

**Supplementary information for “Geodesic hybridization and
lifetime engineering of non-Hermitian microcavity photons in
topologically curved space”**

Yongsheng Wang¹, Yuhao Ren², Xiaoxuan Luo¹, Bo Li¹, Zaoyu Chen¹, Zhenzhi Liu¹, Fu Liu¹, Yin
Cai¹, Yanpeng Zhang¹, Jin Liu² and Feng Li¹

*¹Key Laboratory for Physical Electronics and Devices of the Ministry of Education & Shaanxi
Key Lab of Information Photonic Technique, School of Electronic Science and Engineering,
Faculty of Electronic and Information Engineering, Xi'an Jiaotong University, Xi'an 710049,
China*

*²State Key Laboratory of Optoelectronic Materials and Technologies, School of Physics, Sun Yat-
sen University, Guangzhou, China*

1. PSOS graphs and side-viewed ray trajectories and wave simulations of modes.

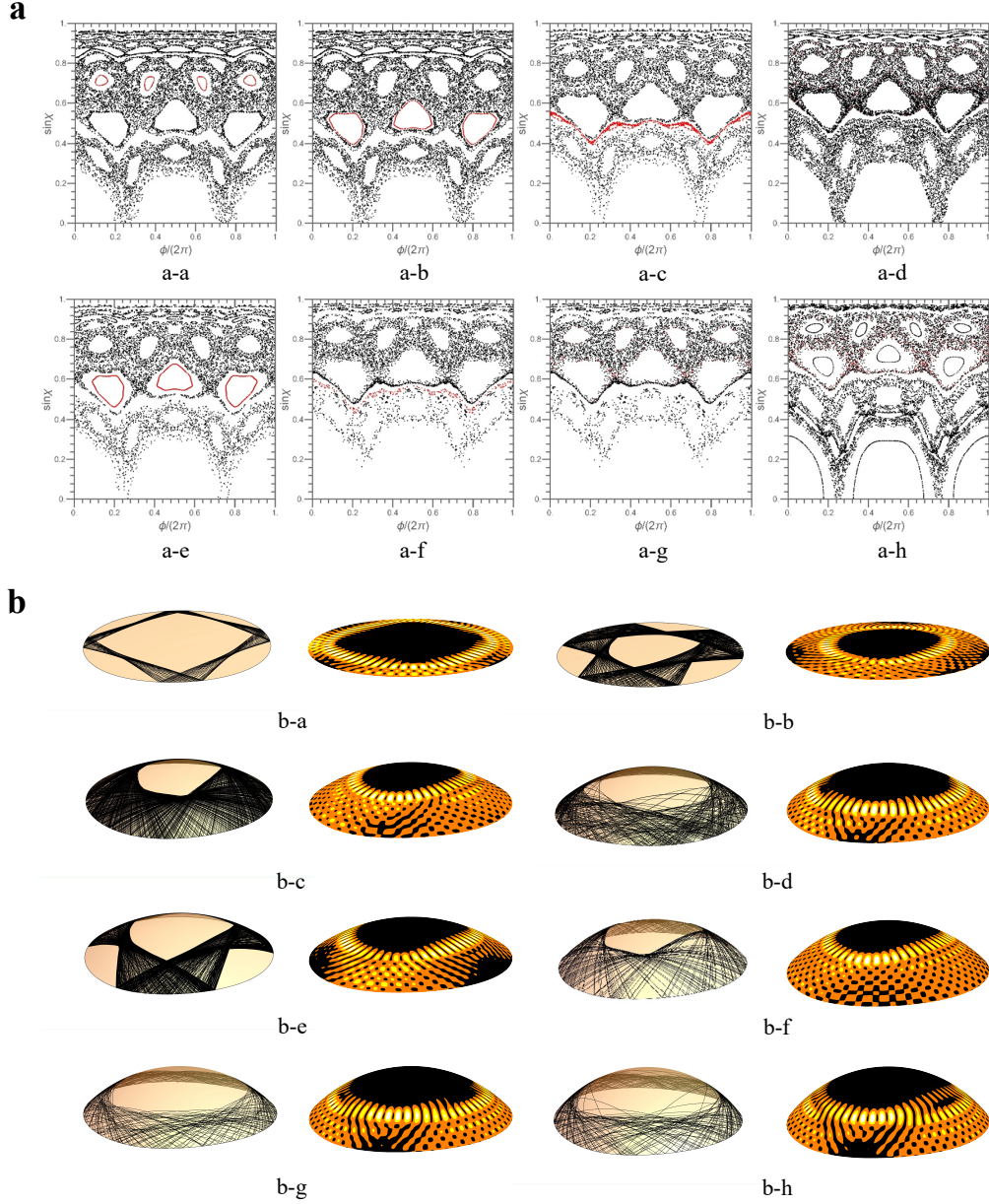


Fig. S1: Ray Dynamics and wave optics simulations of modes a-h in Fig. 3 of the main text. **a** The PSOSs of the curved microcavities supporting modes a-h [graphs from a-a to a-h], while the points corresponding to the trajectories in Fig. 3c of the main text are colored red, showing clearly the origin of each mode. **b** The side views of the light trajectories [left panels of b-a to b-h] and electric field distributions [right panels of b-a to b-h] of modes a-h.

