

The interactive dynamics of a binary asteroid and ejecta after medium kinetic impact

Yunfeng Gao¹, Bin Cheng^{2†} and Yang Yu^{1*†}

^{1*}Beihang University, Beihang University, D411 New Main Building, Beijing, 100191, Beijing, China.

²Tsinghua University, Tsinghua University, Mengminwei Sci. & Tech. Building, Beijing, 100084, Beijing, China.

*Corresponding author(s). E-mail(s): yuyang.thu@icloud.com;
Contributing authors: gaoyunfeng@buaa.edu.cn;
chengbin171@163.com;

[†]These authors contributed equally to this work.

Abstract

Kinetic impacts on a binary asteroid will produce ejecta debris as well as causing a disturbance to the spin-orbit motion of the binary components. Understanding the complex interactions between the ejecta and the binary system can be crucial to both theoretical study and mission design. In this paper, we develop a detailed model for the dynamical evolution of ejecta within a binary asteroid system, in which we combined various perturbations on the ejecta and considered the collisional process with the asteroid. Taking an example scenario like the DART impact, we calculated the fate of ejecta from size 0.01 mm to 10 m and analyzed the distribution of their final deposition locations. The results show that ejecta with diameter less than 0.1 mm escape quickly under the influence of solar radiation pressure, while ejecta of larger size survive longer, i.e., a few cycles in the binary system for more than 400 days. We analyzed the influence of the coefficient of restitution between ejecta and asteroid, and found more damped collisions would allow the ejecta to survive longer in orbit. We also studied the effect of the tumbling rotation of the secondary on the distribution of ejecta on the surface of the secondary, the results suggest that a tumbling rotational state causes obvious changes in Dimorphos' surface slope, which, however, are not sufficient for triggering surface landslides. In addition, the influence of the internal structure of the primary on the evolution of the ejecta is studied,

and we show it has little effect on the evolution of the ejecta but does affect the distribution of ejecta on the surface of the primary. This is due to the change of the distribution of relative acceleration on the surface of the primary, which changes the regions where the ejecta can remain.

Keywords: Asteroid, Ejecta, Dynamics, Collisional physics

1 Introduction

Binary asteroids account for approximately 16% of near-Earth asteroids [Margot et al \(2002\)](#) and are one of the popular targets for deep space exploration. Except the singleton asteroids that are confirmed to be active in the solar system, such as (6478) Gault, (101955) bennu, (136108) Haumea, etc, NASA's Double Asteroid Redirection Test (DART) mission, launched on November 24, 2021, plans to impact the secondary (Dimorphos) of binary asteroid system 65803 Didymos in late 26 September 2022 using a 563-kg impactor at 6.14 km/s, which is also expected to produce an artificial ejection event in this two-body system as well as to change their orbit-spin states ([Cheng et al \(2016\)](#); [Michel et al \(2016\)](#); [Cheng et al \(2018\)](#); [Rainey et al \(2019, 2020\)](#); [Rivkin et al \(2021\)](#); [Agrusa et al \(2022\)](#)). More fantastically, ESA's Hera mission will arrive in the Didymos system in 4 years later to observe the results of the impact ([Agrusa et al \(2022\)](#)). Evolution of the ejecta bears both theoretical and practical significances to the mission, which can help to understand the general laws of the geological evolution of binary asteroids and reveal the special mechanical behavior of interplanetary materials in complex environment. In addition to observation-based study, a theoretical analysis based on numerical models are also used to track and analyze the evolution of ejecta near asteroids which helps to assess to understand the ejecta environment posterior to DART impact and provides safety assurance for Hera mission.

There are still some intriguing questions worth discussing on the behaviors of ejecta. First, The ejecta that stay near orbit for a long time may pose a threat to future spacecraft observations. The size and velocity of the ejecta may significantly affect the evolution of ejecta. Studying whether there will be ejecta of certain sizes at certain velocities that can survive in orbit for a long time after impact is critical for predicting whether the spacecraft will be in danger. Second, [Agrusa et al \(2021\)](#) found that Dimorphos is prone to unstable spins in its long-axis direction after DART's impact, which may change the dynamical environment of Dimorphos' surface. Third, the internal structure of the asteroid is not well understood, its possible heterogeneous internal structure may affect the distribution of the ejecta, which, in turn, allows for an indirect way to infer the internal mass distribution of the primary. In order to discuss these questions, we established a complete dynamic model, including considering the various forces on ejecta, using a gravitational field

model that can describe asteroids with uneven internal mass distribution, establishing a fast collision detection method between ejecta and asteroid, and accounting for the uncertainty of rebound velocity of ejecta that re-impact the asteroid surface.

In this paper, we focus on the model building to study the evolution of ejecta in the binary system under the complex dynamical perturbations of the Solar system and calculate its final distribution. The paper is organized as follows. In Section 2, we introduce the interactive dynamics modeling, includes dynamic models of binary asteroids and ejecta, ejecting model and impact reflection model. In Section 3, we presents the simulation schemes, results and discussions, includes the evolution of ejecta with different sizes and different coefficient of restitution of the surface of asteroid, the influence of the tumbling rotational state of the secondary on its surface dynamic environment and the influence of the internal structure of the primary on the distribution of ejecta. Our conclusion is given in Section 4. These simulations are performed on a large scale computing cluster using CUDA and OpenMP parallelism techniques.

2 Interactive dynamics modeling

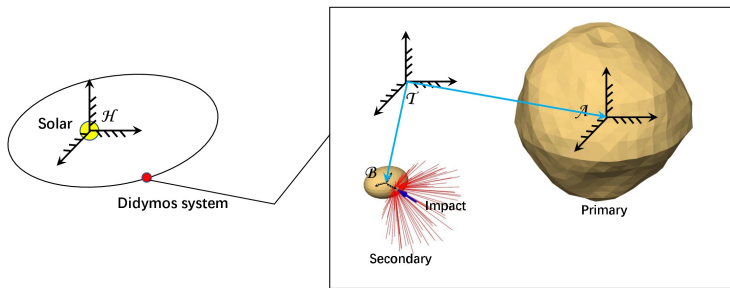


Fig. 1 System configuration and 4 coordinate frames used for dynamics: $\mathcal{H}$ is Heliocentric ecliptic J2000; $\mathcal{T}$ is orbit translational frame; $\mathcal{A}$ is primary body-fixed frame; $\mathcal{B}$ is secondary body-fixed frame.

In this paper, we take the example of the binary asteroid system Didymos to introduce the modelling of interaction dynamics. Its main purpose is the description and presentation of our model which includes five tasks: (1) Build a dynamic model of the motion of a binary system, which is only

affected by mutual gravity and the solar, since the ejecta has almost no effect on the motion of binary system. (2) Build a dynamic model of the motion of the ejecta, which is affected by multiple forces of solar system. (3) Calculate the size and velocity of the ejecta formed after the impact event. (4) Build a secondary collision ejection model between ejecta and asteroid. (5) Set the stopping conditions of the simulation. The configuration of the system and coordinate is shown in Fig.1. Four coordinate systems are adopted: Heliocentric ecliptic J2000 $\mathcal{H}$; orbit translational frame $\mathcal{T}$; primary body-fixed frame $\mathcal{A}$; secondary body-fixed frame $\mathcal{B}$.

2.1 Binary dynamics modeling

The shape model of binary asteroid constructed by [Benner et al \(2010\)](#) is adopted to describe primary (Didymos), and the shape model constructed by [Michel et al \(2016\)](#) is adopted to describe secondary (Dimorphos). The system property values can be accessed in [Naidu et al \(2020\)](#). We set the impact direction to be along the direction of the secondary's orbit (the direction that reduces secondary's orbit velocity), with the impact occurring at the centre of the primary. Here we assume a kinetic energy transfer efficiency of 1 (see [Rivkin et al \(2021\)](#) for specific description), which corresponds to a velocity change of 4 mm/s to the asteroid. For the description of the gravitational field of small bodies, as we are studying the effects of the internal structure of asteroid, we need to choose models that is convient to change the internal structure of asteroid in terms of gravitational field modelling. Therefore, we use the finite element method for calculating the full 2-body problem proposed by [Yu et al \(2019\)](#).

The essence of this method is to divide two rigid bodies into multiple elements, derive the expression for the gravitational potential between each two elements, and then perform the summation. Assuming that both asteroids are rigid bodies and there are N_1 nodes in the primary and N_2 nodes in secondary, for each two nodes α and β (α is one node of the primary, β is one node of the secondary), we need to calculate the gravitational attraction force $\mathbf{f}_{\alpha\beta}$ on the nodes α , torque $\mathbf{t}_\alpha$ on the primary and torque $\mathbf{t}_\beta$ on the secondary. Then the summation is performed and we get the gravitational attraction force $\mathbf{F}_{PS}$ on the primary, torque $\mathbf{T}_P$ on the primary and torque $\mathbf{T}_S$ on the secondary. The specific formula can be found in [Yu et al \(2019\)](#).

