Supplementary Information: Flow Characterization and Aerosol Dispersion of Wind Instruments

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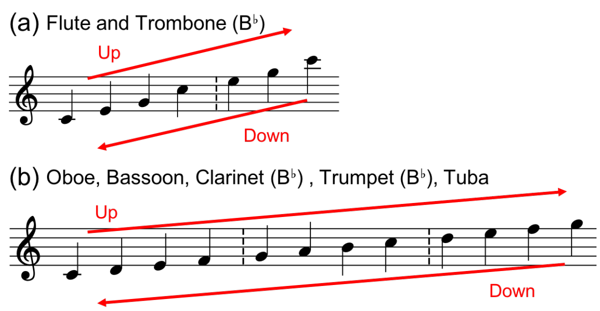
1 Musical notes



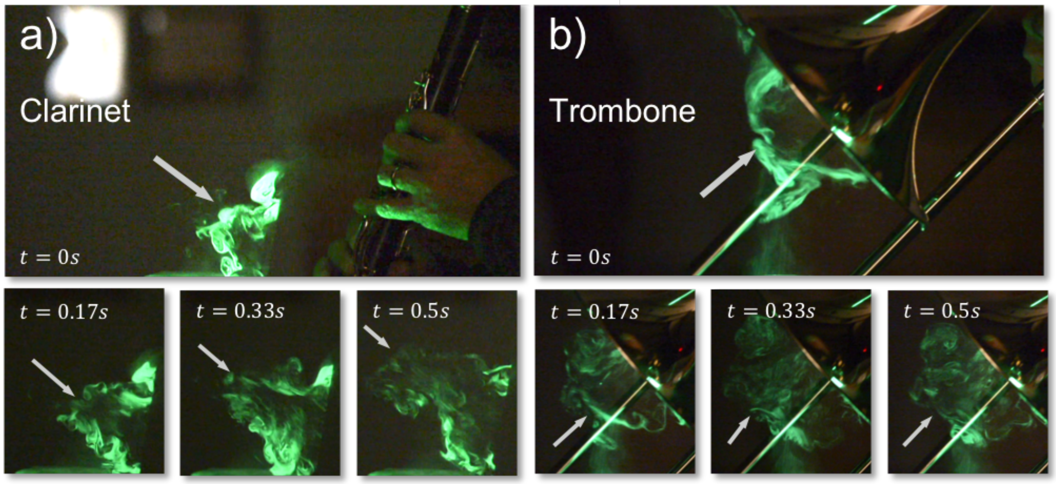
Supplementary Fig. 1 – Cycles played by musicians during the aerosol and flow velocimetry acquisition. Flute and trombone followed arpeggios scales in (**a**). Oboe, Bassoon, Clarinet, Trumpet and Tuba followed heptatonic major scales in (**b**).

In order to characterize both flow structures and aerosol production, each musician was recorded while playing a few cycles up and down along frequencies covering up the whole usable range of the instrument, at a moderate tempo and at mezzo forte dynamical range. Each musician was free to choose the note pattern for his cycle. Flute and trombone followed arpeggios over C and B-flat respectively (Supplementary Fig. 1a). Oboe, Bassoon, Clarinet, Trumpet and Tuba followed heptatonic major scales in C of B-flat (Supplementary Fig. 1b).

2 Jet formation from instruments

The flow exiting an opening from a wind instrument is characterized by transient ejections at high velocity or jets. Supplementary Fig. 2a depicts the evolution of a jet exiting a clarinet tone hole, and Supplementary Fig. 2b shows the jet exiting a trombone bell; both are revealed by an artificial fog. The front of the jet, indicated by a gray arrow, moves away perpendicularly from the opening.

In the following we consider the analytical solution of a laminar axisymmetric jet to describe the our observations.



Supplementary Fig. 2 – Times lapses for the expulsion of jets from clarinet tone hole (**a**) and trombone bell (**b**). Frame series depict the emission of jets structures above a clarinet tone holes in **a** and at a trombone bell in **b**. The arrow show the front of the jet displacement with time.

