Supplement

1 Model Description

Here we describe the details about our planetary population synthesis model, which is based on the planetesimal-driven core accretion scenario. The model includes the evolution of the central star (§ 1.1), the evolution and radial structure of the protoplanetary disc (§ 1.2), the growth of solid cores (§ 1.3), the accumulation and loss of atmospheres (§ 1.4), orbital migration (§ 1.6), and dynamical interactions between protoplanets, including resonance trapping (§ 1.7), and orbital instabilities for multi-protoplanet systems and their outcomes (§ 1.8). In addition, we describe the calculation method for the radius of the isolated planet during its thermal evolution in § 1.5 and the initial conditions and parameters for generating the planetary populations in § 1.9.

1.1 Stellar Evolution

From the table provided by Baraffe et al. (1998), we take the stellar radius R_*, luminosity L_* and effective temperature T_* as a function of stellar mass M_* and age for the solar metallicity with He abundance of $Y = 0.275$. Those data are used for calculating the protoplanetary disc temperature, planetary equilibrium temperature, and atmospheric photoevaporation rate in our planetary population synthesis calculations.

1.2 Protoplanetary Disc

Our planetary population synthesis calculations need the surface density distributions of gas and solids (or planetesimals) in the protoplanetary disc and their evolution, and also need the disc midplane temperature. Those are calculated in a similar way to Emsenhuber et al. (2021a), which is summarised below.

1.2.1 Gas disc profile and evolution

(i) Initial profiles

The initial profile of disc gas surface density is assumed as (Veras & Armitage, 2004; Andrews et al., 2010)

$$\Sigma_{\rm g}^{(t=0)} = \Sigma_{\rm g0}\left(\frac{r}{r_0}\right)^{-q_{\rm g}} \exp\left(-\left(\frac{r}{r_{\rm disc}}\right)^{2-q_{\rm g}}\right)\left(1-\sqrt{\frac{r_{\rm in}}{r}}\right), \tag{1}$$

where $\Sigma_{\rm g0}$ is the gas surface density at orbital radius $r = r_0$, $r_{\rm disc}$ is the gas disc characteristic radius beyond which the surface density decays exponentially, and $r_{\rm in}$ is the inner edge radius. This is the so-called self-similar solution with the turbulent viscosity $\nu_{\rm acc} \propto r^{q_{\rm g}}$ (Lynden-Bell & Pringle, 1974; Hartmann et al., 1998), including the smooth cutoff at $r = r_{\rm in}$. We set $q_{\rm g} = 0.9$, which is inferred from observations of protoplanetary discs (Andrews et al., 2010), and $r_0 = 1$ au. The surface density at $r = r_0$, $\Sigma_{\rm g0}$, is calculated from the total disc gas mass $M_{\rm disc}$ as

$$\Sigma_{\rm g0} = \frac{2-q_{\rm g}}{2\pi} M_{\rm disc} r_0^{-q_{\rm g}} r_{\rm disc}^{q_{\rm g}-2}. \tag{2}$$

This is obtained by integrating Eq. (1) multiplied by $2\pi r$ (i.e., $2\pi r \Sigma_{\rm g}^{(t=0)}$) from $r = 0$ to $r_{\rm disc}$, ignoring the exponential and inner edge cutoff terms.

(ii) Viscous diffusion

The evolution of the disc gas surface density $\Sigma_{\rm g}$ is assumed to occur via radial viscous diffusion, photo-evaporation, and absorption by embedded planets and is expressed as (e.g., Lynden-Bell & Pringle, 1974)

$$\frac{\partial \Sigma_{\rm g}}{\partial t} - \frac{3}{r}\frac{\partial}{\partial r}\left(r^{1/2}\frac{\partial}{\partial r}\left(\nu_{\rm acc}\Sigma_{\rm g} r^{1/2}\right)\right) = -\dot{\Sigma}_{\rm pe} - \dot{\Sigma}_{\rm planet}, \tag{3}$$

where $-\dot{\Sigma}_{\rm pe}$ and $-\dot{\Sigma}_{\rm planet}$ are the sink terms due to photo-evaporation and absorption by planets, respectively. We adopt the α-prescription for the turbulent viscosity: $\nu_{\rm acc} = \alpha_{\rm acc} c_{\rm s} H_{\rm disc}$ (Shakura & Sunyaev, 1973), where $c_{\rm s} = \sqrt{k_{\rm B} T_{\rm disc}/(\mu m_{\rm H})}$ is the isothermal sound speed and $H_{\rm disc} = c_{\rm s}/\Omega_{\rm K}$ is the disc scale height. Here μ is the mean molecular weight, which is assumed to be 2.34 for gas with solar abundances, $T_{\rm disc}$ is the midplane temperature calculated below, k_B and m_H are the Boltzmann constant ($= 1.38 \times 10^{-16}\,\mathrm{erg\,K^{-1}}$) and the hydrogen atomic mass ($= 1.66 \times 10^{-24}$ g), respectively, and Ω_K is the Keplerian frequency. We treat $\alpha_{\rm acc}$ as an input parameter.

Equation (3) is solved with a log-uniform grid with 500 points ($N_{\rm grid,disc}$) extending from $r_{\rm in}$ to $r_{\rm max} = 1000$ au. The boundary conditions are $\Sigma_g(r_{\rm in}) = 0$ and $\Sigma_g(r_{\rm max}) = 0$.

(iii) Photo-evaporation

We consider photo-evaporation processes caused by UV irradiation from the central star (internal photo-evaporation) and from nearby massive stars (external photo-evaporation). The total photo-evaporation rate $\dot{\Sigma}_{\rm pe}$ is determined by the sum of the two photo-evaporation rates. The external photo-evaporation rate is calculated by the prescription from Matsuyama et al. (2003). The model assumes that the FUV photons from nearby massive stars evaporate the disc gas uniformly only in the regions exterior to the effective gravitational radius. The gravitational radius for the dissociated gas is defined as

$$r_{\rm g,I} = \frac{GM_*}{c_{\rm s,I}^2}, \tag{4}$$

where G is the gravitational constant ($= 6.67 \times 10^{-8}\,\mathrm{cm^3\,g^{-1}\,s^2}$); $c_{\rm s,I}$ is the sound speed with temperature of 1×10^3 K and mean molecular weight of 1.35 (Matsuyama et al., 2003), given that the hydrogen is fully-dissociated and the He/H ratio is equal to solar. Assuming that the disc gas is uniformly removed only from the regions exterior to the effective gravitational radius $\beta_{\rm I} r_{\rm g,I}$, the external photo-evaporation rate is given by

$$\dot{\Sigma}_{\rm pe,ext} = \begin{cases} 0 & \text{for} \quad r < \beta_{\rm I} r_{\rm g,I} \\ \dfrac{\dot{M}_{\rm wind}}{\pi(r_{\rm max}^2 - \beta_{\rm I}^2 r_{\rm g,I}^2)} & \text{otherwise,} \end{cases} \tag{5}$$

where we assume $r_{\rm max} = 1000$ au and regard $\dot{M}_{\rm wind}$ as an input parameter providing the total mass loss rate. For the effective gravitational radius, analytical estimates (e.g., Liffman, 2003) and numerical results (e.g., Begelman et al., 1983; Adams et al., 2004; Font et al., 2004) show that $\beta_{\rm I}$ =0.1–0.2 would be appropriate. Here we set $\beta_{\rm I} = 0.14$ following Emsenhuber et al. (2021a).

Next, the internal photo-evaporation rate is calculated from the following equation (e.g., Hollenbach et al., 1994; Clarke et al., 2001)

$$\dot{\Sigma}_{\rm pe,int}(r) = \begin{cases} 0 & \text{for} \quad r < \beta_{\rm II} r_{\rm g,II} \\ 2c_{\rm s,II} n_0(r) m_{\rm H} & \text{otherwise.} \end{cases} \tag{6}$$

Here, $r_{\rm g,II}$ and $c_{\rm s,II}$ are the gravitational radius and sound speed for ionised gas of temperature 1×10^4 K and mean molecular weight 0.68 (assuming the fully ionised hydrogen and the He/H ratio of solar; Hollenbach et al., 1994; Liffman, 2003; Matsuyama et al., 2003), respectively. We also set $\beta_{\rm II} = 0.14$, as well as $\beta_{\rm I}$. The number density of the ionised hydrogen n_0 is given by

$$n_0(r) = 3.1 \times 10^5 \left(\frac{r_{\rm g,II}}{1\ \mathrm{au}}\right) \left(\frac{\Phi}{10^{40}\ \mathrm{s^{-1}}}\right)^{1/2} \left(\frac{r}{1\ \mathrm{au}}\right)^{-5/2}, \tag{7}$$

following the fitting formula derived in Hollenbach et al. (1994), with Φ the ionising EUV photon luminosity. Finally, the total photo-evaporation rate is given by

$$\dot{\Sigma}_{\rm pe} = \dot{\Sigma}_{\rm pe,ext} + \dot{\Sigma}_{\rm pe,int}. \tag{8}$$

(iv) Absorption by planets

The sink term for absorption of disc gas by planets $\dot{\Sigma}_{\rm planet}$ is simply given by

$$\dot{\Sigma}_{\rm planet} = \frac{\dot{M}_{\rm env}}{4\pi a_{\rm p} R_{\rm H}}, \tag{9}$$

where $\dot{M}_{\rm env}$ is the runaway gas accretion rate of the planet given by Eq. (46), and $a_{\rm p}$ and $R_{\rm H}$ are the semi-major axis and Hill radius of the planet, respectively. Equation (9) is calculated for grids inside the Hill radius of each planet.

