

Supplementary information for

Coherent Surface Plasmon Polariton Amplification via Free Electron Pumping

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Methods

Experimental setup

The experiments were performed with a Ti: Sapphire laser system, which delivers 8 mJ, 30 fs, p-polarized, 800 nm-centered (bandwidth spanning 750–850 nm) laser pulses at 1 kHz repetition rate. Supplementary Fig.S1 depicts the optical beam path and the pump-probe designs for the two different sets of SPPs measurements. As schematically shown in Fig.S1, the pulsed laser beams are split into two arms with 91:9 energy ratio respectively for the pump and the probe utilities. The pump pulse that undergoes a beam expansion and tight focusing by an $f/2$ off-axis parabolic mirror, is then irradiated onto a 50- μm -diameter iron metal wire to excite the THz SPPs seed and the simultaneously generated copropagating electrons, as described in detail in ref¹. Here, inside the 5- μm -radius spot region over the target surface, approximately 3 mJ energy is deposited, resulting in about $1.27 \times 10^{17} \text{ W cm}^{-2}$ power density on the wire surface. During the whole experiment, the excitation and the detection of the THz SPPs were carried out in single-shot mode in a vacuum chamber under the pressure of $5 \times 10^{-3} \text{ Pa}$. Meanwhile, the wire target is moved endwise after each shot, so that the laser pulse can always interact with a fresh area.

To measure the spatiotemporally resolved THz SPPs evolution, we implemented a 1 mm-thick, (111)-cut terbium gallium garnet ($\text{Tb}_3\text{Ga}_5\text{O}_{12}$, TGG) single crystal which is commonly used in optical isolators and magnetic field characterizations. The TGG crystal is constructive for identifying magnetic field profiles since it is transparent to infrared light and has a very large Verdet constant of 80 rad/Tm at 800 nm². Thus, by examining the magneto-optic rotations of the probe polarization at different time delays, we can reveal the SPPs evolutions from its different spatial slices acquired at different TGG distances. In this experiment, the 10 mm $\times$ 20 mm $\times$ 1 mm TGG crystal is placed in the near-field of the THz SPPs respectively at 500, 750, and 1000 μm , standing parallel to the wire axis. The probe beam with 0.10 mJ cm⁻² fluence hence traverses through the THz SPPs, the TGG crystal, a 3-fold magnification $4f$ imaging system, a Glan prism, a polarizer, and is then captured by two separate cameras for recording both the p- and the s-polarization snapshots.

The electric field waveform of the THz SPPs is obtained by diverging the probe beam from orthogonal to an approximately parallel geometry. In this measurement, the probe beam is positively chirped from 30 fs to 8.66 ps using a 50-cm-long SF11 glass rod. This ensures the probe pulse duration exceeds that of the SPPs, such that the entire waveform can be imprinted on the

polarization changes of the probe beam as it traverses through the 1-mm-thick electro-optic crystal (here we used the zinc telluride, ZnTe)³. The ZnTe that sits closely adjacent to the wire surface is placed 3 cm downstream of the pump focus with its (001) crystallographic axis aligned perpendicular to the wire. To supplement the magnetic measurement, the probe beam is focused by a convex lens directly onto the contacting region between the ZnTe crystal and the wire. In both the magnetic and the electric field measurements, the probe beams are temporally overlapped with the THz SPPs by controlling the probe delay time with a motorized stage.

SPPs dynamics reconstruction

In a paramagnetic medium like TGG, the magneto-optical induced Faraday polarization changes ψ to a linearly-polarized light beam that propagates along the x-axis can be expressed as:

$$\psi = \int_0^L V B_x(x, t) dz \quad (1)$$

where V and L respectively denote the Verdet constant and the thickness of the medium, and $B_x(x, t)$ is the longitudinal field component of the external magnetic field $B(r, t)$. Since the femtosecond probe pulse duration is much shorter than the THz SPPs, the magnetic field can reasonably be regarded as invariant as it traverses through the TGG crystal, so that the Faraday polarization angle can be simply written as $\psi = VL B_x$.

Accordingly, we can acquire the instantaneous spatial magnetic field profiles on the TGG crystal from the polarization changes of the probe beam. Supplementary Fig.S2 presents the magnetic field evolutions on different TGG distances in the near field of the THz SPPs respectively at $d = 500, 750$, and $1000 \mu\text{m}$, with each pixel derived from the p- and the s- components of the probe beam considering the projection angle of the magnetic field on the TGG crystal. To illustrate this more explicitly, this is because the transverse magnetic field in the x-y plane has an angular distribution that is symbolic of the TM_{01} mode, as schematically shown in Fig.S5a. Such detection geometry would involve only the $B_x = B \cos \phi = B \cos(\arctan(d/h))$, at a specific height h above the horizontal central plane. As a result, the instantaneous magnetic field can be deduced as:

$$B(r, t) = \frac{1}{VL \cos \phi} \arctan \sqrt{\frac{[I_{ss}] - [I_{sb}]}{[I_{ps}] - [I_{pb}]}} \quad (2)$$

77 where $[I_{ss}]$, $[I_{sb}]$, $[I_{ps}]$ and $[I_{pb}]$ respectively stands for the s- and p-polarized signals (that is, with
78 the pump beam) and background (without the pump beam) matrix, each acquired by the CCDs.

