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System Dynamics Modeling of Quality Indicators and Cost of Quality in Stainless Steel Cold Rolling Plants

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Case Report

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System Dynamics Modeling of Quality Indicators and Cost of Quality in Stainless Steel Cold Rolling Plants

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Abstract

This study examines five operational and one economic quality indicators from a stainless steel cold rolling facility in Türkiye over 2019–2025. First-order adjustment stocks are calibrated from observations, and a conditional policy simulation incorporates prevention effort, delayed quality responses and cost feedback. Historical calibration is separated from policy assumptions. Seven annual observations cannot identify causal coefficients; consequently, policy outputs are scenarios rather than validated forecasts or savings. Appendices provide parameter calculations, annual simulations and sensitivity analysis.

Keywords: System dynamics, cost of quality, cold rolling, feedback, scenario analysis.

1. Introduction

The cost of quality encompasses not only the monetary consequences of final product nonconformities but also prevention and appraisal activities. Therefore, cost minimisation is not equivalent to simply reducing the quality control budget (Schiffauerova and Thomson, 2006; Tsai, 1998). In cold rolling, geometric and surface nonconformities may arise at different stages of the production process (Roberts, 1978; Lenard, 2014). Although rework consumes production capacity, it may reduce the incidence of final product nonconformities; these two effects must therefore be distinguished.

1.1. Research Questions

What historical trends do the indicators exhibit? How can delayed adjustment be represented? Under what conditions do hypothetical prevention policies have a favourable impact on costs?

2. Literature Review

System dynamics aims to explain behaviour over time through stocks, flows, delays, and feedback mechanisms (Forrester, 1961; Sterman, 2000). Richardson (1991) discusses the conceptual foundations of feedback, while Meadows (2008) emphasises the importance of system structure in shaping behaviour. Causal-loop models alone do not constitute quantitative evidence of causality; quantitative modelling requires an explicit distinction between stocks and flows (Morecroft, 2015; Ford, 2010). Model boundary adequacy and structural validity should be assessed beyond the model's ability to reproduce observed behaviour (Barlas, 1996; Forrester and Senge, 1980).

In quality economics, the prevention–appraisal–failure (PAF) classification constitutes one of the fundamental analytical frameworks (Feigenbaum, 1991; Juran and Godfrey, 1999; Crosby, 1979). Mandal et al. (2002) examined manufacturing quality within a system dynamics framework. Kiani et al. (2009)

developed a feedback model for the components of the cost of quality. Mahanty et al. (2012) investigated deterioration, quality perception, and investment policies from a dynamic perspective. Alglawe et al. (2019) incorporated opportunity costs into the PAF framework within the automotive supply chain. Therefore, the application of system dynamics to quality costs is not novel in itself; rather, the contribution of the present study lies in the integrated assessment of six indicators from the same manufacturing facility within a transparent conditional modelling framework.

Zhu et al. (2020) adopted a data-driven approach to investigate the impact of rework loops on the performance of multistage manufacturing systems. Colledani and Angius (2020) jointly evaluated traceability and rework decisions in relation to manufacturing performance. Oberegger et al. (2020) employed system dynamics to examine the return on quality investments in the electronics industry. Zhou et al. (2022) proposed a dynamic framework for mitigating the effects of rework in engineer-to-order supply chains. Gejo-García et al. (2022) integrated digital twin technology with system dynamics to evaluate manufacturing systems. Although these studies address different manufacturing processes, their model coefficients have not been directly transferred to the cold rolling facility examined in the present study. Krynke (2026) investigated the trade-off between quality and cost using discrete-event simulation, an approach that complements system dynamics.

3. Data and Methodology

Annual values extracted from the charts in the internal report dated 22 January 2026 were used (Anonymous Facility, 2026, pp. 6–7, 9, 11, 13, 15). The rates are expressed in percentage points, while CoQ/Ton is expressed in USD per ton of packaged output. Operational categories may overlap and have different denominators; therefore, the individual rates cannot be aggregated to obtain an overall defect rate.

Table 1. Quality Indıcators

Year	NonPrime	ReWork	2.Quality	Scrap	Claim	CoQ/T
2019	4.774	10.374	1.056	3.730	2.549	186.196
2020	4.536	5.807	0.431	3.202	1.159	97.234
2021	1.947	9.976	0.424	3.910	0.609	84.279
2022	2.207	14.630	0.333	4.445	0.894	99.885
2023	3.149	16.556	0.605	4.961	1.183	105.051
2024	1.723	22.150	0.690	3.990	1.050	72.470
2025	0.856	22.656	0.525	4.028	0.406	75.993

For reasons of commercial confidentiality, the economic analysis is based exclusively on the cost of quality per ton of packaged output. The initial cost components in the policy model are approximate values obtained by scaling the internal cost distribution to a per-ton basis. Absolute annual costs and production volumes have not been disclosed.

$$z_0 = 75{,}993 \text{ USD/ton} \quad (1)$$

3.1. Historical Behaviour Model

For each indicator, the perceived performance stock, x, is expressed in the same units as the observed value, y. The fixed target, m, is defined as the arithmetic mean of the observations over the 2019–2023 training

period. The adjustment coefficient, k, is constrained to the interval [0, 1] and estimated by minimising the four one-step prediction errors over the training period. This coefficient does not represent a structural physical parameter.

$$dx_i/dt = (m_i - x_i)/\tau_i \quad (2)$$

$$x_{i,t+1} = x_{i,t} + k_i(m_i - x_{i,t}) \quad (3)$$

$$k_i = \mathrm{clip}[\ \Sigma(m_i - y_{i,t})(y_{i,t+1} - y_{i,t}) \,/\, \Sigma(m_i - y_{i,t})^2,\ 0,\ 1\ ] \quad (4)$$

The discrete-time equation represents an annual adjustment model, with its continuous-time counterpart established through the relationship $k = 1 - \exp(-1/\tau)$. When $k = 0$, no adjustment occurs, whereas the limiting case of $k = 1$ represents instantaneous adjustment. One-step-ahead forecasts for 2024 and 2025 are evaluated using the actual observed value from the preceding year. The model is benchmarked against the persistence model, defined as $y_{t+1} = y_t$.