1.2.2 Disc temperature

The disc midplane temperature $T_{\rm disc}$ is determined by the combination of viscous heating and direct and indirect stellar irradiation and can be expressed as (e.g., Emsenhuber et al., 2021a)

$$\sigma T_{\rm disc}^4 = \sigma T_{\rm vis}^4 + \sigma T_{\rm irr}^4 + \sigma T_{\rm eq}^4 \exp(-\tau_r), \tag{10}$$

where $T_{\rm vis}$ and $T_{\rm irr}$ are the temperatures that would be achieved if only viscous heating or indirect stellar irradiation, respectively, was available, and $T_{\rm eq}$ is the equilibrium temperature due to the direct stellar irradiation through the disc midplane, and τ_r is the radial optical depth at the disc midplane.

The temperature $T_{\rm vis}$ is given by (e.g., Nakamoto & Nakagawa, 1994; Hueso & Guillot, 2005)

$$\sigma T_{\rm vis}^4 = \frac{1}{2}\left(\frac{3}{8}\tau_{\rm R} + \frac{1}{2\tau_{\rm P}}\right)\dot{E}, \tag{11}$$

where $\tau_{\rm R}(=\kappa_{\rm R}\Sigma_{\rm g})$ and $\tau_{\rm P}$ are the Rosseland and Planck mean optical depths, respectively, and $\dot{E}$ is the viscous energy generation rate per unit surface area. The Rosseland mean opacity $\kappa_{\rm R}$ is given by the analytical fitting of the opacity of dust grains provided by Bell & Lin (1994). We ignore the gas opacity and simply set $\kappa_{\rm R} = 0\ {\rm cm^2/g}$ after the evaporation of dust grains. Also, we simply set $\tau_{\rm P} = \max(2.4\tau_{\rm R}, 0.5)$ (Nakamoto & Nakagawa, 1994; Hueso & Guillot, 2005; Suzuki et al., 2016). The minimum value of 0.5 is adopted so that the coefficient of $\dot{E}$ in Eq. (11) approaches to unity in the optically thin limit. The viscous heating rate $\dot{E}$ is

$$\dot{E} = \frac{9}{4}\nu_{\rm acc}\Sigma_{\rm g}\Omega_{\rm K}^2. \tag{12}$$

The temperature due to indirect irradiation $T_{\rm irr}$ is given by (Kusaka et al., 1970; Adams et al., 1988; Ruden & Pollack, 1991)

$$T_{\rm irr}^4 = T_*^4\left[\frac{2}{3\pi}\left(\frac{R_*}{r}\right)^3 + \frac{1}{2}\left(\frac{R_*}{r}\right)^2\frac{H_{\rm disc}}{r}\left(\frac{{\rm d}\ln H_{\rm disc}}{{\rm d}\ln r} - 1\right)\right]. \tag{13}$$

For computational simplicity, we adopt a constant value of ${\rm d}\ln H_{\rm disc}/{\rm d}\ln r = 9/7$, which is the approximate equilibrium solution for a flaring disc (Chiang & Goldreich, 1997). Finally, $T_{\rm eq}$ and τ_r are given by

$$T_{\rm eq}^4 = \frac{L_*}{16\pi\sigma r^2}, \tag{14}$$

and

$$\tau_r = \int_0^r \kappa_{\rm R}\rho_{\rm disc}\,{\rm d}r\,, \tag{15}$$

where $\rho_{\rm disc} = \Sigma_{\rm gas}/\sqrt{2\pi}H_{\rm disc}$ is the gas density at the disc midplane.

Since all of the temperatures above change with time, the H_2O snowline, defined as the radial location for $T_{\rm disc} = 170$ K, moves in the protoplanetary disc as follows. In the early stage of the disc evolution, the disc midplane temperature $T_{\rm disc}$ is almost equal to $T_{\rm vis}$ and generally much higher than $T_{\rm eq}$ because of the large viscous heating rate. Thus, the snowline locates farther than the present position. As the disc evolves, $T_{\rm disc}$ becomes comparable to $T_{\rm irr}$, which is much smaller than $T_{\rm vis}$, and the snowline moves inward on a $\sim$Myr timescale. Then, at the timing of the disc gas dissipation, $T_{\rm disc}$ jumps up to $T_{\rm eq}$ in a quite short time (typically $\sim 10^4$ yrs). Since $T_{\rm eq} > T_{\rm irr}$ holds in many cases, the snowline moves outward in this stage. Finally, the snowline slightly moves according to the evolution of the stellar luminosity (generally inward around M dwarfs).

1.2.3 Distribution and evolution of planetesimals

We set the initial distribution of planetesimals (or solids) in terms of surface density as

$$\Sigma_{\rm s}^{(t=0)} = \eta_{\rm ice}\Sigma_{\rm rock}^{(t=0)}, \tag{16}$$

where $\eta_{\rm ice}$ is the enhancement factor associated with H_2O condensation (see below) and $\Sigma_{\rm rock}^{(t=0)}$ is the initial surface density of rocks, which is given by

$$\Sigma_{\rm rock}^{(t=0)} = \Sigma_{\rm s0}\left(\frac{r}{r_0}\right)^{-q_{\rm s}}\exp\left[-\left(\frac{r}{r_{\rm solid}}\right)^2\right]\left(1-\sqrt{\frac{r_{\rm in}}{r}}\right); \tag{17}$$

Σ_{s0} is the initial surface density of rocks at $r = r_0$, and $r_{\rm solid}$ is the characteristic radius of the existence region of planetesimals (or called the planetesimal disc). We set $q_{\rm s} = 1.5$ (Birnstiel et al., 2012; Birnstiel & Andrews, 2014) and $r_{\rm solid} = 0.5 r_{\rm disc}$ (Ansdell et al., 2018). Similarly to Σ_{g0}, Σ_{s0} is calculated by

$$\Sigma_{s0} = \frac{2 - q_{\rm s}}{2\pi \eta_{\rm ice}} M_{\rm solid} r_0^{-q_{\rm s}} r_{\rm solid}^{q_{\rm s}-2}. \tag{18}$$

Here $M_{\rm solid}$ is the total mass of planetesimals initially existing in the entire protoplanetary disc, which is given by

$$M_{\rm solid} = 10^{[{\rm Fe/H}]} Z_\odot M_{\rm disc}, \tag{19}$$

where [Fe/H] is the disc metallicity relative to the solar one, which is assumed equal to the central star's one, and $Z_\odot$ is the solar metallicity, which is set to 0.015 (Lodders, 2003). The enhancement factor $\eta_{\rm ice}$ is given by

$$\eta_{\rm ice} = \begin{cases} 1 & \text{for} \quad r < r_{\rm ice}, \\ 2 & \text{for} \quad r > r_{\rm ice}, \end{cases} \tag{20}$$

where $r_{\rm ice}$ is the snowline position.This is similar to the prescription in Ida et al. (2013), but we assume $\eta_{\rm ice} = 2$ beyond the snowline (Lodders, 2003), instead of 4.2 in their original prescription. We ignore snowlines for condensates other than H_2O.