The model has no restrictions on the shape and internal mass distribution of an asteroid and has acceptable accuracy in the mutual gravitational potential description between two unconstrained gravitational bodies, therefore avoiding the problem of nonconvergence caused by traditional series. Here $\mathbf{f}_{\alpha\beta}$, $\mathbf{t}_\alpha$ and $\mathbf{t}_\beta$ are calculated $N_1 \times N_2$ times respectively. We use GPU parallel technology to accelerate it and the relevant C++ code can be found in [Gao et al \(2022\)](#). In addition, we added the solar tidal forces $\mathbf{F}_{\odot,P}$ and $\mathbf{F}_{\odot,S}$ acting on primary and secondary in the dynamic model, which are calculated by the following equations

$$\begin{aligned} \mathbf{F}_{\odot,P} &= GM_{\odot}M_P \left(\frac{\mathbf{R}}{|\mathbf{R}|^3} - \frac{\mathbf{R} + \mathbf{R}_P}{|\mathbf{R} + \mathbf{R}_P|^3} \right) \\ \mathbf{F}_{\odot,S} &= GM_{\odot}M_S \left(\frac{\mathbf{R}}{|\mathbf{R}|^3} - \frac{\mathbf{R} + \mathbf{R}_S}{|\mathbf{R} + \mathbf{R}_S|^3} \right) \end{aligned} \quad (1)$$

where $\mathbf{R}$ is the vector from the Sun to the binary mass center, $M_{\odot}$ is the mass of solar, M_P and M_S is the mass of the primary and the secondary, respectively. $\mathbf{R}_P$ and $\mathbf{R}_S$ indicate vectors from the system mass center to the primary and secondary, respectively.

2.2 Ejecta ballistic modeling

Ejecta from Dimorphos are influenced by multiple forces, including the gravitational attraction of the binary system, solar gravity, solar radiation pressure, Poynting-Robertson drag, solar wind drag, gravitational perturbations from other planets, etc. For the calculation of the gravitational force of binary system on ejecta, we simplify the gravitational calculation method for full two-body mentioned in Section 2.1 to calculate the force of a rigid body on a mass point, as shown in the following formula

$$\begin{aligned} \mathbf{F}_{PE} &= G \sum_{\alpha=1}^{N_1} w_{A\alpha} \sigma(\rho_{P\alpha}) m_i \frac{\mathbf{r}_{P\alpha} - \mathbf{r}_i}{|\mathbf{r}_{P\alpha} - \mathbf{r}_i|^3} \\ \mathbf{F}_{SE} &= G \sum_{\beta=1}^{N_2} w_{B\beta} \sigma(\rho_{S\beta}) m_i \frac{\mathbf{r}_{S\beta} - \mathbf{r}_i}{|\mathbf{r}_{S\beta} - \mathbf{r}_i|^3} \end{aligned} \quad (2)$$

where $\mathbf{F}_{PE}$ and $\mathbf{F}_{SE}$ represent the gravitational forces of primary and secondary on ejecta, respectively, $\mathbf{r}_{P\alpha}$ is the position of node α in the primary in the world coordinate system $\mathcal{T}$, m_i and $\mathbf{r}_i$ is the mass and position in the frame $\mathcal{T}$ of ejecta i , $w_{P\alpha}$ is the nodal weights of node α in the primary, $\sigma(\rho_{P\alpha})$ is the density at node α in the primary, replacing the angle labels with S and β corresponds to the secondary.

Since the DART mission impact time has been largely determined, we can easily calculate the tidal forces of the solar and other planets. Fig.2 shows the ratio of planetary tides (magnitude) to solar tidal forces from Sep. 1, 2022 to Sep. 1, 2032. The result shows planetary tidal perturbations are much smaller than solar tidal perturbations in 10 years by at least two orders of magnitude. The planetary tidal disturbance is greatest when Didymos system is close to Earth (4,October,2022), and its ratio to the solar tide is 0.005746. Therefore, our calculations do not take into account the effects of planetary tidal perturbations on ejecta.

Regarding the solar radiation pressure, we introduce β to denote the ratio of the solar radiation pressure to the solar gravitational force (Burns et al (1979))

$$\beta = \frac{3\epsilon S_{\odot} \text{AU}^2}{2cGM_{\odot}\rho d} \quad (3)$$

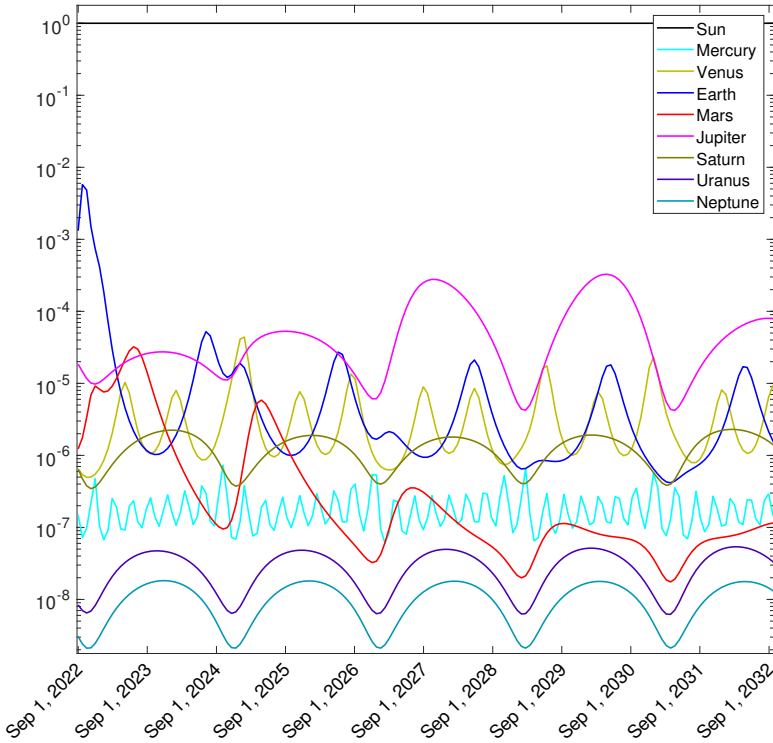


Fig. 2 The scaled planetary tides (magnitude) from Sep. 1, 2022 to Sep. 1, 2032, compared with the solar tide. The dashed line segment marks the encounter date (also supposed as the impact time), and the unified solar tide value 10^0 is identified with the bold baseline.

where ϵ is the reflection coefficient, we take 1.15 here. $S_{\odot} = 1360 \text{ W/m}^2$, d and ρ is diameter and density of ejecta. For Poynting-Robertson drag and solar wind drag, the results in Yu et al (2017) show they have little effect on ejecta, which are ignored in our works.

Through the above, the dynamic equation of ejecta i in the coordinate system $\mathcal{T}$ is:

$$\ddot{\mathbf{r}}_i = \frac{\mathbf{F}_{\odot,T} + \mathbf{F}_{\odot,R} + \mathbf{F}_{PE} + \mathbf{F}_{SE}}{m_i} \quad (4)$$

where $\mathbf{F}_{\odot,T}$ is the tidal force and $\mathbf{F}_{\odot,R}$ is the radiation pressure of the solar on ejecta i in $\mathcal{T}$.