79 We note that in Fig.S2-S5, the magnetic fields sliced at different TGG distances have distinct
80 evolution patterns in terms of their appearance time, strength, moving speed, and even the rising
81 time durations (defined as from 10% to 90% of the maximum). The specific comparisons of the
82 three different sets of data are presented in Fig.2b in the main text and Fig.S5b. Qualitatively, these
83 differences arise as a natural process of the SPPs expansion, which evolves from the initial point-
84 like laser focus spot to the propagating THz SPPs over the wire surface, just as analogous to the
85 radiation emitted by a moving point charge. We simulated this process in Fig.3d-f in the main text,
86 where the SPPs formation, amplification, and part of the propagation processes are exemplified,
87 giving lucid support to the experimental variances we observed.

88 As a result, to reconstruct the THz SPPs dynamic, one needs to coordinate different magnetic
89 field profiles in the whole space to depict the precise spatiotemporal evolutions of the THz SPPs.
90 However, this is a formidable task since it demands immense available experiment space and
91 laborious optical path adjustment. Here, we have made a simple assumption that the THz SPPs are
92 rotational symmetric owing to the TM_{01} mode. Based on this assumption, we can extract the
93 properties of the THz SPPs by fitting the near field data acquired from different TGG distances to
94 the surface of the wire. The total magnetic energy evolution in the detection zone (from 500 to
95 2000 μm to the wire surface, considering the thickness of the TGG) is deduced by integrating the
96 magnetic field over this volume (Fig.2b). It should be noted that, the THz SPPs contained beyond
97 this zone, especially within the 500 μm distance, is not detectable due to intense scattering light
98 interruptions by the pump light. Consequently, the measured magnetic energy evolution in Fig.2b
99 only records part of the amplification process, whose maximum field is acquired at a time ($\tau_d =$
100 4.27 ps) that corresponds to that of the electric field characterization with a rising duration of 4.43
101 ps.

103 **SPPs spectrum analysis**

104 The THz SPPs excited by the femtosecond laser pulse is in nature a half-cycle wave packet
105 without the inverse field part⁴. Microscopically, this originates from the net electron flow (current)
106 that moves along the wire, which gives rise to a nonvanishing net direct current (DC) component
107 in the SPPs. However, when these THz SPPs are emitted from a photonic structure (i.e., the metal

wire in our case), the DC current would turn into a varying one and emit alternating electromagnetic fields into the free space. In this respect, such a scenario resembles exactly the situation in a radiating antenna, where the shifting current at the transmitting end radiates electromagnetic waves. We henceforth refer to the frequency of the emitting THz electromagnetic waves as that of the half-cycle THz SPPs.

Essentially, the far-field electromagnetic spectrum can be extracted from the near field SPPs field. In our analysis, we completed this process by taking the time derivative of the SPPs wave packet that is taken by the TGG at different time delays. Hence, the electric field radiated by a traveling current must have the following form⁵,

$$\vec{\mathcal{E}}(\vec{r}, t) \propto \frac{1}{r} \frac{d\Gamma\left(t - \frac{r}{c}\right)}{dt} \vec{\theta} \quad (3)$$

where Γ is the envelope of the current, $\vec{\theta}$ is the angle of observation. By doing so, the current changes of the instantaneous THz SPPs can be deduced supposing that it is immediately emitted into the free space. And consequently, the corresponding frequency spectrum can be acquired via the conventional Fourier transformations.

Semiclassical calculation of the electron-SPPs interactions

In this section, we develop the theory for the semiclassical description of the electron-SPPs interactions and elaborate on their numerical calculation methods.

The wave function for an electron pulse in free space can be written as,

$$\Psi(x, t, \tau) = \exp\left[ik_0 z - i\frac{U_0}{\hbar}(t - \tau)\right] \phi\left[z - \frac{\hbar k_0}{m}(t - \tau)\right] \quad (4)$$

where ϕ describes the envelope of the free electron and the wave function that is located at $z = 0$ when $t = \tau$, and τ is the time delay introduced to the electron wave function. Hence, the time evolution of the electron wave function during the interaction of an electromagnetic field, such as the case depicted in Fig.4a, can be expressed by the Schrodinger equation of the form,

$$i\hbar \frac{\partial \Psi}{\partial t} = \frac{(p + qA)^2}{2m} \Psi \quad (5)$$

The solution of this equation, as detailed in ref ⁶, can be derived based on the following transportation equation:

$$135 \quad v_e \frac{\partial \phi}{\partial z} + \frac{\partial \phi}{\partial t} = i v_e f(z, t) \phi \quad (6)$$

