quad (20)$$

where $\dot{I}_{r,f}^s$ is the ionospheric delay rate, which can be accessible through derivation, or simply doing difference between epochs:

$$\dot{I} = \frac{I(t + \Delta t) - 2I(t) + I(t - \Delta t)}{2\Delta t} \quad (21)$$

where t is the current epoch; Δt is the sampling interval. This formula actually calculates the average ionospheric delay rate over a period of time. Thus, the Δt should be as small as possible to ensure that the ionospheric delay rate can be obtained.

Since ionospheric errors are frequency-dependent, the ionospheric delays of ORBCOMM satellites signals (~137 MHz) are 2~3 orders of magnitude greater than that of GPS L1 signal (1575.42 MHz) (Khalife and Kassas 2019b). Besides, ORBCOMM satellites travel at high speed and their elevation angles change very fast, which leads to over 18 m/s in the ionospheric delay over 100 minutes (Kassas et al. 2019). Moreover, the ionospheric delay rates are affected by solar activities, orbital altitudes, and signal frequencies. The following section analyzes the ionospheric delay rates. The simulation strategies are shown in Table 4. Among them, the ionospheric delays are calculated using the NeQuick-G model (Aragon-Angel et al. 2019), and the ionospheric delay rates are obtained by doing difference between epochs using equation (21). The a_{i0} , a_{i1} , a_{i2} are three coefficients to represent solar activity, which are transmitted to the users in the Galileo navigation broadcast message.

Table 4 Ionospheric delay rates simulation strategies

Items	Strategy
Date	April 1 st
Solar activity	High (236.831641, -0.393629, 0.004028)
coefficients (a_{i0} , a_{i1} , a_{i2})	Medium (121.129893, 0.351254, 0.013464)
	Low (2.580271, 0.127628, 0.025275)
Satellite orbital altitude	350, 400, 450, 500, 550, 1000, 2000, 5000, 10000, 20000, 30000 km
Station	Static, BLH=(358.72°, 16.25°, 78.11 m)
Cut-off elevation angle	5°
Signal frequency	123 MHz, 1.5 GHz, 5 GHz, 15 GHz, 20 GHz, 35 GHz
Ionospheric delay	NeQuick-G model
Ionospheric delay rate	Doing difference between epochs, $\Delta t = 0.1$ s

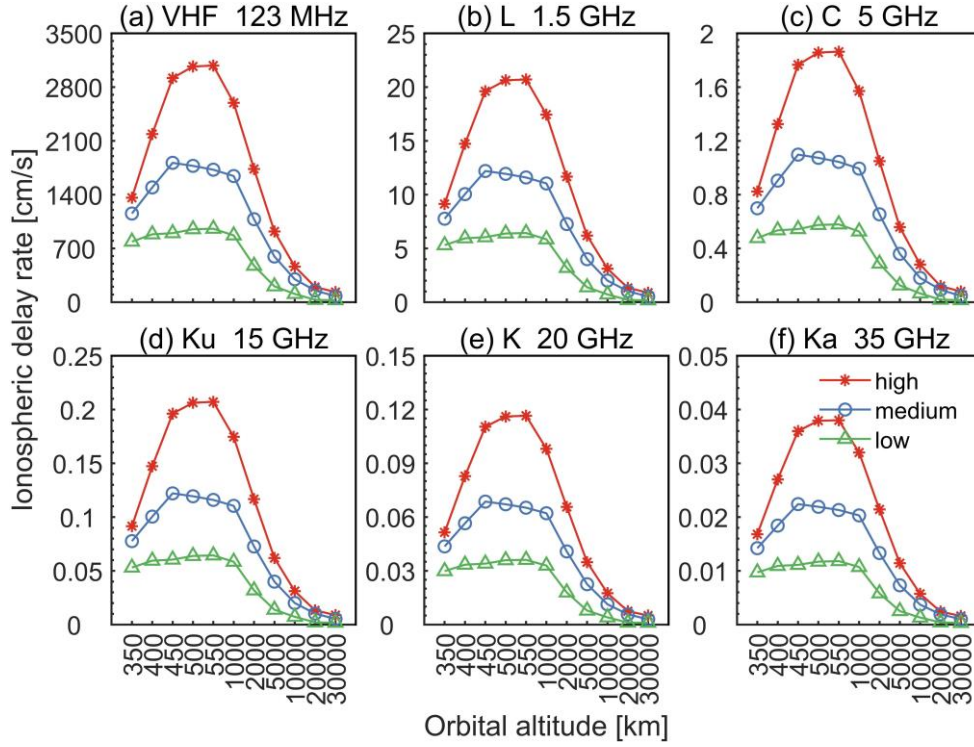


Fig. 2 Ionospheric delay rates of different signals under different solar activities