2.3 Impact ejection modeling

Here an improved scheme proposed by Housen and Holsapple (2011); Holsapple and Housen (2012); Yu and Michel (2018) is applied to the discretization of the ejecta (for the specific formula, please refer to the appendix in Yu and Michel (2018)). Assuming a medium kinetic impact, for DART mission, we got

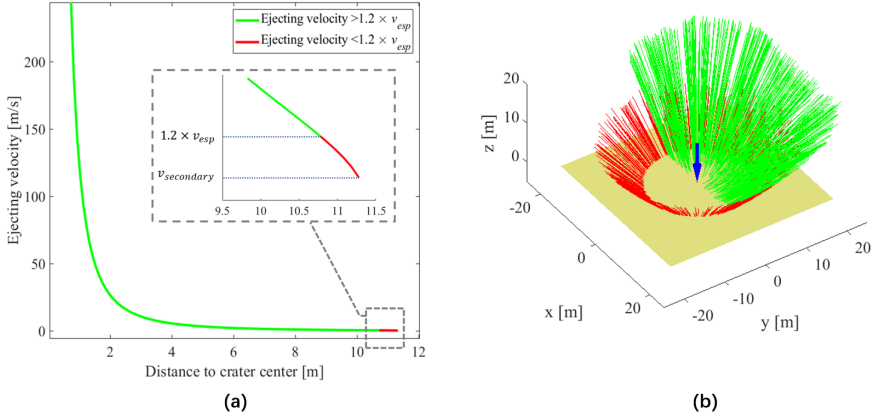


Fig. 3 Selection of simulation samples. (a) is the relationship between the ejecting velocity and the distance of the ejecting point from the center of the crater. Red indicates that the ejecting velocity is less than $1.2 \times v_{esp}$, which is the part that will be simulated. Green indicates the ejecting velocity greater than $1.2 \times v_{esp}$, which will not be simulated. Note that the minimum ejecting velocity is the orbital velocity of the secondary. (b) is the diagram of the impact ejecting, the red is the simulated part, and the green is the unsimulated part (we hide the green part where $y < 0$ for better expression).

the results shown in Table 1. The radius of the impact crater is expected to be 11.412m, the total mass of the ejecta is 9.567×10^5 kg, and the velocity distribution of ejecta is from 0m/s to 247.387m/s. Here we assume a ejecta size between 1mm - 10m and the total number of ejecta generated by the model is 6.759×10^9 . The escape velocity v_{esp} at the point of impact is 0.245m/s, which is lower than the ejecting velocity of the majority of ejecta, meaning that the majority of ejecta ejected will escape. For the simulation we focus on ejecta with an initial ejection velocity of less than $1.2 \times v_{esp}$, which corresponds to ejecta ejected between 10.827m and 11.412m from the centre of the crater. The total number of this part of the ejecta is 9.875×10^8 and the mass is 1.398×10^5 kg, representing 14.61% of the total mass of all the ejecta. Fig.3a shows the relationship between the ejecting velocity and the distance of the ejecting point from the center of the crater. Red indicates that the ejecting velocity is less than $1.2 \times v_{esp}$, which is the part that we will simulate. Green indicates the ejecting velocity greater than $1.2 \times v_{esp}$, which will not be simulated because at this ejecting velocity, the ejecta would quickly escape from the binary system. Fig.3b gives a diagram of the impact ejecting, again in red for the simulated part and in green for the non-simulated part. Only the ejecta at the edge of the crater has a small enough ejecting velocity that we will use it as a sample for simulation. For impact ejection events, the duration of ejecta ejecting from the crater is mostly a few hundred milliseconds, much smaller than our simulation time. Therefore, in our work, all ejecta are considered to be ejected from the crater at the same time.

Table 1 Impact ejection parameters.

Radius of crater	11.412 m
Velocity of ejecta	< 247.287 m/s
Simulation size of ejecta	1 mm – 10 m
Total mass of ejecta	9.567×10^5 kg
Total number of ejecta	6.759×10^9
Mass of ejecta < $1.2 \times v_{esp}$	1.398×10^5 kg
Number of ejecta < $1.2 \times v_{esp}$	9.875×10^8

2.4 Second collision modeling

Some of the ejecta will collide with binary asteroids during the motion, here we propose a quick collision detection model that can efficiently locate on which face of the polyhedron the ejecta is projected without using an traversal approach. Our collision detection model is suitable for that any ray from the center of mass of the asteroid (called O here and after) has only one intersection with the asteroid model outline. Assuming that the line connecting the ejecta E and the center of mass of the asteroid O is OE , and its intersection with the asteroid's outline is P , the principle of our collision detection model is to compare the lengths of OE and OP .

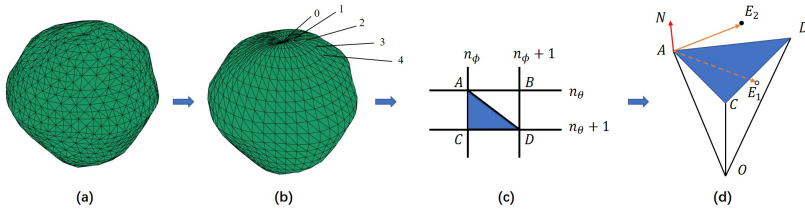


Fig. 4 Collision detection model. (a) is the original polyhedron model; (b) is the new polyhedral model generated by interpolation of the original model; (c) is to detect which triangle the projection point P is on; (d) is to detect whether the ejecta E is inside or outside the quadrangular pyramid $OACD$, where E_1 is inside the polyhedron, E_2 is outside the polyhedron.

First, the vertex coordinates of the polyhedral model are interpolated to generate a polyhedral model with uniform distribution of latitude and longitude, and the longitude lines and latitude lines are numbered (Fig.4a and Fig.4b). Then convert the Cartesian coordinates of the position of ejecta to spherical coordinates (θ_e, ϕ_e, r_e) , and use Eq.10 to calculate which of the two lines of longitude and latitude the projection of the ejecta on the model surface lies between (Fig.4c).

$$\begin{aligned} n_\theta &= f \text{loor} \left(\frac{\theta_e}{\Delta\theta} \right), \\ n_\phi &= f \text{loor} \left(\frac{\phi_e}{\Delta\phi} \right) \end{aligned} \quad (5)$$

Here we can determine that the ejecta is projected within $\square ABCD$, and the next task is to determine whether it is projected within $\triangle ABD$ or $\triangle ACD$.

Assume that the projection point is P . P falling within triangle $\triangle ACD$ needs to satisfy:

$$\begin{aligned} (\overrightarrow{AP} \times \overrightarrow{AC}) \cdot (\overrightarrow{AD} \times \overrightarrow{AC}) &> 0 \\ (\overrightarrow{CP} \times \overrightarrow{CB}) \cdot (\overrightarrow{CA} \times \overrightarrow{CB}) &> 0 \\ (\overrightarrow{DP} \times \overrightarrow{DA}) \cdot (\overrightarrow{DC} \times \overrightarrow{DA}) &> 0 \end{aligned} \quad (6)$$

Assuming that the projection point is inside triangle $\triangle ACD$, we finally need to determine whether ejecta E is inside the quadrilateral cone $OACD$ by

$$\overrightarrow{AE} \cdot \vec{N} < 0 \quad (7)$$

where $\vec{N}$ is the normal vector of $\triangle ACD$. As shown in Fig.4d, $\overrightarrow{AE}_2$ projects positive on $\vec{N}$, outside the polyhedron; $\overrightarrow{AE}_1$ projects negative on $\vec{N}$, inside the polyhedron.

We then processed the collision, and since the surface characteristics of the Didymos are not well known and the incident and reflected velocities of the ejecta are most likely not specular reflection relations, we considered the diffusive of the ejecta according to the Cercignani-Lampis-Lord (CLL) model (Cercignani and Lampis (1971)). Assuming the incident velocity of the ejecta is $(u_i, v_i, 0)$, where u_i is the normal velocity with respect to the plane and v_i is the corresponding tangential velocity. The reflected velocity consists of two components, specular velocity $(u_s, v_s, 0)$ and diffusive velocity (u_d, v_d, w_d) , the direction of w_d is perpendicular to u_d and v_d . The specular velocity is calculated by

$$u_s = -\alpha_n u_i, v_s = \alpha_t v_i \quad (8)$$

where α_n is normal coefficient of restitution and α_t is tangential coefficient of restitution. The diffusive velocity is calculated by

$$\begin{aligned} u_d &= \sqrt{r^2 + (1 - \alpha'_n) U_i^2 + 2r \sqrt{1 - \alpha'_n} \cdot \cos \theta \cdot U_i} \cdot \sqrt{u_i^2 + v_i^2} \\ v_d &= \left(\sqrt{1 - \alpha'_t} \cdot V_i + r \cos \theta \right) \cdot \sqrt{u_i^2 + v_i^2} \\ w_d &= r \cdot \sin \theta \cdot \sqrt{u_i^2 + v_i^2} \end{aligned} \quad (9)$$

where r and θ is a random number between 0 and 1, and

$$\begin{aligned} \alpha'_n &= \alpha_n \\ \alpha'_t &= \alpha_t (2 - \alpha_t) \\ U_i &= \frac{u_i}{\sqrt{u_i^2 + v_i^2}} \\ V_i &= \frac{v_i}{\sqrt{u_i^2 + v_i^2}} \end{aligned} \quad (10)$$

The final reflection velocity is the vector sum of specular velocity and diffusive velocity

$$(u_r, v_r, w_r) = \mu (u_s, v_s, 0) + (1 - \mu) (u_d, v_d, w_d) \quad (11)$$

where $\mu \in [0, 1]$, when $\mu = 1$, particles complete specular, when $\mu = 0$, particles complete diffusive. Fig.5 shows the reflection of 500 particles impacting on a plane under the same incident conditions.