3.2. Policy Simulation: Feedback and Delays

In the policy experiment, conducted separately from the historical adjustment model, five operational rates, a prevention effect stock, u, and a cumulative unit cost indicator, B, are employed. The initial prevention cost is set at $p_0 = 18.139378$ USD/ton, the failure cost at $F_0 = 56.662653$ USD/ton, and the unclassified cost difference at $d = 1.190969$ USD/ton. The sum of these components amounts to 75.993 USD/ton.

$$F_t = F_0 \,\Sigma\, w_i\, x_{i,t} / x_{i,0}\ ;\ \ \Sigma w_i = 1 \quad (5)$$

$$p_t = p_0 + s + g\ \max(0, F_t - F^*) \quad (6)$$

$$u_{t+1} = u_t + \lambda[(p_t - p_0)/p_0 - u_t] \quad (7)$$

$$x_{i,t+1} = x_{i,t} + r[x_{i,0}\exp(-e_i u_t) - x_{i,t}] \quad (8)$$

$$z_t = p_t + F_t + d \quad (9)$$

$$B_{t+1} = B_t + z_t \quad (10)$$

The cost feedback mechanism increases the prevention budget when F rises, while the delayed effect of prevention reduces failure rates and alleviates budgetary pressure. This balancing feedback loop represents a hypothesis. The rework rate is assumed to decrease in response to prevention activities; however, it may also increase in the short term owing to intensified inspection activities. The vector $w = (0.15, 0.15, 0.10, 0.55, 0.05)$ represents illustrative economic weights rather than actual accounting cost shares. The parameters $e = (0.8, 0.4, 0.6, 0.3, 0.9)$, $\lambda = 0.5$, $r = 0.5$, and $F^* = 50.387161$ USD/ton are hypothetical.

B represents the cumulative sum of annual unit costs rather than actual total expenditure. The initial conditions are $u = 0$ and $B = 0$. The cost in 2026 reflects the initial intervention, whereas the operational rates represent the state of the system at the beginning of the year. The additional prevention expenditures under scenarios S1 and S2 are 2.290325 and 5.725814 USD/ton, respectively. Under scenario S3, the additional prevention expenditure is 1.145163 USD/ton, and the feedback coefficient is set at 0.35.

4. Findings

Table 2. Findings for calculations

Indicator	2019	2025	Change %	k	Test MAE	Persistence Model MAE
Non-Prime	4.774	0.856	-82.070	0.729	1.793	1.147
Rework	10.374	22.656	118.392	0.428	6.424	3.050
Second Quality	1.056	0.525	-50.284	1.000	0.082	0.125
Scrap	3.730	4.028	7.989	0.303	0.357	0.504
Claim	2.549	0.406	-84.072	0.900	0.535	0.389
CoQ/Ton	186.196	75.993	-59.187	1.000	40.297	18.052

The test based on two observations is diagnostic in nature and does not establish general predictive superiority. In particular, the fixed-mean reversion structure struggles to capture the upward trend in the rework rate. A reduction in quality-related nonconformity rates and a reduction in the cost of quality per ton are distinct phenomena and should not be treated as equivalent.

4.1. Non-Prime

The observed values in 2019 and 2025 were 4.774 and 0.856, respectively, corresponding to an absolute change of −3.918 and a relative change of −82.070%. The training-period mean was 3.323, and the estimated adjustment coefficient was 0.729. The one-step-ahead forecasts for 2024 and 2025 were 3.276 and 2.889, respectively, compared with the observed values of 1.723 and 0.856. The mean absolute error (MAE) of the adjustment model was 1.793, compared with 1.147 for the persistence model.

The decline in the non-prime rate indicates an improvement in final product classification performance. However, it should be monitored in conjunction with the increase in the rework rate. Since multiple events may occur within the same coil, a direct conversion rate between these categories cannot be calculated.

4.2. Rework

The observed values in 2019 and 2025 were 10.374 and 22.656, respectively, corresponding to an absolute change of 12.282 and a relative change of 118.392%. The training-period mean was 11.469, and the estimated adjustment coefficient was 0.428. The one-step-ahead forecasts for 2024 and 2025 were 14.379 and 17.579, respectively, compared with the observed values of 22.150 and 22.656. The mean absolute error (MAE) of the adjustment model was 6.424, compared with 3.050 for the persistence model.

The increase in the rework rate may pose operational risks in terms of capacity utilisation, energy consumption, and cycle time. Coils subjected to successful and unsuccessful rework operations should be tracked separately.

4.3. Second quality

The observed values in 2019 and 2025 were 1.056 and 0.525, respectively, corresponding to an absolute change of −0.531 and a relative change of −50.284%. The training-period mean was 0.570, and the estimated adjustment coefficient was 1.000. The one-step-ahead forecasts for 2024 and 2025 were 0.570 and 0.570, respectively, compared with the observed values of 0.690 and 0.525. The mean absolute error (MAE) of the adjustment model was 0.082, compared with 0.125 for the persistence model.

Although the second-quality product rate has declined relative to its initial level, the fluctuations observed after 2022 do not guarantee sustained improvement. Price discounts and product mix determine the associated economic implications.

4.4. Scrap

The observed values in 2019 and 2025 were 3.730 and 4.028, respectively, corresponding to an absolute change of 0.298 and a relative change of 7.989%. The training-period mean was 4.050, and the estimated adjustment coefficient was 0.303. The one-step-ahead forecasts for 2024 and 2025 were 4.685 and 4.008, respectively, compared with the observed values of 3.990 and 4.028. The mean absolute error (MAE) of the adjustment model was 0.357, compared with 0.504 for the persistence model.

