Supplementary material of

On dispersion curve coloring for mechanical metafilters

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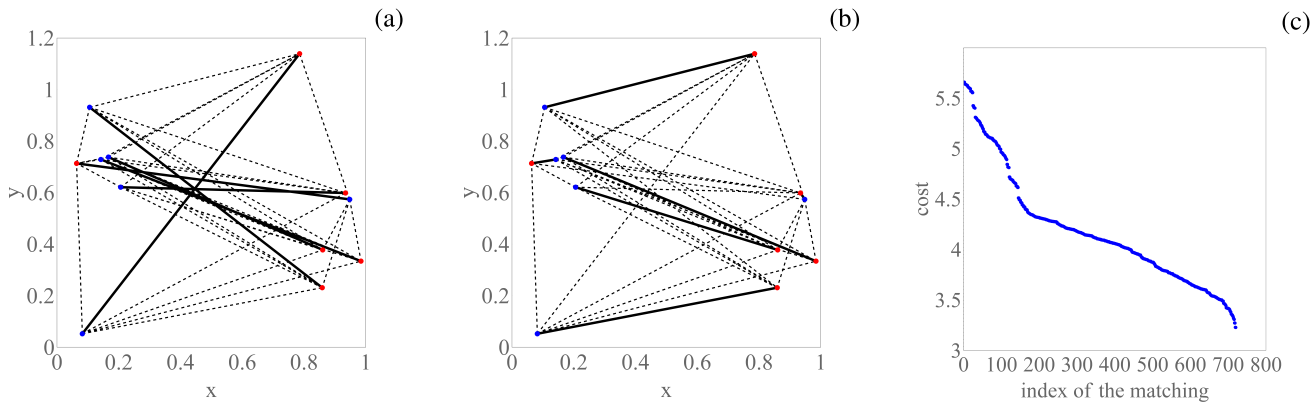
Euclidean bipartite matching problem

The next concepts from graph theory are recalled here (see also[1] for details). Let $G = (V, E, d)$ be a weighted undirected graph with vertex set V, edge set E, and the weight function $d : E \to \mathbb{R}$, where $d(u, v)$ represents the weight of the edge $(u, v) \in E$. A matching $M \subseteq E$ is a subset of edges such that every vertex in V is incident to at most one edge in M. The matching is perfect if every vertex in V is incident to exactly one edge in M. The cost of the matching has the expression $c \doteq \sum_{(u,v)\in M} d(u, v)$. The Euclidean bipartite matching problem is formulated as follows.

Problem 1. *Given a first set R of n red points and a second set B of n blue points in the plane, let $G = (R \cup B, E, d)$ be the complete weighted bipartite graph with the property that there is an edge between any two points if and only if they are of different colors. Additionally, for each $u \in R$ and $v \in B$, the weight $d(u, v)$ of the edge (u, v) is expressed by the Euclidean distance between the points u and v. The objective is to find a perfect matching in this graph having minimum cost c.*

For illustrative purposes, Supplementary Figure 1(a) reports the optimal solution to a two-dimensional version of the Euclidean bipartite matching problem. However, it is worth remarking that in Figure 6(b), its one-dimensional version is considered. Then, Supplementary Figure 1(b) highlights the worst such solution, i.e., the one corresponding to the largest cost. Finally, Supplementary Figure 1(c) reports the costs of all possible matchings.



Supplementary Figure 1. (a) Optimal solution to a two-dimensional version of the Euclidean bipartite matching problem. The edges corresponding to that matching are reported in bold. (b) Corresponding worst solution (i.e., having the largest cost c). The edges corresponding to that matching are reported in bold. (c) Cost of each matching. The costs are reported from the largest one (associated with part (b) of the figure) to the smallest one (associated with part (a) of the figure).

Application of Algorithm 1 to generalized eigenvector curves

In this section, the alternative application of Algorithm 1 to generalized eigenvector curves is briefly outlined. In this case, each such curve lies on a higher-dimensional space than the corresponding dispersion curve. Since there are no intersections of generalized eigenvector curves, at a first sight, in this alternative case the application of Algorithm 1 would be easier. However, this alternative approach is actually not straightforward, due to the following reasons.

- solving the Euclidean bipartite matching subproblems is computationally more expensive with respect to the original application of the algorithm, due to the larger dimension of the space;
- even in the simplest case in which each generalized eigenvalue has multiplicity 1, unit-norm generalized eigenvector curves are defined within a multiplicative complex scalar, which may depend on a parameter (e.g., on the curvilinear coordinate Ξ) when a general-purpose algorithm is applied to find numerically the generalized eigenvectors.

The last issue can be partly solved in the following case, in which one specific component (e.g., the first one) of all the generalized eigenvectors found numerically by changing the curvilinear coordinate, is always different from 0. In this case, by assuming that there exist continuous generalized eigenvector curves, one can pre-process the data by making each such component equal to a positive real number, possibly depending on the curvilinear coordinate. This is simply obtained by multiplying each numerically found generalized eigenvector by a suitable unit-norm complex number, making it possible to remove the phase from that component. With this pre-processing, Algorithm 1 has been applied to the set of generalized eigenvector curves, in the case of the same beam-lattice model considered in the present article (and also in the case of the other models investigated in[2]). Then, dispersion curves have been labeled in the same way as the labeled generalized eigenvector curves. The results found in this way have turned out to be the same as the one reported in Figures 8 and 9, but have required a larger computational effort to be obtained.

References

1. Vaidya, P. M. Geometry helps in matching. *SIAM J. on Comput.* **18**, 1201–1225 (1989).

2. Bacigalupo, A., Gnecco, G., Lepidi, M. & Gambarotta, L. Optimal design of low-frequency band gaps in anti-tetrachiral lattice meta-materials. *Compos. Part B: Eng.* **115**, 341–359 (2017).