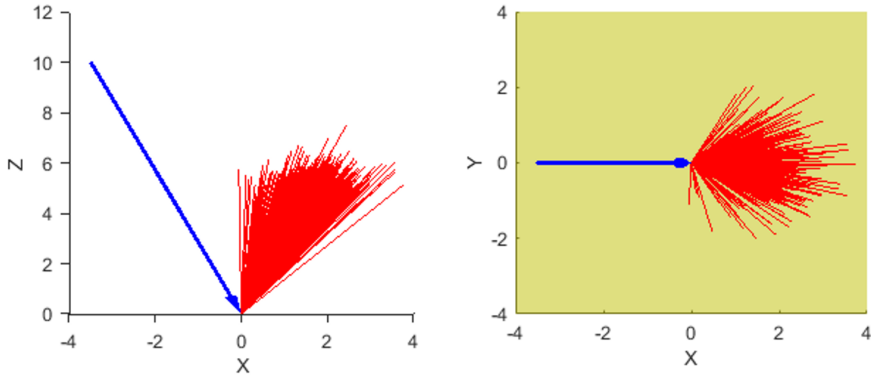


Fig. 5 CLL reflection model, observe the reflection of 500 particles impacting on a plane at the same initial velocity from front view and top view respectively.

The main reason for introducing the CLL collision reflection model is the lack of a complete understanding of the asteroid surface. If a specular reflection model is used, the surface of the asteroid is divided into many surfaces (each surface has a large number of area under the current model) and the reflection model provided by a particular surface is consistent, i.e., some ejecta impact on the same surface with the same output velocity magnitude and similar output velocity direction, and these ejecta reflections will all be in a very similar state. In practice this is rare over a large area, the CLL collision reflection model can provide a terrain-induced uncertainty about the impact.

2.5 Simulation stop condition

The simulation stops when all ejecta escapes the binary asteroids gravitational influence sphere or stop on the surfaces of one of binary asteroids. For the former condition, it is sufficient to check whether the distance of the ejecta relative to the center of mass of the binary asteroid is greater than the radius of the gravitational influence sphere. Here we discuss in detail the conditions for the ejecta to stop on the surface of the asteroid.

Assuming that the ejecta collides with the asteroid, one of the conditions under which it can remain on the surface of the asteroid is that the reflection velocity in the normal direction is extremely small. Due to the low spin angular velocity of the secondary, the ejecta can stay at any position of the surface of

secondary and we can use only one condition to judge the situation from the surface of the secondary. However, the spin angular velocity of the primary is so fast that the ejecta cannot stay in some parts of surfaces, and we can determine whether the ejecta can stay locally by calculating the relative acceleration on the surface of the primary. The relative acceleration vector can be obtained by subtracting the implicated acceleration and Coriolis acceleration from the absolute acceleration. The following vectors are in the $\mathcal{T}$ coordinate system if not otherwise specified. Absolute acceleration can be calculated by

$$\mathbf{a}_a = \frac{\mathbf{F}_{PE} + \mathbf{F}_{SE} + \mathbf{F}_{\odot,E}}{m_e} \quad (12)$$

where $\mathbf{F}_{\odot,E}$ is the combined force of the solar gravitational force and solar radiation pressure on the ejecta. The implicated acceleration can be calculated in the following formula. The position of the ejecta can be expressed as

$$\mathbf{r} = \mathbf{R}_P + \mathbf{r}_e \quad (13)$$

where $\mathbf{R}_P$ is the position of primary, $\mathbf{r}_e$ is the position of the ejecta relative to the primary. According to Eq.13

$$\dot{\mathbf{r}} = \dot{\mathbf{R}}_P + Q\boldsymbol{\omega} \times \mathbf{r}_e \quad (14)$$

where Q is the directional cosine matrix of the primary and $\boldsymbol{\omega}$ is the angular velocity in coordinate system $\mathcal{A}$. Then

$$\mathbf{a}_e = \ddot{\mathbf{r}} = \ddot{\mathbf{R}}_P + Q\boldsymbol{\omega} \times (Q\boldsymbol{\omega} \times \mathbf{r}_e) + (Q\hat{\boldsymbol{\omega}}\boldsymbol{\omega} \times \mathbf{r}_e + Q\boldsymbol{\alpha} \times \mathbf{r}_e) \quad (15)$$

where $\ddot{\mathbf{r}}$ is the acceleration of the primary, $\boldsymbol{\alpha}$ is the angular acceleration of the primary in coordinate system $\mathcal{A}$, and $\hat{\boldsymbol{\omega}}$ is the antisymmetric matrix of $\boldsymbol{\omega}$ ($Q\hat{\boldsymbol{\omega}}\boldsymbol{\omega} = 0$). Thus the relative acceleration

$$\mathbf{a}_r = \mathbf{a}_a - \mathbf{a}_r - \mathbf{a}_C \quad (16)$$

where $\mathbf{a}_C$ is Coriolis acceleration. Assume that the local normal vector is $\mathbf{n}$ (in the coordinate system $\mathcal{T}$), if the ejecta can stay locally, it is necessary to satisfy

$$a_r^n = \mathbf{a}_r \cdot \mathbf{n} < 0 \quad (17)$$

where a_r^n is the projection of the relative acceleration in the n direction.

The specific processing method is shown in Fig.6, where the simulation step size is 1s, the solid line outline indicates that the ejecta is outside the surface of the asteroid, and the dotted line outline indicates the inside of the asteroid surface. Step 1 (Fig.6a) shows the state of the ejecta flying in orbit before the collision; Step 2 (Fig.6b) shows that the ejecta collide with the surface of the asteroid, and here we assume that its reflection velocity is almost zero. At this time, we need to check whether the current surface can the ejecta stay (whether $a_r^n < 0$). If the result is yes, the ejecta is considered to have stopped on the asteroid, and the subsequent position relative to the asteroid surface remains unchanged (Fig.6c); if the result is no, we artificially place the ejecta

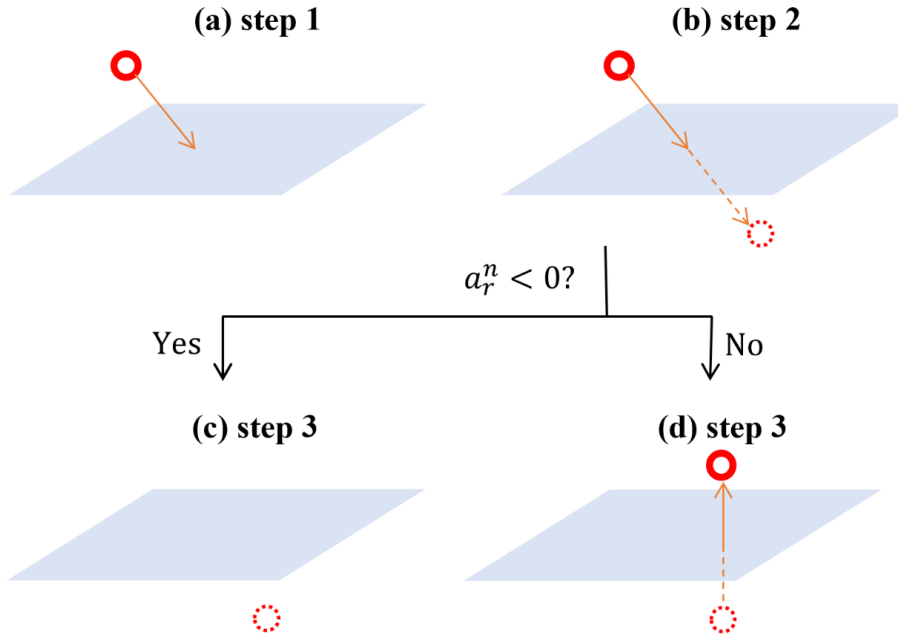


Fig. 6 The specific method of ejecta landing on surface in our model.

on the outside of the asteroid surface (due to the small step size, the artificially raised distance is small), and make its velocity to be 0 relative to the local surface, then it will continue to fly in the orbit (Fig. 6d).