The scrap rate remains above its initial level. The net economic loss cannot be estimated without information on metal recovery value and the underlying causes of scrap generation.

4.5. Claim

The observed values in 2019 and 2025 were 2.549 and 0.406, respectively, corresponding to an absolute change of −2.143 and a relative change of −84.072%. The training-period mean was 1.279, and the estimated adjustment coefficient was 0.900. The one-step-ahead forecasts for 2024 and 2025 were 1.269 and 1.256, respectively, compared with the observed values of 1.050 and 0.406. The mean absolute error (MAE) of the adjustment model was 0.535, compared with 0.389 for the persistence model.

The decline in the claim rate indicates an improvement in customer-related quality performance; however, changes in the sales mix and delays in claim reporting represent alternative explanations.

4.6. CoQ/Ton

The observed values in 2019 and 2025 were 186.196 and 75.993, respectively, corresponding to an absolute change of −110.203 and a relative change of −59.187%. The training-period mean was 114.529, and the estimated adjustment coefficient was 1.000. The one-step-ahead forecasts for 2024 and 2025 were 114.529 and 114.529, respectively, compared with the observed values of 72.470 and 75.993. The mean absolute error (MAE) of the adjustment model was 40.297, compared with 18.052 for the persistence model.

The reduction in the cost of quality per ton indicates an economic improvement normalised by the physical scale of production. However, packaging tonnage and rolling tonnage should not be treated as equivalent.

5. Policy Experiments and Economic Outcomes

Table 3. Economic outcomes for future

Scenario	2030 USD/t	5 years mean USD/t	S0 different USD/t (mean)
S0 Fixed Policy	75.993	75.993	0.000
S1 Increase in Prevention	76.130	77.380	-1.387
S2 High Prevention	76.550	79.540	-3.547
S3 Feedback	75.321	77.610	-1.617

A positive difference represents conditional unit cost savings, whereas a negative difference indicates additional costs. The five-year comparison is based on the arithmetic mean of annual USD/ton values rather than a production-weighted average or total expenditure. During the initial years, prevention expenditure increases immediately, while the quality response occurs with a delay.

S0 – Fixed Policy: The unit cost is 75.993 USD/ton in both the first and final years. In 2030, the scrap and rework rates are 4.028% and 22.656%, respectively. The five-year average unit cost is 75.993 USD/ton. These results are obtained under hypothetical effectiveness assumptions.

S1 – Increase in Prevention Expenditure: The unit cost is 78.283 USD/ton in the first year and 76.130 USD/ton in the final year. In 2030, the scrap and rework rates are 3.925% and 21.885%, respectively. The five-year average unit cost is 77.380 USD/ton. These results are obtained under hypothetical effectiveness assumptions.

S2 – High Prevention Expenditure: The unit cost is 81.719 USD/ton in the first year and 76.550 USD/ton in the final year. In 2030, the scrap and rework rates are 3.776% and 20.786%, respectively. The five-year average unit cost is 79.540 USD/ton. These results are obtained under hypothetical effectiveness assumptions.

S3 – Feedback Policy: The unit cost is 79.335 USD/ton in the first year and 75.321 USD/ton in the final year. In 2030, the scrap and rework rates are 3.885% and 21.589%, respectively. The five-year average unit cost is 77.610 USD/ton. These results are obtained under hypothetical effectiveness assumptions.

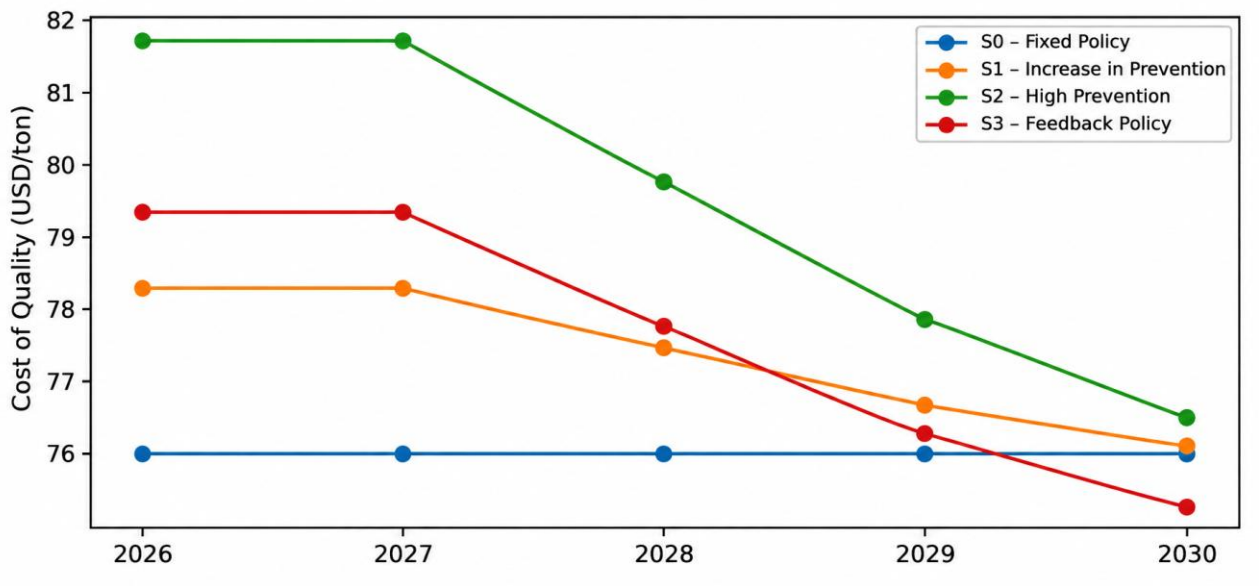

Figure 1. Conditional Policy Simulation (Not Based on Observed Data).