The planetesimal distribution changes due to the accretion and scattering by protoplanets, and also due to the sublimation of ice as the snowline moves. Assuming that $\Sigma_{\rm rock}$ is only affected by the first two processes, we calculate its evolution by

$$\frac{{\rm d}\Sigma_{\rm rock}}{{\rm d}t} = -\frac{\dot{M}_{\rm core} + \dot{M}_{\rm scat}}{2\pi \eta_{\rm ice} a_{\rm p} \Delta a_{\rm FZ}}, \tag{21}$$

where $\dot{M}_{\rm core}$ and $\dot{M}_{\rm scat}$ are the planetesimal accretion and scattering rates by the protoplanet, respectively, and $\Delta a_{\rm FZ}$ is the full-width of the feeding zone of the planet, which is assumed to be 10 $R_{\rm H}$ (Kokubo & Ida, 2002). The prescriptions for $\dot{M}_{\rm core}$ and $\dot{M}_{\rm scat}$ are described in section 1.3. To calculate Eq. (21), we set a log-uniform grid, which is different from that for disc gas, with $N_{\rm grid,solid} = 1000$ extending from $r_{\rm in}$ to $5 r_{\rm solid}$. To evaluate $\Sigma_{\rm s}(t)$, we first integrate Eq. (21) to derive $\Sigma_{\rm rock}(t)$ for all grids inside the feeding zone of each planet. If a grid is in the feeding zone of multiple planets, the right hand side of Eq. (21) is calculated for all the involved planets and the results are summed up. At the same time, $\eta_{\rm ice}(t)$ for each grid is evaluated using $T_{\rm disc}(t)$. Then, the planetesimal surface density is derived by $\Sigma_{\rm s}(t) = \eta_{\rm ice}(t) \Sigma_{\rm rock}(t)$

1.3 Solid Core Growth

We assume that protoplanetary solid cores grow by the accretion of planetesimals. The mass growth rate is given by

$$\frac{{\rm d}M_{\rm core}}{{\rm d}t} = \Omega_{\rm K} \bar{\Sigma}_{\rm s} R_{\rm H}^2 p_{\rm col} \tag{22}$$

with $\bar{\Sigma}_{\rm s}$ the mean surface density of planetesimals in the planet's feeding zone and $p_{\rm col}$ the collision probability of planetesimals; $p_{\rm col}$ is given by (Inaba et al., 2001)

$$p_{\rm col} = \min\left(p_{\rm col,med}, \left(p_{\rm col,high}^{-2} + p_{\rm col,low}^{-2}\right)^{-1/2}\right), \tag{23}$$

with

$$p_{\rm col,high} = \frac{1}{2\pi}\left(\frac{R_{\rm cap}}{R_{\rm H}}\right)^2 \left(\mathcal{F}(i_{\rm plt}/e_{\rm plt}) + \frac{6 R_{\rm H} \mathcal{G}(i_{\rm plt}/e_{\rm plt})}{R_{\rm cap} \tilde{e}_{\rm plt}^2}\right), \tag{24}$$

$$p_{\rm col,med} = \frac{1}{4\pi \tilde{i}_{\rm plt}}\left(\frac{R_{\rm cap}}{R_{\rm H}}\right)^2 \left(17.3 + \frac{232 R_{\rm H}}{R_{\rm cap}}\right), \tag{25}$$

$$p_{\rm col,low} = 11.3\sqrt{R_{\rm cap}/R_{\rm H}}. \tag{26}$$

Here $i_{\rm plt}$ and $e_{\rm plt}$ are respectively the inclination and eccentricity of planetesimals, and $\tilde{i}_{\rm plt} = i_{\rm plt}/h$ and $\tilde{e}_{\rm plt} = e_{\rm plt}/h$ are the reduced inclination and eccentricity, respectively, with $h = R_{\rm H}/a_{\rm p}$. We assume $i_{\rm plt} = e_{\rm plt}/2$, in which case

$\mathcal{F}(i_{\rm plt}/e_{\rm plt}) = 17.3$ and $\mathcal{G}(i_{\rm plt}/e_{\rm plt}) = 38.2$ (see Greenzweig & Lissauer, 1990, 1992; Inaba et al., 2001, for the explicit form of $\mathcal{F}(x)$ and $\mathcal{G}(x)$). The value of $\tilde{e}_{\rm plt}$ around a protoplanet is assumed equal to the equilibrium eccentricity determined by the balance of viscous stirring by the planet and the damping due to the disc gas drag. Following the analytic estimates by Kokubo & Ida (2002), it is expressed as

$$\tilde{e}_{\rm plt} = 4.1 \left(\frac{\rho_{\rm disc}}{1 \times 10^{-9}\,{\rm g/cc}} \right)^{-1/5} \left(\frac{\rho_{\rm plt}}{3\,{\rm g/cm^3}} \right)^{2/15} \left(\frac{a_{\rm p}}{1\,{\rm au}} \right)^{-1/5} \left(\frac{m_{\rm plt}}{10^{20}{\rm g}} \right)^{1/15}, \tag{27}$$

where $\rho_{\rm plt} = 3.0$ g cm^{-3} and $m_{\rm plt} = 10^{20}$ g are the material density and mass of planetesimals. Finally, we calculate the effective capture radius enhanced by the atmospheric gas drag, $R_{\rm cap}$, following Inaba & Ikoma (2003);

$$R_{\rm plt} = \frac{3}{2} \frac{v_{\rm ran}^2 + 2GM_{\rm p}/R_{\rm cap}}{v_{\rm ran}^2 + 2GM_{\rm p}/R_{\rm H}} \frac{\rho(R_{\rm cap})}{\rho_{\rm plt}} R_{\rm cap}, \tag{28}$$

with $R_{\rm plt}$ the planetesimal's radius derived from $m_{\rm plt}$ and $\rho_{\rm plt}$, $v_{\rm ran} = e_{\rm plt} a_{\rm p} \Omega_{\rm K}$ the random velocity of the planetesimals and $\rho(R_{\rm cap})$ the envelope gas density at the radius of $R_{\rm cap}$.

After the disc gas dissipation, some of the planetesimals that encountered a planet are ejected from the system, instead of colliding with the planet. This ejection rate can be estimated by comparing the collisional cross section and the scattering cross section, and is given by (Ida & Lin, 2004)

$$\dot{M}_{\rm scat} = \left(\frac{v_{\rm surf}}{v_{\rm esc}} \right)^4 \dot{M}_{\rm core}, \tag{29}$$

where $v_{\rm esc} = \sqrt{2GM_*/a_{\rm p}}$ is the escape velocity from the host star and $v_{\rm surf} = \sqrt{GM_{\rm p}/R_{\rm cap}}$ is the surface velocity of the planet. This is used for the calculation of the evolution of $\Sigma_{\rm rock}$ (Eq. (21)).

1.4 Atmospheric Accumulation and Loss

1.4.1 Purely hydrostatic equilibrium phase

During vigorous planetesimal accretion, the atmospheric mass is rather small because of significant energy deposition. Then, the atmosphere is in the hydrostatic equilibrium and thermally steady state (e.g., Ikoma et al., 2000). We solve the standard set of equations for stellar structure, namely,

$$\frac{\partial P}{\partial M_R} = -\frac{GM_R}{4\pi R^4}, \tag{30}$$

$$\frac{\partial T}{\partial M_R} = -\frac{GM_R}{4\pi R^4} \frac{T}{P} \nabla, \tag{31}$$

$$\frac{\partial R}{\partial M_R} = \frac{1}{4\pi R^2 \rho}, \tag{32}$$

where P, T, and ρ are the pressure, temperature, and density of the atmosphere (or envelope) gas, respectively, R is the radial distance to the planet's centre, and M_R is the total mass inside the sphere of radius R. In addition to the above equations, we use the ideal equation of state (or the P-T relationship) for chemical equilibrium mixtures composed of H- and O-bearing molecules and He, taking H_2O sublimation/condensation into account (see Kimura & Ikoma, 2020, for the details). Also, ∇ is the temperature gradient, $\mathrm{d}\log T/\mathrm{d}\log P$, for radiative diffusion or convection (dry or moist adiabat).

In the early stages of planetary accretion, the temperature is high enough at the bottom of the atmosphere to keep a global magma ocean (Ikoma & Genda, 2006; Kimura & Ikoma, 2020). The atmosphere-magma interaction produces volatiles, modifying the atmospheric composition significantly. Therefore, we assume that the water-producing reaction effectively occurs and that the produced water vapour is uniformly mixed in the atmosphere. Hereafter, we call such an atmosphere at this phase the *vapour-mixed atmosphere*, and denote its mass by $M_{\rm mix}$.