3 Simulational scheme and numerical results

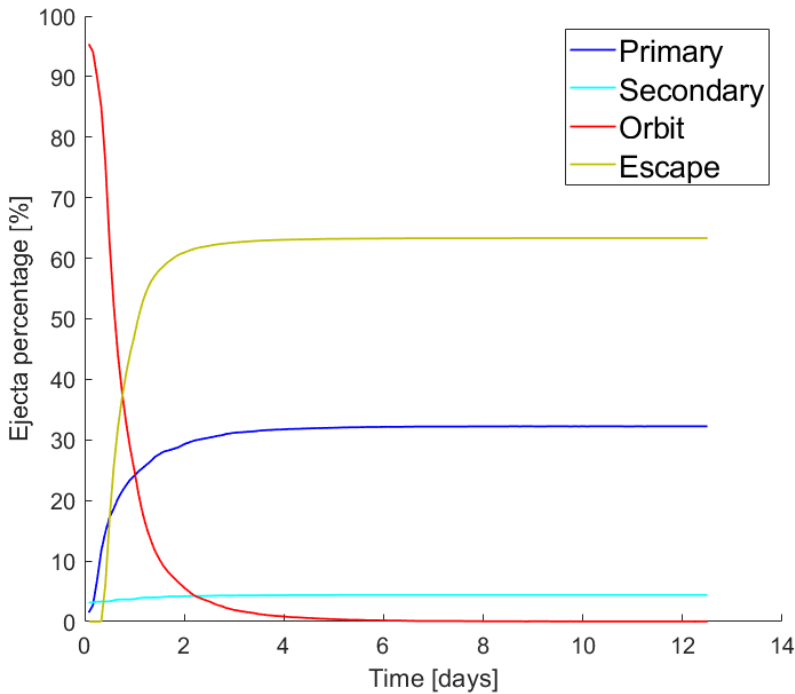
3.1 Ballistic propagation of ejecta in terms of ejecta sizes

In this section, we perform 7 simulations, including 1 general simulation (GS) and 6 special simulations (SS). GS1 follows the ejecta initialization model in Section 2.3, we set the simulation x (x is the distance from ejecting position to the center of crater) to be 10.827 m to 11.412 m (in the case of $x < 10.827$, the initial velocity of the ejecta is greater than $1.2v_{esp}$, which will escape the binary system) and the ejecta size to be 1mm to 10m. We set both the normal coefficient of restitution and tangential coefficient of restitution between ejecta and asteroid to 0.5. Due to computing resource constraints, here we use 1×10^5 particles. In order to study the evolution of ejecta of different sizes (diameter), we performed 6 special simulations (noted as SS1-SS6). For each case, we use 10^5 particles with diameters of uniform distribution within $10^{-5} - 10^{-4}$ m, $10^{-4} - 10^{-3}$ m, $10^{-3} - 10^{-2}$ m, $10^{-2} - 10^{-1}$ m, $10^{-1} - 10^0$ m and $10^0 - 10^1$ m respectively. The specific simulation scheme is shown in Table 2.

Fig. 7 shows the ejecta fate of the (GS1). At the initial moment, a part of the ejecta stay on the surface of Dimorphos because they are too close to the crater

Table 2 Simulation conditions and final ejecta distribution of GS1 and SS1-SS6.

	Size range (m)	Simulation number	Landing on primary	Landing on secondary	Escape
GS1	$10^{-3} - 10$	100000	32.07%	4.49%	63.44%
SS 1	$10^{-5} - 10^{-4}$	100000	0%	3.74%	96.26%
SS 2	$10^{-4} - 10^{-3}$	100000	14.93%	3.89%	81.18%
SS 3	$10^{-3} - 10^{-2}$	100000	44.26%	5.75%	49.99%
SS 4	$10^{-2} - 10^{-1}$	100000	49.51%	12.48%	37.98%
SS 5	$10^{-1} - 1$	100000	29.22%	21.15%	49.63%
SS 6	$1 - 10$	100000	19.04%	25.05%	55.91%

**Fig. 7** Ejecta fate of GS1, the lines of four colors represent the percentage of ejecta staying on primary, staying on secondary, flying in orbit and escaping out of the binary system.

edge so that their initial velocity relative to the secondary are too small, while the other part enter the orbital space of the binary system. Here we define that an ejecta is considered to have escaped if it moves out of the sphere of influence of the binary system. Part of the ejecta entering orbit will gradually escape from the binary system due to its excessive initial velocity or the influence of solar radiation pressure; the other part will experience collision with asteroid and eventually stay on the two asteroids. In our general simulation, the ejecta

all stopped moving within 12 days, and the final distribution is shown in Table 2.

Fig.8 shows the fate of the ejecta of SS1-SS6, and a very clear feature is that as the ejector size becomes larger, the number of escapes decreases significantly and the survival time in orbit becomes longer, the final distribution is shown in Table 2. The size of the ejecta in SS1 is so small that most of them escape out of the binary system quickly under the influence of solar radiation pressure, except for those that stay on the surface of the secondary at the beginning, and no ejecta even land on the surface of the primary. For SS5 and SS6, some of the ejecta survive in orbit for long time, even more than 600 days.

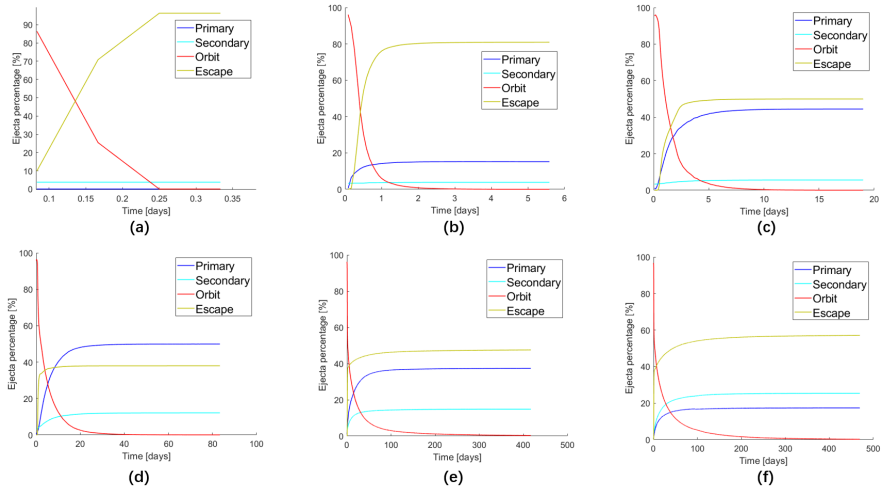


Fig. 8 Ejecta fate of SS1 - SS6. The meanings of different color lines are consistent with Fig.7

In our simulation model, the ejecta size variation affects the magnitude of solar radiation pressure. Here we introduce an index d , which is defined as the critical ejecta diameter d for a certain position, at which the effect of the binary system gravitational force and the solar radiation pressure force is equal. If the ejecta size is smaller than d , the influence of solar radiation pressure dominates, otherwise the gravitational force of the binary asteroids play a major role. Fig.9a shows the distribution of d in the sphere of influence of the binary system (viewed from the normal direction of the orbit), and the farther away from the mass center of binary system, the stronger the effect of solar radiation pressure. Fig.9b shows the distribution of d near center of mass of the binary system. It can be seen that there is a larger d area near Dimorphos, because the gravitational forces in this region from the two asteroids are in opposite direction. Therefore, the overall effect is weak, making the solar radiation pressure dominant. The index d also indicates the minimum size at which a ejecta can survive in orbit for a long time, if the size of the

ejecta is smaller than d , it will finally escape from the binary system under the influence of solar radiation pressure.

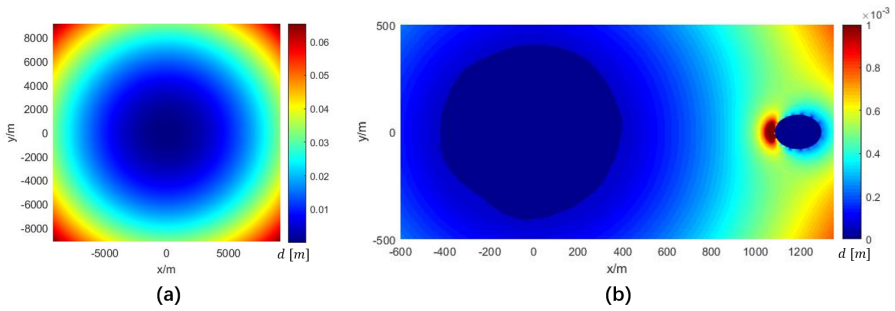


Fig. 9 Distribution of index d . (a) shows the distribution of d within the radius of the binary system influence sphere; (b) shows the distribution of d near the primary and secondary. Note the order of magnitude difference in color between (a) and (b).