The results (**Error! Reference source not found.**) show that the ionospheric delay rates in high solar activity are the largest, while medium and low solar activities follow. The ionospheric delay rates of LEO satellites (350~2000 km altitudes) are much larger than that of satellites at altitudes of 5000~30000 km. Thus, the ionospheric delay rates of LEO satellites should be discussed in detail. Take ORBCOMM satellites (825 km altitude and VHF-band) as an example, its ionospheric delay rate ranges from 9 m/s to 29 m/s under different solar activities. If the LEO satellites broadcast signals in L-band, there will be 3~21 cm/s ionospheric delay rates, which will cause about 6~42 m errors in Doppler positioning. That's to say, the VHF/L-band signals ionospheric delay rates cannot be neglected in LEO positioning. In equation (20), the ionospheric delay rate is inversely proportional to the square of frequency. Thus, the higher the frequency, the smaller the ionospheric delay rate. If the C-band signal is used for LEO navigation, there will be 0.3~2 cm/s ionospheric delay rates, which will cause about 0.6~4 m errors in Doppler positioning. For Ka/K/Ku-band, their ionospheric delay rates are sharply reduced to below 0.21 cm/s due to their high frequencies.

Then, we add the C-band signal ionospheric delay rates into the observation simulation. The results (Fig. 3) show that if the ionospheric errors are not corrected in Doppler positioning, there will be obvious offsets in N and U directions. The offset in U direction is more than 1 m, which is consistent with the above theoretical value.

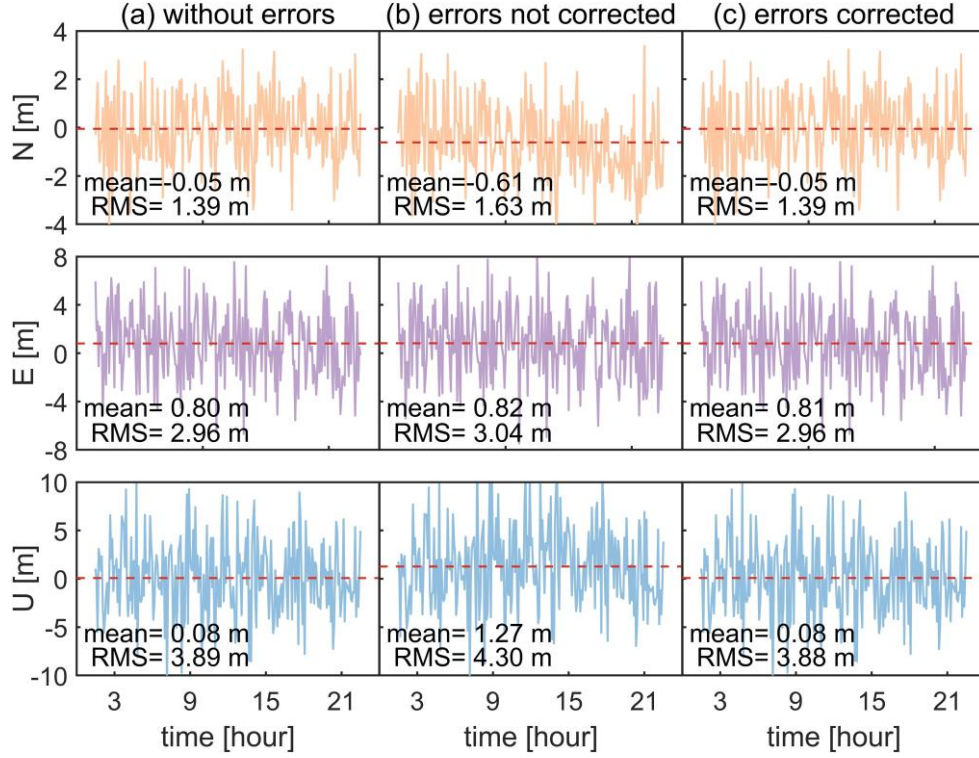


Fig. 3 Doppler positioning offsets caused by C-band signal ionospheric delay rates, three rows represent the positioning accuracy for N, E, and U directions, respectively.

There are many ways to correct the ionospheric delay rate, such as dual-frequency combination of observations, estimated as parameters in positioning or refined ionospheric delay models. For dual-frequency receivers, the ionospheric delay can be mitigated by ionosphere-free (IF) combinations. Similarly, the ionospheric delay rate can also be mitigated using ionosphere-rate-free (IRF) combinations.