We calculate the atmospheric mass $M_{\rm mix}$ in the same way as Kimura & Ikoma (2020), treating the water mass fraction in the vapour-mixed atmosphere $X_{\rm H_2O}$ as an input parameter. We set $T = T_{\rm disc}$ and $P = P_{\rm disc}$ at $R = \min(R_{\rm B}, R_{\rm H})$, where $R_{\rm B}$ and $R_{\rm H}$ are the Bondi and Hill radii, respectively. The luminosity L, which is assumed constant in the entire atmosphere, is equal to that from the solid core; namely,

$$L_{\rm core} = L_{\rm acc} + L_{\rm cool} + L_{\rm radio}, \tag{33}$$

where $L_{\rm acc}$, $L_{\rm cool}$ and $L_{\rm radio}$ are the luminosity due to the planetesimal accretion, the solid core cooling and radioactive decay, respectively. The accretion luminosity

$$L_{\rm acc} = \frac{GM_{\rm core}\dot{M}_{\rm core}}{R_{\rm core}}, \tag{34}$$

where $\dot{M}_{\rm core}$ is the planetesimal accretion rate, and $R_{\rm core}$ is the solid planet radius. By assuming that the core surface temperature $T_{\rm surf}$ and the disc gas temperature $T_{\rm disc}$ are related in the form of $\mathrm{d}T_{\rm surf}/\mathrm{d}\rho_{\rm disc} = T_{\rm disc}/4$, as indicated by the analytical solution of the fully-radiative atmosphere, $L_{\rm cool}$ is given by (Ikoma & Hori, 2012)

$$L_{\rm cool} = \frac{M_{\rm core}C_{\rm rock}T_{\rm disc}}{4\tau_{\rm disc}}, \tag{35}$$

where $C_{\rm rock}$ is the specific heat of rock ($= 1.2 \times 10^7\,\mathrm{erg\,g^{-1}K^{-1}}$), and $\tau_{\rm disc}$ is the disc dissipation timescale. We set $\tau_{\rm disc} = 1 \times 10^4$ yr in our simulations, since disc dissipation occurs on a timescale of $\sim 10^4$–10^5 yr due to photoevaporation in the final stage of disc evolution. Increasing this value by an order of magnitude has little effect on our results. While the above equation was derived assuming a thin radiative atmosphere, the vapour-rich primordial atmosphere considered in this study is thick even in the final stage of the disc evolution, and the lower layer is often convective. Even in that case, the changing rate of temperature at the bottom of the atmosphere is about the same as that at the radiative-convective boundary. Therefore, Eq. (35) is still considered to be a good approximation for describing the cooling luminosity. The radiogenic luminosity $L_{\rm radio}$ is simply set to $2 \times 10^{20}(M_{\rm core}/M_\oplus)$ erg/s (Guillot et al., 1995).

1.4.2 Quasi-static contraction phase

Once planetesimal accretion stops, the vapour-mixed atmosphere contracts gravitationally and further accumulation of disc gas occurs. Thus, we have to integrate the equation of entropy change, in addition to Eqs. (30)-(32);

$$\frac{\partial L_R}{\partial M_R} = \dot{\varepsilon} - T\frac{\mathrm{d}S}{\mathrm{d}t}, \tag{36}$$

where L_R is the total energy flux passing through a spherical surface of radius R (or luminosity), $\dot{\varepsilon}$ is the energy generation rate per unit mass, and S is the specific entropy. We assume that the accumulating disc gas composed predominantly of H and He never mixes with the lower vapour-mixed layer and, thus, the mass of the vapour-mixed atmosphere ($M_{\rm mix}$) is conserved in this phase. Hereafter, the accumulated mass of disc gas is denoted by $M_{\rm HHe}$. Therefore we calculate the quasi-static contraction of the two-layer atmosphere in this phase.

The numerical integration of Eq. (36) is done based on the total energy conservation approximation adopted in several previous studies on the formation of gas giants (Papaloizou & Nelson, 2005; Mordasini et al., 2012; Fortier et al., 2013; Piso & Youdin, 2014; Venturini et al., 2016). The total atmospheric energy conservation between the time $t - \Delta t$ and t being considered, the following relation holds (see Piso & Youdin, 2014, for the derivation):

$$\frac{E_{\rm env}(t) - E_{\rm env}(t-\Delta t)}{\Delta t} = L_{\rm core} + e_{\rm gas}\frac{M_{\rm env}(t) - M_{\rm env}(t-\Delta t)}{\Delta t} - L \tag{37}$$

or

$$\Delta t = \frac{E_{\rm env}(t) - E_{\rm env}(t-\Delta t) - e_{\rm gas}\left[M_{\rm env}(t) - M_{\rm env}(t-\Delta t)\right]}{L_{\rm core} - L} \tag{38}$$

where $e_{\rm gas}$ is the total energy per unit mass (i.e., the sum of the specific internal energy and the gravitational energy) of the disc gas at the outer boundary, $M_{\rm env} = M_{\rm mix} + M_{\rm HHe}$ is the total envelope mass, and $E_{\rm env}$ is the envelope's total energy defined by

$$E_{\rm env} = \int_{M_{\rm core}}^{M_{\rm p}} \left(u - \frac{GM_R}{R}\right)\mathrm{d}M_R\,; \tag{39}$$

u is the specific internal energy. In Eq. (37), we have neglected the work done by the atmospheric surface (i.e. the boundary between the atmosphere and disc gas) for simplicity. This term generally accounts for only a few % relative to the other terms (e.g., Lee et al., 2014) and hardly affects the results of this study.

Once planetesimal accretion is over, the luminosity decreases with the mass growth of the envelope until reaching the critical luminosity below which no hydrostatic solution is found (Ikoma et al., 2000; Hubickyj et al., 2005). Thus, for $L(t) = \max[0.95L(t-\Delta t), L_{\rm core}]$, we integrate Eqs. (30)-(32) to calculate $E_{\rm env}(t)$ and $M_{\rm env}(t)$, assuming $L_R = L(t)$, and thereby calculate Δt from Eq. (38). The assumption of $L_R = L$ is valid until the critical luminosity while invalid after that (Ikoma et al., 2000). The H_2O fraction X_{H_2O} is set to zero for $M_R > M_{\rm core} + M_{\rm mix}$, and is set to the input value otherwise. Once the critical luminosity is reached, we shift to the runaway gas accretion regime (see the following section).

1.4.3 Runaway gas accretion phase

The runaway gas accretion phase is divided into three sub-phases (e.g., Tanigawa & Ikoma, 2007). In the earlier phase the gas accretion is regulated by the Kelvin-Helmholtz contraction of the atmosphere. Numerical simulations show that the contraction timescale is strongly dependent on planet mass (Tajima & Nakagawa, 1997; Ikoma et al., 2000). Here we assume that the growth timescale, $\tau_{\rm KH}$, increases with the cube of planetary mass, following the previous planetary population synthesis models (e.g., Ida et al., 2013).

$$\tau_{\rm KH} = 1 \times 10^9 \left(\frac{M_{\rm p}}{M_\oplus}\right)^{-3} \ {\rm yr}. \tag{40}$$

While Tajima & Nakagawa (1997) showed a stronger dependence, our results are insensitive to the choice of the power index, provided the index is smaller than -3, partly because the duration of this phase is short. When the planet grows large enough to open a gap in the surrounding disc, the growth is limited by the gas supply from the flow in the gap. The supply rate is given by (Tanigawa & Ikoma, 2007; Tanigawa & Tanaka, 2016)

$$\frac{{\rm d}M_{\rm p}}{{\rm d}t} = D\Sigma_{\rm g,gap}, \tag{41}$$

where the coefficient D is empirically given by

$$D = 0.29\left(\frac{H_{\rm disc}}{a_{\rm p}}\right)^{-2}\left(\frac{M_{\rm p}}{M_*}\right)^{4/3} a^2\Omega_{\rm K}, \tag{42}$$

and $\Sigma_{\rm g,gap}$ is the gas surface density at the bottom of the gap, which is also empirically given by (Kanagawa et al., 2015)

$$\Sigma_{\rm g,gap} = \frac{\Sigma_{\rm g}}{1+0.04K}, \tag{43}$$

with K being

$$K = \left(\frac{M_{\rm p}}{M_*}\right)^2\left(\frac{h}{a}\right)^{-5}\alpha_{\rm vis}^{-1}. \tag{44}$$

Here, $\alpha_{\rm vis}$ is a parameter for disc turbulent viscosity, which is not necessarily the same as $\alpha_{\rm acc}$. When the wind driven accretion is dominant in the global angular momentum transfer, $\alpha_{\rm acc}$ is larger than $\alpha_{\rm vis}$ by about one order of magnitude (Simon et al., 2013; Armitage et al., 2013; Hasegawa et al., 2017). Thus, we set $\alpha_{\rm vis} = 0.1\alpha_{\rm acc}$ in this study.

Equation (41) is valid during there is sufficient disc gas outside the gap. As the disc depletes, the gas accretion rate is limited by the global disc accretion rate $\dot{M}_{\rm disc}$ given by

$$\dot{M}_{\rm disc} = 6\pi r^{1/2}\frac{\partial}{\partial r}\left(\Sigma_{\rm g}\nu_{\rm acc}r^{1/2}\right). \tag{45}$$

Considering these processes, the runaway gas accretion rate is calculated by

$$\frac{{\rm d}M_{\rm p}}{{\rm d}t} = \min\left(\frac{M_{\rm p}}{\tau_{\rm KH}}, D\Sigma_{\rm g,gap}, \dot{M}_{\rm disc}\right). \tag{46}$$

Note that the solid accretion never occurs in this phase.

1.5 Atmospheric Thermal Evolution and Loss

1.5.1 Atmospheric Thermal Evolution

The planetary radius $R_{\rm p}$ is expressed by

$$R_{\rm p} = R_{\rm core} + \Delta R_{\rm env}, \tag{47}$$

where $R_{\rm core}$ is the solid core radius and $\Delta R_{\rm env}$ is the thickness of the envelope.

