Supplementary Information: Multiresolution Spectrally Encoded Terahertz Reflection Imaging Through a Highly Diffusive Cloak

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Figure S1(a) shows the schematic of the optical components inside the PHASR scanner housing. Figure S1(b) illustrates the image of the Boehler star without the presence of any scattering layers formed using the normalized extinction coefficients at 0.53 THz. In Fig. S1(b), the petals filled with α-lactose monohydrate are clearly discernible in the absence of any scattering effects. The radius of the unresolved circle in the center represents the diffraction-limited imaging resolution at 0.53 THz, and it serves as the ground truth for comparison purposes in the paper.

Figure S2(a) shows the scattering LDPE powders placed inside a reservoir, having dimensions of 20×20×4.63 mm^3 L×W×H, over the Boehler star. Figure S2(b) shows the image of the Boehler star sample formed using the normalized extinction coefficients at 0.53 THz as the color map, in the presence of the scattering layer with 0.5 g/cm^3 volume density. Figure S2(c) shows the image formed by including only the 3$^{\text{rd}}$ and 4$^{\text{th}}$-level details vectors, i.e., $\tilde{D}_3(f)$ and $\tilde{D}_4(f)$, in the wavelet MRA of the extinction coefficients. Figure S2(d) compares the bimodality coefficient as a function of frequency, $\beta(f)$, for images formed using the extinction coefficients (the orange line) and $\sum_{j=3}^{4}\tilde{D}_j(f)$ (the blue line) in the presence of the scattering layer with 0.5 g/cm^3 particle volume density. The $\beta(f)$ of images formed using the extinction coefficients is less than the 5/9 threshold at all frequencies. In other words, the scattering artifacts prevent the images formed using the extinction coefficients from resolving the Boehler star petals filled with α-lactose powders. On the other hand, in the $\beta(f)$ of the images formed using $\sum_{j=3}^{4}\tilde{D}_j(f)$ the bimodality coefficient surpasses the 5/9 threshold only at 0.53 THz, corresponding to the α-lactose's resonance. Importantly, $\sum_{j=3}^{4}\tilde{D}_j(f)$ is a unique combination, and no other such combination of wavelet details vectors exists that yield the α-lactose's resonant frequency as the only local maximum in $\beta(f)$ surpassing 5/9. The transport mean free path normalized by wavelength, l_{tr}/λ, of the scattering layers with 0.3 and 0.5 g/cm^3 volume density is given in Fig. S2(e). The l_{tr} values are measured using a Boehler star target of petals filled with high-density polyethylene (HDPE) and placed underneath the scattering layers. The black and magenta lines in Fig. S2(e) respectively show the wavelength-normalized propagation distance of one-way (toward the target) and round-trip (back to the detector) travels through the scattering layer. It can be noted that l_{tr} in both scattering regimes is approximately an order of magnitude larger than the wavelength at the peak wavelength of the THz emissions.

Figure S3 shows the results of spectral unmixing of extinction coefficients using principal component analysis (PCA) of the spectral images over the frequency range $f = 0.1 - 1.1$ THz. Figure S3(a) shows that each PC is formed by a linear, weighted combination of all the spectral images along the measurement bandwidth. The weights associated with each frequency in constructing the PCs are determined by the elements of the eigen-vectors of cov(E) (i.e., the elements of each row vector in $\mathbf{U}^T$). Figure S3(b) gives the eigen-vectors corresponding to the first four PCs in the order of capturing a higher variance in the original set. It can be noted that there is no eminent absorption feature in the eigen-vectors of the first (red line, square markers) and second (blue line, diamond markers) PCs. However, the eigen-vector of the third PC (green line, circle markers) has two distinct peaks at 0.53 and 0.6 THz, corresponding to the resonant frequencies of α-lactose and PABA. Similarly, in the fourth eigen-vector (purple line, triangle markers) the image at 0.53 THz has the greatest weight value. Therefore, it can be surmised that the fourth PC is mainly associated to the α-lactose's resonance. Figure S3(c-f) show the images of the PCs formed using the eigen-vectors given in Fig. S3(b). It can be seen that the data variance captured by the first and second PCs does not provide any contrast on any of the petals. Similar patterns were also observable in the fifth and higher PCs, which mainly captured the speckle patterns caused by the scattering. Therefore, while the petals filled with α-lactose and PABA are distinguishable using PC 3 and PC 4, petals filled with the other materials (i.e., HDPE, 3HBA, and riboflavin) could not be resolved using the PCA approach.

