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Sayan Sirimontree
Thammasat University

Chanachai Thongchom
Thammasat University

Peyman Roodgar Saffari
Thammasat University

Pouyan Roodgar Saffari
Thammasat University

Nima Refahati (refahati@damavandiau.ac.ir)
Islamic Azad University of Damavand

Thira Jearsiripongkul
Thammasat University

Suraparb Keawsawasvong
Thammasat University

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Effect of external mean flow on sound transmission loss of double-walled porous functionally graded magneto-electro-elastic sandwich plates

Sayan Sirimontree¹, Chanachai Thongchom¹*, Peyman Roodgar Saffari¹*, Pouyan Roodgar Saffari¹, Nima Refahati²*

¹Department of Civil Engineering, Faculty of Engineering, Thammasat School of Engineering, Thammasat University, Pathumthani 12120, Thailand
²Department of Mechanical Engineering, Damavand Branch, Islamic Azad University, Damavand P.O. Box 3971878911, Iran
³Department of Mechanical Engineering, Faculty of Engineering, Thammasat School of Engineering, Thammasat University, Pathumthani 12120, Thailand

*Corresponding author:

Chanachai Thongchom(Tchanach@engr.tu.ac.th)
Peyman Roodgar Saffari (peyman.saffari1364@gmail.com, rpeyman@engr.tu.ac.th)
Nima Refahati (refahati@damavandiau.ac.ir, nima.refahati1978@gmail.com)

Abstract

In the present study, the sound transmission loss through the air-filled rectangular double-walled sandwich smart magneto-electro-elastic (MEE) plates with porous functionally graded material (PFGM) core layer under initial external electric and magnetic potentials, and external mean airflow is studied using the third-order shear deformation theory (TSDT). Three states of uneven porosity distributions are considered for PFGM core layer which are supposed to vary along the in-plane and thickness directions based on the power-law model. The derivation of vibroacoustic equations in the form of coupled relations is realized by implementing Hamilton’s principle. An analytical approach, i.e. second velocity potential, is exploited to solve them in conjunction with double Fourier series, and the final result is the desired sound transmission loss (STL) equation. The developed solution is investigated in terms of its accuracy and precision via a comparison with other available data in existing research. Parameter studies reveal the impacts of the initial electric and magnetic potentials, porosity distributions, incident angles, acoustic cavity depth on STL through the double-walled sandwich smart MEE plates.

Keywords Sound Transmission Loss, Magneto-electro-elastic Plate, Porosity, Functionally Graded Material, Third-Order Shear Deformation Theory.

I. Introduction

Introducing the concept of porosity in other fields of sciences has resulted in interesting concepts such as metal foams that present various advantages such as superior energy absorption [1] and resistance to heat and impact [2]. The shape of the pores, pore strut or wall arrangement, surface area and surface roughness, in addition to porosity and pore size, determine the behavior of porous materials. It is, thus, mandatory to define the relation between pore features and physical behavior of such materials to be able to use them in different applications or studies. Until now, several experimental and numerical projects investigated the influence of the porous materials on
the dynamic problems, such as, free and forced vibration of beams [3], plates [4], and cylindrical shells [5]. Because of high stiffness-to-weight rations and high strength to-weight in plates, these significant structural components are applied in many engineering applications like aviation, buildings, submarines, and automotive [6–9]. Al Rjoub and Alshatnawi [10] investigated the influence of even porosity distribution on the variations of natural frequencies of functionally-graded material (FGM) cracked plate using The power law model. Formed from a combination of metal and ceramic, FGMs display a uniformly-varying distribution of properties along a desired path in the structure. Muc and Flis [11] studied the flutter characteristics and free vibrations of PFGM plates via the TSDT, analytical and Rayleigh-Ritz methods. Roodgar Saffari et al. [12–15] used NSGT in the framework of the first-order shear deformation shell assumption to study the free vibration of fluid-conveying BN nanotubes. Hashemi et al. [16] used Poincare–Lindstedt method (MPLM) to analyze the nonlinear free vibration behavior of in-plane bi-directional PFGM plate. Quan et al. [17,18] indicated the effect of the porosity coefficient, volume fraction index, and elastic medium on the nonlinear vibration of PFGM sandwich plate subjected to the blast loading. Thongchom et al. [19, 21] investigated tensile strength and modulus of elasticity nanocomposite and sound transmission loss of sandwich cylindrical shell

The capability of smart materials including piezoelectric (PZT), MEE, electrorheological fluid (ERF) to adapt their properties in reaction to environmental factors including electricity, magnetic, heat, and loading is one of the major reasons for the wide range of studies conducted on them [22,23]. MEE composites constitute a motivating type of composite smart materials which offer the advantages of both piezomagnetism and piezoelectricity. The ability provided by converting energy between different phases partly explains the interest of researchers into these materials [24–26]. Vinyas and Kattimani [27] presented a finite element method (FEM) to calculate the static parameters of FG MEE rectangular plates under the thermal environment. A simple velocity feedback control law for vibration suppression skew MEE plates utilizing active constrained layer damping (ACLD) based on the layerwise shear deformation theory was carried out by Vinyas [28]. Also, Vinyas [29] numerically analyzed the effect of porous properties on the frequency response of FGM MEE circular and annular plates taking account the third-order shear deformation theory TSDT and the FEM. Ebrahimi et al. [30] studied the effects of magnetic and electric potentials in conjunction with porosity volume fraction on the variations of FGM-MEE plate resting on elastic substrate applying TSDT. Arshid et al. [31] used the generalized differential quadrature method (GDQM) to investigate the natural frequencies of annular plate made up of FGM MEE under multi physical loads. Hamidi et al. [32] presented the dynamic behavior of MEE multilayer plates on the elastic substrate. Esayas and Kattimani [33] analyzed the influences of the porosity on the nonlinear vibration of FGM-MEE plates constricted layer damping patches. Sh et al. [34] used FSDT and finite element method (FEM) to study the geometrically nonlinear free vibration and transient problem of porous FGM-MEE plates. Dat et al. [35] investigated the effect of temperature increment, magnetic and electric potentials on the vibration behavior of sandwich MEE plate under blast loading.
The study on double-walled structures is inseparable from the research on double-walled plates as they are promising components in rapid transit, aerospace vehicles and double-glazed windows, among others [36]. Outstanding features of double-walled structures in mechanical and acoustic aspects have made them an attractive choice in different fields of science from civil engineering to aerospace [37–39]. The continuous research on their noise cancellation characteristics is one important area where their great potential is being exploited via empirical and analytical means. The literature is rich in studies focusing on the vibroacoustic features of noise transmission of single or double-walled shells and plates. A statistical energy analysis (SEA) was presented by Oliazadeh et al. [40] to estimate sound transmission loss (STL) through single-and double-walled thin rectangular plates considering absorbing material. A common method of evaluation of noise attenuation is the measurement of sound transmission loss. To this aim, the energy of the impinging sound waves is divided over that of the transmitted sound waves. The resulting ratio is usually expressed in dB. They validated their results with the experimental outcomes and demonstrated that filling the acoustic cavity between the double-walled plate with lightweight absorbing material like fiberglass rises the STL at the critical frequency. Xin et al. [41] studied the influence of external mean flow on the changes of STL of double rectangular plate via acoustic velocity potential. The STL through the triple-panel partition is compared with that of a double-panel partition by Xin and Lu [42] for clamped boundary conditions. Talebitooti et al. [43] employed two-variable refined plate theory for predicting the STL cross laminated composite plate in the presence of external mean flow. Based on the viscoelastic Zener model in the framework of FSDT, Amirinezhad et al. [44] analyzed the wave propagation across plate made of polymeric foam. The main results denoted that increasing damping reduces the stiffness and therefore reduces STL in high frequencies. Danesh and Ghadami [45] analytically investigated the effect of electric voltage of piezoelectric materials on the variation of STL through double-walled FGM piezoelectric plate using TSDT. They proved that Using Helium and Hydrogen gases for filling the acoustic cavity between the two piezoelectric plates has a substantial effect on the sound isolation performance. Recently, Hasheminejad and Jamalpoor [46] applied the classical plate theory (CPT) and multi-input multi-output sliding mode control (MIMOSMC) plan to improve the STL of simply supported hybrid smart double sandwich plate including PZT and ERF materials. The enhancement of STL through a double-plate structure around the mass-air-mass resonance frequency is presented by Mao [47] using electromagnetic shunt damper (EMSD). Wang et al. [48] analyzed the effect of the external mean flow on the STL of a metamaterial plate submerged in moving fluids. Based on the TSDT and Rayleigh integral, Gunasekaran et al. [49] investigated the vibroacoustic behavior of FGM graphene reinforced composite plate subjected to the nonuniform edge loads. Ghassabi and Talebitooti [50] applied three-dimensional (3D) theory of elasticity to study the acoustic response of MEE shell structure.