The solid core is assumed to consist of iron, silicate, and, if present, ice. Following Zeng et al. (2019), we calculate the core radius as

$$R_{\rm core} = R_{\rm rock}(1 + 0.55 f_{\rm ice} - 0.14 f_{\rm ice}^2), \tag{48}$$

where $f_{\rm ice}$ is the ice mass fraction and $R_{\rm rock}$ is the pure rocky (iron+silicate) core radius, which is calculated from Fortney et al. (2007) as

$$R_{\rm rock} = (0.0592 f_{\rm rock} + 0.0975)(\log M_{\rm core})^2 + (0.2337 f_{\rm rock} + 0.4938)\log M_{\rm core} + (0.3102 f_{\rm rock} + 0.7932), \tag{49}$$

with $f_{\rm rock}$ being the Si/(Si+Fe) mass ratio set to the Earth-like value of 0.66 in this study.

To evaluate the planetary radius including the envelope, we calculate the quasi-static thermal evolution of the planet after the disc gas dissipation. For numerical convenience, we treat the upper part of the envelope (simply called the atmosphere below) separately from its deeper part (called the envelope), following previous evolution models of ice giant planets (e.g., Kurosaki & Ikoma, 2017). The envelope structure is calculated by directly integrating Eqs. (30)–(32) and Eq. (36) for a given water mass fraction $X_{\rm H_2O}$. For the equation of state, we use SCvH (Saumon et al., 1995) for H-He and SESAME (Lyon & Johnson, 1992) for H_2O and mix H-He and H_2O according to the additive-volume law and the ideal mixing for entropy.

The inner boundary condition for the envelope structure is

$$L_R = L_{\rm core}, \quad R = R_{\rm core} \qquad \text{at } M_R = M_{\rm core}. \tag{50}$$

Here $R_{\rm core}$ is calculated with Eq. (48). The core luminosity $L_{\rm core}$ is the same as Eq. (33) with $L_{\rm acc} = 0$. We also calculate $L_{\rm cool}$ self-consistently during the quasi-static evolution with

$$L_{\rm cool} = -M_{\rm core} C_{\rm rock} \frac{{\rm d}T_{\rm surf}}{{\rm d}t} = -M_{\rm core} C_{\rm rock} \frac{T_{\rm surf}(t) - T_{\rm surf}(t-\Delta t)}{\Delta t} \tag{51}$$

where $T_{\rm surf}$ is the temperature at the solid core surface (i.e. the bottom of the envelope). Since $T_{\rm surf}(t)$ is determined by the envelope structure, we should find a self-consistent value of $L_{\rm cool}$ iteratively.

The outer boundary condition for the envelope structure is given in terms of pressure and temperature, respectively, by

$$P = P_{\rm out}, \quad T = T_{\rm out} \qquad \text{at } M_R = M_{\rm p}. \tag{52}$$

To evaluate $P_{\rm out}$ and $T_{\rm out}$, we calculate the radiative-convective structure of the atmosphere in the same way as Kurosaki & Ikoma (2017). The atmosphere is assumed to be plane parallel and its mass and thickness are negligible compared to the total planetary mass and radius, respectively. Thus, the gravitational acceleration $g = GM_{\rm p}/R_{\rm out}^2$ is constant through the atmosphere, with $R_{\rm out}$ denoting the radius at the boundary between the atmosphere and the envelope (note that it is not equal to the planetary radius $R_{\rm p}$; see below). This boundary is assumed to locate at the radius where the optical depth for the stellar visible radiation, $\tau_{\rm vis}$, is 10, so that the temperature gradient in the envelope is hardly affected by the irradiation from the central star.

The temperature in the radiative region is calculated by the analytical formula given by Matsui & Abe (1986);

$$\sigma T^4 = F_{\rm p} \frac{D\tau_{\rm IR} + 1}{2} + \frac{\sigma T_{\rm eq}^4}{2}\left[1 + \frac{D}{\gamma} + \left(\frac{\gamma}{D} - \frac{D}{\gamma}\right)\exp(-\tau_{\rm vis})\right], \tag{53}$$

where $\tau_{\rm IR}$ is the optical depth for infrared radiation, $F_{\rm p} = L_{\rm p}/4\pi R_{\rm out}^2$ is the net energy flux from the planet, with $L_{\rm p}$ denoting the luminosity at the top of the envelope, $T_{\rm eq}$ is the equilibrium temperature calculated with Eq. (14), $D = 3/2$ is the diffusivity factor arising from the angular dependence of the radiation flux, and $\gamma = \kappa_{\rm vis}/\kappa_{\rm IR}$ with $\kappa_{\rm vis}$ and $\kappa_{\rm IR}$ being the opacities for visible and infrared radiation, respectively. We set $\gamma = 0.1$ following Kurosaki & Ikoma (2017).

The radiative-convective boundary (i.e. tropopause) is determined so that the temperature and the radiation flux connects continuously, using the same numerical procedure as Kurosaki & Ikoma (2017). As a result, $P_{\rm out}$ and $T_{\rm out}$ are determined when $L_{\rm p}$ and $R_{\rm out}$ are given from the envelope structure calculation. Thus, we find a self-consistent set of $(P_{\rm out}, T_{\rm out}, L_{\rm cool}, L_{\rm p}, R_{\rm out})$ at the time t for a given $M_{\rm p}$ with iterations. Then we define the planetary radius $R_{\rm p}$ as the pressure level of 10 mbar, and the envelope thickness is derived simply by $R_{\rm env} = R_{\rm p} - R_{\rm core}$.

If the planet has experienced the runaway accretion phase, however, the thick envelope with nebular composition exists instead of the water-enriched envelope. The radius in this case is calculated with the fitting formula given by Lopez & Fortney (2014) as;

$$R_{\rm env} = 2.06 R_\oplus \left(\frac{M_{\rm p}}{M_\oplus}\right)^{-0.21} \left(\frac{f_{\rm env}}{0.05}\right)^{0.59} \left(\frac{F_{\rm p}}{F_\oplus}\right)^{0.044} \left(\frac{t}{\rm yr}\right)^{-0.11} + R_{\rm corr}, \tag{54}$$

where $f_{\rm env} = M_{\rm env}/M_{\rm p}$ and $F_{\rm p}$ is the stellar insolation. $R_{\rm corr}$ is the correction factor defined as

$$R_{\rm corr} = \frac{9k_{\rm B}T_{\rm eq}}{g\mu_{\rm neb}m_{\rm H}}, \tag{55}$$

with g the gravitational acceleration and $\mu_{\rm neb} = 2.34$ the mean molecular weight of the nebular gas.

1.5.2 Atmospheric Photoevaporation

After the disc dispersal, the atmospheric escape (or photoevaporation) occurs because of stellar XUV irradiation. We assume that the atmospheric photoevaporation starts once the radial optical depth at the planet's location measured from the central star, τ_r, becomes less than unity. We use the fitting formula by Kubyshkina et al. (2018a,b) for the escape rate, which yields higher values of the escape rate than those of the energy-limited escape rate when the escape parameter is small ($\lesssim 10$). Although the atmospheres in our calculations are enriched with water vapour, we simply adopt the formula derived for hydrogen-dominated atmospheres. Since the enrichment with water leads to raising the value of the escape parameter and, thus, reducing the escape rate, our calculations are expected to overestimate the escape rate. The fitting formula is expressed in the form of

$$\dot{M}_{\rm esc} = e^{\beta}\left(\frac{F_{\rm XUV}}{\rm erg\,cm^{-2}\,s^{-1}}\right)^{\alpha_1}\left(\frac{a}{\rm au}\right)^{\alpha_2}\left(\frac{R_{\rm p}}{R_{\oplus}}\right)^{\alpha_3}\Lambda^k, \tag{56}$$

where $\beta, \alpha_1, \alpha_2, \alpha_3, k$ are the fitting coefficients (see Kubyshkina et al., 2018b, for their values), $F_{\rm XUV}$ is the incoming XUV flux, and Λ is the escape parameter. The stellar XUV luminosity, $L_{\rm XUV}$, is taken from the table given by Johnstone et al. (2021). Since the table just gives the best-fit relation between the stellar $L_{\rm XUV}$ and time, we set $L_{\rm XUV} = 10^{\delta \log L_{\rm XUV}} L_{\rm XUV,bf}$ to account for the variation between observed stars, according with Johnstone et al. (2021). Here, $L_{\rm XUV,bf}$ is the best-fit value of XUV luminosity given in the table, and $\delta \log L_{\rm XUV}$ is the deviation factor which follows the normal distribution with $\mu = 0$ and $\sigma = 0.359$.