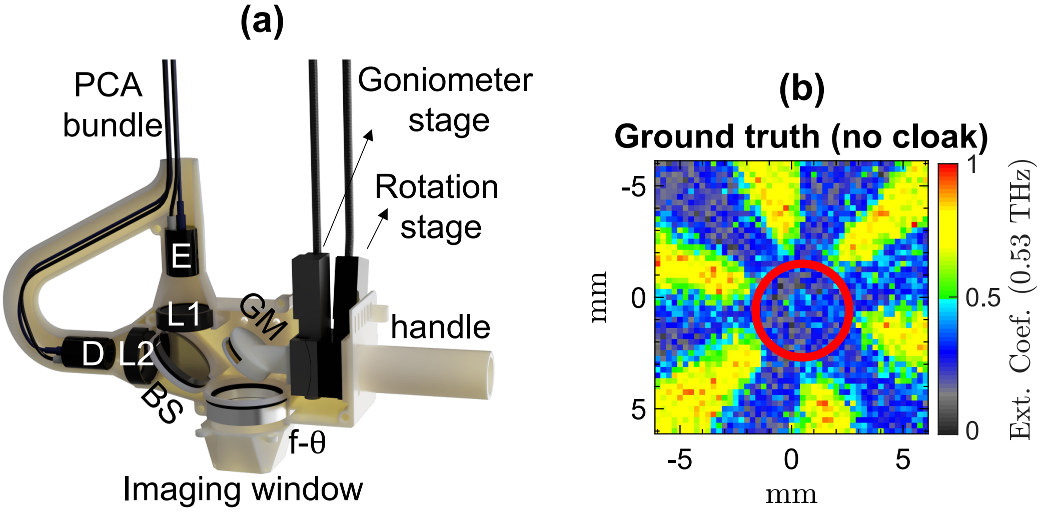

Figure S1: **(a)** shows the schematic of the optical components inside the PHASR scanner housing. **(b)** shows the image of the Boehler star target without a scattering layer formed using the extinction coefficients at 0.53 THz (i.e., the α-lactose's resonance) with $0.25{\times}0.25$ mm^2 pixel area. The radius of the red circle in the center is 2.1 mm, corresponding to 1.11 mm spatial resolution.