136 where $f(z, t) = -\frac{e}{\hbar \omega} E(z, t)$ is real. The result can hence be deduced as,

$$137 \quad \Psi(z, t, \tau) = \exp \left[i k_0 z - i \frac{U_0}{\hbar} (t - \tau) \right] \phi \left[z - \frac{\hbar k_0}{m} (t - \tau) \right] \exp i v_e \int_{t_0}^t f(z + v_e(s - t), s) ds \quad (7)$$

138 Taking the ansatz $t - z/v_e = T$, then we have that

$$139 \quad \Psi(T, \tau) = \exp \left[-i \frac{U_0}{\hbar} (T - \tau) \right] \phi \left[\frac{\hbar k_0}{m} (T - \tau) \right] \exp \left[i v_e \int_{t_0}^T f(v_e(s - T), s) ds \right] \quad (8)$$

140 By setting $t \rightarrow +\infty$ and $t_0 \rightarrow -\infty$, this equation yields an almost identical result to that in ref
 141 ⁶. However, the main difference here is that the time is used as the integral parameter instead of
 142 the distance. Meanwhile, to discuss the impact of time lapping between the electron beam and the
 143 laser beam, a parameter τ is introduced to represent the time delay between the two pulses. The
 144 energy spectrum of the electron can be found by Fourier transformation. To compare with the
 145 experimental results, the single electron energy probability distribution should be convoluted with
 146 the distribution function.

147 Using this modified equation (8), we would like to discuss two different scenarios.

148 A. When compared with the extended "PINEM" theory, using time as the integral parameter
 149 makes it easier for us to contrast the PINEM-type experiment with the typical streaking results
 150 such as that reported in ref ⁷. By assuming no electromagnetic field penetration behind the foil in
 151 the case of ref ⁷, which is equivalent to $f(z, t) = 0$ when $s - T > 0$, then we have:

$$152 \quad f(v_e(s - T), s) = -\frac{eE}{\hbar \omega} \theta(T - s) \exp \left(-\frac{s^2}{2\sigma^2} \right) \cos(\omega s) \quad (9)$$

153 Hence the equation (8) can be rewritten as,

$$154 \quad \Psi(T, \tau) = \exp \left[-i \frac{U_0}{\hbar} (T - \tau) \right] \phi \left[\frac{\hbar k_0}{m} (T - \tau) \right] \exp \left[-i \frac{eE}{\hbar \omega} v_e \int_{-\infty}^T \exp \left(-\frac{s^2}{2\sigma^2} \right) \cos(\omega s) ds \right] \quad (10)$$

155 which has almost the same format as that deduced from strong-field approximation (SFA)⁸. The
 156 energy spectrum of this wave function can be calculated based on Fourier transformation,

157

$$158 \quad S(U, \tau) = C_0 \left| \int_{-\infty}^{+\infty} dT \exp \left[i \frac{U - U_0}{\hbar} T \right] \phi \left[\frac{\hbar k_0}{m} (T - \tau) \right] \exp \left[-i \frac{eE}{\hbar \omega} v_e \int_{-\infty}^T \exp \left(-\frac{s^2}{2\sigma^2} \right) \cos(\omega s) ds \right] \right|^2 \quad (11)$$

It should be noted that, there are two characteristics contained in this expression: Firstly, the energy deviation of the electron beam $U-U_0$ is on the order of the photon energy in ref⁷. Hence the energy spectrum of the electron can be acquired from the Fourier transform of the electron pulse envelope. Secondly, besides the free electron energy term $\exp\left[-i\frac{U_0}{\hbar}T\right]$, the envelope $\phi\left[\frac{\hbar k_0}{m}(T-\tau)\right]$ will determine the energy spread of the electron beam.

Based on these features, we now discuss the influence of the electron pulse width on the final energy distribution.

I) The integral in equation (11) can be carried out by approximation,

$$\int_{-\infty}^T \exp\left(-\frac{s^2}{2\sigma^2}\right) \cos(\omega s) ds \triangleq \frac{1}{\omega} \exp\left(-\frac{s^2}{2\sigma^2}\right) \sin(\omega s) \Big|_{-\infty}^T = \frac{1}{\omega} \exp\left(-\frac{T^2}{2\sigma^2}\right) \sin(\omega T) \quad (12)$$

If the pulse width of the electron is much greater than the period of the laser, the result of equation (11) can be expanded based on the Bessel functions. This gives rise to the discrete electron energy probability distribution that is spanned by the photon energy, as shown in Fig.4d.