1.6 Orbital Migration

The planetary orbital migration rate can be calculated by

$$\frac{{\rm d}a_{\rm p}}{{\rm d}t} = \frac{2\Gamma}{M_{\rm p}a_{\rm p}\Omega_{\rm K}}, \tag{57}$$

where Γ is the total torque acting on the planet. In the type-I regime, the torque is given as the sum of the Lindblad ($\Gamma_{\rm L}$), corotation ($\Gamma_{\rm C}$), and thermal ($\Gamma_{\rm T}$) components:

$$\Gamma = \Gamma_{\rm L} + \Gamma_{\rm C} + \Gamma_{\rm T}. \tag{58}$$

We adopt the formula for the Lindblad torque derived by Jiménez & Masset (2017) through 3D hydrodynamic simulations as

$$\Gamma_{\rm L} = (-2.34 + 1.50\beta_T - 0.10\beta_\Sigma) f(\chi_{\rm P})\Gamma_0, \tag{59}$$

where $\beta_T = {\rm d}\log T_{\rm disc}/{\rm d}\log r$, $\beta_\Sigma = {\rm d}\log \Sigma_{\rm g}/{\rm d}\log r$, $\chi_{\rm P}$ is the thermal diffusion coefficient at the disc midplane, and Γ_0 is the characteristic torque expressed as

$$\Gamma_0 = \left(\frac{M_{\rm p}}{M_*}\right)^2\left(\frac{H_{\rm disc}}{a_{\rm p}}\right)^{-2}\Sigma_{\rm g}a_{\rm p}^4\Omega_{\rm K}^2. \tag{60}$$

The function $f(\chi_{\rm P})$ transitions from $1/\gamma$ ('adiabatic regime') to unity ('locally isothermal regime') as $\chi_{\rm P}$ increases (i.e. the thermal diffusion of the disc gas becomes effective). Here γ is the specific heat ratio of the disc gas. See Jiménez & Masset (2017) for the explicit form of $\chi_{\rm P}$ and $f(\chi_{\rm P})$.

We also use the formula for the corotation torque derived by Jiménez & Masset (2017), which is expressed as the sum of four components:

$$\Gamma_{\rm C} = \Gamma_{\rm C,vor} + \Gamma_{\rm C,ent} + \Gamma_{\rm C,temp} + \Gamma_{\rm C,vv}, \tag{61}$$

where $\Gamma_{\rm C,vor}$, $\Gamma_{\rm C,ent}$, and $\Gamma_{\rm C,temp}$ are the torques arising from the radial gradient of vortensity, entropy, and temperature. The last term $\Gamma_{\rm C,vv}$ is associated with the viscously created vortensity arising during the horse-shoe

U-turns (see Casoli & Masset, 2009; Masset & Casoli, 2009, 2010, for more detailed explanations). Each of the first three torques is calculated by the combination of two terms; for example,

$$\Gamma_{\rm C,vor} = (1-\epsilon_b)\Gamma_{\rm vor,lin} + \epsilon_b \Gamma_{\rm vor,hs}, \tag{62}$$

where $\Gamma_{\rm vor,lin}$ and $\Gamma_{\rm vor,hs}$ are the vortensity-related components of the linear corotation torque and the horseshoe torque, respectively. The horseshoe torque $\Gamma_{\rm vor,hs}$ is the product of the unsaturated horseshoe drag $\Gamma_{\rm vor,uhs}$ and the saturation function $\mathcal{F}_{\rm v}$ (i.e., $\Gamma_{\rm vor,uhs}\mathcal{F}_{\rm v}$); the latter expresses the saturation of horseshoe drag and depends on horseshoe width. The blending coefficient ϵ_b is derived in Masset & Casoli (2010) by fitting their numerical results. Note that the saturation function and the blending coefficient are different among $\Gamma_{\rm C,vor}$, $\Gamma_{\rm C,ent}$ and $\Gamma_{\rm C,temp}$. The final term $\Gamma_{\rm C,vv}$ has only the component of the horseshoe torque (i.e., no linear corotation torque) because this torque is only arising from the horse-shoe region. Again, see Jiménez & Masset (2017) for the explicit forms of these torques.

The thermal torque $\Gamma_{\rm T}$ arises when the horseshoe gas cools during the U-turn by thermal diffusion (cold torque) and when the horseshoe gas is heated by the luminous planet (heating torque). We calculate the thermal torque using the analytical formulation by Masset (2017);

$$\Gamma_{\rm T} = 1.61\frac{\gamma-1}{\gamma}\eta\left(\frac{H_{\rm disc}}{\lambda_c}\right)\left(\frac{L_{\rm p}}{L_{\rm c}}-1\right)\Gamma_0, \tag{63}$$

where $\eta = -\beta_\Sigma/3 - \beta_T/6 + 1/2$, $\lambda_c = \sqrt{\chi_{\rm P}/(q\Omega_{\rm K}\gamma)}$ with $q = M_{\rm p}/M_*$, $L_{\rm p}$ is the planetary luminosity, and $L_{\rm c} = 4\pi G M_{\rm p}\chi_{\rm P}\rho_{\rm disc}/\gamma$ is the critical planetary luminosity above which the positive heating torque exceeds the negative cold torque. Note that, as discussed in Masset (2017), the thermal torque acts effectively only when $M_{\rm p} < \chi_{\rm P}c_{\rm s}/G$ is satisfied. This condition means that the thermal diffusion timescale across the Bondi radius is shorter than the acoustic timescale. If $M_{\rm p} > \chi_{\rm P}c_{\rm s}/G$, we set $\Gamma_{\rm thermal} = 0$.

When the planet grows massive enough to open a gap in the disc, its orbital migration transitions to the type-II regime. Recently, Kanagawa et al. (2018) found that the migration rate in this regime can be expressed in the similar way as that in type I regime by replacing gas surface density $\Sigma_{\rm g}$ with $\Sigma_{\rm gap}$ and removing the corotation torque. To connect the both regimes smoothly, we express the total torque in the type II regime as

$$\Gamma = \frac{\Gamma_{\rm L} + (\Gamma_{\rm C}+\Gamma_{\rm T})\exp(-K/K_t)}{1+0.04K}, \tag{64}$$

where $K_t = 20$ is the typical value of K where the disc gas gap becomes deep enough (Kanagawa et al., 2018). This is similar to the formula given in Kanagawa et al. (2018), but the formulae of $\Gamma_{\rm L}$ and $\Gamma_{\rm C}$ are different, and $\Gamma_{\rm T}$ is also added. Since Eq. (64) becomes equal to Eq. (58) when K is small (i.e., the planet mass is small), we always use Eq. (64) in the calculations of this study.

1.7 Resonance trapping

Planets or planetary embryos with converging orbits can be captured in mean-motion resonances with each other; this phenomenon is called resonance trapping (Murray & Dermott, 1999). In this study, the resonance trapping process is included in a similar way to Ida & Lin (2010) and Ida et al. (2013). The treatment differs depending on whether the pair includes a giant planet or not—the definition of a 'giant planet' is given in Section 1.8, while an 'embryo' refers to a celestial body other than the giant planet.

1.7.1 Case with two embryos

When the orbits of two adjacent embryos (denoted by i and j) are converging and get close enough to each other, their separation ($b = |a_i - a_j|$) expands impulsively at every conjunction. The change in separation via the orbital repulsion is estimated by a linear analysis as (Goldreich & Tremaine, 1982; Hasegawa & Nakazawa, 1990)

$$\Delta b = 30\left(\frac{b}{r_{\rm H}}\right)^{-5} r_{\rm H}, \tag{65}$$

where $r_{\rm H} = ((M_i+M_j)/3M_*)^{1/3}a_{ij}$ with $a_{ij} = \sqrt{a_i a_j}$. The time interval between two conjunctions (i.e., the synodic period) is approximately given by

$$T_{\rm syn} \simeq \frac{2\pi}{({\rm d}\Omega_{\rm K}/{\rm d}r)b} \simeq \frac{4\pi a_{ij}}{3b\Omega_{\rm K}}. \tag{66}$$

Therefore, the expansion rate of the separation is found to be

$$\frac{\mathrm{d}b}{\mathrm{d}t} \sim \frac{\Delta b}{T_{\mathrm{syn}}} = 7\left(\frac{b}{r_{\mathrm{H}}}\right)^{-4}\left(\frac{r_{\mathrm{H}}}{a_{ij}}\right)^{2} v_{\mathrm{K}}. \tag{67}$$

When the converging speed of the two orbits Δv_{mig} ($\equiv |(\dot{a}_j)_{\mathrm{mig}} - (\dot{a}_i)_{\mathrm{mig}}|$) becomes equal to $\mathrm{d}b/\mathrm{d}t$, the two embryos are assumed to stop approaching, ending up captured in a mean-motion resonance with each other. The resultant separation is, thus,

$$b_{\mathrm{trap}} = 0.16\left(\frac{M_i + M_j}{M_{\oplus}}\right)^{1/6}\left(\frac{\Delta v_{\mathrm{mig}}}{v_{\mathrm{K}}}\right)^{-1/4} r_{\mathrm{H}}. \tag{68}$$

Note that we do not specify in which mean motion resonance those two embryos are really captured, but assume the mean motion resonance with the width nearly equal to b_{trap}. In the planetary population synthesis calculations, we calculate b_{trap} for all the converging pairs. If $b < b_{\mathrm{trap}}$ is satisfied, the pair is treated as captured in the resonance. If $b_{\mathrm{trap}} < 2\sqrt{3}r_{\mathrm{H}}$, the two embryos or planets collide and merge with each other.