5.1. Effectiveness Sensitivity Analysis

Table 4. Sensitivity Analysis

Effectiveness Multiplier	S1 Average Savings (USD/t)	S2 Average Savings (USD/t)	S3 Average Savings (USD/t)
0.000	-2.290	-5.726	-3.342
0.500	-1.833	-4.603	-2.455
1.000	-1.387	-3.547	-1.617
1.500	-0.952	-2.552	-0.822
2.000	-0.527	-1.614	-0.068

When prevention effectiveness is zero, additional prevention expenditure merely increases costs. As effectiveness increases, failure costs decrease. The results are sensitive to the calibration of model coefficients; the sensitivity ranges should not be interpreted as confidence intervals. A practical approach for management is to conduct a controlled pilot study to quantify the actual relationship between prevention expenditure and its effectiveness.

6. Discussion

Unlike the Markov approach, the present study employs continuous-valued performance stocks, delays, and budgetary feedback mechanisms. However, the historical fitting submodel and the policy model serve different purposes; the k coefficients obtained from the former cannot be interpreted as measures of prevention effectiveness. Within the framework established by Sterman (2000) and Barlas (1996), structural and behavioural validity should be assessed separately. Under the fixed policy, the equilibrium conditions $x = x_0$ and $u = 0$ are maintained; in the zero-effectiveness test, x remains unchanged; and all weights are positive

and sum to unity. These algebraic checks contribute to the internal consistency of the model but do not establish its empirical validity.

Kiani et al. (2009) and Mahanty et al. (2012) represent direct precedents for investigating the delayed economic effects of quality investments. Alglawe et al. (2019) additionally considers opportunity costs. In the present study, lost sales, capacity-related opportunity costs, and customer satisfaction stocks were not incorporated into the model because they were not measured. In light of Zhu et al. (2020) and Zhou et al. (2022), the rework loop could be further extended by incorporating physical material flow data. In particular, data on first-pass yield, the number of repeated rolling operations, and product quality classification after rework should be collected.

7. Conclusion and Managerial Recommendations

The study demonstrates that improvements in final quality indicators may coincide with an increase in the rework burden. This finding does not establish causality but reflects simultaneously observed trends in different directions. Management should not evaluate performance solely on the basis of Non-Prime or Claim rates; scrap, rework, unit costs, and capacity consumption should be monitored within the same report.

The economic justification for prevention investment depends on whether the avoided failure costs exceed the additional expenditure. Owing to delays, the economic effects of increasing the annual budget may not materialise within the first year. Comparing fixed and feedback-based budget policies highlights the trade-off between short-term expenditure increases and long-term quality improvements. Before simulation results are used to inform investment decisions, effectiveness coefficients should be empirically estimated through pilot studies.

Cause-based Pareto analysis is recommended for scrap, while first-pass yield and post-rework product classification should be monitored for rework. For claims, customer records should be matched with the corresponding production dates. Quality costs should be aligned with accounting classifications, actual cost shares should be determined for each category, and double counting of the same event should be avoided. Establishing a monthly time series comprising at least 84 observations would provide a stronger foundation for estimating delay parameters and conducting temporal validation. The model could subsequently be disaggregated by product family, production line, and quality defect category.

The scientific contribution of this study lies in the explicit separation of descriptive analysis derived from actual annual indicators and hypothetical dynamic policy experiments. No claim is made regarding the development of a new system dynamics theory or the first application of this approach in the literature. The limitations of the study include its single-facility scope, short time series, unmeasured causal coefficients, unit-cost scale, potentially overlapping rates, and validation based on only two observations.

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Appendix A. Explicit Mathematical Solution of the Historical Model

Training sample 2019–2023. Objective function $J(k)=\Sigma[b_t-ka_t]^2$. Derivative $-2\Sigma a_t b_t+2k\Sigma a_t^2=0$; second derivative $2\Sigma a_t^2$ is positive. Owing to the boundary constraint, the solution is clipped to the interval [0, 1].

Non-Prime

m = (4.774 + 4.536 + 1.947 + 2.207 + 3.149) / 5 = 3.323

Year	a=m−y	b=y after−y	a×b	a²
2019	-1.451	-0.238	0.345	2.107
2020	-1.213	-2.589	3.141	1.472
2021	1.376	0.260	0.358	1.892
2022	1.116	0.942	1.051	1.245

k = clip(4.895477/6.715740,0,1) = 0.728956.

2024: 3.149 + 0.728956 × (3.322600 − 3.149) = 3.275547. 2025: 1.723 + 0.728956 × (3.322600 − 1.723) = 2.889038.

MAE = (|3.275547−1.723| + |2.889038−0.856|)/2 = 1.792792.

Rework

m = (10.374 + 5.807 + 9.976 + 14.630 + 16.556) / 5 = 11.469

Year	a=m−y	b=y after−y	a×b	a²
2019	1.095	-4.567	-4.999	1.198
2020	5.662	4.169	23.603	32.054
2021	1.493	4.654	6.947	2.228
2022	-3.161	1.926	-6.089	9.994

k = clip(19.461876/45.474168,0,1) = 0.427977.

2024: 16.556 + 0.427977 × (11.468600 − 16.556) = 14.378712. 2025: 22.150 + 0.427977 × (11.468600 − 22.150) = 17.578612.

MAE = (|14.378712−22.150| + |17.578612−22.656|)/2 = 6.424338.

Second quality

m = (1.056 + 0.431 + 0.424 + 0.333 + 0.605) / 5 = 0.570

Year	a=m−y	b=y after−y	a×b	a²
2019	-0.486	-0.625	0.304	0.236
2020	0.139	-0.007	-0.001	0.019
2021	0.146	-0.091	-0.013	0.021
2022	0.237	0.272	0.064	0.056

k = clip(0.354045/0.332988,0,1) = 1.000000.