II) Otherwise, if the pulse width of the electron is shorter than the period of the laser, or the electron energy spread is greater than the energy of a photon, the discrete electron energy ladders would be replaced with another interesting phenomenon. To illustrate this, we consider an electron pulse with a rectangular envelope as follows,

$$\phi\left[\frac{\hbar k_0}{m}(T-\tau)\right] \propto \theta\left(T-\tau-\frac{w_e}{2}\right) + \theta\left(T-\tau+\frac{w_e}{2}\right) \quad (13)$$

where w_e is the pulse width of the electron beam. Here, since the electron pulse width is shorter than the period of the laser, the integral result of equation (12) can be seen as constant (in fact a function of the time delay), then we have the probability distribution for electron energy as:

$$S(U, \tau) = C_0 \left| \frac{\int_{-\infty}^{+\infty} dT \exp\left[i\frac{U-U_0}{\hbar}T\right] \left[\theta\left(T-\tau-\frac{w_e}{2}\right) + \theta\left(T-\tau+\frac{w_e}{2}\right) \right]}{\exp\left[-i\frac{eEv_e}{\hbar\omega^2} \exp\left(-\frac{\tau^2}{2\sigma^2}\right) \sin(\omega\tau)\right]} \right|^2 \quad (14)$$

This result shows that if the electron acquires the energy

$$\frac{eEv_e}{\hbar\omega^2} \exp\left(-\frac{\tau^2}{2\sigma^2}\right) \sin(\omega\tau) \quad (15)$$

then the Fourier Transformation thus can be taken as

$$S(U, \tau) \propto \delta \left(U - U_0 - \exp \left(-\frac{\tau^2}{2\sigma^2} \right) \sin(\omega\tau) \right) \quad (16)$$

where δ is the Delta function. And the line width of $S(U, \tau)$ is proportional to $\frac{1}{w_e}$, corresponding exactly to the result in Fig.4c.

Due to the shorter electron pulse width than the period of the laser, on the one hand, this electron energy spectrum can be seen as the direct result of the Ehrenfest theorem; on the other hand, we can safely say that the electron pulse measures the laser pulse. Since the phase of the laser field is acquired in the interaction, it is intriguing to consider what would happen if there were only one photon in the laser pulse. This question can be answered immediately by the fact that the phase of the photon and photon number operator do not commute⁹,

$$[\tilde{n}, \tilde{\phi}] = 1 \quad (17)$$

The above equation shows that when there is only one photon in the pulse, the phase of the laser field will have substantial uncertainty, which shall lead to strong deviating results from what we have derived above. In fact, most of the PINEM or PINEM like experiments are carried out in the vicinity of a material (metal, dielectrics, etc.), where the photons can be seen as SPPs or phonon polariton. As demonstrated in ref¹⁰, when few photons are presented, the spontaneous radiation of the electron becomes dominant; thus, the electron is expected to lose energy most of the time.

B. In this part, we will discuss the interaction of the electron beams with the SPPs near a metallic material, such as the metal wire used in our experiment. When a propagating SPPs interacts with a copropagating electron beam along the wire, it is of interest to check in what circumstances the results would be different from that in ref⁶. Because the electric field therein has the following form,

$$f \propto \exp[-(z - vt)^2 / (2\Delta^2)] \exp[-z^2 / (2\sigma^2)] \quad (18)$$

This equation indicates that the electric field does not strictly propagate along the z-direction, as determined from the second exponential term in the equation. To be consistent with our discussion, we use an expression for f as follows,

$$f(z, t) = -\frac{eE}{\hbar\omega} \theta(z) \exp \left[-\frac{(z - v_{en}t)^2}{2\sigma_{en}^2} \right] \exp \left(-\frac{z}{\Gamma} \right) \text{real} \left\{ \exp \left[i\hbar \left(\frac{z}{v_{ph}} - \omega t \right) \right] \right\} \quad (19)$$

where v_{en} is the envelope velocity of SPPs, v_{ph} is the phase velocity of the SPPs, and Γ is the dissipation rate of the SPPs along the metal wire. The Heaviside θ function is used here to model the coupling process of a free space electromagnetic field to a metal wire. An error function can be used to make this process more gradual, but this will make the calculation rather complex. The real process may be very complex, but our result can reveal a qualitative interpretation of this process. Equation (19) gives the general expression for SPPs propagating along a metal wire. By substitution, the integral in equation (8) can be written as,

$$\int_{t_0}^t f(v_e(s-T), s) ds = -\frac{eE}{\hbar\omega} \int_T^{+\infty} ds \cdot \exp\left[-\frac{(s-r_{en}T)^2}{2\sigma_{en}^2\eta_{en}^2}\right] \exp\left(-\frac{s-T}{\Gamma}\right) \text{real}\left\{\exp\left[i\omega\left(\frac{v_e(s-T)}{v_{ph}} - s\right)\right]\right\} \quad (20)$$

where we have set $\frac{1}{\eta_{en}} = v_e - v_{en}$, and $r_{en} = \frac{v_{en}}{v_e - v_{en}}$. To explicitly check the frequency dependence of this integral, we can set $\mu = s - T$, then

$$\int_{t_0}^t f(v_e\mu, \mu + T) ds = -\frac{eE}{\hbar\omega} \int_0^{+\infty} d\mu \cdot \exp\left[-\frac{(\mu + Tl_{en})^2}{2\sigma_{en}^2\eta_{en}^2}\right] \exp\left(-\frac{\mu}{\Gamma}\right) \text{real}\left\{\exp\left[i\omega\left(\frac{v_e\mu}{v_{ph}} - \mu - T\right)\right]\right\} \quad (21)$$

where $l_{en} = 1 - r_{en} = \frac{v_e - 2v_{en}}{v_e - v_{en}}$. It can be easily checked that if we set $v_{en} = 0$, $v_{ph} \gg v_e$ and change the integral direction, then we can get the results in case 1.