Despite the growing attention to different aspects of single or double-walled plates with respect to their noise transmission, the authors found no relevant study on the STL of a finite double-walled simply supported sandwich PFGM-MEE plate under the action of harmonic plane sound waves in the presence of external mean flow based on the assumptions of TSDT. The current
research tries to cover this gap and provide insightful explanations on the behavior of such structures. The material characteristics of PFGM core layer change slowly through the thickness via power-law scheme. Using Hamilton’s principle and TSDT, the coupled vibroacoustic equations are derived. One of the methods to solve the derived vibroacoustic equations is the sound velocity potential approach. This method is employed here to solve the coupled equations of the considered double-walled plate with the equations of acoustic cavity included.

2. Preliminary formulations

The problem schematic is displayed in Fig. 1. One should note the employed Cartesian coordinates defined for the double-walled rectangular sandwich MEE (a × b) plate encompassing the acoustic cavity baffled in a wall of infinite dimensions. Also, a plane sound wave of time-harmonic nature, with the azimuth angle \(0° \leq \alpha \leq 360°\) and elevation angle \(0° \leq \beta \leq 90°\), impinges the top surface of upper MEE plate. It is supposed that each sandwich MEE plate is composed of two identical BaTiO3-CoFe2O4 piezomagnetic layers with uniform thickness \(h_m\) and PFGM core layer with uniform thickness \(h_c\). Furthermore, each MEE plate is subjected to electric \(\Upsilon(x, y, z, t)\) and magnetic \(\psi(x, y, z, t)\) potentials. As can be seen, the acoustic cavity depth is denoted with the symbol \(L\). As can be seen, an external mean flow passes in the incident field with a constant velocity \(V\).

2.1. Constitutive relations

To derive the dynamic equations governing the motion, based on the TSDT which considers both rotary inertia and shear deformation in the transverse direction, the classical displacement field \((U, V, W)\) in Cartesian coordinates for each sandwich MEE plate is employed in the form [51]

\[
\begin{align*}
U_i(x, y, z, t) &= u_{0i}(x, y, t) + z\theta_{xi}(x, y, t) - \frac{4z^3}{3h^2}\left(\theta_{xi}(x, y, t) + \frac{\partial W_{0i}(x, y, t)}{\partial x}\right), \\
V_i(x, y, z, t) &= v_{0i}(x, y, t) + z\theta_{yi}(x, y, t) - \frac{4z^3}{3h^2}\left(\theta_{yi}(x, y, t) + \frac{\partial W_{0i}(x, y, t)}{\partial y}\right), \\
W_i(x, y, z, t) &= W_{0i}(x, y, t),
\end{align*}
\]

where \(i = 1, 2\), in plane deflections of the mid-surface along \(x\) and \(y\) directions presented with \(u_0\) and \(v_0\), respectively. Furthermore \(\theta_y\) and \(\theta_x\) are the rotations of the middle plane along \(x\) and \(y\) directions, respectively, and \(w\) denotes the lateral plate displacement. Regarding the linear strain-displacement relation, the normal \((\varepsilon_{xx}, \varepsilon_{yy})\) and shear \((\gamma_{xz}, \gamma_{yz}, \gamma_{xy})\) strain components of each sandwich MEE plate are defined as
\[\begin{align*}
\varepsilon_{xii} &= \frac{\partial u_{oi}}{\partial x} + z \frac{\partial \theta_{xi}}{\partial x} - C_1 z^3 \left( \frac{\partial \theta_{xi}}{\partial x} + \frac{\partial^2 W_{oi}}{\partial x^2} \right), \\
\varepsilon_{yii} &= \frac{\partial v_{oi}}{\partial y} + z \frac{\partial \theta_{yi}}{\partial y} - C_1 z^3 \left( \frac{\partial \theta_{yi}}{\partial y} + \frac{\partial^2 W_{oi}}{\partial y^2} \right), \\
\gamma_{xyi} &= \frac{\partial u_{oi}}{\partial y} + \frac{\partial v_{oi}}{\partial x} + z \left( \frac{\partial \theta_{xi}}{\partial y} + \frac{\partial \theta_{yi}}{\partial x} \right) - C_1 z^3 \left( \frac{\partial \theta_{xi}}{\partial y} + \frac{\partial \theta_{yi}}{\partial x} + 2 \frac{\partial^2 W_{oi}}{\partial x \partial y} \right), \\
\gamma_{xzi} &= (1 - 3 C_1 z^2) \left( \theta_{xi} + \frac{\partial W_{oi}}{\partial y} \right), \\
\gamma_{yz} &= (1 - 3 C_1 z^2) \left( \theta_{yi} + \frac{\partial W_{oi}}{\partial x} \right)
\end{align*}\]

where \(i = 1, 2\) and \(C_1 = \frac{4}{3h^2}\). The classical constitutive stress-strain relations including normal \((\sigma_{xx}, \sigma_{yy})\) and shear \((\tau_{xz}, \tau_{yz}, \tau_{xy})\) stress components for the PFGM core layer in each sandwich plate are presented as

\[\begin{bmatrix}
\sigma_{xii} \\
\sigma_{yii} \\
\tau_{xzi} \\
\tau_{xyi}
\end{bmatrix}_{\text{PFGM}} =
\begin{bmatrix}
Q_{11}(z) & Q_{12}(z) & 0 & 0 & 0 \\
Q_{12}(z) & Q_{22}(z) & 0 & 0 & 0 \\
0 & 0 & Q_{66}(z) & 0 & 0 \\
0 & 0 & 0 & Q_{44}(z) & 0 \\
0 & 0 & 0 & 0 & Q_{55}(z)
\end{bmatrix}_{\text{PFGM}}
\begin{bmatrix}
\varepsilon_{xii} \\
\varepsilon_{yii} \\
\gamma_{xzi} \\
\gamma_{xyi}
\end{bmatrix}
\]

\[\begin{aligned}
Q_{11} &= \frac{E(z)}{1-\theta^2(z)}, & Q_{22} &= \frac{E(z)}{1-\theta^2(z)}, & Q_{12} &= \frac{\theta(z) E(z)}{1-\theta^2(z)}, & Q_{66} &= G_{12}(z), & Q_{44} &= G_{23}(z), & Q_{55} &= G_{13}(z).
\end{aligned}\]

where \(i = 1, 2\). \(E\) and \(\theta\) refer to Young modulus and Poisson’s ratio, respectively. To achieve FG-like properties, the bottom and top surfaces of the core layer are purely made from metal and ceramic, respectively. As already described, three porosity distribution patterns are considered for this study. Accordingly, the corresponding values of elastic modulus, mass density and Poisson’s ratio based on the rule of mixture with three different types of uneven porosity distributions are described as [52]

**Type A:**

\[\begin{aligned}
E(z) &= (E_c - E_m)(z/h + 0.5)^p + E_m - (1 - e^{-0.5z})(E_c + E_m)(1 - 2|z|/h_c), \\
\rho(z) &= (\rho_c - \rho_m)(z/h + 0.5)^p + \rho_m - (1 - e^{-0.5z})(\rho_c + \rho_m)(1 - 2|z|/h_c), \\
\theta(z) &= (\theta_c - \theta_m)(z/h + 0.5)^p + \theta_m - (1 - e^{-0.5z})(\theta_c + \theta_m)(1 - 2|z|/h_c),
\end{aligned}\]