The IRF combination $\dot{P}_{IRF}$ is:

$$\dot{P}_{IRF} = \frac{1}{f_1^2 - f_2^2} (f_1^2 \cdot \dot{P}_{r,1}^s - f_2^2 \cdot \dot{P}_{r,2}^s) \quad (22)$$

where f_1 and f_2 are the frequencies of signals, $\dot{P}_{r,1}^s$ and $\dot{P}_{r,2}^s$ are pseudorange rates on the different frequencies. When the noise at different frequencies are the same ($\delta_{\dot{\rho}} = \delta_{\dot{\rho}_1} = \delta_{\dot{\rho}_2}$), the combination noise is:

$$\delta_{\dot{\rho}_{IRF}} = \sqrt{\frac{f_1^4 + f_2^4}{(f_1^2 - f_2^2)^2}} \cdot \delta_{\dot{\rho}} \quad (23)$$

If f_1 and f_2 are GPS-L1 and GPS-L2, the combination noise is about 2.978 times larger than $\delta_{\dot{\rho}}$ (Li and Huang 2013). When the frequencies are 5 and 7 GHz in C-band, the combination noise will be amplified by about 2.292 times. Although the dual-frequency combination of observations can correct

the ionospheric delay rate, the combination noise is magnified.

Tropospheric delay rates

The tropospheric delay (T_r^s) is:

$$T_r^s = m(E) \cdot ZTD \quad (24)$$

where $m(E)$ is the mapping function related to the elevation angle E . ZTD is the Zenith Total Delay, which can be corrected by models like Hopfield, Saastamoinen or others. Similarly, the tropospheric delay rate can be derived by doing the difference between epochs:

$$\dot{T}_r^s = \frac{T_r^s(t + \Delta t) - 2T_r^s(t) + T_r^s(t - \Delta t)}{2\Delta t} \quad (25)$$

where t is the current epoch; Δt is the sampling interval. When Δt is small enough, $\dot{T}_r^s$ can be considered as the instantaneous tropospheric delay rate. For geodetic measurements, it is usually assumed that the tropospheric delay is elevation dependent and varies little over a short period of time (Bevis et al. 1992). In this case, the tropospheric delay rate mainly depends on the mapping function, which is related to the elevation angle rate. In a 10 minutes duration, the elevation angle rate of ORBCOMM satellites are 60 times larger than that of GPS (Kassas et al. 2019). Therefore, it is necessary to analyze the tropospheric delay rates of satellites with different orbital altitudes. The simulation strategies are shown in Table 5.

Table 5 Tropospheric delay rates simulation strategies

Items	Strategy
Date	April 1 st , 2022
Station	Static, BLH=(358.72°, 16.25°, 78.11 m)
Satellite orbital altitude	350, 400, 450, 500, 550, 1000, 2000, 5000, 10000, 20000, 30000 km
Cut-off elevation angle	7°
Tropospheric delay	ZTD: Saastamoninen+GPT3; $m(E)$:VMF3 mapping function
Tropospheric delay rate	Difference between epochs, $\Delta t=0.1s$