We can always set the time delay between the two pulses to make sure the lower limit of the integral in equation (21) to be $-\infty$. Thus, the situation can be divided into three scenarios.

I) When $v_e > v_{en}$, the electron beam will catch up with the SPPs. Then equation (21) can be written in a form that resembles a Fourier Transform,

$$-\frac{eE}{\hbar\omega} \exp(-i\omega T) \int_{-\infty}^{+\infty} d\mu \cdot \exp\left[-\frac{(\mu + Tl_{en})^2}{2\sigma_{en}^2\eta_{en}^2}\right] \exp\left(-\frac{\mu}{\Gamma}\right) \exp\left[i\omega\left(\frac{v_e}{v_{ph}} - 1\right)\mu\right] \quad (22)$$

where $\omega(\frac{v_e}{v_{ph}} - 1)$ is just the frequency we get when we carry out the integration. If Γ is very large, then the result is a Gaussian function with the form,

$$-\frac{eE}{\hbar\omega}C\exp(-i\omega T)\exp\left[-\left(\frac{v_e}{v_{ph}}-1\right)^2/2\Sigma^2\right] \quad (23)$$

This expression shows that when the velocity mismatch between the electron and the SPPs is very large ($v_e \gg v_{ph}$ or $v_{ph} \gg v_e$), the outcome would vanish to zero, as analogous to the typical PINEM results. Importantly, when $v_e = v_{ph}$, the electron will propagate in phase with the surface electromagnetic field. This would give rise to substantial energy exchange between electrons and the surface electromagnetic modes.

It should be noted that, when the electron pulse width is shorter than the laser period, a similar energy spectrum that measures the phase of the laser beam is acquired (equation (14)). When $v_e = v_{ph}$, electrons are in phase with the surface electromagnetic field. Consequently, pure energy gain can be achieved in certain phases to form an SPPs-based free electron accelerator, like the dielectric laser accelerator (DLA). But maintaining the electrons in the acceleration phase is challenging, especially in the optical frequency which requires the electrons' pulse width to be small as the attosecond scale. In the opposite case, shear deceleration can also be realized where the declined electron energy is transferred into the surface electromagnetic, laying the ground for the SPPs amplification reported here. Based on equation (14), the energy fluctuation resembles exactly the case of the rabbi oscillation in a two-level system. Hence, by controlling the entering phase of the electrons inside the SPPs field, vast kinetic energy of the electron can be constructively emitted into the surface electromagnetic field. In fact, this can be realized on the assumption that $v_e = v_{en}$, which we will discuss in the next paragraph.

II) As emphasized above, the condition $v_e = v_{en}$ is critical for the realization of the concept for the SPPs light source. Under such a scenario, the first exponential term in equation (21) can be extracted out of the integral. Then,

$$\int_{t_0}^t f(v_e\mu, \mu + T) ds = -\frac{eE}{\hbar\omega} \exp\left[-\frac{(v_e T)^2}{2\sigma_{en}^2} - i\omega T\right] \int_0^{+\infty} d\mu \cdot \exp\left(-\frac{\mu}{\Gamma}\right) \cdot \text{real}\left\{\exp\left[i\omega\left(\frac{v_e\mu}{v_{ph}} - \mu\right)\right]\right\} \quad (24)$$

The result takes the form that resembles the forced oscillation,

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$$-\frac{eE}{\hbar\omega} \frac{\exp\left[-\frac{(v_e T)^2}{2\sigma_{en}^2} - i\omega T\right]}{\frac{1}{\Gamma} - i\omega\left(\frac{v_e}{v_{ph}} - 1\right)} \quad (25)$$

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where $\frac{1}{\Gamma}$ is very small. As a result, enormous energy modulation can be acquired when the phase match between the electron beam and the SPP is maintained and supposes no dissipation. This forms the grounds for the SPPs laser that is based on free-electron energy oscillation.

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Meanwhile, besides the electrons, the laser beam also has a great impact on the electron energy spectrum. The eventual energy divergence is defined as the sum of the electron beam and the laser beam, that is $\Delta E = \Delta E_{e0} + \Delta E_{l0}$, where ΔE_{e0} is the energy divergence of the original electron beam and ΔE_{l0} is energy divergence of the laser beam. Numerical calculation is carried out to support this result as shown in Fig.4.

263

III) The same calculation can be carried out for the case of $v_e < v_{en}$, in which the time delay τ must be negative to make sure the electrons can interact with the surface electromagnetic field. The result resembles that given in I). Meanwhile, when the phase velocity of the surface electromagnetic field is equal to the velocity of the electron beam, such a scenario would just yield a result similar to ref⁶.

