myjournal manuscript No.
(will be inserted by the editor)

Supplementary materials for "Experimental study on dynamics of the multi-individual clapping interacting system"

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Received: date / Accepted: date

Abstract This supplement to our main manuscript shows the detailed algorithms of three analysis methods involved in manuscript, and some supplementary results.

Keywords Synchronization mechanism · Multi-individual clapping system · Dynamics · Clapping interaction

1 Detrended Fluctuation Analysis (DFA) and Detrended Cross-Correlations Analysis (DCCA).

DCCA [1] works on two time series, $x(i)$ and $y(i)$ of the equal length n, the modified version of DCCA consists of the following steps: (1) global detrending: fitting a polynomial $x^*(i)$ of degree m to $x(i)$ and a polynomial $y^*(i)$ of degree m to $y(i)$, where typically $m = 1,2,3,4,5$. (2) we calculate the cumulative sum of the $x(i)$ and $y(i)$:

$$X(t) = \sum_{i=1}^{t}(x(i) - x^*(i)), \quad Y(t) = \sum_{i=1}^{t}(y(i) - y^*(i)), \quad t = 1,2,\cdots,n. \tag{1}$$

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(3) Dividing $X(t)$ and $Y(t)$ into windows of size s. The covariance in each window is calculated as follows:

$$F^2_{DCCA}(s) = \frac{1}{n-ks}\sum_{t=1}^{n-ks}[X(t)-X_s(t)][Y(t)-Y_s(t)], \tag{2}$$

where the $X_s(t)$ and $Y_s(t)$ denote theoretical values obtained from regression of the $X(t)$ and $Y(t)$ within each window s, respectively. This calculation is repeated over all possible window lengths s. A power law is expected as follows: $F_{DCCA}(s) \sim s^{\lambda}$, where the λ is the scaling exponent and corresponds to the slope of log-log plot of $F_{DCCA}(s)$ as a function of s. The two series exhibit persistent LRCCs as $\lambda > 0.5$[2], i.e. each series has a long memory of the previous value of both its own and the other one. $\lambda = 0.5$ indicates the absence of LRCC, which means the detrended covariance oscillates around zero as a function of the time scale s [1,2]. When we input the $x(i)$ instead of the $y(i)$, DCCA reduces to DFA as follows:

$$F_{DFA}(s) = \sqrt{\frac{1}{n-ks}\sum_{t=1}^{n-ks}[X(t)-X_s(t)]^2}, \tag{3}$$

then a power law is expected as follows: $F_{DFA}(s) \sim s^{\alpha}$, where the α is the scaling exponent and corresponds to the slope of log-log plot of $F_{DFA}(s)$ as a function of s. As $\alpha > 0.5$, large (small) values are more likely to be followed by the large (small) ones, which leads to persistent long-range correlations (LRCs), otherwise, the time series are in an anti-persistent correlation if $\alpha < 0.5$. Typically, as $\alpha = 0.5$, the time series is white noise and exhibit the absence of LRCs [2,3]. In both analysis methods (DCCA and DFA), we considered window lengths ranging from $s = 10$ to $s = N/2$.

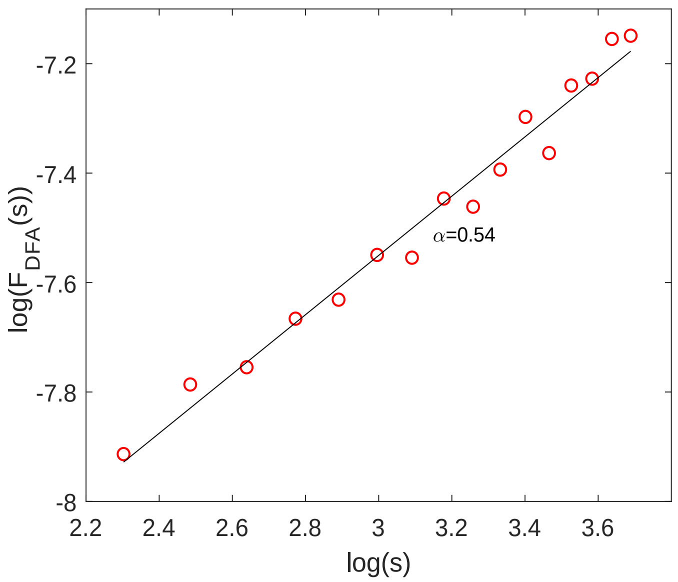

Fig. S1. (color online) Average DFA diffusion plots for the mean time series of 1000 synthetically generated time series of total length 80(red circles) and fixed scaling(0.5). The DFA exponent for the mean time series of 1000 synthetically generated time series is 0.54. The mean and standard deviation of DFA exponents of the 1000 synthetically generated time series are 0.52 and 0.15.