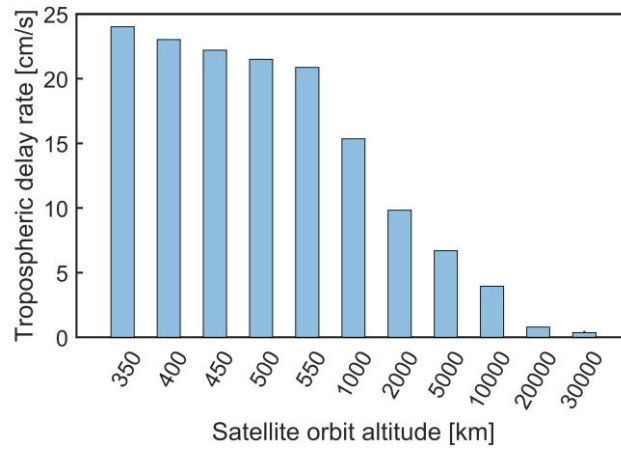


Fig. 4 Tropospheric delay rates of satellites with different orbital altitudes

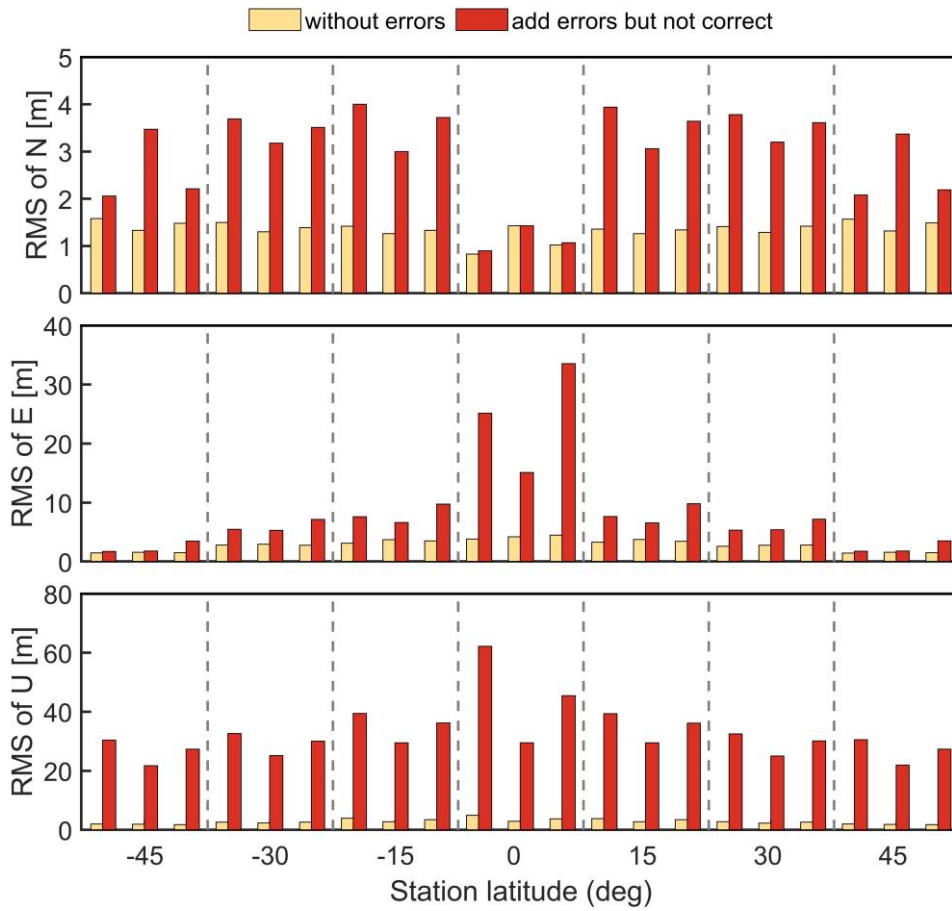


Fig. 5 Doppler positioning errors caused by tropospheric delay rates

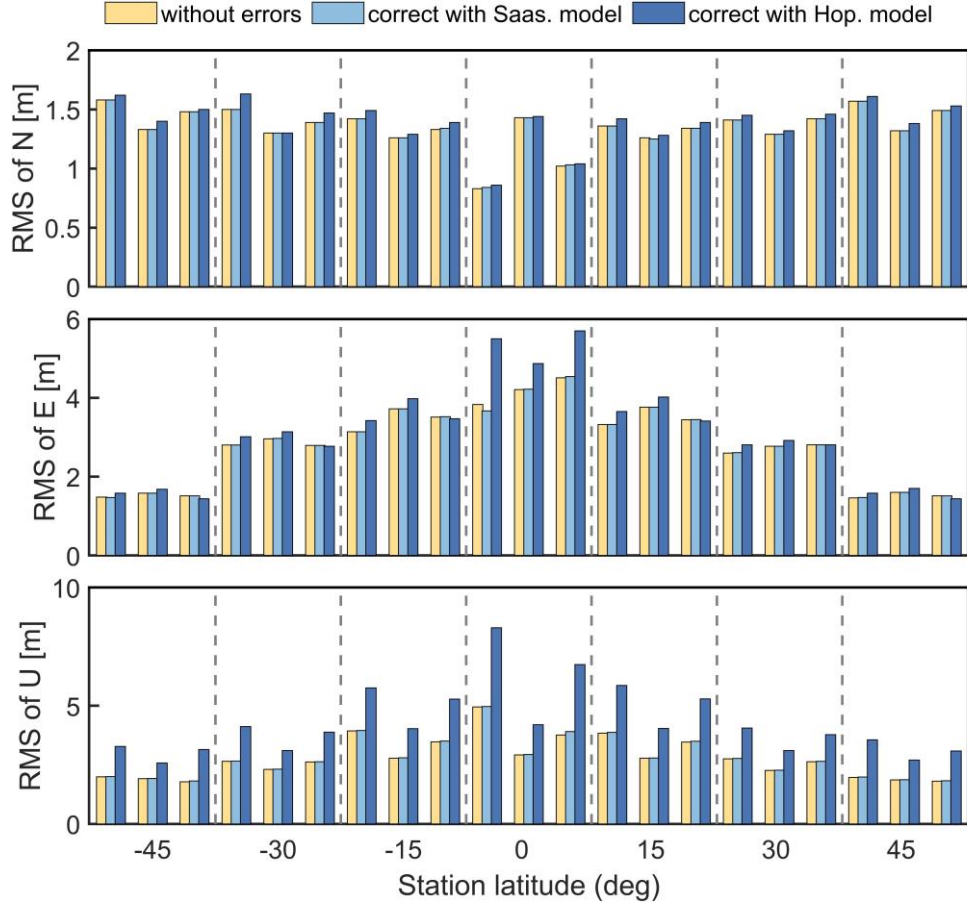


Fig. 6 Tropospheric error corrections using different models

Results (Fig. 4) show that the tropospheric delay rates are decreased with the increasing orbital altitudes. For LEO satellites with altitudes of 350~550 km, their tropospheric delay rates are 21~25 cm/s, which will cause 42~50 m errors in Doppler positioning. Then, the tropospheric delay rates are added into the observation simulation. The positioning errors caused by tropospheric errors before and after error corrections are shown in Fig. 5 and Fig. 6. If there are no error corrections, the positioning