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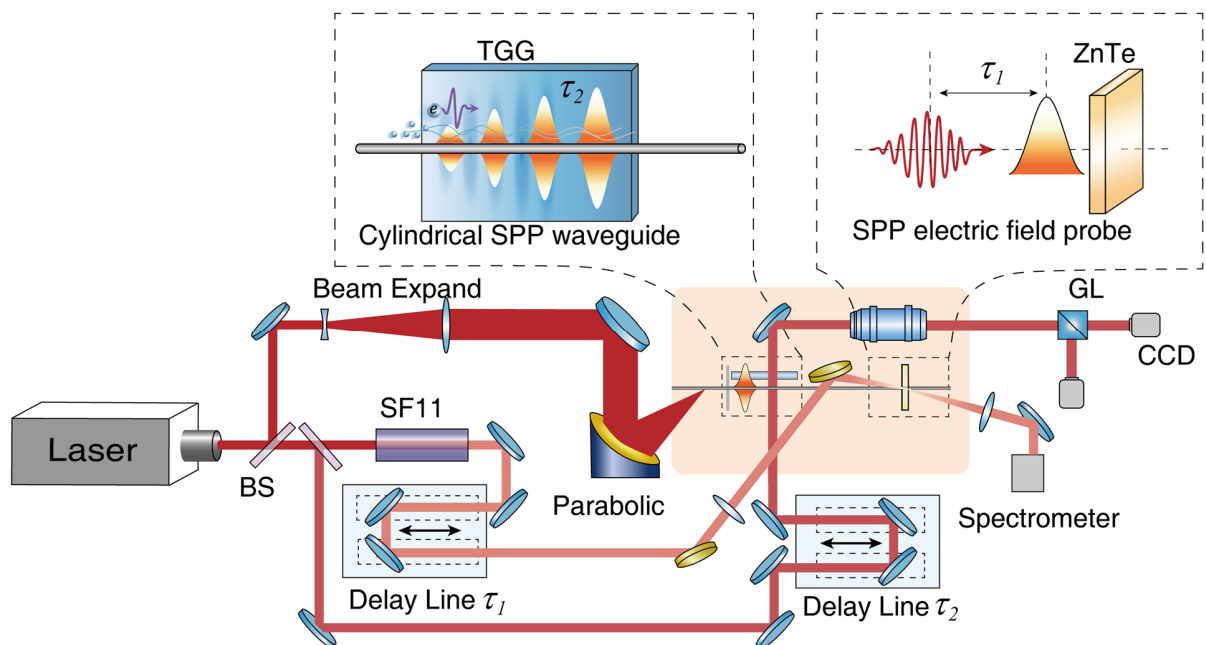


Fig. S1. Experimental setup. A femtosecond laser (red) splits into two arms. The upper arm with 91% of the laser energy is tightly focused by a parabolic mirror to excite the SPPs seed and the copropagating electron beam. The bottom arm can be switched between two optical paths to serve our two different sets of SPPs measurements, whose specific measurement configurations at the near-field of the SPPs are enlarged in the two insets displayed above.