2 Windowed detrended cross-correlation (WDCC)

WDCC [4,5] works on two time series, $x(i)$ and $y(i)$ of the equal length n. The main principle is to compute the cross-correlation function, from lag $-k_m$ to lag k_m ($k_m = 10$ in this paper), considering windows of length $L(L = 15$ in this paper). WDCC consists of the following steps: (1) the data in each window $[x(k_m+k+1),\ x(k_m+L+1+k)]$, $k=-k_m,\cdots,0,\cdots,k_m$(or $[y(k_m+k+1),y(k_m+L+1+k)]$) is detrended before calculating cross-correlation. As shown in Fig. S3, we illustrated the effect of detrending within a window L. (2) Calculate the cross-correlation $r(k)$ between the first interval $[x(k_m+1),x(k_m+L+1)]$ and $[y(k_m+k+1),y(k_m+L+1+k)]$. (3) The first interval is then lagged by one point, and a second cross-correlation function between $[x(k_m+2),x(k_m+L+2)]$ and $[y(k_m+k+1),y(k_m+L+1+k)]$ is computed. This process is repeated up to the last interval $[x(n-k_m-L-1),x(n-k_m-1)]$. Note that in the present work we use a sliding window, as initially proposed by Boker et al.[5] We consider that this method provides a more complete picture of cross-correlations between the two series. (4) The correlation coefficients $r(k)$ are transformed in z-Fisher scores $Z_r(k)$, $Z_r(k) = log[(1+r(k))/(1-r(k))]/2$. (5) Finally, the $Z_r(k)$ coefficients are averaged over all windows and backward transformed in correlation metrics: $\bar{r}(k) = (exp(2\overline{Z_r}(k))-1)/(exp(2\overline{Z_r}(k))+1)$.

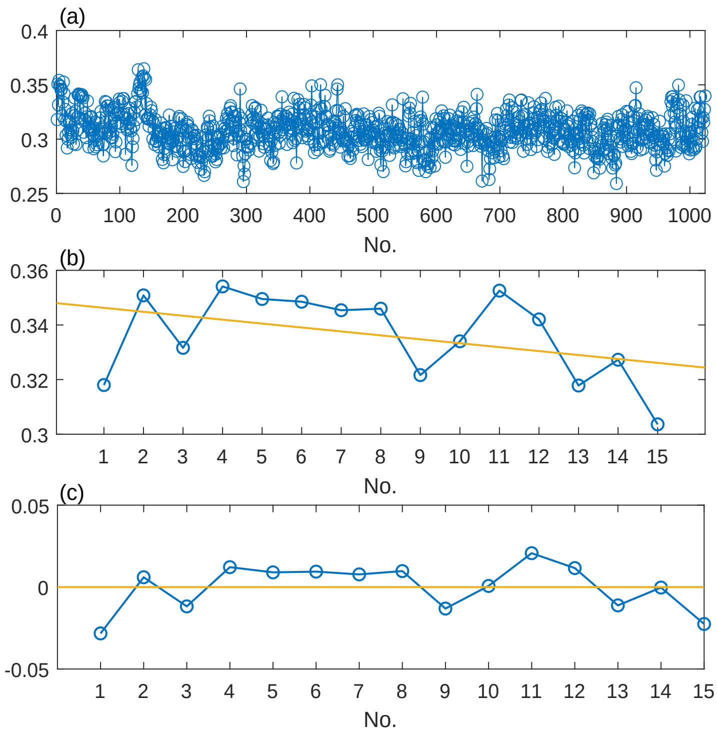

Fig. S2. (color online) (a) The simulated series with DFA exponent $\alpha = 0.9$, $N = 1024$. (b) A window of 15 point extracted from the series of graph (a). The yellow line represents the linear trend within the window. (c) The same window, after detrending.

3 The extended version of autoregressive fractionally integrated moving average (EARFIMA)

We generated inherent period series of three individuals (Fig. S3(a)) and inherent period series of four individuals (Fig. S4(a)) by the extended version of autoregressive fractionally integrated moving average (EARFIMA) process (see our manuscript for the detailed algorithm). The arbitrary two inherent period series generated by the EARFIMA procedure exhibit LRCCs, which is consistent with the autoregressive fractionally integrated moving average process [1]. As shown in Fig. S3, and Fig. S4, the DCCA exponents $\lambda > 0.5 \pm 0.15$.