**Type B:**

\[\begin{aligned}
E(z) &= (E_c - E_m)(z/h + 0.5)^p + E_m - \log(1 + 0.5z)(E_c + E_m)(1 - 2|z|/h), \\
\rho(z) &= (\rho_c - \rho_m)(z/h + 0.5)^p + \rho_m - \log(1 + 0.5z)(\rho_c + \rho_m)(1 - 2|z|/h), \\
\theta(z) &= (\theta_c - \theta_m)(z/h + 0.5)^p + \theta_m - \log(1 + 0.5z)(\theta_c + \theta_m)(1 - 2|z|/h),
\end{aligned}\]
\[ \rho(z) = (\rho_c - \rho_m)(z/h + 0.5)^p + \rho_m - \log(1 + 0.5\zeta)(\rho_c + \rho_m)(1 - 2|z|/h), \]  
(5) \]

\[ \vartheta(z) = (\vartheta_c - \vartheta_m)(z/h + 0.5)^p + \vartheta_m - \log(1 + 0.5\zeta)(\vartheta_c + \vartheta_m)(1 - 2|z|/h) \]

Type C:
\[ E(z) = (E_c - E_m)(z/h + 0.5)^p + E_m - 0.5\zeta(E_c + E_m), \]
\[ \rho(z) = (\rho_c - \rho_m)(z/h + 0.5)^p + \rho_m - 0.5\zeta(\rho_c + \rho_m), \]  
(6) \]
\[ \vartheta(z) = (\vartheta_c - \vartheta_m)(z/h + 0.5)^p + \vartheta_m - 0.5\zeta(\vartheta_c + \vartheta_m), \]

in which m and c signify, respectively, metal and ceramic phases. Also, \( \rho \) is the mass density. The always-positive gradient index \( p \) is used in this study to determine the changes of a specific property in the thickness direction. The greater the gradient index, the more metallic the structure. Furthermore, \( \zeta \) expresses the porosity coefficient. It is noteworthy that for each MEE plate, the basic relations including electric displacement and magnetic induction, and stress tensor can be expressed as [53]

\[
\begin{pmatrix}
\sigma_{x\xi} \\
\sigma_{y\eta} \\
\tau_{y\xi} \\
\tau_{x\xi} \\
\tau_{x\eta}
\end{pmatrix}
= 
\begin{pmatrix}
c_{11} & c_{12} & 0 & 0 & 0 \\
c_{12} & c_{22} & 0 & 0 & 0 \\
0 & 0 & c_{44} & 0 & 0 \\
0 & 0 & 0 & c_{55} & 0 \\
0 & 0 & 0 & 0 & c_{66}
\end{pmatrix}
\begin{pmatrix}
\varepsilon_{x\xi} \\
\varepsilon_{y\eta} \\
\gamma_{y\xi} \\
\gamma_{x\xi} \\
\gamma_{x\eta}
\end{pmatrix}
- 
\begin{pmatrix}
0 & 0 & e_{31} \\
0 & 0 & e_{32} \\
0 & e_{24} & 0 \\
e_{15} & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
E_{x\xi} \\
E_{y\eta} \\
E_{y\xi} \\
E_{x\xi} \\
E_{x\eta}
\end{pmatrix}
- 
\begin{pmatrix}
0 & 0 & f_{31} \\
0 & 0 & f_{32} \\
0 & f_{24} & 0 \\
f_{15} & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
H_{x\xi} \\
H_{y\eta} \\
H_{y\xi} \\
H_{x\xi} \\
H_{x\eta}
\end{pmatrix}
\]
\[
\begin{bmatrix}
D_{xi} \\
D_{yi} \\
D_{zi}
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & e_{31} \\
0 & 0 & e_{32} \\
e_{15} & 0 & 0
\end{bmatrix}^T
\begin{bmatrix}
\varepsilon_{xxi} \\
\varepsilon_{yyi} \\
Y_{yzi} \\
Y_{xzi} \\
Y_{xyi}
\end{bmatrix}
+ \begin{bmatrix}
\kappa_{11} & 0 & 0 \\
0 & \kappa_{22} & 0 \\
0 & 0 & \kappa_{33}
\end{bmatrix}
\begin{bmatrix}
E_{xi} \\
E_{yi} \\
E_{zi}
\end{bmatrix} +
\begin{bmatrix}
\mu_{11} & 0 & 0 \\
0 & \mu_{22} & 0 \\
0 & 0 & \mu_{33}
\end{bmatrix}
\begin{bmatrix}
\mathcal{H}_{xi} \\
\mathcal{H}_{yi} \\
\mathcal{H}_{zi}
\end{bmatrix},
\]
(7)

\[
\begin{bmatrix}
B_{xi} \\
B_{yi} \\
B_{zi}
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & f_{31} \\
0 & 0 & f_{32} \\
f_{15} & 0 & 0
\end{bmatrix}^T
\begin{bmatrix}
\varepsilon_{xxi} \\
\varepsilon_{yyi} \\
Y_{yzi} \\
Y_{xzi} \\
Y_{xyi}
\end{bmatrix}
+ \begin{bmatrix}
\mu_{11} & 0 & 0 \\
0 & \mu_{22} & 0 \\
0 & 0 & \mu_{33}
\end{bmatrix}
\begin{bmatrix}
E_{xi} \\
E_{yi} \\
E_{zi}
\end{bmatrix} +
\begin{bmatrix}
\gamma_{11} & 0 & 0 \\
0 & \gamma_{22} & 0 \\
0 & 0 & \gamma_{33}
\end{bmatrix}
\begin{bmatrix}
\mathcal{H}_{xi} \\
\mathcal{H}_{yi} \\
\mathcal{H}_{zi}
\end{bmatrix},
\]

where \( i = 1,2 \). Furthermore, \([\mathbf{B}]\) and \([\mathbf{D}]\) indicate the magnetic induction and electric displacement, respectively. Furthermore, \([\gamma], [\mu], [\kappa], [f], [\varepsilon]\), and \([c]\) are magnetic, magnetoelectric, dielectric, piezomagnetic, piezoelectric, and the elastic constant matrices, respectively. Also, the magnetic and electric fields relating to the magnetic and electric potentials, respectively, are demonstrated with \([\mathcal{H}]\) and \([\mathcal{E}]\). To satisfy Maxwell’s equations in the proposed procedure, two assumptions are made: the magnetic field is the negative gradient of \(\psi(x,y,z,t)\) and the electric field is the negative gradient of \(\mathcal{Y}(x,y,z,t)\). Accordingly, one can write

\[
E_j = -\partial \mathcal{Y}/\partial j, \quad \mathcal{H}_j = -\partial \psi/\partial j, \quad (j = x, y, z).
\]

(8)

In view of the boundary conditions at upper and bottom surfaces of each MEE layer, it is possible to combine linear and cosine variations to explicitly describe the electric and magnetic potentials as in [54]

\[
\mathcal{Y}(x, y, z, t) = -\cos\left\{\frac{\pi [z \pm (hc + h_m)]}{h_m}\right\} \mathcal{Y}(x, y, t) + \frac{2[z \pm (hc + h_m)]}{h_m} \phi_0,
\]