accuracies in all three directions will deteriorate sharply. Among them, the U direction errors are increased by 20~60 m. Compared the results with and without corrections, the Saastamoninen and Hopfield model are able to correct the tropospheric errors effectively.

Zero-Difference Doppler Positioning

In this section, we analyze the performance of the ZD Doppler positioning based on the principle of positioning using Doppler observations. The ephemeris and observations simulation, and positioning processing strategies are shown in Table 2.

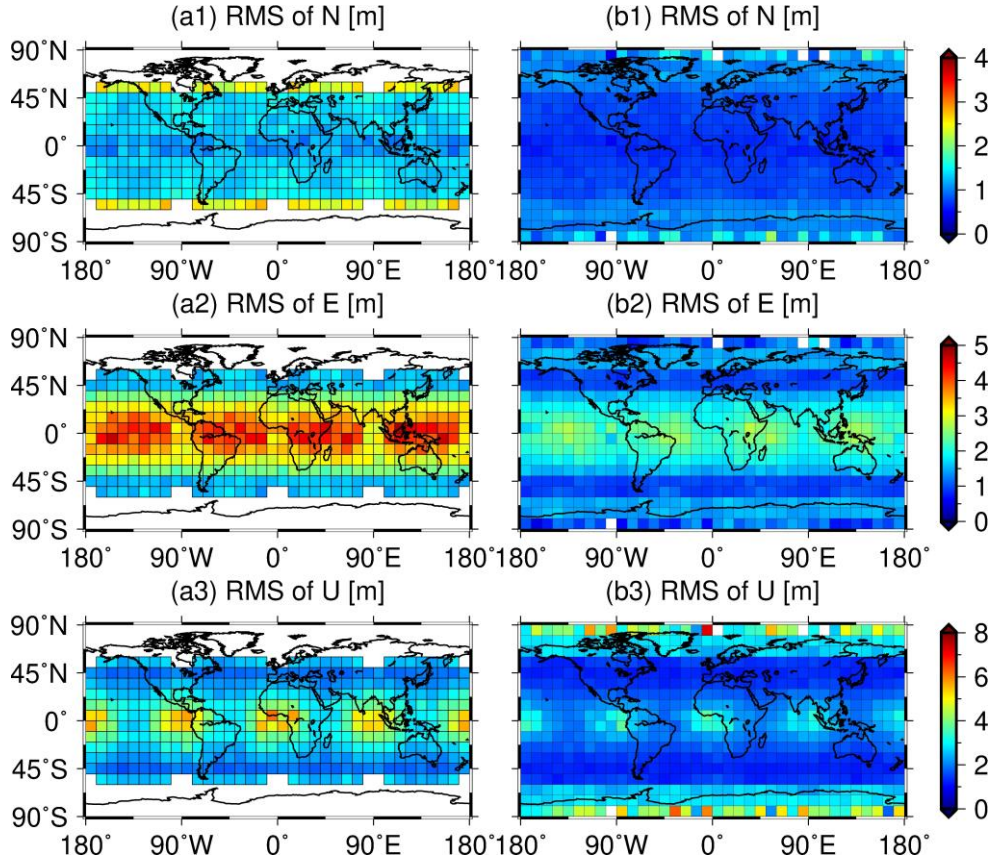


Fig. 7 ZD Doppler positioning results with Constellation A (left) and Constellation B (right). Three rows represent the positioning accuracy in N, E, and U directions, respectively. The white-colored zones indicate that there are no solution for these stations.