2024: 0.605 + 1.000000 × (0.569800 − 0.605) = 0.569800. 2025: 0.690 + 1.000000 × (0.569800 − 0.690) = 0.569800.

MAE = (|0.569800−0.690| + |0.569800−0.525|)/2 = 0.082500.

Scrap

m = (3.730 + 3.202 + 3.910 + 4.445 + 4.961) / 5 = 4.050

Year	a=m−y	b=y after−y	a×b	a²
2019	0.320	-0.528	-0.169	0.102
2020	0.848	0.708	0.600	0.718
2021	0.140	0.535	0.075	0.019
2022	-0.395	0.516	-0.204	0.156

k = clip(0.302012/0.996399,0,1) = 0.303103.

2024: 4.961 + 0.303103 × (4.049600 − 4.961) = 4.684752. 2025: 3.990 + 0.303103 × (4.049600 − 3.990) = 4.008065.

MAE = (|4.684752−3.990| + |4.008065−4.028|)/2 = 0.357343.

Claim

m = (2.549 + 1.159 + 0.609 + 0.894 + 1.183) / 5 = 1.279

Year	a=m−y	b=y after−y	a×b	a²
2019	-1.270	-1.390	1.766	1.613
2020	0.120	-0.550	-0.066	0.014
2021	0.670	0.285	0.191	0.449
2022	0.385	0.289	0.111	0.148

k = clip(2.001788/2.224463,0,1) = 0.899897.

2024: 1.183 + 0.899897 × (1.278800 − 1.183) = 1.269210. 2025: 1.050 + 0.899897 × (1.278800 − 1.050) = 1.255896.

MAE = (|1.269210−1.050| + |1.255896−0.406|)/2 = 0.534553.

CoQ/Ton

m = (186.196 + 97.234 + 84.279 + 99.885 + 105.051) / 5 = 114.529

Year	a=m−y	b=y after−y	a×b	a²
2019	-71.667	-88.962	6375.640	5136.159
2020	17.295	-12.955	-224.057	299.117
2021	30.250	15.606	472.082	915.062
2022	14.644	5.166	75.651	214.447

k = clip(6699.315333/6564.785150,0,1) = 1.000000.

2024: 105.051 + 1.000000 × (114.529000 − 105.051) = 114.529000. 2025: 72.470 + 1.000000 × (114.529000 − 72.470) = 114.529000.

MAE = (|114.529000−72.470| + |114.529000−75.993|)/2 = 40.297500.

Appendix B. Initial Economic Value

Initial unit cost of quality in 2025: 75.993 USD per ton of packaged output. Absolute costs and production tonnage have not been disclosed.

Appendix C. Explicit Calculations for the Policy Simulation

F_0, for S1=56.662653; p_0=18.139378; additional prevention expenditure =2.290325; p=20.429703; z=p+F+d=78.283325 USD/ton. After u=0,5×(2.290325/18.139378)=0,063131. At the first step, x remains unchanged because u = 0.

S0 Fixed Policy

Year	NonPrime	ReWork	2.Quality	Scrap	Claim	u
2026	0.856	22.656	0.525	4.028	0.406	0.000
2027	0.856	22.656	0.525	4.028	0.406	0.000
2028	0.856	22.656	0.525	4.028	0.406	0.000
2029	0.856	22.656	0.525	4.028	0.406	0.000
2030	0.856	22.656	0.525	4.028	0.406	0.000

Year	F USD/t	p USD/t	USD/t	B (total unit cost)
2026	56.663	18.139	75.993	75.993
2027	56.663	18.139	75.993	151.986
2028	56.663	18.139	75.993	227.979
2029	56.663	18.139	75.993	303.972
2030	56.663	18.139	75.993	379.965

2026: F=56.662653 × (0.15×0.856000/0.856 + 0.15×22.656000/22.656 + 0.10×0.525000/0.525 + 0.55×4.028000/4.028 + 0.05×0.406000/0.406) = 56.662653;
z=18.139378+56.662653+1.190969=75.993000 USD/ton. B=75.993000.

Following Year:

Non-Prime=0.856000+0,5×(0.856×exp(−0.8×0.000000)−0.856000)=0.856000;
Rework=22.656000+0,5×(22.656×exp(−0.4×0.000000)−22.656000)=22.656000;
Second Quality=0.525000+0,5×(0.525×exp(−0.6×0.000000)−0.525000)=0.525000;
Scrap=4.028000+0,5×(4.028×exp(−0.3×0.000000)−4.028000)=4.028000;
Claim=0.406000+0,5×(0.406×exp(−0.9×0.000000)−0.406000)=0.406000;
u=0.000000+0,5×((18.139378−18.139378)/18.139378−0.000000)=0.000000.

2027: F=56.662653 × (0.15×0.856000/0.856 + 0.15×22.656000/22.656 + 0.10×0.525000/0.525 + 0.55×4.028000/4.028 + 0.05×0.406000/0.406) = 56.662653;
z=18.139378+56.662653+1.190969=75.993000 USD/ton. B=151.986000.

Following Year:

Non-Prime=0.856000+0,5×(0.856×exp(−0.8×0.000000)−0.856000)=0.856000;
Rework=22.656000+0,5×(22.656×exp(−0.4×0.000000)−22.656000)=22.656000;
Second Quality=0.525000+0,5×(0.525×exp(−0.6×0.000000)−0.525000)=0.525000;
Scrap=4.028000+0,5×(4.028×exp(−0.3×0.000000)−4.028000)=4.028000;

Claim=0.406000+0,5×(0.406×exp(−0.9×0.000000)−0.406000)=0.406000;
u=0.000000+0,5×((18.139378−18.139378)/18.139378−0.000000)=0.000000.