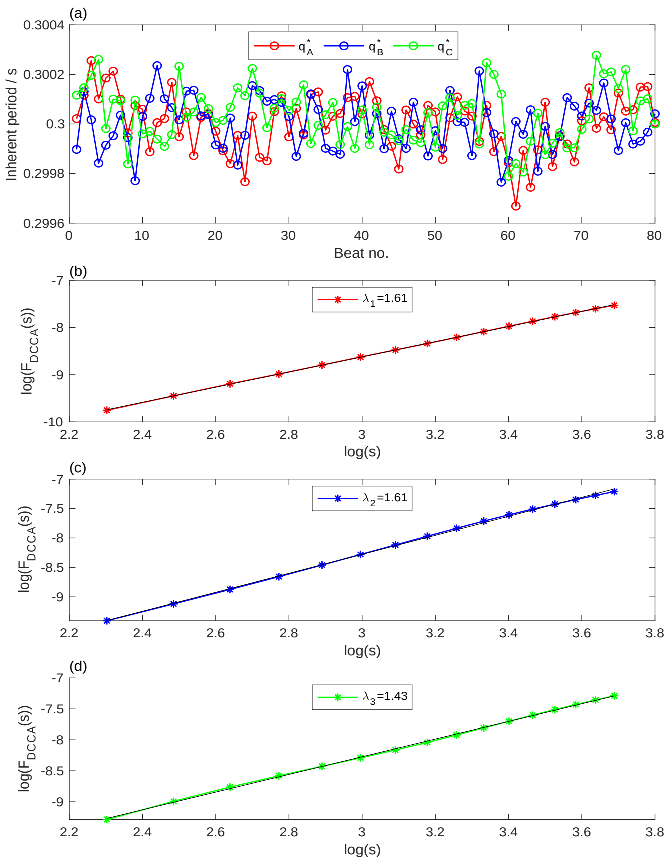

Fig. S3. (color online) Three inherent period series q_A^*, q_B^*, q_C^* simulated by the EARFIMA procedure exhibit LRCCs. (a) q_A^*, q_B^*, q_C^* exhibit slowly varying trends. (b)-(d) Evidence of the LRCCs between the inherent period series simulated by the EARFIMA procedure. The DCCA exponents $\lambda_1 = 1.61 > 0.5 \pm 0.15$, $\lambda_2 = 1.61 > 0.5 \pm 0.15$, $\lambda_3 = 1.43 > 0.5 \pm 0.15$. The initial conditions of EARFIMA are as follows: q_A^*, q_B^*, q_C^* simulated by the EARFIMA procedure with $d = 0.4, W = 0.8, \varepsilon_k \sim N(0,1), \delta_k = 0.0001$.

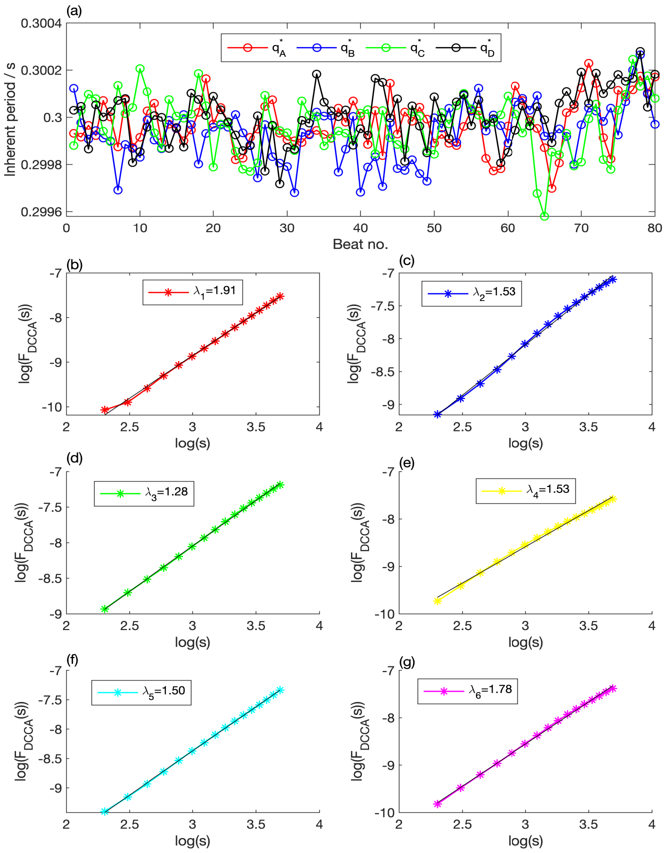

Fig. S4. (color online) Four inherent period series q_A^*, q_B^*, q_C^*, and q_D^* simulated by the EARFIMA procedure exhibit LRCCs. (a) q_A^*, q_B^*, q_C^*, and q_D^* exhibit slowly varying trends. (b)-(g) Evidence of the LRCCs between the inherent period series simulated by the EARFIMA procedure. The DCCA exponents $\lambda_1 = 1.91 > 0.5 \pm 0.15$, $\lambda_2 = 1.53 > 0.5 \pm 0.15$, $\lambda_3 = 1.28 > 0.5 \pm 0.15$, $\lambda_4 = 1.53 > 0.5 \pm 0.15$, $\lambda_5 = 1.50 > 0.5 \pm 0.15$, $\lambda_6 = 1.78 > 0.5 \pm 0.15$. The initial conditions of EARFIMA are as follows: q_A^*, q_B^*, q_C^*, and q_D^* simulated by the EARFIMA procedure with $d = 0.4, W = 0.8, \varepsilon_k \sim N(0,1), \delta_k = 0.001$.

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