2. Curvature-mediated photon lifetime engineering.

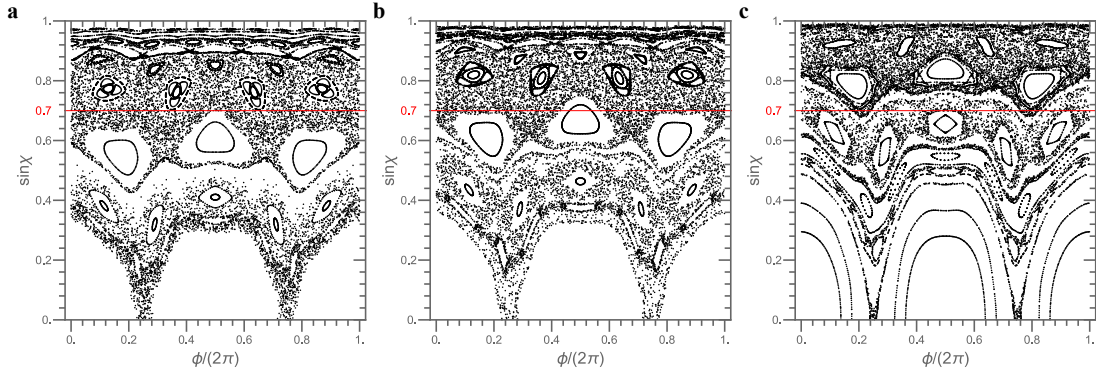


Fig. S2: PSOs of the Face cavity for $k=0.6$ (a), 0.8 (b) and 1.1636 (c). The red horizontal line represents $\sin\chi \approx 0.7$. It can be seen the shape of the islands (e.g. the 4-period) can change when they are pushed upwards to higher $\sin\chi$ by the space curvature.

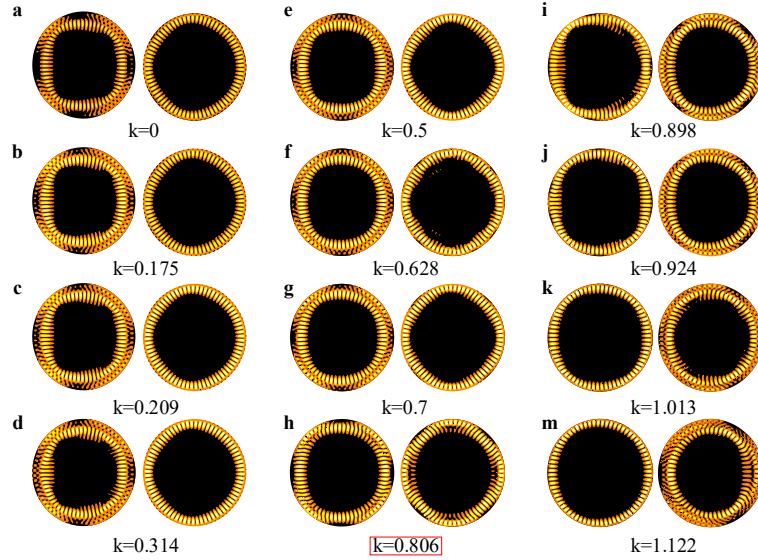


Fig. S3: Mode spatial profiles of the 4-period island (left) and the WGM-chaotic (right) modes at different values of k , while $k=0.806$ corresponds to the situation where the two bare modes are in resonance.

3. The fabrication of curved surface.

The sample of the curved surface is fabricated in 5 steps, which is illustrated in Fig. S4.

- (1) The GaAs substrate is cleaned with acetone, isopropanol and deionized water for 10min, and bake at 150 °C for 10min after the cleaning.
- (2) Spin coat the substrate with MaN-2410 photoresist at a speed of 3000 rpm, and bake at 90°C for 2min, reaching a thickness of about 1 micron.
- (3) Define the circular-shaped pattern using electron beam lithography (EBL), and then develop with MaD-525 to transfer the pattern to the photoresist.
- (4) Perform the reflow process: bake the sample at 160 °C for 5min after the development, and the photoresist melts into a hemispherical shape on the substrate surface.
- (5) Etch the sample using ICP until the photoresist is just fully removed. The shape and morphology of the photoresist is then transfer onto the GaAs substrate. The resulting substrate

displays a morphology of spherical surface, with a sharp circular boundary.

The SEM images of the sample is shown in Fig. S5 and Fig. S6.