2028: F=56.662653 × (0.15×0.856000/0.856 + 0.15×22.656000/22.656 + 0.10×0.525000/0.525 + 0.55×4.028000/4.028 + 0.05×0.406000/0.406) = 56.662653;
z=18.139378+56.662653+1.190969=75.993000 USD/ton. B=227.979000.

Following Year:

Non-Prime=0.856000+0,5×(0.856×exp(−0.8×0.000000)−0.856000)=0.856000;
Rework=22.656000+0,5×(22.656×exp(−0.4×0.000000)−22.656000)=22.656000;
Second Quality=0.525000+0,5×(0.525×exp(−0.6×0.000000)−0.525000)=0.525000;
Scrap=4.028000+0,5×(4.028×exp(−0.3×0.000000)−4.028000)=4.028000;
Claim=0.406000+0,5×(0.406×exp(−0.9×0.000000)−0.406000)=0.406000;
u=0.000000+0,5×((18.139378−18.139378)/18.139378−0.000000)=0.000000.

2029: F=56.662653 × (0.15×0.856000/0.856 + 0.15×22.656000/22.656 + 0.10×0.525000/0.525 + 0.55×4.028000/4.028 + 0.05×0.406000/0.406) = 56.662653;
z=18.139378+56.662653+1.190969=75.993000 USD/ton. B=303.972000.

Following Year:

Non-Prime=0.856000+0,5×(0.856×exp(−0.8×0.000000)−0.856000)=0.856000;
Rework=22.656000+0,5×(22.656×exp(−0.4×0.000000)−22.656000)=22.656000;
Second Quality=0.525000+0,5×(0.525×exp(−0.6×0.000000)−0.525000)=0.525000;
Scrap=4.028000+0,5×(4.028×exp(−0.3×0.000000)−4.028000)=4.028000;
Claim=0.406000+0,5×(0.406×exp(−0.9×0.000000)−0.406000)=0.406000;
u=0.000000+0,5×((18.139378−18.139378)/18.139378−0.000000)=0.000000.

2030: F=56.662653 × (0.15×0.856000/0.856 + 0.15×22.656000/22.656 + 0.10×0.525000/0.525 + 0.55×4.028000/4.028 + 0.05×0.406000/0.406) = 56.662653;
z=18.139378+56.662653+1.190969=75.993000 USD/ton. B=379.965000.

S1 Increase in Prevention Expenditure

Year	NonPrime	ReWork	2.Quality	Second	Claim	u
2026	0.856	22.656	0.525	4.028	0.406	0.000
2027	0.856	22.656	0.525	4.028	0.406	0.063
2028	0.835	22.374	0.515	3.990	0.395	0.095
2029	0.814	22.094	0.506	3.953	0.384	0.110
2030	0.799	21.885	0.498	3.925	0.376	0.118

Year	F USD/t	p USD/t	USD/t	B (total unit cost)
2026	56.663	20.430	78.283	78.283
2027	56.663	20.430	78.283	156.567
2028	55.871	20.430	77.492	234.059
2029	55.090	20.430	76.711	310.770

2030	54.510	20.430	76.130	386.900

2026: F=56.662653 × (0.15×0.856000/0.856 + 0.15×22.656000/22.656 + 0.10×0.525000/0.525 + 0.55×4.028000/4.028 + 0.05×0.406000/0.406) = 56.662653;
z=20.429703+56.662653+1.190969=78.283325 USD/ton. B=78.283325.

Following Year:

Non-Prime=0.856000+0,5×(0.856×exp(−0.8×0.000000)−0.856000)=0.856000;
Rework=22.656000+0,5×(22.656×exp(−0.4×0.000000)−22.656000)=22.656000;
Second Quality=0.525000+0,5×(0.525×exp(−0.6×0.000000)−0.525000)=0.525000;
Scrap=4.028000+0,5×(4.028×exp(−0.3×0.000000)−4.028000)=4.028000;
Claim=0.406000+0,5×(0.406×exp(−0.9×0.000000)−0.406000)=0.406000;
u=0.000000+0,5×((20.429703−18.139378)/18.139378−0.000000)=0.063131.

2027: F=56.662653 × (0.15×0.856000/0.856 + 0.15×22.656000/22.656 + 0.10×0.525000/0.525 + 0.55×4.028000/4.028 + 0.05×0.406000/0.406) = 56.662653;
z=20.429703+56.662653+1.190969=78.283325 USD/ton. B=156.566651.

Following Year:

Non-Prime=0.856000+0,5×(0.856×exp(−0.8×0.063131)−0.856000)=0.834921;
Rework=22.656000+0,5×(22.656×exp(−0.4×0.063131)−22.656000)=22.373521;
Second Quality=0.525000+0,5×(0.525×exp(−0.6×0.063131)−0.525000)=0.515243;
Scrap=4.028000+0,5×(4.028×exp(−0.3×0.063131)−4.028000)=3.990215;
Claim=0.406000+0,5×(0.406×exp(−0.9×0.063131)−0.406000)=0.394787;
u=0.063131+0,5×((20.429703−18.139378)/18.139378−0.063131)=0.094697.

2028: F=56.662653 × (0.15×0.834921/0.856 + 0.15×22.373521/22.656 + 0.10×0.515243/0.525 + 0.55×3.990215/4.028 + 0.05×0.394787/0.406) = 55.871487;
z=20.429703+55.871487+1.190969=77.492160 USD/ton. B=234.058811.

Following Year:

Non-Prime=0.834921+0,5×(0.856×exp(−0.8×0.094697)−0.834921)=0.814234;
Rework=22.373521+0,5×(22.656×exp(−0.4×0.094697)−22.373521)=22.093695;
Second Quality=0.515243+0,5×(0.525×exp(−0.6×0.094697)−0.515243)=0.505622;
Scrap=3.990215+0,5×(4.028×exp(−0.3×0.094697)−3.990215)=3.952697;
Claim=0.394787+0,5×(0.406×exp(−0.9×0.094697)−0.394787)=0.383809;
u=0.094697+0,5×((20.429703−18.139378)/18.139378−0.094697)=0.110480.