After trapped in the resonance, the pair migrates keeping the ratio of their orbital periods (i.e., the ratio of their semi-major axes) unchanged. In that case, their migration rates are different from Eq. (57). We calculate the loss of the angular momenta of the resonantly trapped planets via the interaction between the respective planets and the disc, and, then, redistribute the loss to both planets so that the planets migrate with the fixed semi-major axis ratio. The migration rate of the resonantly trapped planet i denoted by $v_{\mathrm{mig,trap}}$ and that of the planet j by $C_{ij}v_{\mathrm{mig,trap}}$ ($C_{ij} \equiv a_j/a_i$), that angular momentum loss rate, $\dot{\mathcal{L}}_{\mathrm{trap}}$, can be expressed as

$$\dot{\mathcal{L}}_{\mathrm{trap}} = \frac{\mathrm{d}}{\mathrm{d}t}\left(M_i\sqrt{GM_*a_i} + M_j\sqrt{GM_*a_j}\right) \tag{69}$$

$$= \frac{M_i\sqrt{GM_*a_j} + C_{ij}M_j\sqrt{GM_*a_i}}{2a_{ij}} v_{\mathrm{mig,trap}}. \tag{70}$$

Here we assume that the eccentricities of both planets are negligible. Thus, $v_{\mathrm{mig,trap}}$ becomes

$$v_{\mathrm{mig,trap}} = \frac{2a_{ij}(\Gamma_i + \Gamma_j)}{M_i\sqrt{GM_*a_j} + C_{ij}M_j\sqrt{GM_*a_i}}, \tag{71}$$

where Γ_i and Γ_j are the torques exerted on the planets i and j, respectively, by disc gas, calculated with Eq. (64).

The orbital migration terminates at the disc inner edge. The subsequent planets moving inward are trapped in the resonance, and multiple planets line up near the disc inner edge. However, if a heavy enough planet joins such a resonance chain, the planet pushes planets ahead of itself and, in particular, the innermost one into the disc cavity. Following Ida et al. (2013), we adopt the condition for this 'leakage' of the planet based on Ogihara et al. (2010); namely, once the following condition is satisfied, the planet at the disc edge halts the migration of the subsequent planet:

$$\frac{e_1}{2\pi}\frac{M_1\mathcal{L}_1}{\tau_e(M_1)} - \sum_{i=1}^{N_{\mathrm{edge}}}\frac{M_i\mathcal{L}_i}{2\tau_{\mathrm{mig}}(M_i)} > 0, \tag{72}$$

where M_i is the mass of the i-th planet ($i = 1$ is the planet at the inner edge), $\mathcal{L}_i \sim \sqrt{GM_*a_i}$ is the orbital angular momentum, τ_{mig} is the migration timescale, τ_e is the eccentricity damping timescale due to migration, which is given by Tanaka & Ward (2004) as

$$\tau_e = \frac{1}{0.78}\left(\frac{M_{\mathrm{p}}}{M_*}\right)^{-1}\left(\frac{\Sigma_{\mathrm{g}}a^2}{M_*}\right)^{-1}\left(\frac{h}{a}\right)^{4}\Omega_{\mathrm{K}}^{-1}, \tag{73}$$

e_1 is the eccentricity of the inner-most planet, which is fitted by ref. Ogihara et al. (2010) as

$$e_1 = 0.02\left(\frac{\tau_e/\tau_{\mathrm{mig}}}{10^{-3}}\right)^{1/2}, \tag{74}$$

and, finally, N_{edge} is the number of planets trapped near the disc inner edge. If Eq. (72) is not satisfied, the inner-most planet is pushed into the disc cavity. We assume that the semi-major axis ratio for this pushed-out planet and the next planet is kept constant, until orbital crossing or a giant impact occurs.

1.7.2 Case including giant planets

The resonance trapping conditions for a pair of an embryo and a giant planet and that of two giant planets are the same as in the two-embryo case. The difference is in the outcome that happens when $b_{\rm trap} < 2\sqrt{3}r_{\rm H}$. Note that these cases are rare in planetary synthesis models for M dwarfs because giant planets are rarely formed.

A pair of two giant planets, if $b_{\rm trap} < 2\sqrt{3}r_{\rm H}$, undergo close encounters. The post-process is calculated following the procedure for dynamical interactions between protoplanets (see Section 1.8 (ii)).

For a pair of an embryo and a giant planet, the trapping condition is calculated based on Shiraishi & Ida (2008), which gives a criterion for an embryo to enter the feeding zone of a giant planet. This condition is expressed by comparing the decreasing rate in the separation of two bodies relative to the Hill radius of the giant planet $v_{\rm H}$ and the eccentricity damping rate of the embryo $v_{\rm damp}$. Given that an embryo of mass $M_{\rm E}$ and semi-major axis $a_{\rm E}$ interacts with a giant planet of $M_{\rm G}$ and $a_{\rm G}$, $v_{\rm H}$ is given by

$$v_{\rm H} = \frac{{\rm d}\tilde{b}^2}{{\rm d}t} = \frac{2\tilde{b}}{h_{\rm G}}\frac{a_{\rm E}}{a_{\rm G}}\left(\frac{\dot{a}_{\rm E}}{a_{\rm E}} - \frac{\dot{a}_{\rm G}}{a_{\rm G}}\right) - \frac{2\tilde{b}^2}{3}\frac{\dot{M}_{\rm G}}{M_{\rm G}}, \tag{75}$$

where $\tilde{b} = (a_{\rm E} - a_{\rm G})/h_{\rm G}a_{\rm G}$ with $h_{\rm G} = (M_{\rm G}/3M_*)^{1/3}$. On the other hand, $v_{\rm damp}$ is expressed as

$$v_{\rm damp} = -\frac{(e_{\rm E}/h_{\rm G})}{\tau_{\rm damp}} \tag{76}$$

with the damping timescale given by (Artymowicz, 1993; Iwasaki et al., 2002)

$$\tau_{\rm damp} = 400\ {\rm yr}\left(\frac{H_{\rm disc}/a}{0.03}\right)^4\left(\frac{M_{\rm E}}{M_\oplus}\right)^{-1}\left(\frac{\Sigma_{\rm g}}{1000\ {\rm g/cm^2}}\right)^{-1}\left(\frac{a_{\rm E}}{1\ {\rm au}}\right)^{-1/2}\left(\frac{M_*}{M_\odot}\right)^{-1/2}. \tag{77}$$

If $|v_{\rm H}| > |v_{\rm damp}|$ is satisfied, the embryo enters the feeding zone of the giant planet and undergoes close encounters. The post-process is also calculated following the procedure given in Section 1.8 (ii).

1.8 Dynamic interaction of multi-body systems and its outcome

As the disc gas depletes, the typical timescale for eccentricity damping via gas drag becomes larger than the timescale for the orbit destabilisation and crossing of the closest pair to happen (called the orbital crossing timescale). In this case, orbital crossing occurs between the pair. We use the semi-analytical model from Ida & Lin (2010) and Ida et al. (2013) regarding the orbital repulsion, merging events, and gravitational scatterings to calculate the resultant semi-major axis and planetary mass (see Ida & Lin, 2010; Ida et al., 2013, for the details). Here we briefly summarise the numerical procedure.

The treatment of dynamical interactions differs depending on the number of giant planets in the system $N_{\rm giant}$: (i) $N_{\rm giant} = 0, 1$, (ii) $N_{\rm giant} = 2$, and (iii) $N_{\rm giant} \geq 3$. Here we define 'giant planets' as the planets that satisfy both conditions (1) $M_{\rm p} > 30M_\oplus$ and (2) $e_{\rm esc} > 1$, where

$$\begin{aligned} e_{\rm esc} &= \frac{v_{\rm esc}}{v_{\rm K}} = \frac{\sqrt{2GM_{\rm p}/R_{\rm p}}}{\sqrt{GM_*/a_{\rm p}}} \\ &= 1.6\left(\frac{M_{\rm p}}{M_{\rm J}}\right)^{1/3}\left(\frac{\bar{\rho}}{1\ {\rm g/cm^3}}\right)^{1/6}\left(\frac{a_{\rm p}}{1\ {\rm au}}\right)^{1/2}\left(\frac{M_*}{M_\odot}\right)^{-1/2}, \end{aligned} \tag{78}$$

with $M_{\rm J}$ the Jovian mass and $\bar{\rho}$ the mean density of the planet.