(9)

\[
\Psi(x, y, z, t) = -\cos\left\{\frac{\pi [z \pm (hc + h_m)]}{h_m}\right\} \Psi(x, y, t) + \frac{2[z \pm (hc + h_m)]}{h_m} \psi_0,
\]

where \(\phi_0\) and \(\psi_0\) denote the determine the initial electric and magnetic potentials on the upper and lower surfaces of each MEE layer, respectively. Furthermore, \(\Psi\) and \(\mathcal{Y}\) refer to the two-dimensional magnetic and electric potentials. The vibroacoustic governing equations of motion for double-walled sandwich MEE plate can be obtained using Hamilton’s principle as follows
where $\vec{V}$, $\vec{W}$, and $\vec{K}$ are the virtual strain energy, the work done by external forces (the work applied by the incidence sound wave and initial electric and magnetic potentials) and kinetic energy, respectively. By taking into account FSDT (Eq. (1)), the kinetic energy for double-walled sandwich MEE plate is expressed as

$$\vec{K} = \sum_{i=1}^{2} \int_{A_i} \left\{ f_{-h_c/2} - f_{-h_m-h_c/2} \rho_m \left[ (\dot{U}_i)^2 + (\dot{V}_i)^2 + (\dot{W}_i)^2 \right] dz + f_{h_c/2}^{h_m+h_c/2} \rho_m \left[ (\dot{U}_i)^2 + (\dot{V}_i)^2 + (\dot{W}_i)^2 \right] dz \right\} dA_i,$$

where $\rho_m$ is the mass density of each MEE layer, and $A$ demonstrates the of cross-sectional area. The strain energy double-walled sandwich MEE plate is presented as

$$\bar{V} = \sum_{i=1}^{2} \int_{A_i} \left\{ f_{-h_c/2} \left[ \sigma_{xxi} \varepsilon_{xxi} + \sigma_{yyi} \varepsilon_{yyi} + \tau_{yzi} \gamma yzi + \tau_{xzi} \gamma xzi + \tau_{xyi} \gamma yxi \right]ight. \\
\left. - D_{xi} E_{xi} - D_{yi} E_{yi} - D_{zi} H_{xi} - B_{yi} H_{yi} - B_{zi} H_{zi} \right| \right| dz + f_{h_c/2}^{h_m+h_c/2} \left[ \sigma_{xxi} \varepsilon_{xxi} + \sigma_{yyi} \varepsilon_{yyi} + \tau_{yzi} \gamma yzi + \tau_{xzi} \gamma xzi + \tau_{xyi} \gamma yxi \right] dz + f_{-h_c/2}^{h_m+h_c/2} \left[ \sigma_{xxi} \varepsilon_{xxi} + \sigma_{yyi} \varepsilon_{yyi} + \tau_{yzi} \gamma yzi + \tau_{xzi} \gamma xzi + \tau_{xyi} \gamma yxi \right] dz \right\} dA_i.$$

The work of nonconservative forces, based on the sound velocity potential, is defined as

$$\bar{W} = \int_{A_1} \left\{ j \omega \rho_0 (\Gamma_1 - \Gamma_2) W_{01} + (N_E + N_M) \left[ \left( \frac{\partial W_{01}}{\partial x} \right)^2 + \left( \frac{\partial W_{01}}{\partial y} \right)^2 \right] \right\} dA_1 + \\
\int_{A_2} \left\{ j \omega \rho_0 (\Gamma_2 - \Gamma_3) W_{02} + (N_E + N_M) \left[ \left( \frac{\partial W_{02}}{\partial x} \right)^2 + \left( \frac{\partial W_{02}}{\partial y} \right)^2 \right] \right\} dA_2,$$

where $N_E = \int_{-h_m-h_c/2}^{-h_c/2} 2 e_{31} \phi_0 / h_m dz + \int_{h_c/2}^{h_m+h_c/2} 2 e_{31} \phi_0 / h_m dz$ and $N_M = \int_{-h_m-h_c/2}^{-h_c/2} 2 f_{31} \psi_0 / h_m dz + \int_{h_c/2}^{h_m+h_c/2} 2 f_{31} \psi_0 / h_m dz$. Also, $\omega$ is the angular frequency and $\rho_0$ expresses air density. Furthermore $\Gamma_1$, $\Gamma_2$, and $\Gamma_3$ express the velocity potentials in the sound incident area, acoustic cavity, and the transmitted acoustic area. The next step is to find the velocity potential. To this aim, the sound waves of any magnitude (positive and negative) are superposed in each zone. The results is written as [55]

$$\Gamma_1(x, y, z; t) = 1 e^{-j(k_{1x} x + k_{1y} y + k_{1z} z - \omega t)} + T_1 e^{-j(k_{1x} x + k_{1y} y - k_{1z} z - \omega t)},$$

$$\Gamma_2(x, y, z; t) = T_2 e^{-j(k_{2x} x + k_{2y} y + k_{2z} z - \omega t)} + T_3 e^{-j(k_{2x} x + k_{2y} y - k_{2z} z - \omega t)},$$

(10)
\[ \Gamma_3(x, y, z; t) = T_4 e^{-j(k_3x + k_3y + k_3z - \omega t)} \]

in which \( I \) refers to the incident sound amplitude and \( T_1 \) is the unknown modal coefficient of reflected sound wave in the negative-going incident region. Also \( T_2 \) and \( T_3 \), respectively, denote the unknown modal coefficients related with positive-going acoustic cavity and negative-going acoustic cavity. Furthermore, the unknown modal coefficient in positive-going transmitted wave is expressed with \( T_4 \) and \( j = \sqrt{-1} \), and

\[
k_{1x} = k_1 \sin \beta_1 \cos \alpha, \quad k_{1y} = k_1 \sin \beta_1 \sin \alpha, \quad k_{1z} = k_1 \cos \beta_1,
\]

\[
k_{2x} = k_2 \sin \beta_2 \cos \alpha, \quad k_{2y} = k_2 \sin \beta_2 \sin \alpha, \quad k_{2z} = k_2 \cos \beta_2,
\]

\[
k_{3x} = k_3 \sin \beta_3 \cos \alpha, \quad k_{3y} = k_3 \sin \beta_3 \sin \alpha, \quad k_{3z} = k_3 \cos \beta_3.
\]

where \( k_1 = \frac{\omega}{c_0(1 + M \sin \beta_1 \cos \alpha)} \), \( k_2 = k_3 = \frac{\omega}{c_0} \) are the air acoustic wavenumbers. Furthermore, \( c_0 \) states the air sound velocity and \( M = \frac{v}{c_0} \) refers to the Mach number of the external flow. Moreover, in the acoustic cavity and the transmitted fluid region, the directions of sound propagation are as [55]

\[
\beta_2 = \arcsin \left( \frac{\sin \beta_1}{1 + M \sin \beta_1 \cos \alpha} \right), \quad \beta_3 = \arcsin \left( \frac{\sin \beta_1}{1 + M \sin \beta_1 \cos \alpha} \right)
\]

Next, substituting Eqs. (11), (12), and (13) into Hamilton’s principle (Eq. 10), after performing the required integration by parts in time and space, one ultimately obtains the general vibroacoustic equations of motion for the top and bottom sandwich MEE plates as

\[
\delta u_{0i}: \frac{\partial N_{xxi}}{\partial x} + \frac{\partial N_{xyi}}{\partial y} = I_0 \ddot{u}_{0i} + I_1 \dot{\theta}_{xi} - I_3 C_1 \frac{\partial \ddot{W}_{0i}}{\partial x},
\]

\[
\delta v_{0i}: \frac{\partial N_{xyi}}{\partial x} + \frac{\partial N_{yyi}}{\partial y} = I_0 \ddot{v}_{0i} + I_1 \dot{\theta}_{yi} - I_3 C_1 \frac{\partial \ddot{W}_{0i}}{\partial y},
\]

\[
\delta W_{0i}: \frac{\partial Q_{xzi}}{\partial x} + \frac{\partial Q_{yzi}}{\partial y} - 3C_1 \left( \frac{\partial R_{xzi}}{\partial x} + \frac{\partial R_{yzi}}{\partial y} \right) + C_1 \left( \frac{\partial^2 P_{xzi}}{\partial x^2} + 2 \frac{\partial^2 P_{xzi}}{\partial x \partial y} + \frac{\partial^2 P_{yzi}}{\partial y^2} \right) = q_i + N_0 \left( \frac{\partial^2 \ddot{W}_{0i}}{\partial x^2} + \frac{\partial^2 \ddot{W}_{0i}}{\partial y^2} \right),
\]

\[
I_0 \ddot{W}_{0i} + I_3 C_1 \left( \frac{\partial u_{0i}}{\partial x} + \frac{\partial v_{0i}}{\partial y} \right) - I_6 C_1 \left( \frac{\partial^2 \ddot{W}_{0i}}{\partial x^2} + \frac{\partial^2 \ddot{W}_{0i}}{\partial y^2} \right) + I_4 C_1 \left( \frac{\partial \theta_{xi}}{\partial x} + \frac{\partial \theta_{yi}}{\partial y} \right),
\]