3 Schlichting Jet Solution

We consider a steady laminar round air jet entering a large stagnant volume of air from a circular opening. The jet flow is treated to be axisymmetric, with an axial velocity component v_z in the jet flow direction z, and a radial velocity component v_r in the direction r normal to the jet flow. The Navier-Stokes equation for incompressible Newtonian fluid in axial direction z can be expressed as:

$$v_r \frac{\partial v_z}{\partial r} + v_z \frac{\partial v_z}{\partial z} = \frac{\nu}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v_z}{\partial r} \right), \tag{1}$$

where ν is the kinematic viscosity of the air. Here, the pressure of the ambient air is treated as a constant, and buoyancy contribution are ignored. An additional equation that the flow needs to satisfy is the continuity equation:

$$\frac{\partial v_z}{\partial z} + \frac{\partial v_r}{\partial r} + \frac{v_r}{r} = 0. \tag{2}$$

The solutions of eqn.(1) and eqn.(2) can be expressed in terms of an axisymmetric stream function, $\psi = \psi(r, z)$:

$$v_r = -\frac{1}{r} \frac{\partial \psi}{\partial z}, \quad v_z = \frac{1}{r} \frac{\partial \psi}{\partial r}. \tag{3}$$

We expect a similarity solution of the equation in the following form:

$$\psi(r, z) = \nu z F(\eta), \tag{4}$$

where the similar variable $\eta = r/z$, and $F(\eta)$ is a function of η. By applying chain rule to eqn.(3), we can rewrite the velocity components in terms of $F(\eta)$ and its derivative $F'(\eta)$:

$$v_z = \frac{\nu}{z} \frac{F'(\eta)}{\eta}, \quad v_r = \frac{\nu}{z} \left(F'(\eta) - \frac{F(\eta)}{\eta} \right). \tag{5}$$

By substituting eqn.(5) into eqn.(1), we obtain a ordinary differential equation (ODE) that $F(\eta)$ needs to satisfy:

$$\frac{d}{d\eta}\left(\frac{FF'}{\eta}-\frac{F'}{\eta}+F''\right)=0, \tag{6}$$

which can be integrated once to get another ODE:

$$FF'=F'-\eta F''. \tag{7}$$

A particular solution of eqn.(7) is:

$$F(\xi)=\frac{\xi^2}{1+\xi^2/4}, \tag{8}$$

where $\xi=a\eta$, and a is an integration constant to be determined. The solutions for velocity components v_z and v_r obtained from eqn.(5) are:

$$v_z=\frac{\nu}{z}\frac{a^2}{\left(1+\xi^2/4\right)^2},\quad v_r=\frac{\nu}{z}\frac{a(\xi-\xi^3/4)}{\left(1+\xi^2/4\right)^2}. \tag{9}$$

To determine the integration constant a, we calculate the momentum flux of the jet J:

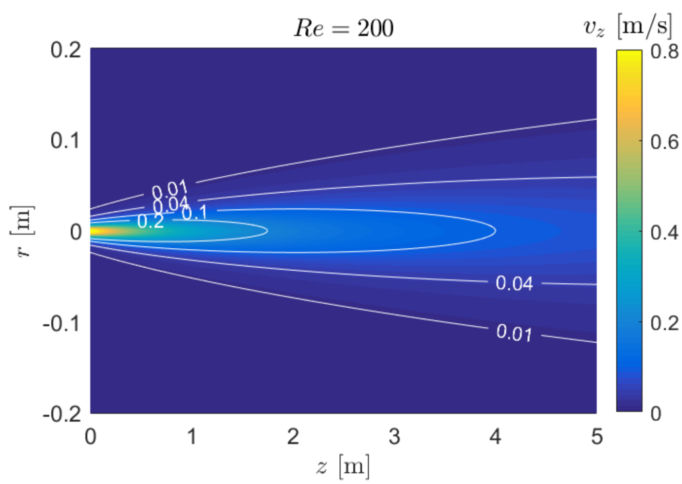
$$J=2\pi\rho\int_0^{+\infty}v_z^2 r\,dr=\frac{16\pi}{3}a^2\nu^2. \tag{10}$$

The Reynolds number of the jet can be defined from the momentum flux J, and the integration constant a can be expressed in terms of the Reynolds number:

$$Re=\frac{1}{\nu}\left(\frac{J}{2\pi\rho}\right)^{1/2},\quad a^2=\frac{3}{8}Re^2. \tag{11}$$