2029: F=56.662653 × (0.15×0.814234/0.856 + 0.15×22.093695/22.656 + 0.10×0.505622/0.525 + 0.55×3.952697/4.028 + 0.05×0.383809/0.406) = 55.090391;
z=20.429703+55.090391+1.190969=76.711063 USD/ton. B=310.769874.

Following Year:

Non-Prime=0.814234+0,5×(0.856×exp(−0.8×0.110480)−0.814234)=0.798912;
Rework=22.093695+0,5×(22.656×exp(−0.4×0.110480)−22.093695)=21.885142;
Second Quality=0.505622+0,5×(0.525×exp(−0.6×0.110480)−0.505622)=0.498475;
Scrap=3.952697+0,5×(4.028×exp(−0.3×0.110480)−3.952697)=3.924691;
Claim=0.383809+0,5×(0.406×exp(−0.9×0.110480)−0.383809)=0.375691;
u=0.110480+0,5×((20.429703−18.139378)/18.139378−0.110480)=0.118371.

2030: F=56.662653 × (0.15×0.798912/0.856 + 0.15×21.885142/22.656 + 0.10×0.498475/0.525 + 0.55×3.924691/4.028 + 0.05×0.375691/0.406) = 54.509544;
z=20.429703+54.509544+1.190969=76.130217 USD/ton. B=386.900091.

S2 High Prevention Expenditure

Year	NonPrime	ReWork	2.Quality	Second	Claim	u
2026	0.856	22.656	0.525	4.028	0.406	0.000
2027	0.856	22.656	0.525	4.028	0.406	0.158
2028	0.805	21.963	0.501	3.935	0.379	0.237
2029	0.757	21.286	0.478	3.843	0.354	0.276
2030	0.722	20.786	0.462	3.776	0.335	0.296

Year	F USD/t	p USD/t	USD/t	B (total unit cost)
2026	56.663	23.865	81.719	81.719
2027	56.663	23.865	81.719	163.438
2028	54.734	23.865	79.791	243.228
2029	52.866	23.865	77.922	321.150
2030	51.494	23.865	76.550	397.700

2026: F=56.662653 × (0.15×0.856000/0.856 + 0.15×22.656000/22.656 + 0.10×0.525000/0.525 + 0.55×4.028000/4.028 + 0.05×0.406000/0.406) = 56.662653;
z=23.865192+56.662653+1.190969=81.718814 USD/ton. B=81.718814.

Following Year:

Non-Prime=0.856000+0,5×(0.856×exp(−0.8×0.000000)−0.856000)=0.856000;
Rework=22.656000+0,5×(22.656×exp(−0.4×0.000000)−22.656000)=22.656000;
Second Quality=0.525000+0,5×(0.525×exp(−0.6×0.000000)−0.525000)=0.525000;
Scrap=4.028000+0,5×(4.028×exp(−0.3×0.000000)−4.028000)=4.028000;
Claim=0.406000+0,5×(0.406×exp(−0.9×0.000000)−0.406000)=0.406000;
u=0.000000+0,5×((23.865192−18.139378)/18.139378−0.000000)=0.157828.

2027: F=56.662653 × (0.15×0.856000/0.856 + 0.15×22.656000/22.656 + 0.10×0.525000/0.525 + 0.55×4.028000/4.028 + 0.05×0.406000/0.406) = 56.662653;
z=23.865192+56.662653+1.190969=81.718814 USD/ton. B=163.437627.

Following Year:

Non-Prime=0.856000+0,5×(0.856×exp(−0.8×0.157828)−0.856000)=0.805232;
Rework=22.656000+0,5×(22.656×exp(−0.4×0.157828)−22.656000)=21.962955;
Second Quality=0.525000+0,5×(0.525×exp(−0.6×0.157828)−0.525000)=0.501283;
Scrap=4.028000+0,5×(4.028×exp(−0.3×0.157828)−4.028000)=3.934863;
Claim=0.406000+0,5×(0.406×exp(−0.9×0.157828)−0.406000)=0.379119;
u=0.157828+0,5×((23.865192−18.139378)/18.139378−0.157828)=0.236742.

2028: F=56.662653 × (0.15×0.805232/0.856 + 0.15×21.962955/22.656 + 0.10×0.501283/0.525 + 0.55×3.934863/4.028 + 0.05×0.379119/0.406) = 54.734414;
z=23.865192+54.734414+1.190969=79.790575 USD/ton. B=243.228203.

Following Year:

Non-Prime=0.805232+0,5×(0.856×exp(−0.8×0.236742)−0.805232)=0.756769;
Rework=21.962955+0,5×(22.656×exp(−0.4×0.236742)−21.962955)=21.285976;
Second Quality=0.501283+0,5×(0.525×exp(−0.6×0.236742)−0.501283)=0.478382;
Scrap=3.934863+0,5×(4.028×exp(−0.3×0.236742)−3.934863)=3.843353;
Claim=0.379119+0,5×(0.406×exp(−0.9×0.236742)−0.379119)=0.353604;
u=0.236742+0,5×((23.865192−18.139378)/18.139378−0.236742)=0.276199.

2029: F=56.662653 × (0.15×0.756769/0.856 + 0.15×21.285976/22.656 + 0.10×0.478382/0.525 + 0.55×3.843353/4.028 + 0.05×0.353604/0.406) = 52.866024;
z=23.865192+52.866024+1.190969=77.922185 USD/ton. B=321.150388.