\[
C_1 I_4 \frac{\partial \ddot{W}_{0i}}{\partial x}.
\]
\[ \delta \theta_{yi} + \frac{\partial M_{xyi}}{\partial y} - C_1 \left( \frac{\partial P_{xyi}}{\partial y} + \frac{\partial P_{xyi}}{\partial x} \right) - Q_{yzi} + 3 C_1 R_{yzi} = \bar{I}_1 \nu_{oi} + \bar{I}_2 \dot{\theta}_{yi} - \]

\[ C_1 \bar{I}_4 \frac{\partial \dot{w}_{oi}}{\partial y}, \]

\[ \delta \bar{V}_i : \int_{-h_m-h_c/2}^{-h_c/2} \left( \frac{\partial D_{xli}}{\partial x} \cos \left( \frac{\pi [z+(h_c+h_m)/2]}{h_m} \right) \right) + \frac{\partial D_{yli}}{\partial y} \cos \left( \frac{\pi [z+(h_c+h_m)/2]}{h_m} \right) + \]

\[ \frac{\pi}{h_m} D_{zli} \sin \left( \frac{\pi [z+(h_c+h_m)/2]}{h_m} \right) \right) dz + \int_{h_c/2}^{h_m+h_c/2} \left( \frac{\partial D_{xli}}{\partial x} \cos \left( \frac{\pi [z-(h_c+h_m)/2]}{h_m} \right) \right) + \]

\[ \frac{\partial D_{yli}}{\partial y} \cos \left( \frac{\pi [z-(h_c+h_m)/2]}{h_m} \right) + \]

\[ \frac{\pi}{h_m} D_{zli} \sin \left( \frac{\pi [z-(h_c+h_m)/2]}{h_m} \right) \right) dz + \int_{h_c/2}^{h_m+h_c/2} \left( \frac{\partial D_{xli}}{\partial x} \cos \left( \frac{\pi [z-(h_c+h_m)/2]}{h_m} \right) \right) + \]

\[ \frac{\partial D_{yli}}{\partial y} \cos \left( \frac{\pi [z-(h_c+h_m)/2]}{h_m} \right) + \]

\[ \frac{\pi}{h_m} D_{zli} \sin \left( \frac{\pi [z-(h_c+h_m)/2]}{h_m} \right) \right) = 0, \]

\[ \delta \bar{\Psi}_i : \int_{-h_m-h_c/2}^{-h_c/2} \left( \frac{\partial B_{xli}}{\partial x} \cos \left( \frac{\pi [z+(h_c+h_m)/2]}{h_m} \right) \right) + \frac{\partial B_{yli}}{\partial y} \cos \left( \frac{\pi [z+(h_c+h_m)/2]}{h_m} \right) + \]

\[ \frac{\pi}{h_m} B_{zli} \sin \left( \frac{\pi [z+(h_c+h_m)/2]}{h_m} \right) \right) dz + \int_{h_c/2}^{h_m+h_c/2} \left( \frac{\partial B_{xli}}{\partial x} \cos \left( \frac{\pi [z-(h_c+h_m)/2]}{h_m} \right) \right) + \]

\[ \frac{\partial B_{yli}}{\partial y} \cos \left( \frac{\pi [z-(h_c+h_m)/2]}{h_m} \right) + \]

\[ \frac{\pi}{h_m} B_{zli} \sin \left( \frac{\pi [z-(h_c+h_m)/2]}{h_m} \right) \right) dz = 0, \]

where \( i = 1,2 \) and \( q_1 = j \omega \rho_0 (\Gamma_1 - \Gamma_2), q_2 = j \omega \rho_0 (\Gamma_2 - \Gamma_3). \) Furthermore, the resultant momentum \( N_{xx}, N_{xy}, N_{yy}, M_{xx}, M_{xy}, M_{yy}, Q_{xx}, Q_{yx} \) and mass inertia terms \( I_0, I_1, I_2, I_3, I_4, I_6 \) are written as

\[ \{N_{xxi}, N_{yyi}, N_{xyi} \} = \int_{-h_c/2}^{-h_c/2} \left\{ \sigma_{xxi}, \sigma_{yyi}, \tau_{xyi} \right\}_m dz + \]

\[ \int_{h_c/2}^{h_c/2} \left\{ \sigma_{xxi}, \sigma_{yyi}, \tau_{xyi} \right\}_{PFGM} dz + \int_{h_c/2}^{h_c/2+h_m} \left\{ \sigma_{xxi}, \sigma_{yyi}, \tau_{xyi} \right\}_m dz, \]

\[ \{M_{xxi}, M_{yyi}, M_{xyi} \} = \int_{-h_c/2}^{-h_c/2} \left\{ \sigma_{xxi}, \sigma_{yyi}, \tau_{xyi} \right\}_m dz + \]

\[ \int_{h_c/2}^{h_c/2} \left\{ \sigma_{xxi}, \sigma_{yyi}, \tau_{xyi} \right\}_{PFGM} dz + \int_{h_c/2}^{h_c/2+h_m} \left\{ \sigma_{xxi}, \sigma_{yyi}, \tau_{xyi} \right\}_m dz, \]

\[ \{P_{xxi}, P_{yyi}, P_{xyi} \} = \int_{-h_c/2}^{-h_c/2} \left\{ \sigma_{xxi}, \sigma_{yyi}, \tau_{xyi} \right\}_m z^3 dz + \]

\[ \int_{h_c/2}^{h_c/2} \left\{ \sigma_{xxi}, \sigma_{yyi}, \tau_{xyi} \right\}_{PFGM} z^3 dz + \int_{h_c/2}^{h_c/2+h_m} \left\{ \sigma_{xxi}, \sigma_{yyi}, \tau_{xyi} \right\}_m z^3 dz, \]

\[ \{Q_{xxi}, Q_{yyi} \} = \left[ \int_{-h_c/2}^{-h_c/2} \left\{ \tau_{xzi}, \tau_{yzi} \right\}_m dz + \int_{h_c/2}^{h_c/2} \left\{ \tau_{xzi}, \tau_{yzi} \right\}_{PFGM} dz + \right. \]

\[ \left. \int_{h_c/2}^{h_c/2+h_m} \left\{ \tau_{xzi}, \tau_{yzi} \right\}_m dz \right]. \]
\[ R_{xz_i}, R_{yzi} = \left[ \int_{-h_c/2}^{h_c/2} \tau_{xz_i}, \tau_{yzi} \right] \frac{1}{m} z^2 dz + \int_{-h_c/2}^{h_c/2} \frac{1}{m} \tau_{xz_i}, \tau_{yzi} PFGM z^2 dz + \int_{h_c/2}^{h_c/2 + h_m} \tau_{xz_i}, \tau_{yzi} \frac{1}{m} z^2 dz \]

\[ \{ l_0, l_1, l_2, l_3, l_4, l_6 \} = \int_{-h_m - h_c/2}^{-h_c/2} \rho_m \{ 1, z, z^2, z^3, z^4, z^6 \} dz + \int_{-h_c/2}^{h_c/2 + h_m} \rho(z) \{ 1, z, z^2, z^3, z^4, z^6 \} PFGM dz + \int_{h_c/2}^{h_c/2 + h_m} \rho_m \{ 1, z, z^2, z^3, z^4, z^6 \} dz, \]

\[ \bar{I}_1 = l_1 - l_3 C_1, \bar{I}_2 = l_2 - 2 C_1 l_4 + C_2^2 l_6, \bar{I}_4 = l_4 - C_1 l_6. \]

where \( i = 1, 2 \). Finally, substituting Eq. (18) (with respect to Eqs. (3) and (4)) into Eq. (17), the equilibrium equation in terms of displacement is derived and detailed in Appendix A.