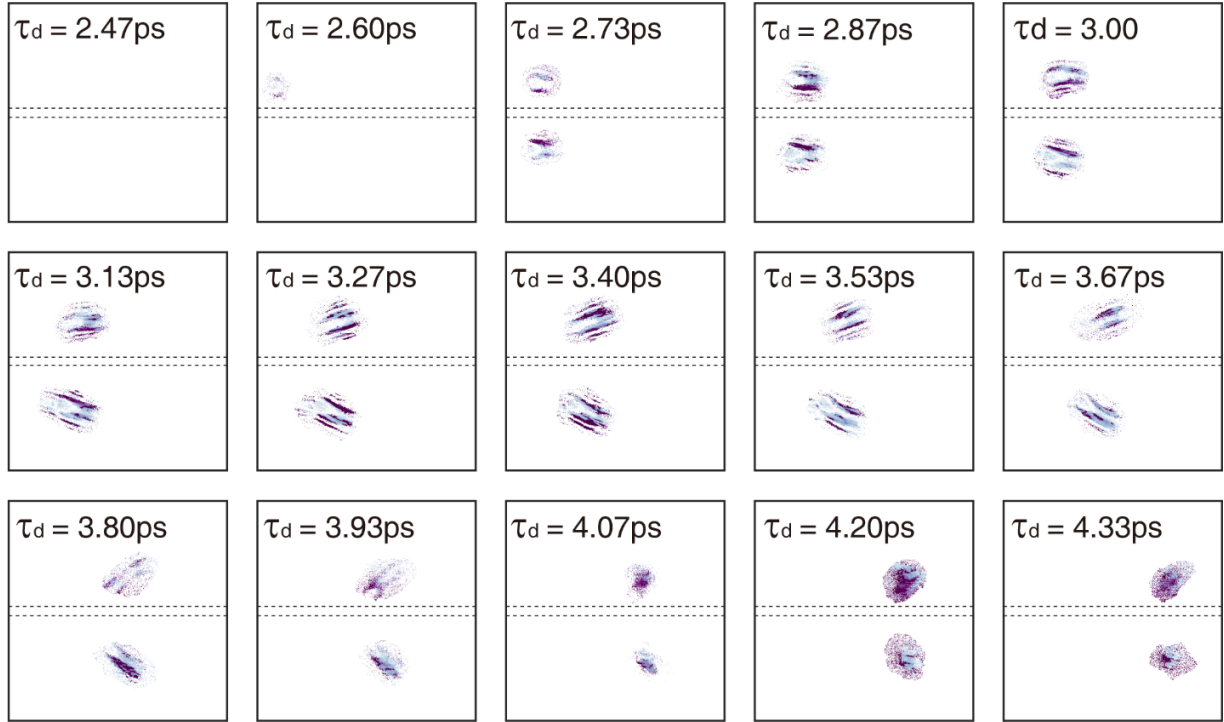


Fig. S2. Magnetic field evolutions of the THz SPPs on the 500 μm -distant TGG. The metal wire borders are marked by the dashed lines. The times are in units of ps.

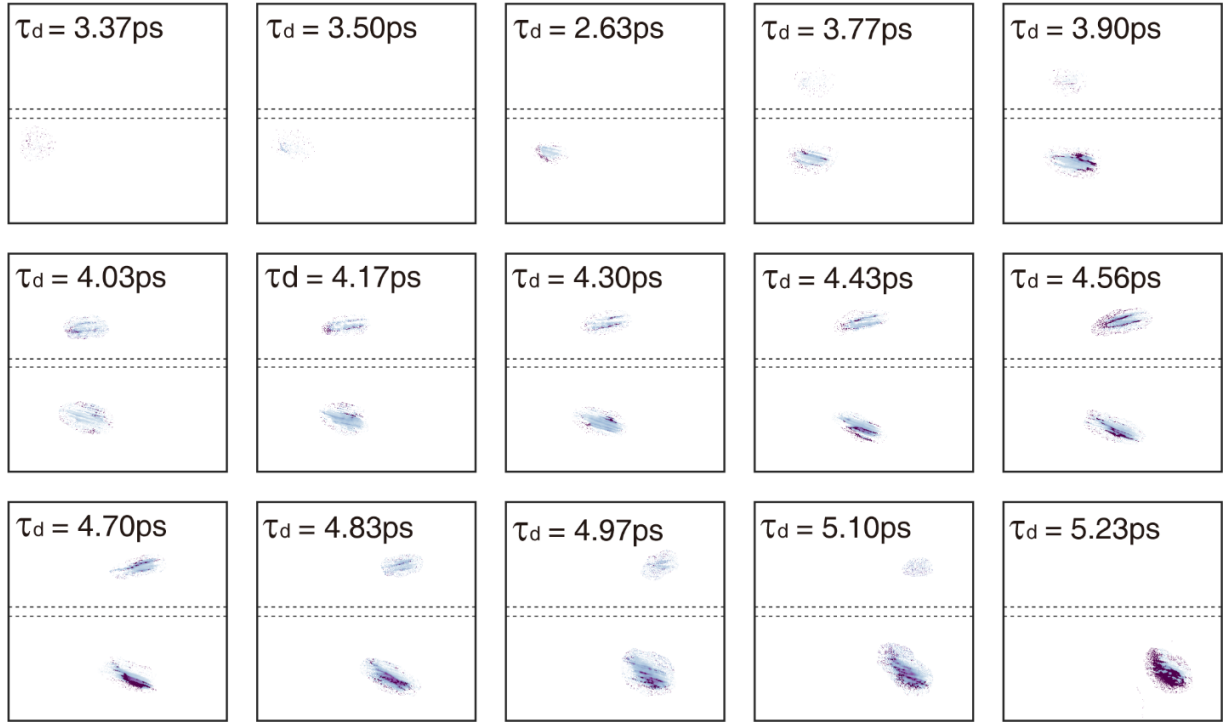


Fig. S3. Magnetic field evolutions of the THz SPPs on the 750 μm -distant TGG. The metal wire borders are marked by the dashed lines. The times are in units of ps.