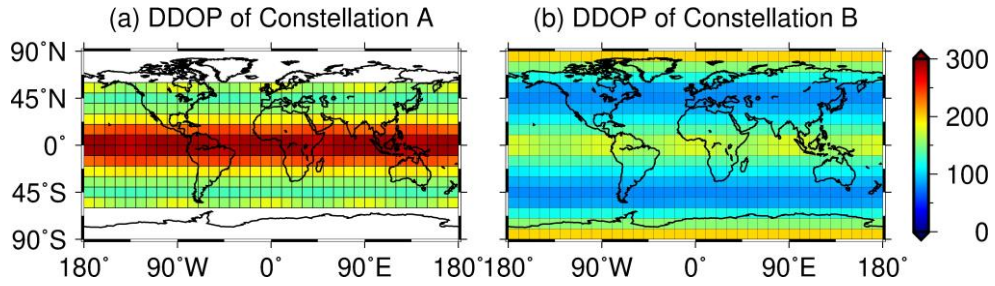


Fig. 8 DDOPs of global stations with Constellation A (a) and Constellation B (b).

It can be seen from Fig. 7 and Fig. 8 that the positioning error RMSs and DDOPs of stations in the different latitude zones are different. Specifically, the RMSs and DDOPs of stations in mid latitude zones are the smallest, followed by the low and high latitude zones. This phenomenon is thought to be related to the constellation configuration (Reid et al. 2018; Zhang et al. 2022). Then, we calculate the average RMSs of global stations. The average RMSs in N, E, U directions are 1.48, 2.73, 2.83 m with Constellation A, and 0.89, 1.53, 2.19 m with Constellation B.

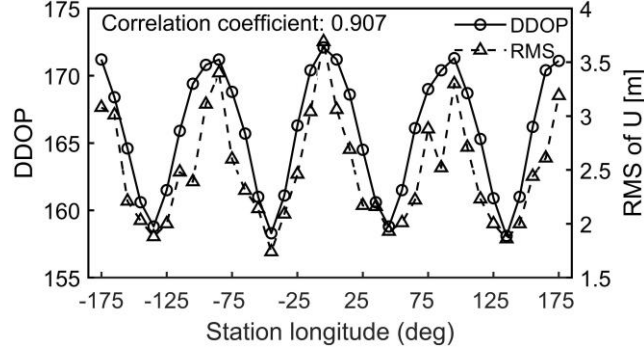


Fig. 9 DDOPs and U-RMSs of stations in the 5 ° N latitude

Besides, the results show that the spatial distribution of DDOPs are consistent with the RMSs. Then, we calculate the correlation between DDOPs and U-RMSs of stations in the 5° latitude. The correlation reaches up to 0.907. Thus, the distribution characteristics of DDOPs can reflect the influence of different constellations on ZD Doppler positioning.

Between-Satellite-Difference Doppler Positioning

This section analyzes the DDOP and accuracy of the BSD Doppler positioning. The data simulation and processing strategies are basically the same as above shown in Table 2. Note that the receiver clock is eliminated. Here, 22 stations are evenly chosen from the global stations, and the reference satellite is chosen with its maximum elevation angle. The results of the chosen 22 stations using ZD and BSD methods are shown in Fig. 10 and Table 6.

Table 6 Average RMS and DDOP of the chosen 22 stations with Constellation A

Positioning algorithm	N-RMS/m	E-RMS/m	U-RMS/m	DDOP
ZD Doppler positioning	1.44	2.91	2.95	201.78
BSD Doppler positioning	1.37	2.58	2.84	176.97