To analytically solving the governing equation of motion (A1-A7), the boundary conditions of all four edges of each sandwich MEE plate are supposed to be simply supported. Therefore, the displacement and moment conditions are presented as

\[ u_{0i}(x, 0, t) = u_{0i}(x, b, t) = v_{0i}(0, y, t) = v_{0i}(a, y, t) = 0, \]

\[ W_{0i}(x, 0, t) = W_{0i}(x, b, t) = W_{0i}(0, y, t) = W_{0i}(a, y, t) = 0, \]

\[ \theta_{xi}(x, 0, t) = \theta_{xi}(x, b, t) = \theta_{yi}(0, y, t) = \theta_{yi}(a, y, t) = 0, \]

\[ \bar{\gamma}_i(x, 0, \tau) = \bar{\gamma}_i(x, b, t) = \bar{\Psi}_i(0, y, t) = \bar{\Psi}_i(a, y, t) = 0, \]

\[ \bar{\gamma}_i(0, y, \tau) = \bar{\gamma}_i(a, y, t) = \bar{\Psi}_i(x, 0, t) = \bar{\Psi}_i(x, b, t) = 0, \]

\[ M_{xxi}(0, y, t) = M_{xxi}(a, y, t) = M_{yyi}(x, 0, t) = M_{yyi}(x, b, t) = 0, \]

\[ P_{xxi}(0, y, t) = P_{xxi}(a, y, t) = P_{yyi}(x, 0, t) = P_{yyi}(x, b, t) = 0, \]

where \( i = 1, 2 \). To satisfy simply supported boundary condition, based on Navier-solution approach, the following deflection components for each sandwich MEE plate are defined

\[ \{ u_{0i}, \theta_{xi} \} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \cos(m \pi x / a) \sin(n \pi y / b) \{ \bar{u}_i, \bar{\theta}_xi \} e^{j \omega t}, \]

\[ \{ v_{0i}, \theta_{yi} \} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sin(m \pi x / a) \cos(n \pi y / b) \{ \bar{v}_i, \bar{\theta}_yi \} e^{j \omega t}, \]

\[ \{ W_{0i}, \bar{\gamma}_i, \bar{\Psi}_i \} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sin(m \pi x / a) \sin(n \pi y / b) \{ \bar{w}_i, \bar{\gamma}_i, \bar{\Psi}_i \} e^{j \omega t}, \]

where \( i = 1, 2 \) and \( m \) and \( n \) are, respectively, the half wave numbers along \( x \) and \( y \) directions. Furthermore, \( \bar{u}_i, \bar{\theta}_xi, \bar{v}_i, \bar{\theta}_yi, \bar{w}_i, \bar{\gamma}_i, \bar{\Psi}_i \) signify the unknown modal coefficients related to the upper and bottom sandwich MEE plates, respectively.

2.2. Acoustic model
One can define the velocity potential at this stage for the three acoustic zones (\(\Gamma_1\): incident zone, \(\Gamma_2\): enclosure, and \(\Gamma_3\): transmission zone) using the transverse modal function \(A_{mn} = \sin(m\pi x/a)\sin(n\pi y/b)\) with respect to Eq. (14) for the sandwich panel according to [55]

\[
\Gamma_1(x, y, z; t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} I_{mn} A_{mn}(x, y) e^{-j(k_1z-\omega t)} + \\
\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} T_1 A_{mn}(x, y) e^{-j(-k_1z-\omega t)},
\]

(21)

\[
\Gamma_1(x, y, z; t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} I_{mn} A_{mn}(x, y) e^{-j(k_2z-\omega t)} + \\
\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} T_2 A_{mn}(x, y) e^{-j(-k_2z-\omega t)},
\]

\[
\Gamma_1(x, y, z; t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} I_{mn} A_{mn}(x, y) e^{-j(k_3z-\omega t)}
\]

To describe the modal amplitude of the plate corresponding to impingent wave, one should exploit the ordinary orthogonality equations of modal functions. The details of this procedure are expressed as

\[
I_{mn} = 4(I_0/ab) \int_a^b \int_0^b e^{-j(k_1x+k_1y)} \sin(m\pi x/a) \sin(n\pi y/b) dy dx,
\]

which \(I_0\) denotes the amplitude of incident wave.

To determine the coefficients \((T_1, T_2, T_3, T_4)\), it is imperative to concurrently satisfy the continuity conditions for the normal velocity components at the boundary of fluid and structure for the top and bottom surfaces of each sandwich MEE plate as

\[
\rho_0 j\omega \frac{\partial r_1}{\partial z} \bigg|_{z = L + 2h_m + h_c} = \rho_0 \left( \frac{\partial}{\partial t} + V \frac{\partial}{\partial x} \right)^2 W_{01}, \quad \rho_0 j\omega \frac{\partial r_2}{\partial z} \bigg|_{z = L} = \rho_0 \frac{\partial^2 w_{01}}{\partial t^2},
\]

(22)

Substituting equations (20) and (21) in equation (22) yields the next important equations:

\[
T_1 = I_{mn} e^{-2j(k_1z(L + 2h_m + h_c))} - (\omega - V k_{1z})^2 \frac{\bar{w}_1 e^{-jk_1z(L + 2h_m + h_c)}}{k_{1z} k_{1z}}, \quad T_2 = \\
\omega \frac{(\bar{w}_1 - \bar{w}_2 e^{jk_{2z}L})}{k_{2z}(e^{-jk_{2z}L} - e^{jk_{2z}L})}, \quad T_3 = \omega \frac{(\bar{w}_1 - \bar{w}_2 e^{-jk_{2z}L})}{k_{2z}(e^{jk_{2z}L} - e^{-jk_{2z}L})}, \quad T_4 = \omega \frac{\bar{w}_2 e^{-jk_{3z}L}}{k_{3z}}.
\]

(23)

To derive the matrix format of the equilibrium equations, one should implement Eqs. (20)–(21) in the developed governing relations described in (A1-A7). The results are presented here as in


\[
\begin{bmatrix}
\mathcal{L}_{1,1} & \mathcal{L}_{1,2} & \mathcal{L}_{1,3} & \mathcal{L}_{1,4} & \mathcal{L}_{1,5} & \mathcal{L}_{1,6} & \mathcal{L}_{1,7} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\mathcal{L}_{2,1} & \mathcal{L}_{2,2} & \mathcal{L}_{2,3} & \mathcal{L}_{2,4} & \mathcal{L}_{2,5} & \mathcal{L}_{2,6} & \mathcal{L}_{2,7} & 0 & 0 & 0 & 0 & 0 & 0 \\
\mathcal{L}_{3,1} & \mathcal{L}_{3,2} & \mathcal{L}_{3,3} & \mathcal{L}_{3,4} & \mathcal{L}_{3,5} & \mathcal{L}_{3,6} & \mathcal{L}_{3,7} & 0 & 0 & 0 & 0 & 0 & 0 \\
\mathcal{L}_{4,1} & \mathcal{L}_{4,2} & \mathcal{L}_{4,3} & \mathcal{L}_{4,4} & \mathcal{L}_{4,5} & \mathcal{L}_{4,6} & \mathcal{L}_{4,7} & 0 & 0 & 0 & 0 & 0 & 0 \\
\mathcal{L}_{5,1} & \mathcal{L}_{5,2} & \mathcal{L}_{5,3} & \mathcal{L}_{5,4} & \mathcal{L}_{5,5} & \mathcal{L}_{5,6} & \mathcal{L}_{5,7} & 0 & 0 & 0 & 0 & 0 & 0 \\
\mathcal{L}_{6,1} & \mathcal{L}_{6,2} & \mathcal{L}_{6,3} & \mathcal{L}_{6,4} & \mathcal{L}_{6,5} & \mathcal{L}_{6,6} & \mathcal{L}_{6,7} & 0 & 0 & 0 & 0 & 0 & 0 \\
\mathcal{L}_{7,1} & \mathcal{L}_{7,2} & \mathcal{L}_{7,3} & \mathcal{L}_{7,4} & \mathcal{L}_{7,5} & \mathcal{L}_{7,6} & \mathcal{L}_{7,7} & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
\vec{u}_1 \\
\vec{v}_1 \\
\vec{w}_1 \\
\vec{\theta}_x \\
\vec{\theta}_y \\
\vec{\gamma}_1 \\
\vec{\psi}_1 \\
\vec{\psi}_2 \\
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
F \\
0 \\
0 \\
\end{bmatrix}
\]  

(24)

where \(\mathcal{L}_{i,j}\) and \(F\) are expressed in Appendix B.