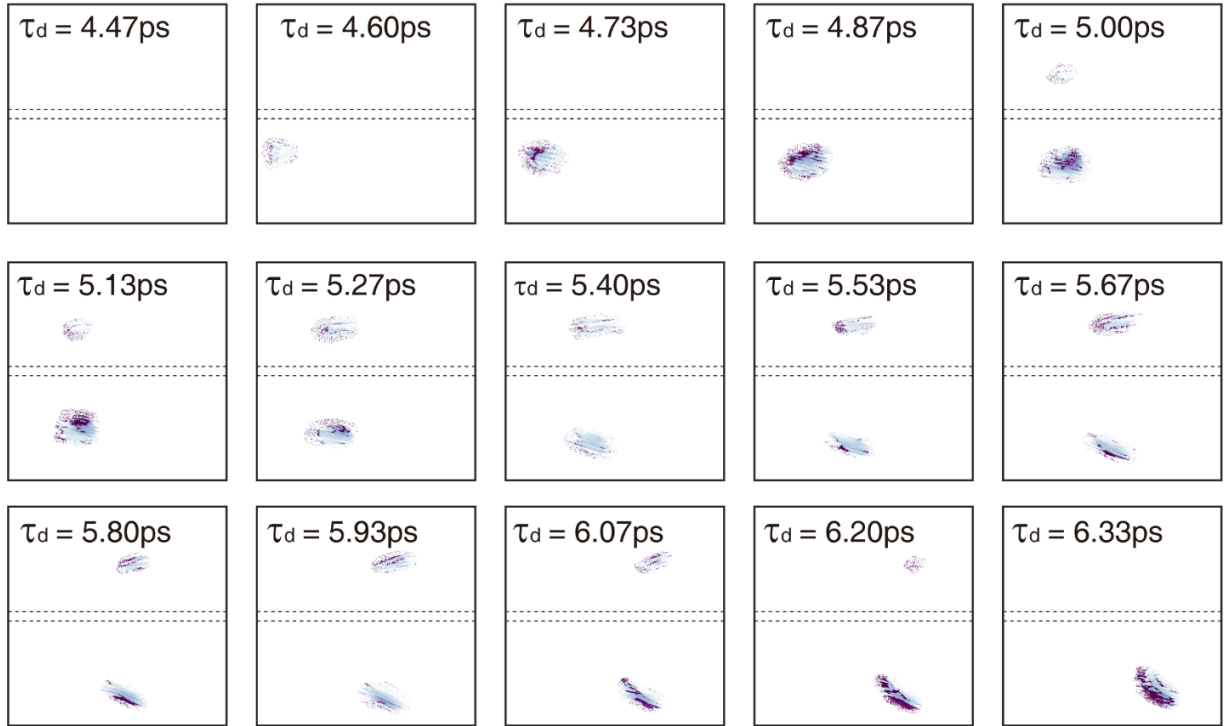


Fig. S4. Magnetic field evolutions of the THz SPPs on the 1000 μm -distant TGG. The metal wire borders are marked by the dashed lines. The times are in units of ps.

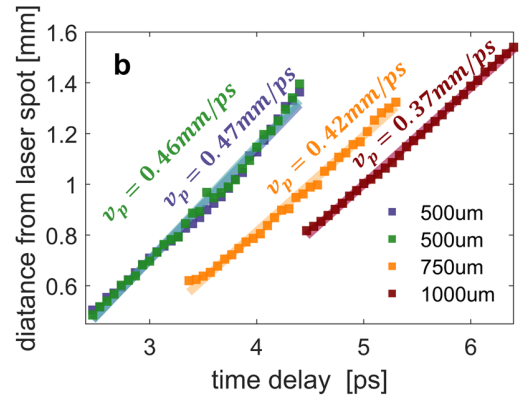
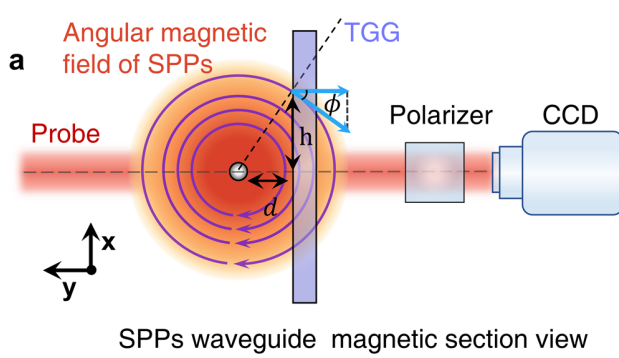


Fig. S5. SPPs' magnetic field dynamics analysis. **a**, Illustration of the magnetic field of the TM_{01} mode on a metallic wire structure. The wire is aligned along the z-axis (perpendicular to the paper plane). Hence, only the $B_x = B \cos \phi = B \cos(\arctan(d/h))$ component of the magnetic field can be imprinted on the polarization changes of the probe light. **b**, The propagation velocity of the magnetic profiles on different TGG distances. Noteworthy, the superluminal propagation speed results from the oblique irradiance angle of the SPPs when it encounters the TGG surface, giving rise to the large phase velocity on the TGG.

296 **Captions for Movies S1**

297 **Movie S1. Video showing the evolution dynamics of the THz SPPs on the 500- μm TGG**
298 **distances.** The video is composed of the p- (a) and s-polarized (b) snapshots of the magnetic field
299 profiles, and the magnetic field evolutions (c-d) with varying time delays. The contours in (d) mark
300 both the profile and the strength variances of the magnetic field that are acquired from Movie S1
301 (c). When laser pump illuminates the wire (whose edge is marked by the gray dotted lines) from
302 the left fringe of each frame, the THz SPPs are excited and propagate along the wire toward the
303 right side. Here, the first THz SPPs signal appears around $\tau_d = 2.47$ ps and keeps growing until it
304 reaches a relatively slow growth rate and stable profile during $3.60 \text{ ps} < \tau_d < 4.40 \text{ ps}$. The further
305 rising of the THz SPPs is impeded by the dephasing between the electron and the THz SPPs.
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References and Notes

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