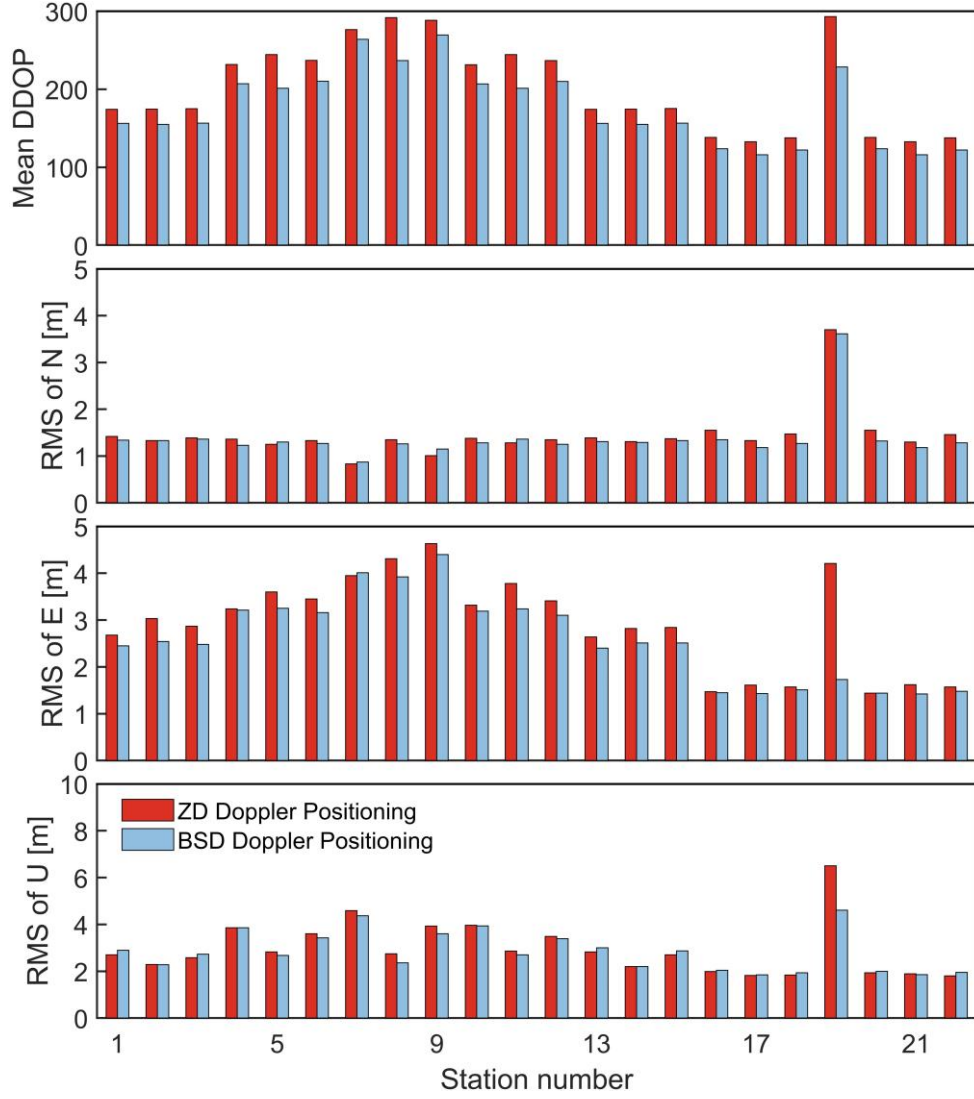


Fig. 10 Positioning results of chosen 22 stations using two methods with Constellation A

Results show that with Constellation A, the performance of BSD Doppler positioning is better than ZD Doppler positioning. The average DDOP is reduced by about 12.3%. The positioning accuracies in N, E, and U directions are improved by 16%, 4%, and 14%, respectively. The reasons for this are as follows: (i) by doing the single difference between satellites, the receiver clock drift is eliminated, and the error sources related to satellites are also diminished; (ii) the item $\Delta v_{x_1} \cdot (\vec{e}_{x_1}^2 - 1) / \rho_1$ in matrix G is in the order of 10^{-4} , which is far less than the last column c . As for the G^{BSD} matrix, the last column related to the receiver clock is eliminated. Then, each term of the G^{BSD} matrix is in the same order of magnitude, thus reducing the DDOP value.

When it comes to reducing DDOP, the satellite selection algorithm can also be used. Given that there are a large number of visible satellites, one feasible way is to select a group of satellites from all

visible satellites to minimize DDOP.