2.3. STL formulation

Inversing the power transmission coefficient, actually yields a highly-used quantity known as sound transmission loss or STL usually described in dB. Regarding the double-panel domain, one can obtain [46]

\[
STL = 10 \log_{10} \left( \frac{\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} |I_{mn} + T_1|^2}{\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} |T_4|^2} \right),
\]  

(25)

3. Numeric investigation

The numerical findings are provided here. Before presenting them, however, it is best to compare sample results to the available data in certain studies. However, Table 1 lists different parameters employed hereafter (not including the verification part).

3.1. Convergence checking

As delineated earlier, owing to the use of double Fourier series as the employed solution approach, one needs to select an adequate number of modes for the series to attain an accurate answer. To guarantee such a condition, a trial-and-error scheme is implemented. For this purpose, the values of \(m\) and \(n\) are augmented and the stability in the estimated STL is examined. For a normal impinging sound wave, and type C of the porosity configurations, and different frequencies, Fig. 2 demonstrates the convergence of obtained STL in the double-walled sandwich MEE square plate when \(a = b = 0.8m, L = 3cm, h_m = 0.5mm, h_c = 1mm, \beta_1 = 30^\circ, \phi_0 = 0, \psi_0 = 0, e_0 = 0.1, M = 0, p = 1\). As evident in the figure, 1600 terms \((m = 40\) and \(n = 40)\) appear to secure the convergence of results.
3.2. Verification study

A series of comparisons are made here to show whether the developed procedure is accurate enough.

As a first verification study, by disregarding the properties of PFGM core in their respective systems, the first nondimensional natural frequency \( \tilde{\omega}_{11} = \omega_{11} a^2 \sqrt{\rho_m/c_{11}} \) of a rectangular MEE plate for different aspect ratios are obtained in Table 2, which are then compared and validated against the results reported in Refs. [56,57].

In another comparison investigation, by neglecting MEE layers, sound wave, porosity, the variation of the first three dimensionless natural frequencies of the FGM plate \((a/b = 1, a = 10h)\) are calculated based on the present model and then compared with the results of Ref. [58] as presented in Table 3. The comparison made in this table indicates that the current results are in good agreement with analytical predictions of Ref. [58].

Finally, by eliminating MEE layers, for the normal incident sound \((\beta_1 = 0^\circ)\), the STL through double-walled elastic square plate is obtained by the presented formulation and is compared with those predicted based CPT by Ref. [59] in Fig. 3 when \(E = 70GPa, \rho = 2700Kg/m^3, \vartheta = 0.3, a = b = 0.3m, L = 80mm, h_c = 1mm\). It is observed that an acceptable agreement exists between the results.

3.3. Main results

The proper understanding of the impacts of effective parameters on the STL in the double-walled sandwich MEE-PFGM plate is realized here using a series of numerical studies. The considered quantities include plate and cavity dimensions, initial magnetic and electric potentials, elevation angle.

Fig. 4. depicts the STL curves against the frequency interval through single/double-walled sandwich MEE-PFGM plate of finite/infinite extent for when \(a = b = 0.8m, L = 3cm, h_m = 0.5mm, h_c = 1mm, \beta_1 = 30^\circ, \phi_0 = 0, \psi_0 = 0, e_0 = 0.1, M = 0, p = 1\). It should be noted that the “mass-air-mass” resonance dip \(f_m\) in Fig. 4 is marked which is a unique phenomenon owned by the double-panel system and can be approximately predicted by the formula [53]

\[
f_m = \left( \frac{1}{2\pi \cos \beta_2} \right) \left( \rho_0 \frac{c_0^2}{\left[(I_{plate1} + (I_0)_{plate2})/L\right][((I_0)_{plate1} \times (I_0)_{plate2})]} \right)^{1/2}.
\]

One should note that the condition for mass-air-mass resonance is the resonance of two MEE-PFGM panels over the stiffness of the splitting layer of air, resulting in a frequency interval of unsatisfactory STL. Furthermore, at higher frequencies, the resonance dips indicated with \(f_s\) are the related to the standing-wave resonance phenomenon (i.e. \(f_s = \frac{ic_0}{2L\sin \beta_2}\)) that happen when “the distance difference between the routes that the two intervening waves pass through is the multiple of the half wavelength of the incidence sound wave” [45]. It can be observed that the number of
resonances of the finite double plate partitions surpasses that of the finite single plate. This is attributed to the effects of acoustic cavity between the two plates. In addition, the dips prior to “mass-air-mass” resonance represent the system natural frequencies which are the same for double and single panels irrespective of the acoustic cavity. It is also of central importance that greater plate dimensions result in smoother STL response such that the number of dips/peaks reaches a minimum. Also, Regarding the STL-frequency plot of the double plate with infinite dimensions, an upper bound for the partitions of finite size is dictated after the “mass-air-mass” resonance dip, as the panel mode-dominated STL disappears for this special case. Nonetheless, for frequencies smaller than the “mass-air-mass” resonance dip, the noise-cancellation efficiency of the structure with infinite dimensions is not up to par compared to the finite system. This is a direct result of boundary constraints.

Fig. 5 indicates the variations of the STL across the sandwich double MEE-PFGM plate versus different values of the material gradient index for type C of the porosity configurations when $a = b = 0.8 \text{m}, L = 3 \text{cm}, h_m = 0.5 \text{mm}, h_c = 1 \text{mm}, \beta_1 = 30^\circ, \phi_0 = 0, \psi_0 = 0, e_0 = 0.1, M = 0$. As a result, the change from ceramic to metallic state, which indicates a growth in FGM index and a decrease in stiffness, reduces the dimensionless natural frequency. Furthermore, before the “mass-air-mass” resonance dip, an increase in the material gradient index improves the STL performance. Moreover, by increasing the material gradient index, the location of the “mass-air-mass” resonance dip shifts downward which is linked to the increased equivalent mass density (see equation (26)).

Figure 6a and b describes the effects of different porosity coefficients for all types of porosity distribution on the variation of STL through double-walled sandwich MEE-PFGM plate when $a = b = 0.8 \text{m}, L = 3 \text{cm}, h_m = 0.5 \text{mm}, h_c = 1 \text{mm}, \beta_1 = 30^\circ, \phi_0 = 0, \psi_0 = 0, M = 0, p = 1$. However, it can be observed that with increasing porosity coefficient, STL curves of the system decreases in all cases of porosity distributions. This behavior is attributed to the reduction in structural stiffness of the system with increasing porosity. However, it seems that different porosity configurations have a little effect on the changes of the STL.

The influence of the external flow Mach number on the STL through the sandwich double MEE-PFGM plate for type C of the porosity configurations is displayed in Fig. 7 when $a = b = 0.8 \text{m}, L = 3 \text{cm}, h_m = 0.5 \text{mm}, h_c = 1 \text{mm}, \beta_1 = 30^\circ, \phi_0 = 0, \psi_0 = 0, e_0 = 0.1$. It can be observed that by increasing the external flow Mach number, the STL value significantly improves in the resonances. This behavior is due to this fact that a convective fluid loading is exerted on the structure because of the presence of external mean flow, which decreases the sound energy transmitted through the panel system and increases the sound energy reflected.

Fig. 8 displays the effects of the external electric potential on the changes of STL across double-walled sandwich MEE-PFGM plate for type C of the porosity configurations when $a = b = 0.8 \text{m}, L = 3 \text{cm}, h_m = 0.5 \text{mm}, h_c = 1 \text{mm}, \beta_1 = 30^\circ, \psi_0 = 0, e_0 = 0.1, M = 0$. Notably, soundproofing efficiency can be enhanced by increasing the external voltage. This is particularly
true for low-frequency intervals of “mass-air-mass” resonance. Furthermore, as can be expected the, the location of “mass-air-mass” resonance is independent of applied electric potential.

In order to study the effect of external magnetic potential on the STL curves of double-walled sandwich MEE-PFGM plate, Fig. 9 is presented for type C of the porosity configurations when \(a = b = 0.8\text{m}, L = 3\text{cm}, h_\text{m} = 0.5\text{mm}, h_\text{c} = 1\text{mm}, \beta_1 = 30^\circ, \phi_0 = 0, M = 0, e_0 = 0.1\). As can be seen, the values of ST can be enhanced by increasing the external magnetic potential specially before “mass-air-mass” resonance dip. This is particularly true for low-frequency intervals of “mass-air-mass” resonance. Furthermore, as can be expected the, the location of “mass-air-mass” resonance is independent of initial magnetic potential. A careful examination of this figure reveals that the effect of external magnetic potential on the improvement of STL is more significant than that of electric voltage.