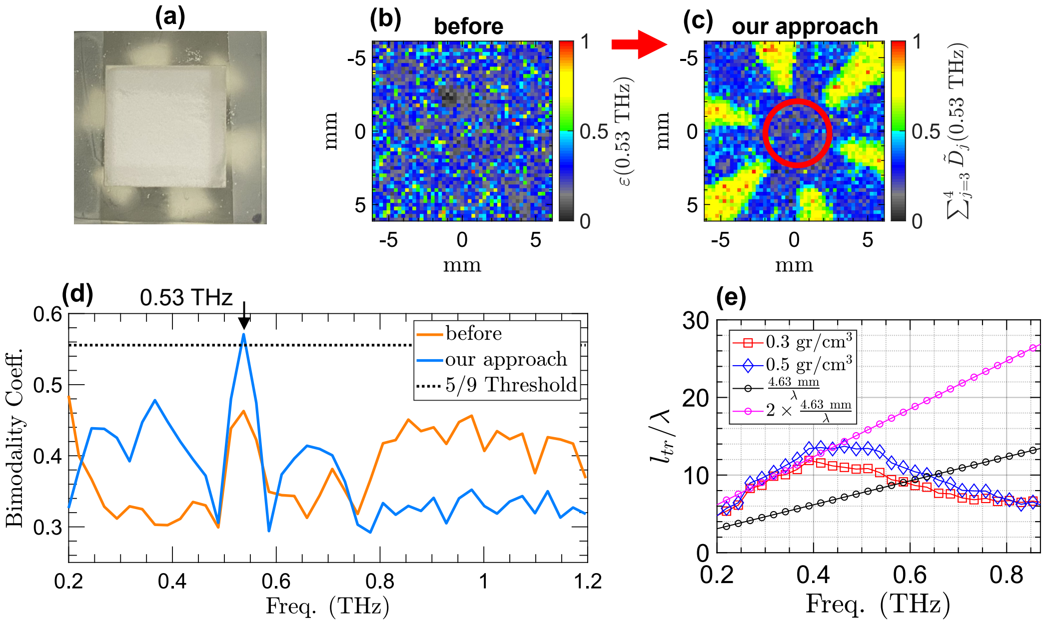

Figure S2: **(a)** shows the Boehler star covered with an optically-opaque scattering layer with a fixed thickness of about 4.63 mm. The scattering layer is made of low-density polyethylene (LDPE) with 180 μm mean particle-size. **(b)** shows the image of the α-lactose Boehler star covered with a scattering layer of 0.5 g/cm^3 volume density. The color axis is given by the measured extinction coefficients at 0.53 THz. **(c)** shows the image of the sample in **(b)** reconstructed using $\sum_{j=3}^{4} D_j(0.53\ \mathrm{THz})$ as the color map. **(d)** shows the bimodality coefficient, β, for images formed using the extinction coefficients (the orange line) and $\sum_{j=3}^{4} D_j(f)$ (the blue line), given the scattering layer with 0.5 g/cm^3 volume density. In **(d)**, the only β value greater than 5/9 threshold (the horizontal black dashed line) is located at 0.53 THz (i.e., the α-lactose's resonance). **(e)** shows the transport mean free path, l_{tr}, of the scattering layers of 0.3 and 0.5 g/cm^3 scattering volume density normalized by the wavelength. The l_{tr} values are measured using a Boehler star target of petals filled with high-density polyethylene (HDPE) and placed underneath the scattering layers. The black and magenta lines respectively show the wavelength-normalized propagation distance of one-way (toward the target) and round-trip (back to the detector) travels through the scattering layer.

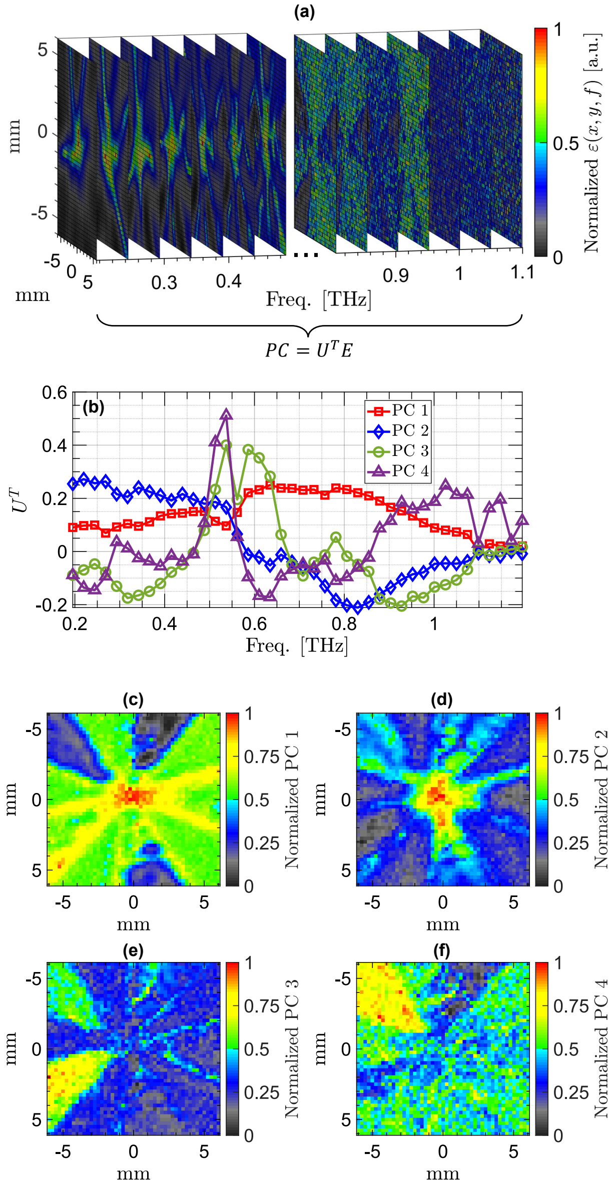

Figure S3: **(a)** shows how a principal component (PC) can be formed using a linear, weighted combination of the spectral images along the measurement bandwidth, where the weight associated with each spectral image is determined by the eigen-vectors of cov(E), i.e., the row vectors in $\mathbf{U}^T$. **(b)** shows the eigen-vectors of the first four PC bands in the order of capturing a higher variance. While there is no eminent absorption feature in the eigen-vectors of the first and second PCs, distinct spectral peaks at 0.53 and 0.6 THz (i.e., resonant frequencies of α-lactose and PABA) are discernible in the eigen-vectors of the third and fourth PCs. **(c-f)** show the images of the first four PCs in the order of capturing a higher variance. The third and fourth PCs yield the petals of α-lactose and PABA.