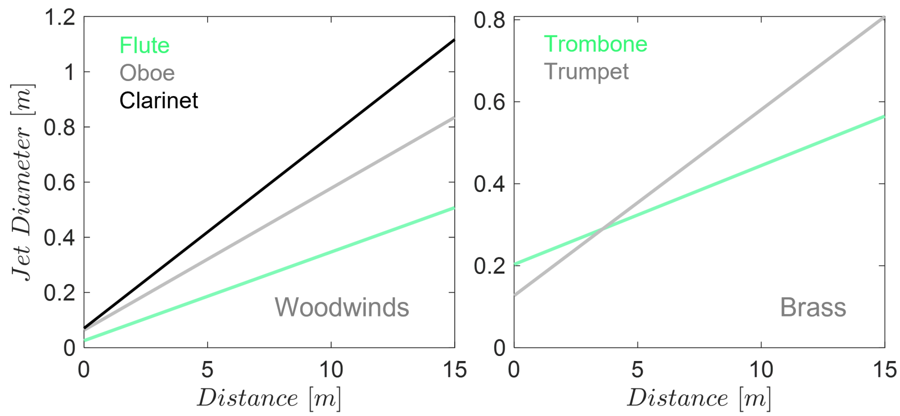
We can rewriting eqn. (9) in terms of the Reynolds number:

$$v_z=\frac{\nu}{z}\frac{3Re^2/4}{\left(1+3Re^2r^2/32z^2\right)^2},\quad v_r=\frac{\nu}{z}\frac{\left(3Re^2r/8z+9Re^4r^3/256z^3\right)}{\left(1+3Re^2r^2/32z^2\right)^2}. \tag{12}$$



Supplementary Fig. 3 – Velocity contour of axial velocity v_z for a Schlichting jet at a Reynolds number $Re=200$. The jet propagate with a decaying axial velocity and a growing diameter.

The axial velocity v_z from the jet solution is shown is Supplementary Fig. 3. We can see that the jet propagate with a decaying axial velocity and a growing diameter. We can define the edge of the jet as $v_r = 0$, and calculate the diameter of the jet; the growth of jet diameter for different instruments is shown in Supplementary Fig. 4. We find that the jet diameter grows with the axial distance as: $D \sim z/Re$. The larger the Reynolds number is, the longer distance the jet can propagate without the diameter expanding.



Supplementary Fig. 4 – The growth of jet diameter for different instruments. The larger the Reynolds number is, the slower the jet diameter expands.

4 The Stokes number

The Stokes number, characterizing the behaviors of particles suspended in fluid flows, is the ratio of the relaxation time of a particle t_p to the characteristic time of a flow $t_f = L/\overline{U}$:

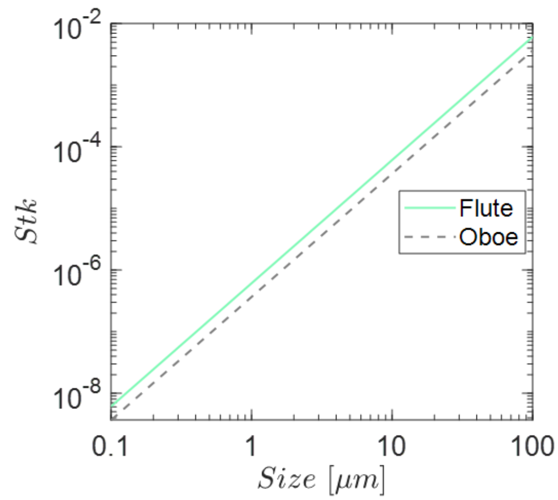
$$Stk = \frac{t_p \overline{U}}{L}, \tag{13}$$

where $\overline{U}$ is the mean velocity of the flow over a propagation distance L. Since the particle Reynolds number, $Re_p = \overline{U} d_p/\nu_{air} \sim 0.1$, is much below unity, the flow surrounding the particles can be treated as Stokes flow. Here, d_p is the diameter of the particle, and ν_{air} is the kinematic viscosity of the air. Therefore, the relaxation time of a particle, t_p, can be selected as the Stokes time, the characteristic time scale for a particle to accelerate or decelerate in a Stokes flow:

$$t_p = \frac{\rho_w d_p^2}{18\mu_{air}}, \tag{14}$$

where ρ_w is the density of water, since the particles we consider here are water droplets.