Sensitivity of initial position error

In GNSS positioning, even if the receiver's initial position value is set to be 0, the solution can still get convergent. However, the situation is different with LEO satellites due to their lower orbital height than GNSS. In LEO-Doppler positioning, the nonlinear least-square estimation need an appropriate initial position value to ensure convergence. For example, when using ORBCOMM satellite (825 km altitude) and Iridium Next satellite (780 km altitude), the receiver's position estimate was initialized at around 20~200 km away from the true position (Khalife and Kassas, 2019a; Farhangian and Landry, 2020; Orabi et al., 2021; Jiang et al., 2022). However, the influences of different orders of initial position value have not been considered in detail. In the following part, the LEO-based Doppler positioning is tested for its sensitivity to initial position error. In the test, the receiver's actual position is (6123320.353, -136480.099, 1773905.419) km in ECEF, and the initial errors are added in all directions. The test time is set to be from 1:30 to 22:30 with an interval of 30 s. Thus, there are 2521 epochs in total. The LEO satellite's attitude is 550 km, and the positioning results with different initial position errors are shown in Table 7.

Table 7 Positioning results with different initial receiver position errors

Initial error [km]	RMS-N [m]	RMS-E [m]	RMS-U [m]	Solution epochs
0	2.451	0.449	1.488	2521
10	2.451	0.449	1.488	2521
100	2.451	0.449	1.488	2521
300	2.452	0.448	1.489	2511
350	2.446	0.449	1.477	1824
400	2.507	0.378	1.548	82
405	2.509	0.538	1.716	5
410	No solution			
450	No solution			

The results show that the LEO-based Doppler positioning is sensitive to the initial position error. With an initial position error of 300 km or larger, the iteration process maybe divergent, resulting in the number of solution epochs decreases. With 410 km initial error, the iterative algorithm cannot converge

to the actual position, resulting in no solution. Therefore, to ensure convergence, the initialization issue should be paid more attention to in LEO-based Doppler positioning.

Conclusion

In this paper, we firstly derive the Doppler measurement equation and propose a Doppler-only point-solution algorithm for GNSS-like navigation systems operated in LEO. In this system, LEO satellites are assumed to be modified to support navigation capabilities, and their positions, velocities and clock drifts could be obtained from ephemeris. Assuming that LEO satellites are equipped with CSACs, their clock drifts are in the order of 10^{-12} s/s. The proposed Doppler-only point-solution algorithm can estimate the receiver clock drift, position and velocity simultaneously.

Secondly, we discuss the main error sources in LEO-based Doppler positioning. As one of the main error sources, satellites' position error of several meters and velocity error of several centimeters per second will reduce the positioning accuracy significantly. For example, the orbital errors of 300 m and 3 m/s will increase the positioning error by about 50~77 times. The ionospheric and tropospheric delay rates of LEO satellites are far greater than that of GNSS satellites. Besides, the ionospheric delay rates are affected by solar activities and signal frequencies. If the LEO satellites broadcast signals in C-band, their ionospheric delay rates will be 0.3~2 cm/s, which will cause about 0.6~4 m offsets in positioning if not corrected. The tropospheric delay rates of LEO satellites with altitudes of 350~550 km are 21~25 cm/s. If there are no error corrections, the positioning accuracy will deteriorate sharply by dozens of meters. Besides, it must be noted that there may exist mismatch between the true atmospheric effects and the models used in simulation.

Subsequently, we analyze the performance of ZD and BSD Doppler positioning methods. The ZD Doppler positioning can achieve a meter-level accuracy, with a global average RMS better than 3 m. This is comparable with pseudorange-based GNSS. By doing the single difference between satellites, the performance of BSD Doppler positioning is better than ZD Doppler positioning. As DDOP is reduced by 12.3%, the positioning accuracy gets improved by about 10%. These all benefit from the diminished errors and improved design matrix structure.

Afterwards, the LEO-based Doppler positioning is tested for its sensitivity to initial position error. If the initial position errors are larger than 300 km and the attitude of LEO satellite is 550 km, the solution may diverge from the actual position terribly, and the number of solution epochs will decrease sharply.

Thus, an appropriate initialization must be provided to ensure convergence in LEO positioning.

Owing to the fact that there are sufficient visible satellites with LEO constellations, the satellite selection algorithm will be developed to improve the positioning accuracy. Besides, given the encouraging results shown in this study, additional work should focus on designing the navigation payloads and signals to enable LEO satellites to support navigation capabilities.

Statements and Declarations

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All authors certify that they have no affiliations with or involvement in any organization or entity with any financial interest or non-financial interest in the subject matter or materials discussed in this manuscript.

Author contributions

All authors contributed to the study conception and design. Material preparation, data collection and analysis were performed by Chuang Shi, Yulu Zhang and Zhen Li. The first draft of the manuscript was written by Yulu Zhang and all authors commented on previous versions of the manuscript. All authors read and approved the final manuscript.

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