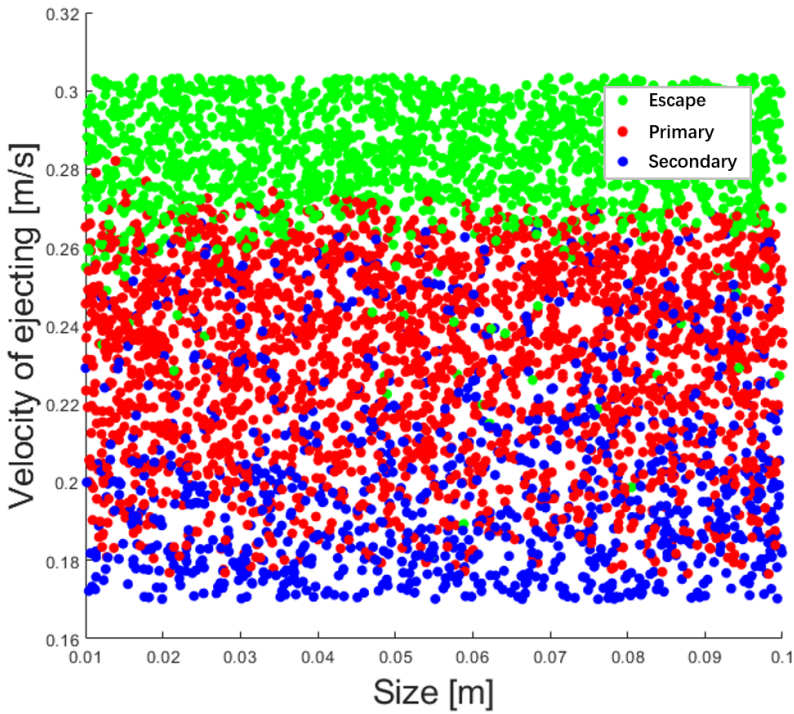


Fig. 10 Final distribution of 5000 random ejecta with different velocity in SS4.

We randomly selected 5000 ejecta as a sample to observe the effect of initial ejecting velocity on the final distribution (Fig.10). The green dots represent the ejecta escaping from the binary system, the red dots represent those landing on the surface of the primary, and the blue dots indicate those landing on the surface of the secondary. It can be clearly seen that ejecta with small enough initial ejecting velocity fall directly on the surface of the secondary, while ejecta with large enough initial ejecting velocity escape, and other ejecta will eventually land on the surface of the primary or secondary after orbital motions. The distribution of green dots can also prove that our initial ejecta velocity threshold of less than $1.2 \times v_{esp}$ is reasonable.

3.2 Coefficient of restitution influence

Uncertain mechanical properties of asteroid surface regolith would introduce different coefficients of restitution. With a small coefficient of restitution, ejecta lose more energy and are more likely to land on the asteroid surface after collision compared to a larger coefficient of restitution. Under the simulation conditions of GS1 and SS1-SS6, we adjusted the tangential coefficient of restitution and the normal coefficient of restitution to be 0.05, and perform 7 simulations (denoted as GS2 and SS7-SS12).

The final distribution of the ejecta for GS2 and SS7-SS12 is given in Table 3, and the ejecta fate of GS2 is shown in Fig.11. The results show that when the coefficient of restitution becomes small, the proportion of ejecta landing on the asteroid is larger. In addition, a smaller coefficient of restitution leads to a lower reflection velocity of the ejecta, which further reduces the survival time of the ejecta in the orbit. Fig.12 shows the orbital survival times of ejecta of different sizes under these two collision coefficients of restitution. For ejecta of size $10^{-5} - 10^{-4}$ m, the coefficient of restitution has no effect on orbital survival time, because the ejecta are so small that they are quickly cleared by the solar radiation pressure and escape the binary system without colliding with the asteroid surface. For larger size ejecta, a larger coefficient of restitution increases the number of collisions and thus makes the orbital survival time longer. Therefore, the time of ejecta survive in orbit is sensitive to the surface properties of the asteroid.

Table 3 Simulation conditions and final ejecta distribution of GS2 and SS7-SS12.

	Size range (m)	Simulation number	Landing on primary	Landing on secondary	Escape
GS 2	$10^{-3} - 10$	100000	30.70%	4.71%	64.59%
SS 7	$10^{-5} - 10^{-4}$	100000	0%	3.63%	96.37%
SS 8	$10^{-4} - 10^{-3}$	100000	26.12%	4.50%	69.38%
SS 9	$10^{-3} - 10^{-2}$	100000	42.01%	5.07%	52.92%
SS 10	$10^{-2} - 10^{-1}$	100000	53.59%	8.33%	38.08%
SS 11	$10^{-1} - 1$	100000	37.61%	15.75%	46.64%
SS 12	$1 - 10$	100000	23.31%	19.94%	56.75%

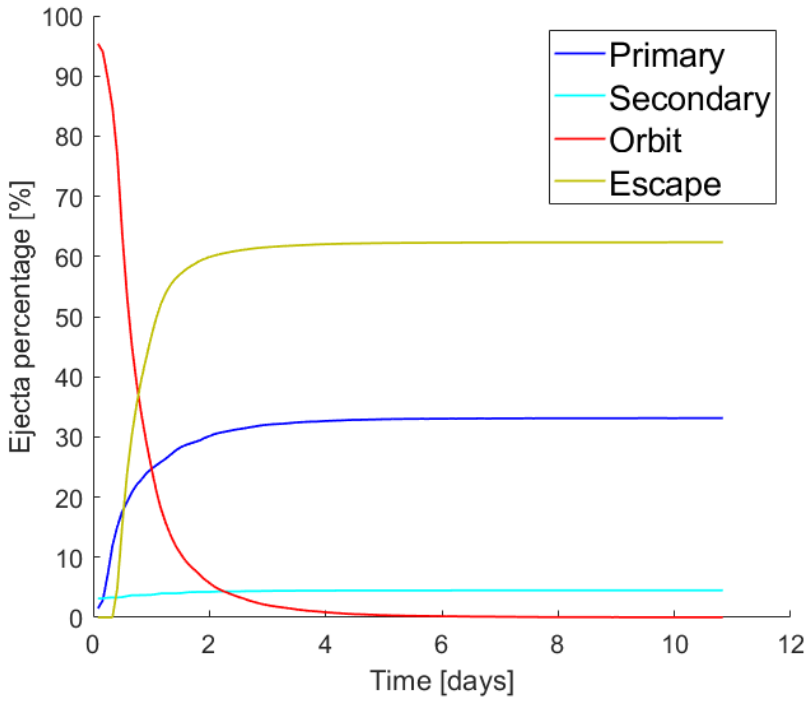


Fig. 11 Ejecta fate of GS2, the meanings of different color lines are consistent with Fig.7

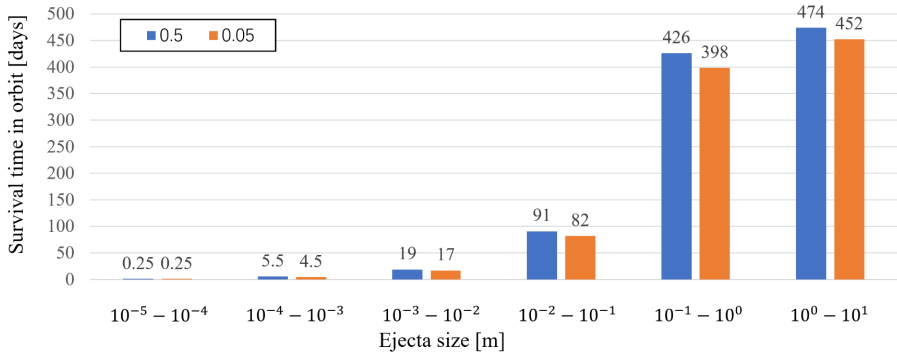


Fig. 12 Orbital survival times of ejecta with different sizes. Blue indicates that the tangential/normal coefficient of restitution are both 0.5, and orange indicates that the tangential/normal coefficient of restitution are both 0.05.

3.3 Dependence on the tumbling rotational state of Dimorphos

The present observations show that Dimorphos is in tidal self-locking with Didymos, i.e. the angular velocity of the rotation of Dimorphos is equal to the angular velocity of the revolution. Impact can destabilize this state, and it was found that Dimorphos is more prone to destabilization in a rotational state oriented around its long axis (Agrusa et al (2021)). This tumbling rotational state is related to the shape of the secondary and the momentum transfer efficiency. In this section we discuss the effect that the tumbling rotational state of the secondary would have on the ejecta on the surface. For the momentum transfer efficiency, we take 3 here under the applicable model described above. We assume that the impact takes place at the extreme edge of the long axis of the secondary (which can cause the largest change in the angular velocity of the secondary) and that the change in angular velocity is

$$\Delta\omega_S = \frac{3m_k U_k l}{J_S^Z} \quad (18)$$

where l is the arm of force of the impact to the centre of mass of Dimorphos (here we take the longest semi-long axis of the secondary), J_S^Z is the rotational inertia around its axis of rotation and U_k is impact velocity of kinetic impactor and m_k is the mass of kinetic impactor. For the description of the attitude of the secondary, we use the 1-2-3 Euler angles, defined as ϕ , θ and ψ respectively, and the configuration of the system is shown in Fig.13a. Using the parameters of Eq.18, the impact may cause a 50% change in angular velocity, so we changed the ratio of the rotation angular velocity to the revolution angular velocity of the Dimorphos from 1:1 to 1.5:1. The variation of the Euler angle is shown in Fig.13b, which shows that at about 8th day the secondary librates more than 90° around its long axis (ϕ), this is a situation we call 'flip'. We use the simulation conditions of GS2 to calculate the evolution of the ejecta in this state, denoted as SS13.