(i) If the system has no or only one giant planet, we first calculate the orbital crossing timescale for every adjacent pair of embryos (i, j). The timescale $\tau_{\rm cross}$ follows the fitting formula given by Zhou et al. (2007) as

$$\log\left(\frac{\tau_{\rm cross}}{T_{\rm K}}\right) = A + B\log\left(\frac{b}{2.3r_{\rm H}}\right), \tag{79}$$

where $T_{\rm K}$ is the Keplerian period at the semi-major axis of $a = \sqrt{a_j a_i}$, $b = |a_i - a_j|$, $r_{\rm H} = ((M_i + M_j)/3M_*)^{1/3}\min(a_i, a_j)$, and

$$A = -2.0 + e_0 - 0.27\log\mu + 0.51i_0 + 0.19i_0\log\mu + 0.03i_0(\log\mu)^2, \tag{80}$$

$$B = 18.7 + 1.1\log\mu - (16.8 + 1.2\log\mu)e_0 - 0.28i_0 + 0.19i_0\log\mu, \tag{81}$$

$$e_0 = \frac{1}{2}\frac{e_i + e_j}{b}a, \quad \mu = \frac{1}{2}\frac{M_i + M_j}{M_*}. \tag{82}$$

Here i_0 is the mean inclination of the two planets in a unit of degree. The inclination of a planet in a unit of radian is set to the half of the eccentricity. The planetary eccentricity before any orbital crossing events is assumed to follow the Rayleigh distribution with root mean square (rms) of $\sigma = (M_i/3M_*)^{1/3}$. Then, after the time interval equal to $\tau_{\rm cross}$, the pair (i, j) undergoes orbital crossing or sometimes ends up merging. Their resultant masses, semi-major axes, and eccentricities are calculated following the procedure presented in Ida & Lin (2010). They are evaluated so that each of the total mass, orbital energy, and Laplace-Runge-Lenz vector is conserved. Finally, if the orbit of the embryo is within $3.5R_{\rm H}$ of the giant planet, the embryo is scattered by the giant planet, and its semi-major axis and eccentricity are modified again, following Ida et al. (2013).

(ii) If the system has two giant planets (1 and 2) and their separation is $b = |a_1 - a_2| < 2\sqrt{3}r_{\rm H}$, with $r_{\rm H} = ((M_1 + M_2)/3M_*)^{1/3}\sqrt{a_1 a_2}$, then, the orbital instability occurs. The orbital elements after the instability are calculated following 'Two Giants Case' in Ida et al. (2013). If the orbital instability occurs, all other embryos are assumed to be ejected from the system. Otherwise, the interactions between embryos are calculated in the same way as in Case (i).

(iii) If the system has more than two giant planets, the orbital crossing timescale is calculated for every pair of giant planets using Eq. (79). If any of the derived $\tau_{\rm cross}$ is larger than the total integration time, the giant planets do not interact with each other, and only the interaction between embryos are calculated following Case (i). Otherwise, the orbital instability occurs after $\tau_{\rm cross}$, and the resultant masses and orbital elements are derived from the 'Three Giants Case' model in Ida et al. (2013). All the other embryos are ejected from the system in this case.

1.9 Initial Conditions and Parameters

To start the planetary population synthesis simulations, we perform random samplings of the initial mass $M_{\rm disc}$, radius $r_{\rm disc}$, metallicity [Fe/H], and inner edge radius $r_{\rm in}$ of the protoplanetary gas disc, the external photo-evaporation rate $\dot{M}_{\rm wind}$, and the initial masses and semi-major axes of embryos in the following way. We use the default subroutine `random_number` in FORTRAN90 to generate random numbers of uniform distribution in $[0, 1]$. To generate random numbers following the normal and Rayleigh distributions, we use Monty Python method and inverse transform method, respectively.

1.9.1 Initial conditions for protoplanetary disc

We determine the initial properties of the protoplanetary disc by scaling recent observation results for stars of $\sim 1\ M_\odot$. We adopt the fitting formula for the disc gas mass for $1M_\odot$ stars, the log-normal distribution with the mean $\log(\mu/M_*) = -1.49$ and the standard deviation $\sigma = 0.35$, which Emsenhuber et al. (2021b) derived from observational results of Tychoniec et al. (2018). The minimum and maximum disc masses are set to $4 \times 10^{-3} M_\odot$ and $0.16M_\odot$, respectively, which roughly correspond to the lightest and heaviest samples in Tychoniec et al. (2018). We also assume that the mean, minimum, and maximum disc masses are proportional to the stellar masses (Andrews et al., 2013). The disc gas radius is calculated with

$$r_{\rm disc} = 10\left(\frac{M_{\rm disc}}{2 \times 10^{-3} M_\odot}\right)^{0.625} \text{au}, \tag{83}$$

which is taken from the observational trend derived in Andrews et al. (2010).

The disc metallicity [Fe/H] follows the normal distribution with $\mu = -0.02$ and $\sigma = 0.22$ derived from Santos et al. (2005). The range of the value is limited to $-0.6 <$[Fe/H]< 0.5. We use the same distribution regardless of the stellar mass.

We assume that the disc inner edge locates at the corotation radius where the Kepler rotation period is equal to the rotation period of the central star. Here the stellar rotation period is assumed to follow the log-normal distribution with $\log(\mu[\text{days}]) = 0.676$ and $\sigma = 0.306$ based on the observational results of young stellar objects by Venuti et al. (2017). The minimum of $r_{\rm in}$ is set to the initial stellar radius R_*. We also use the same distribution for all stellar types.

The external photo-evaporation rate $\dot{M}_{\rm wind}$ generally depends on the population of nearby massive stars. Following Burn et al. (2021), we set the distribution of $\dot{M}_{\rm wind}$ so that the mean value of the resultant disc lifetime locates at $\sim$3Myr (Mamajek, 2009; Ansdell et al., 2018) and that it has a deviation of about half an order, regardless of the stellar mass. Since the disc gas radius $r_{\rm disc}$ is determined only by the disc mass $M_{\rm disc}$, the disc lifetime depends on $M_{\rm disc}$ and $\dot{M}_{\rm wind}$ for given $\alpha_{\rm acc}$. Then we find that, for a star of $0.3M_\odot$ and $\alpha_{\rm acc} = 2\times10^{-3}$, the log-normal distribution with $\log(\mu\ [{\rm M_\odot/yr}]) = -6.0$ and $\sigma = 0.5$ accounts for the above distribution. Also, the stellar mass dependence of $\dot{M}_{\rm wind} \propto M_*^{1.4}$ is found to be suitable for the star in the range of $0.1M_\odot \le M_* \le 0.5M_\odot$.

1.9.2 Initial conditions for planetary embryos

Initially, 50 planetary embryos with mass of 0.01 $M_\oplus$ are placed log-uniformly from $r_{\rm in}$ to $r_{\rm solid}$. Here the initial separations of all the adjacent embryos are larger than the feeding zone width ($10r_{\rm H}$) for the local isolation mass $M_{\rm iso}$ given by (Kokubo & Ida, 2002)

$$M_{\rm iso} = 0.16\left(\frac{\Sigma_{\rm s}}{10\ {\rm g\,cm^{-2}}}\right)^{3/2}\left(\frac{a}{1\ {\rm au}}\right)^{3/4}\left(\frac{M_*}{M_\odot}\right)^{1/2} M_\oplus. \quad (84)$$

Therefore, in quite massive solid discs, the initial number of planetary embryos can be smaller than 50.

1.9.3 Input parameters

The parameters and their fiducial values are summarised in Table.1. These values are used in our calculations unless otherwise mentioned.

Table 1: Parameters used in calculations

Symbol	Meaning	Value	Eq. mainly used
$\alpha_{\rm acc}$	parameter for effective turbulent viscosity	2.0×10^{-3}	Eq. (3)
$\alpha_{\rm vis}$	parameter for turbulent viscosity	2.0×10^{-4}	Eq. (44)
$N_{\rm grid,disc}$	number of grids for gas disc	500	
$N_{\rm grid,solid}$	number of grids for solid disc	1000	
$r_{\rm max}$	outer boundary radius for gas disc	1000 au	Eq. (5)
$r_{\rm solid}$	solid disc radius	$0.5r_{\rm disc}$	Eq. (17)
Φ	ionising EUV photon luminosity	$1.0\times10^{40}\ {\rm s^{-1}}$	Eq. (7)
$m_{\rm plt}$	planetesimal mass	1.0×10^{20} g	Eq. (27)
$\rho_{\rm plt}$	planetesimal material density	$3.0\ {\rm g\,cm^{-3}}$	Eq. (27)
$C_{\rm rock}$	specific heat of solid core for constant volume	$1.2\times10^{7}\ {\rm erg/(g\,K)}$	Eq. (35)
K_t	typical value of K for which the corotation torque becomes ineffective	20.0	Eq. (64)

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