Depicted in Fig. 10 is the influence of air cavity depth on the STL curves of double-walled sandwich MEE-PFGM plate for type C of the porosity configurations when \(a = b = 0.8\text{m}, L = 3\text{cm}, h_\text{m} = 0.5\text{mm}, h_\text{c} = 1\text{mm}, \beta_1 = 30^\circ, \psi_0 = 0, \phi_0 = 0, M = 0, e_0 = 0.1\). As previously expressed, the acoustic cavity has no effect on the resonances of double plate in the frequency region prior to “mass-air-mass” resonance. Adding to the depth of air cavity significantly alters the behavior of STL with respect to frequency. For those frequencies greater than the “mass-air-mass” resonance, one observes that greater depths are accompanied with higher STLs. The “mass-air-mass” resonance dips tend to move downwards as the air cavity deepens. This behavior is explained by the smaller equivalent stiffness associated with cavities.

Fig. 11 indicates the variations of STL double-walled sandwich MEE-PFGM plate for different values of elevation angle and type C of the porosity configurations when \(a = b = 0.8\text{m}, L = 3\text{cm}, h_\text{m} = 0.5\text{mm}, h_\text{c} = 1\text{mm}, \psi_0 = 0, \phi_0 = 0, M = 0, e_0 = 0.1\). The direct relationship between the elevation angle and the “mass-air-mass” resonance is also observed here. Reduced values of the elevation angle of impinging sound waves arguably improve the noise-cancellation of the studied system for all frequencies. Also noteworthy is that a large elevation angle for an incoming sound wave leads to its easier transmission through the double-walled sandwich plate. In addition, since the plate mode is shown to be independent of impingent elevation angle, dips prior to the “mass-air-mass” resonance remains constant with respect to elevation angle.

Fig. 12 exhibits the effects of sandwich MEE-PFGM plate thickness on STL curves for type C of the porosity configurations when \(a = b = 0.8\text{m}, L = 3\text{cm}, \beta_1 = 30^\circ, \psi_0 = 0, \phi_0 = 0, M = 0, e_0 = 0.1\). Clearly, the noise-cancellation behavior of the plate is highly affected by the ratio of sandwich plate thickness. As expected from the mass law, the STL improves from both the increased thickness of single panel and the coupling impacts of air cavity, with the latter playing a more prominent role. Furthermore, the declining move of “mass-air-mass” resonance with growing thickness originates from the higher plate surface density.
4. Conclusions

The Third-order shear deformation theory and three cases of uneven porosity distribution pattern are applied to study the sound transmission loss through the air-filled double-walled sandwich magneto-electro-elastic plate with porous functionally graded material core layer subjected to the initial external electric and magnetic potentials, and external mean airflow. The material characteristics of PFGM core layer change slowly through the thickness via power-law scheme. The coupled vibroacoustic governing equations are obtained using Hamilton’s principle in conjunction with the normal fluid/structure velocity components. The effects of different significant parameters on the sound transmission loss of the structure over certain frequency intervals, particularly the important region of “mass-air-mass” resonance, are investigated. Some of the most important results of this study are listed in the following.

- The values of ST can be enhanced by increasing the external electric and magnetic potentials specially before “mass-air-mass” resonance dip.
- By increasing the external flow Mach number, the STL value significantly improves in the resonances.
- With increasing porosity coefficient, STL values of the system decreases in all cases of porosity distributions.
- Before the “mass-air-mass” resonance dip, an increase in the material gradient index improves the STL performance.
- STL improves from both the increased thickness of single panel and the coupling impacts of air cavity, with the latter playing a more prominent role.
- The “mass-air-mass” resonance dips tend to move downwards as the air cavity deepens. This behavior is explained by the smaller equivalent stiffness associated with cavities.
Table caption

**Table 1.** Material properties of the MEE-PFGM plate and acoustic medium.

**Table 2.** Comparison of dimensionless natural frequency of a MEE plate.

**Table 3.** Comparison of dimensionless natural frequency of a FGM plate.

Figure Caption

**Fig. 1.** The schematic of double-walled MEE-PFGM plate under incidence wave and external mean airflow.

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**Fig. 3.** Comparison study of STL curves for double-walled elastic plate.

**Fig. 4.** The variations of STL through single/double-walled MEE-PFGM plate versus plate dimensions.

**Fig. 5.** Effect of the material gradient index on the STL through double-walled MEE-PFGM plate.

**Fig. 6.** The STL of double-walled MEE-PFGM plate (a): against the porosity distribution models; (b) against the porosity coefficient.

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Data Availability

The datasets generated during the current study are available from the below listed authors on reasonable request:

Peyman Roodgar Saffari: rpeyman@engr.tu.ac.th

Nima Refahati: refahati@damavandiau.ac.ir

Pouyan Roodgar Saffari: poyan.safari31@gmail.com
References


[57] S. Soni, N.K. Jain, P. V Joshi, Analytical modeling for nonlinear vibration analysis of partially


| Table 1. Material properties of the MEE-PFGM plate and acoustic medium. |
|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| Properties (MEE Layer) | BaTiO$_3$ − CoFe$_2$O$_4$ | | |
| Elastic (GPa) | $c_{11} = 226, c_{12} = 125, c_{22} = 226,$ | $e_{31} = -2.2, e_{32} = -2.2,$ | $\kappa_{11} = 5.64, \kappa_{22} = 5.64,$ |
| | $c_{44} = 44.2, c_{55} = 44.2, c_{66} = 51$ | $e_{24} = 5.8, e_{15} = 5.8$ | $\kappa_{33} = 6.35$ |
| Piezoelectric (C m$^{-2}$) | $e_{31} = 290.1, e_{32} = 290.1,$ | $\mu_{11} = 5.367, \mu_{11} = 5.367,$ | $\gamma_{11} = -297, \gamma_{22} = -297,$ |
| | $e_{24} = 275, e_{15} = 275$ | $\mu_{33} = 2737.5$ | $\gamma_{33} = 83.5$ |
| Dielectric (10$^{-9}$C V$^{-1}$ m$^{-1}$) | $\kappa_{11} = 5.64, \kappa_{22} = 5.64,$ | $\kappa_{11} = 5.64, \kappa_{22} = 5.64,$ | $\kappa_{33} = 6.35$ |
| Piezomagnetic (N A$^{-1}$ m$^{-1}$) | $f_{31} = 290.1, e_{32} = 290.1,$ | $\mu_{11} = 5.367, \mu_{11} = 5.367,$ | $\mu_{33} = 2737.5$ |
| | $e_{24} = 275, e_{15} = 275$ | | |
| Magnetoelectric (10$^{-12}$ N S V$^{-1}$ C$^{-1}$) | | | |
| Magnetic (10$^{-6}$ N s$^2$ C$^{-2}$ ) | $\gamma_{11} = -297, \gamma_{22} = -297,$ | | |
| Mass density (Kg m$^{-3}$) | | | | $\rho_m = 5550$ |

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<th>Properties (FGM Core)</th>
<th>Si3N4 (ceramic)</th>
<th>SUS304 (metal)</th>
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<td>$\rho = 2370$</td>
<td>$\rho = 8166$</td>
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<td>Properties (Acoustic medium)</td>
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<td>Sound Speed (m s$^{-1}$)</td>
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<tr>
<td>Mass density (Kg m$^{-3}$)</td>
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Table 2. Comparison of dimensionless natural frequency of a MEE plate.
Table 3. Comparison of dimensionless natural frequency of a FGM plate.

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<tr>
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<th>Ref. [57] (CPT)</th>
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<td>0.366</td>
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<td>p</td>
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<td>$\omega_1$</td>
<td>$\omega_2$</td>
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<td>------------</td>
<td>------------</td>
<td>------------</td>
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<tr>
<td></td>
<td>Ref. [50]</td>
<td>0.049</td>
<td>0.116</td>
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</table>
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Supplementary Files

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- AppendixSupplementary.docx