Table 4 gives the final distribution of the ejecta and their orbital survival time. The results show that the secondary tumbling rotational state does not significantly affect the final distribution of the ejecta, because the increase in the spin angular velocity caused by its tumbling rotation does not make its surface ejecta degree of shedding. However, this tumbling rotational state may trigger mass movement from on surface of the secondary, which we can study by calculating the relative acceleration of the material on the surface of the secondary with respect to local surface (please refer to Section 2.1 for the specific calculation formula).

The distribution of the relative acceleration from the surface of the secondary in the case of self-locking to the primary is given in Fig.14a (note that here we have converted the relative acceleration $\mathbf{a}_r$ to the coordinate system $\mathcal{B}$.), the arrows indicate the tendency of the matter to move and the colour

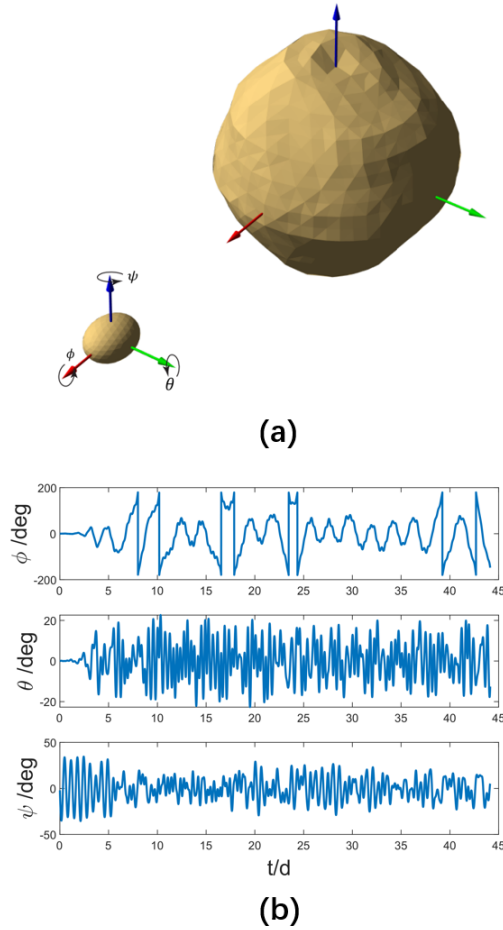


Fig. 13 Explanation of the parameters of tumbling rotational state of the secondary. (a) Diagram of the 1-2-3 Euler angles; (b) Variation of the three Euler angles with time for a ratio of the angular velocity of spin to the angular velocity of revolution of 1.5:1.

Table 4 Simulation conditions and final ejecta distribution of GS2 and SS13.

	Simulation number	Landing on primary	Landing on secondary	Escape	Orbital survival time
GS 2	100000	30.70%	4.71%	64.59%	10.75 days
SS 13	100000	30.91%	4.82%	64.27%	10.75 days

represents how stable the ejecta is in the local area, with warmer colours indicating that the ejecta is more likely to stay in place; cooler colours indicating that the ejecta is more likely to slide. The specific concrete definitions of the

colour values are as follows:

$$\lambda = \arctan \left| \frac{a_r^t}{a_r^n} \right| \quad (19)$$

where a_r^t is the projection of the relative acceleration in the local surface.

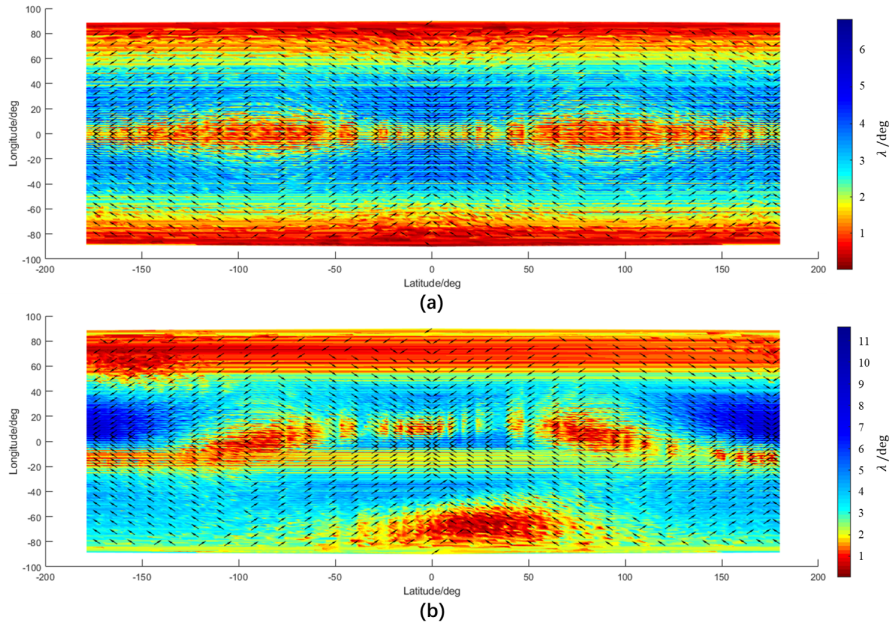


Fig. 14 Distribution of the relative acceleration on the surface of the primary, where the arrows indicate the direction of the relative acceleration and the colours indicate the magnitude of λ . (a) A moment when the secondary is tidally locked to the primary; (b) A moment when the secondary is unstably spinning.

The results show that in the latitudinal direction there is a tendency for material to slip towards the equator; in the longitudinal direction there is a tendency for material to slip towards the two apices of the long axes of secondary. The boundary between the eastward and westward slip of material occurs mainly at longitudes of $\pm 90^\circ$, 0° and $\pm 180^\circ$. The region with the greatest tendency to slip is distributed in the north direction and south direction at two apices of the long axes.

Fig.14b shows the relative acceleration from the surface of the secondary at a given moment in the 'flip' state. The results show a clear change in the distribution of λ , with the material on the surface of the secondary becoming more susceptible to sliding, and a change in the location of the most susceptible regions. There is also a change in the boundary between the eastward and westward sliding of the material, which would allow the surface shape to change over the long evolution of the secondary.

3.4 Dependence on the internal structure of primary

The internal structure of the primary of the Didymos system has so far been little studied, and its plausible effect may have on the deposition of ejected material is unknown. In this section, we focus on the influence of the internal mass distribution of the primary on the fate of the ejecta. We keep the mass constant and adjust the internal structure of the primary, using four models: (a) Internal density distribution is uniform. (b) The density decreases from the inside to the outside, with the center density set to 3.2g/cm^2 (equivalent to the density of granite) and the surface density set to 1.6g/cm^2 (equivalent to the density of sand soil); (c) the internal structure is hollow. (d) The internal structure contains some holes (we implement it by removing some elements and replace them with voids); The four models correspond to Fig.15a, b, c and d respectively.

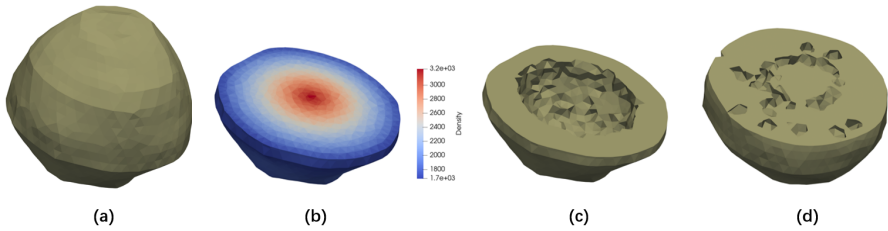


Fig. 15 The internal structure of the primary. (a) Uniform internal mass distribution; (b) Density decreases from inside to outside; (c) Hollow structure; (d) Internal randomly distributed holes.

We started SS14 - SS16 separately, using the same initial conditions for each simulations, to calculate the evolution of 100,000 ejecta with a size distribution ranging from 1 mm to 10 m. Table 5 gives the final distribution of the ejecta and the orbital survival time. The results show that the internal structure of the primary has no significant effect on the evolution of the ejecta. The number of ejecta landing on the surface of the primary is almost identical.

The primary of Didymos system has a very fast spin rate, causing ejecta in some regions to be thrown off the surface and into orbit. The internal structure of the primary can have an effect on the distribution of these regions, resulting in a different distribution of ejecta on the surface of the primary. We analysed the mechanical environment at the surface of the four primary models shown in Fig.15 to study the regions where ejecta can rest on the surface of the primary.