We calculate the Stokes number using eqn.(13-14), for aerosol particles of a size varying from 1 to 100 μm, for flute, the instruments of the highest $\overline{U}$, and oboe, the instruments of the lowest $\overline{U}$. As shown in Supplementary Fig. 5, the Stokes number Stk ranges from 10^{-8} to 10^{-2} for different particle diameters. A large Stokes number of $Stk \sim 1$ indicates the particles are inertial and likely to detach from the flow; and a small Stk suggests the particles will faithfully follow the flow. Here, the highest Stokes number of 0.01 indicates that the aerosol particles act as ideal tracers of the flow.



Supplementary Fig. 5 – The Stokes number Stk for aerosol particle sizes ranging from 0.1 to 100 μm, for flute, the instrument with the highest velocity, and oboe, the instruments with the lowest velocity. The Stokes number for all other instruments lie in between the two curves.

5 Particle settling time

We consider the setting of aerosol particles due to gravity, in the absence of a flow. The net force of buoyancy and gravity is:

$$F_b = \frac{\pi}{6}(\rho_w - \rho_{air})d_p^3. \tag{15}$$

Since the particle Reynolds number is low, we can model the air drag force by the Stokes drag:

$$F_d = 3\pi\mu_{air}d_p. \tag{16}$$

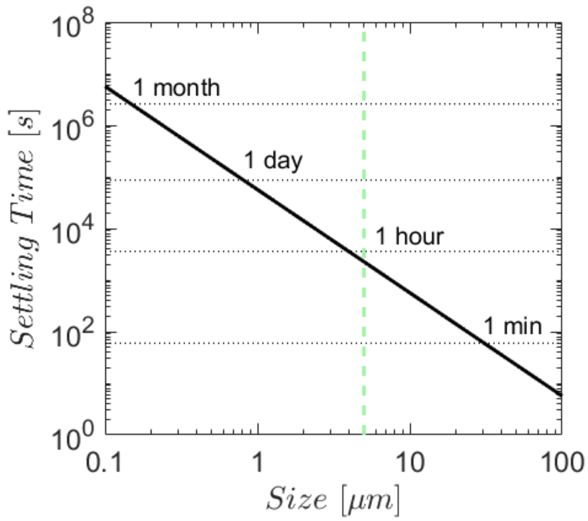
Taken together, a balance between these two forces yields the settling speed of aerosol particles:

$$V_s = \frac{gd_p^2(\rho_w - \rho_{air})}{18\mu_{air}}. \tag{17}$$

As shown in eqn. (17), the settling speed of aerosol particles is largely dependent on the particle size. We can treat the settling of aerosol particles without flow as a sedimentation at a constant speed V_s. And we can calculate the settling time of particles, t_s, for a certain settling height ΔH:

$$t_s = \frac{\Delta H}{V_s} = \frac{18\mu_{air}\Delta H}{gd_p^2(\rho_w - \rho_{air})}. \tag{18}$$

We can see in eqn. (18) that the settling time is inversely proportional to the square of particles size; the larger the particle is, the faster they settle. For the particle to settle the height of an average adult, ΔH = 1.7 m, the settling time is plotted in Supplementary Fig. 6. We can see that the settling time for particles of size ≤ 10 μm varies from an hour to a month. But for particles at the size of 50 to 100 μm, the settling time can be less than one minute, which is comparable to the time scale of the flow coming out from the instruments.



Supplementary Fig. 6 – The particle settling time due to gravity as a function of particle size. The settling time for smaller particles of a size ≤ 10 μm varies from an hour to a month; but the settling time for larger particles ≥ 30 μm can be less than one minute.

Supplementary Movies

Supplementary Movie 1. Shortened real-time movie (duration 20 s) of the play of clarinet with the sound in the background.

Supplementary Movie 2. Shortened real-time movie (duration 20 s) of the play of flute with the sound in the background.

Supplementary Movie 3. Shortened real-time movie (duration 20 s) of the play of trombone with the sound in the background.