Following Year:

Non-Prime=0.756769+0,5×(0.856×exp(−0.8×0.276199)−0.756769)=0.721533;
Rework=21.285976+0,5×(22.656×exp(−0.4×0.276199)−21.285976)=20.786129;
Second Quality=0.478382+0,5×(0.525×exp(−0.6×0.276199)−0.478382)=0.461603;
Scrap=3.843353+0,5×(4.028×exp(−0.3×0.276199)−3.843353)=3.775523;
Claim=0.353604+0,5×(0.406×exp(−0.9×0.276199)−0.353604)=0.335123;
u=0.276199+0,5×((23.865192−18.139378)/18.139378−0.276199)=0.295928.

2030: F=56.662653 × (0.15×0.721533/0.856 + 0.15×20.786129/22.656 + 0.10×0.461603/0.525 + 0.55×3.775523/4.028 + 0.05×0.335123/0.406) = 51.493794;
z=23.865192+51.493794+1.190969=76.549955 USD/ton. B=397.700343.

S3 Feedback Policy

Year	NonPrime	ReWork	2.Quality	Second	Claim	u
2026	0.856	22.656	0.525	4.028	0.406	0.000
2027	0.856	22.656	0.525	4.028	0.406	0.092
2028	0.826	22.246	0.511	3.973	0.390	0.138
2029	0.796	21.842	0.497	3.919	0.374	0.150
2030	0.778	21.589	0.488	3.885	0.364	0.145

Year	F USD/t	p USD/t	USD/t	B (total unit cost)
2026	56.663	21.481	79.335	79.335
2027	56.663	21.481	79.335	158.669
2028	55.517	21.080	77.788	236.458
2029	54.393	20.687	76.271	312.728
2030	53.690	20.440	75.321	388.050

2026: F=56.662653 × (0.15×0.856000/0.856 + 0.15×22.656000/22.656 + 0.10×0.525000/0.525 + 0.55×4.028000/4.028 + 0.05×0.406000/0.406) = 56.662653;
z=21.480963+56.662653+1.190969=79.334585 USD/ton. B=79.334585.

Following Year:

Non-Prime=0.856000+0,5×(0.856×exp(−0.8×0.000000)−0.856000)=0.856000;
Rework=22.656000+0,5×(22.656×exp(−0.4×0.000000)−22.656000)=22.656000;
Second Quality=0.525000+0,5×(0.525×exp(−0.6×0.000000)−0.525000)=0.525000;
Scrap=4.028000+0,5×(4.028×exp(−0.3×0.000000)−4.028000)=4.028000;
Claim=0.406000+0,5×(0.406×exp(−0.9×0.000000)−0.406000)=0.406000;
u=0.000000+0,5×((21.480963−18.139378)/18.139378−0.000000)=0.092109.

2027: F=56.662653 × (0.15×0.856000/0.856 + 0.15×22.656000/22.656 + 0.10×0.525000/0.525 + 0.55×4.028000/4.028 + 0.05×0.406000/0.406) = 56.662653;
z=21.480963+56.662653+1.190969=79.334585 USD/ton. B=158.669170.

Following Year:

Non-Prime=0.856000+0,5×(0.856×exp(−0.8×0.092109)−0.856000)=0.825596;
Rework=22.656000+0,5×(22.656×exp(−0.4×0.092109)−22.656000)=22.246233;
Second Quality=0.525000+0,5×(0.525×exp(−0.6×0.092109)−0.525000)=0.510886;
Scrap=4.028000+0,5×(4.028×exp(−0.3×0.092109)−4.028000)=3.973110;
Claim=0.406000+0,5×(0.406×exp(−0.9×0.092109)−0.406000)=0.389850;
u=0.092109+0,5×((21.480963−18.139378)/18.139378−0.092109)=0.138163.

2028: F=56.662653 × (0.15×0.825596/0.856 + 0.15×22.246233/22.656 + 0.10×0.510886/0.525 + 0.55×3.973110/4.028 + 0.05×0.389850/0.406) = 55.517338;
z=21.080103+55.517338+1.190969=77.788410 USD/ton. B=236.457580.

Following Year:

Non-Prime=0.825596+0,5×(0.856×exp(−0.8×0.138163)−0.825596)=0.796012;
Rework=22.246233+0,5×(22.656×exp(−0.4×0.138163)−22.246233)=21.842058;
Second Quality=0.510886+0,5×(0.525×exp(−0.6×0.138163)−0.510886)=0.497060;
Scrap=3.973110+0,5×(4.028×exp(−0.3×0.138163)−3.973110)=3.918783;
Claim=0.389850+0,5×(0.406×exp(−0.9×0.138163)−0.389850)=0.374189;
u=0.138163+0,5×((21.080103−18.139378)/18.139378−0.138163)=0.150141.

2029: F=56.662653 × (0.15×0.796012/0.856 + 0.15×21.842058/22.656 + 0.10×0.497060/0.525 + 0.55×3.918783/4.028 + 0.05×0.374189/0.406) = 54.393129;
z=20.686629+54.393129+1.190969=76.270727 USD/ton. B=312.728307.

Following Year:

Non-Prime=0.796012+0,5×(0.856×exp(−0.8×0.150141)−0.796012)=0.777565;
Rework=21.842058+0,5×(22.656×exp(−0.4×0.150141)−21.842058)=21.588738;
Second Quality=0.497060+0,5×(0.525×exp(−0.6×0.150141)−0.497060)=0.488417;
Scrap=3.918783+0,5×(4.028×exp(−0.3×0.150141)−3.918783)=3.884689;
Claim=0.374189+0,5×(0.406×exp(−0.9×0.150141)−0.374189)=0.364436;
u=0.150141+0,5×((20.686629−18.139378)/18.139378−0.150141)=0.145284.

2030: F=56.662653 × (0.15×0.777565/0.856 + 0.15×21.588738/22.656 + 0.10×0.488417/0.525 + 0.55×3.884689/4.028 + 0.05×0.364436/0.406) = 53.689810;
z=20.440468+53.689810+1.190969=75.321247 USD/ton. B=388.049554.