Table 5 Simulation conditions and final distribution of GS2 and SS14-SS16.

	Simulation number	Landing on primary	Landing on secondary	Escape	Orbital survival time
GS 2	100000	30.70%	4.71%	64.59%	10.75 days
SS 14	100000	30.84%	4.66%	64.50%	10.75 days
SS 15	100000	30.83%	4.69%	64.48%	10.75 days
SS 16	100000	30.33%	4.68%	64.99%	10.75 days

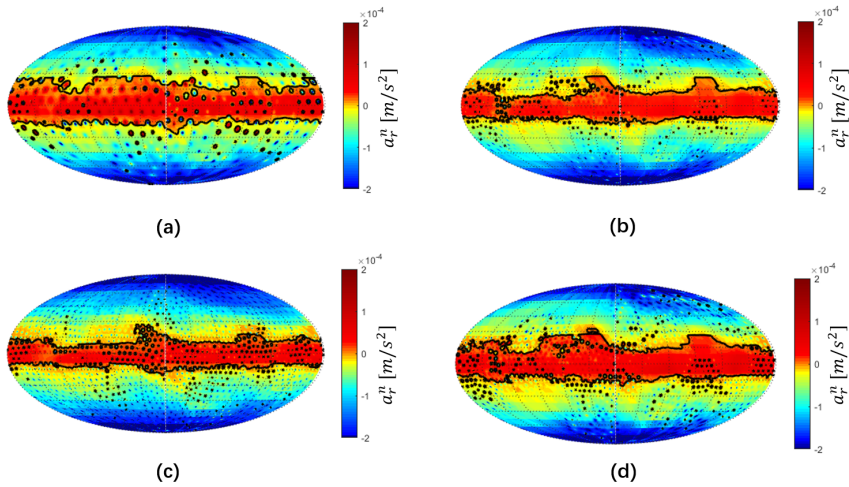


Fig. 16 Distribution of the normal component of the relative acceleration on the surface of the primary for the four internal structures corresponding to Fig.15, where the regions surrounded by the black line indicates that the ejecta cannot stay local.

Using the method of Section 2.5, we calculated the local normal component a_r^n of the relative acceleration on the surface of the primary, which if positive indicates that the ejecta cannot stay here and will take off. The distribution of a_r^n for the four models of the primary described above is given in Fig.16. The area surrounded by black lines is the area where the ejecta cannot stay. There are some small black "circles" in Fig.16, which are due to the gravitational field errors generated by the vertices of the polyhedron model. This will be corrected later when a more accurate model is obtained. The results show that ejecta close to the equator cannot stay on the surface. The distribution of regions differs under the four models, suggesting that the internal structure of the primary has an influence on the deposition distribution of the ejecta on the surface of the primary.

4 Conclusions

We developed a detailed model for numeric simulation of the post-impact motion of a binary-ejecta system. For the interaction between ejecta and two asteroids, we propose a fast collision detection model, which can greatly improve the efficiency of collision detection. CLL model is used for collision processing, taking into account the diffusiveness of ejecta. With large-scale simulations, we get the evolution of ejecta with diameters from 0.01 mm to 10 m. The ejecta with lower velocity will land directly on the surface of the secondary, while others will enter the orbit of the binary system and eventually escape from the binary system or land on the surface of the two asteroids. We found that the ejecta less than 0.1 mm in diameter is more susceptible to escape quickly under the influence of solar radiation pressure, while the larger

size ejecta will survive in the orbit for a long time. We examined the survival time of the ejecta in orbit with two coefficients of restitution, and the results show that a large coefficient of restitution results in a longer survival time of the ejecta in orbit. We analysed the effect of the tumbling rotational state of the secondary on the distribution of material on the surface of the secondary after the impact. This state results in a significant change in surface slope of the secondary but without causing the material to slip. Besides, we analysed the effect of the internal structure of the primary on the distribution of the ejecta on its surface. The result show the area in which the ejecta can remain varies with its internal structure. In the future, when finer asteroid surface models are obtained, we can add asteroid surface material migration models to our models to study the evolutionary behaviour of binary surface features after impact.

Acknowledgments. We acknowledge Hera WG3 group for constructive conversations and useful discussions. Y.Y. acknowledges financial support provided by the National Natural Science Foundation of China Grant No. 12022212.

Declaration. The financial support of our research is provided by the National Natural Science Foundation of China Grant No. 12022212.

We declare that the authors have no competing interests as defined by Springer, or other interests that might be perceived to influence the results and/or discussion reported in this paper.

We confirm that the manuscript has been read and approved by all named authors and that there are no other persons who satisfied the criteria for authorship but are not listed. We further confirm that the order of authors listed in the manuscript has been approved by all of us. Yunfeng Gao and Yang Yu wrote the main manuscript text and prepared figures 1-16. The research work is discussed by all authors. All authors reviewed the manuscript.

The results/data/figures in this manuscript have not been published elsewhere, nor are they under consideration by another publisher. Our dynamic models and data are free to use. If you want to use it, please contact the author by email and explain the purpose.

We confirm that we have given due consideration to the protection of intellectual property associated with this work and that there are no impediments to publication, including the timing of publication, with respect to intellectual property. In so doing we confirm that we have followed the regulations of our institutions concerning intellectual property.

We understand that the Corresponding Author is the sole contact for the Editorial process. He is responsible for communicating with the other authors about progress, submissions of revisions and final approval of proofs.

References

Agrusa H, Richardson D, Barbee B, et al (2022) Predictions for the dynamical state of the didymos system before and after the planned dart impact. LPI

Contributions 2678:2447

Agrusa HF, Gkolias I, Tsiganis K, et al (2021) The excited spin state of dimorphos resulting from the dart impact. *Icarus* 370:114,624

Benner LA, Margot J, Nolan M, et al (2010) Radar imaging and a physical model of binary asteroid 65803 didymos. In: AAS/Division for Planetary Sciences Meeting Abstracts# 42, pp 13–17

Burns JA, Lamy PL, Soter S (1979) Radiation forces on small particles in the solar system. *Icarus* 40(1):1–48

Cercignani C, Lampis M (1971) Kinetic models for gas-surface interactions. *Transport theory and statistical physics* 1(2):101–114

Cheng A, Michel P, Jutzi M, et al (2016) Asteroid impact & deflection assessment mission: kinetic impactor. *Planetary and space science* 121:27–35

Cheng AF, Rivkin AS, Michel P, et al (2018) Aida dart asteroid deflection test: Planetary defense and science objectives. *Planetary and Space Science* 157:104–115

Gao Y, Yu Y, Cheng B, et al (2022) Accelerating the finite element method for calculating the full 2-body problem with cuda. *Advances in Space Research* 69(5):2305–2318

Holsapple KA, Housen KR (2012) Momentum transfer in asteroid impacts. i. theory and scaling. *Icarus* 221(2):875–887

Housen KR, Holsapple KA (2011) Ejecta from impact craters. *Icarus* 211(1):856–875

Margot JL, Nolan M, Benner L, et al (2002) Binary asteroids in the near-earth object population. *Science* 296(5572):1445–1448

Michel P, Cheng A, Küppers M, et al (2016) Science case for the asteroid impact mission (aim): a component of the asteroid impact & deflection assessment (aida) mission. *Advances in Space Research* 57(12):2529–2547

Naidu S, Benner L, Brozovic M, et al (2020) Radar observations and a physical model of binary near-earth asteroid 65803 didymos, target of the dart mission. *Icarus* 348:113,777

Rainey ES, Stickle AM, Cheng AF, et al (2019) Impact modeling for the double asteroid redirection test mission. In: Hypervelocity Impact Symposium, American Society of Mechanical Engineers, p V001T04A003

- Rainey ES, Stickle AM, Cheng AF, et al (2020) Impact modeling for the double asteroid redirection test (dart) mission. *International Journal of Impact Engineering* 142:103,528
- Rivkin AS, Chabot NL, Stickle AM, et al (2021) The double asteroid redirection test (dart): planetary defense investigations and requirements. *The Planetary Science Journal* 2(5):173
- Yu Y, Michel P (2018) Ejecta cloud from the aida space project kinetic impact on the secondary of a binary asteroid: Ii. fates and evolutionary dependencies. *Icarus* 312:128–144
- Yu Y, Michel P, Schwartz SR, et al (2017) Ejecta cloud from the aida space project kinetic impact on the secondary of a binary asteroid: I. mechanical environment and dynamical model. *Icarus* 282:313–325
- Yu Y, Cheng B, Hayabayashi M, et al (2019) A finite element method for computational full two-body problem: I. the mutual potential and derivatives over bilinear tetrahedron elements. *Celestial Mechanics and Dynamical Astronomy* 131(11):1–21