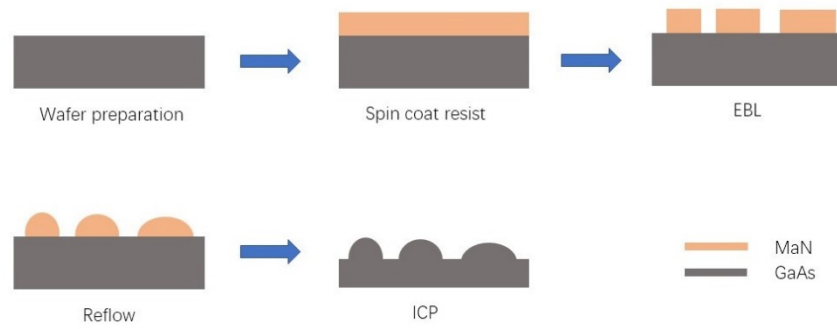


Fig. S4: Fabrication process of the curved surface on GaAs substrate.

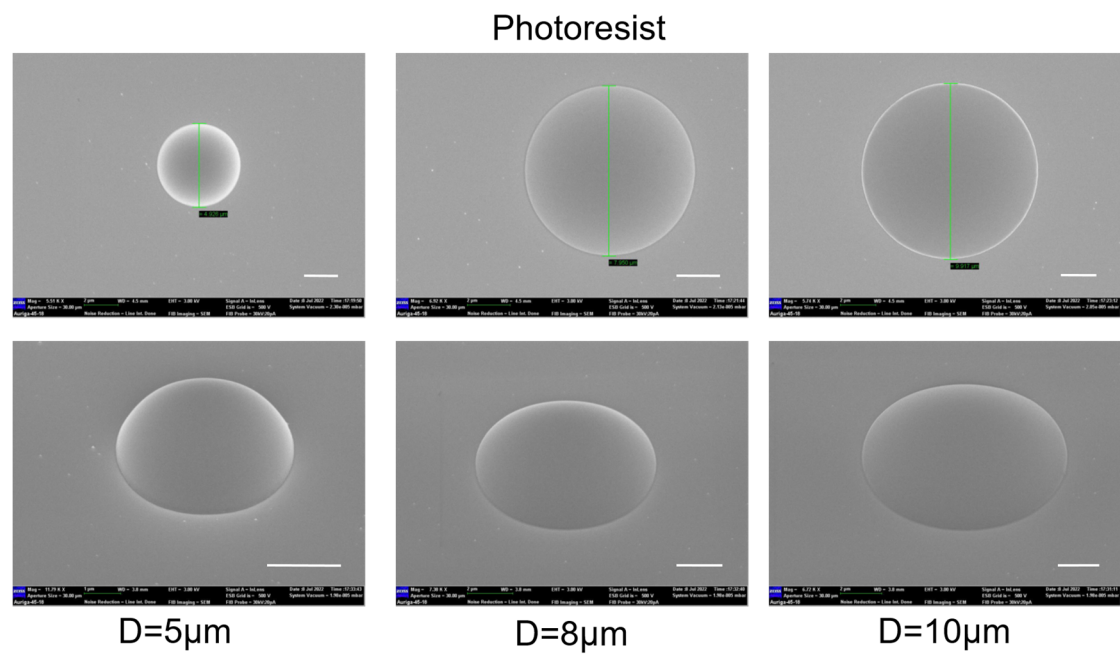


Fig.S5: The photoresist morphology after reflow (left to right: photoresist diameter of 5 μ m, 8 μ m, and 10 μ m), top view (upper panels) and the 45° side view (lower panels). The white horizontal bars in all figures represent 2 μ m.

After removal of photoresist

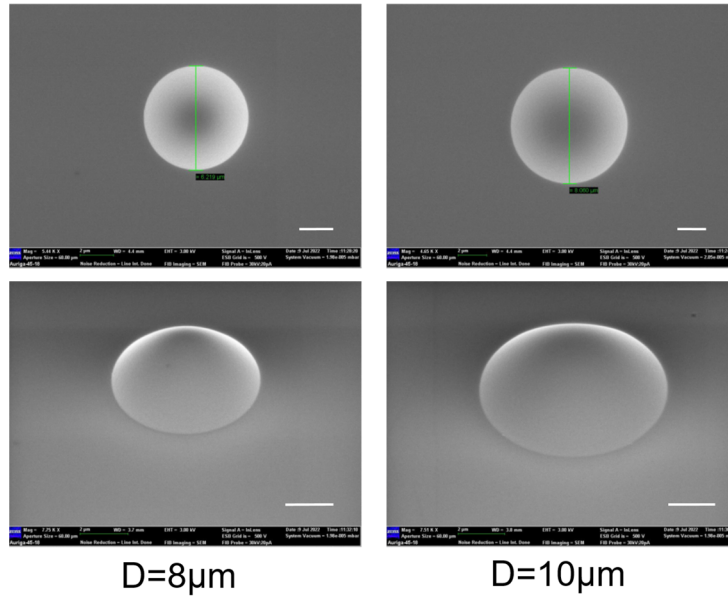


Fig.S6: The structure morphology after removing the photoresist with ICP etching (left to right: photoresist diameter of 8μm and 10μm, which yields 6.2μm and 8μm in diameter after the ICP etching), the top view (upper panels), and the 45° side view (lower panels). The white horizontal bars in all figures represent 2μm.