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Contextual assessment of customer trust and loyalty on E-commerce platforms based on Z-numbers

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Research Article

Keywords: E-commerce, customer loyalty, customer trust, Z-numbers, decision support systems

Posted Date: November 24th, 2026

DOI: https://doi.org/10.21203/rs.3.rs-11837015/v1

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Additional Declarations: The authors declare potential competing interests as follows: Ramiz Alekperov is employed by the company that owns or operates Aptekonline.az, one of the platforms evaluated in this study. He worked as a consultant on the platform until 2025, including during the data collection period (2023–2024), and has had no involvement with the platform since then. To limit potential bias, the platform data were collected from publicly accessible content, the linguistic assessments were provided by independent experts who participated anonymously, and the results are cross-checked against alternative decision methods (Table 13). In addition, all Aptekonline.az inputs and calculations were independently re-checked by the co-author Shahin Dadashov, who has no employment, consulting or financial relationship with Aptekonline.az or its operating company. The authors have no other competing interests to declare.

Contextual assessment of customer trust and loyalty on E-commerce platforms based on Z-numbers

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Abstract. Customer loyalty on e-commerce platforms depends on trust, yet existing assessment methods rarely account for the reliability of expert judgments. This paper proposes the Customer Trust–Loyalty Reliability Index (CTLRI), a decision-support framework based on Z-numbers. Trust and loyalty indicators are aggregated with Aliev Z-number arithmetic, which keeps the assessment and reliability components separate. Platforms are ranked by their Z-distance to a fixed universal ideal profile and classified into three risk levels, and the distance is decomposed into assessment and reliability deficiencies. A Z-SWOT module offers a diagnostic interpretation of strategic position without altering the ranking. Applied to seven pharmaceutical e-commerce platforms from Azerbaijan and international markets, the framework ranks Aptekonline.az first, followed by Amazon. Comparison with TOPSIS and VIKOR variants yields a consistent ranking, while CTLRI adds platform-specific managerial diagnostics.

Keywords: E-commerce; customer loyalty; customer trust; Z-numbers; decision support systems

1 Introduction

The rapid development of information and communication technologies over the past decade has transformed e-commerce into one of the fastest-growing sectors of the global economy. The widespread expansion of Internet infrastructure, advances in mobile technologies, the evolution of digital payment systems, and continuous improvements in logistics services have fundamentally changed consumer purchasing behavior. Modern consumers increasingly perform the entire purchasing process—from information search and product comparison to selection and payment—through electronic platforms. This digital transformation has not only created new business opportunities but has also intensified competition among e-commerce platforms seeking to attract and retain customers in an increasingly competitive marketplace [1,2].

In such a competitive environment, the sustainable growth of e-commerce enterprises depends not only on acquiring new customers but also on maintaining long-term relationships with existing ones. Consequently, customer loyalty has become one of the most important strategic indicators of platform competitiveness, profitability, and sustainable development. Previous studies have consistently demonstrated that retaining existing customers is considerably less costly than acquiring new ones and that increased customer loyalty directly contributes to higher sales, profitability, customer lifetime value, and market share [1,3–6]. Therefore, developing reliable and objective methods for evaluating customer loyalty has become an important research topic from both theoretical and practical perspectives.

Customer loyalty is no longer regarded as a simple indicator of repeated purchasing behavior. Instead, it represents a multidimensional construct influenced by numerous interrelated factors, including customer trust, satisfaction, perceived service quality, product perception, recommendation intention, and long-term customer–platform relationships — relationships extensively documented across the loyalty, trust, and e-commerce literature. In the e-commerce environment, the evaluation of loyalty becomes even more challenging because customer opinions are inherently uncertain, originate from heterogeneous information sources, and often exhibit varying degrees of reliability.

To address this problem, a wide variety of analytical approaches have been proposed. These methodological streams — reviewed in detail in the subsections below — collectively represent the landscape of customer loyalty assessment. Although statistical and machine learning techniques perform effectively when processing large-scale datasets, they generally do not explicitly represent assessment reliability as an independent modeling component, and rarely account for the varying confidence levels associated with different sources of customer information. Similarly, conventional fuzzy-based approaches successfully represent uncertainty but typically ignore the reliability associated with expert judgments or customer evaluations. Consequently, existing approaches remain limited in their ability to simultaneously represent uncertainty and the reliability of assessments within real-world e-commerce decision environments — as demonstrated across foundational fuzzy set theory, fuzzy decision-making, and early Z-number studies.

The introduction of the Z-number concept by Zadeh provides an effective mathematical framework for overcoming these limitations [53]. A Z-number consists of two complementary components: the first represents the uncertain value of an assessment, while the second quantifies the reliability of that assessment. By simultaneously modeling uncertainty and reliability within a unified framework, Z-numbers provide richer descriptive capabilities than conventional fuzzy models and have become increasingly attractive for solving complex decision-making problems under uncertainty — as demonstrated across the growing body of Z-number theory and its extensions.

However, a systematic review of the existing literature reveals an important research gap. Current Z-number-based studies mainly focus on multi-criteria decision-making, risk analysis, expert systems, and uncertainty modeling — as demonstrated across the growing body of Z-number theory and its extensions, whereas studies in e-commerce primarily emphasize customer behavior prediction, recommendation systems, customer segmentation, and marketing analytics — encompassing e-loyalty models, data mining, RFM analysis, and recommender systems. To the best of our knowledge, no existing study simultaneously **(i)** models customer trust and customer loyalty within a unified Z-number framework, **(ii)** explicitly incorporates the reliability of assessments throughout the evaluation process, **(iii)** integrates the strategic position of an e-commerce platform using a Z-SWOT model, and **(iv)** constructs a universal ideal customer loyalty profile for distance-based evaluation. Addressing this gap provides the primary motivation for the proposed CTLRI framework and defines the boundaries of its methodological contribution.

To address these limitations, this paper proposes a **novel contextual decision-making framework** for customer trust and loyalty assessment based on Z-number

theory. In the proposed framework, customer trust and customer loyalty are first represented as independent Z-numbers and subsequently integrated into a unified trust–loyalty indicator. Next, the strategic position of the platform is evaluated using a Z-SWOT model, and the resulting strategic information is incorporated into the decision-making process. Finally, a universal ideal loyalty profile is fixed for cross-platform comparability, and the actual platform performance is evaluated by measuring its Z-distance from this ideal, with Z-SWOT providing context-aware diagnostic interpretation of the observed gaps.

The main scientific contributions of this study can be summarized as follows:

1. A reliability-preserving weighting mechanism: criterion weights are applied exclusively to the assessment component (A), while the reliability component (B) is propagated through Aliev Z-number arithmetic without semantic modification — preserving expert confidence throughout the entire computational pipeline.
2. A unified Z-number integration of customer trust and loyalty into a single composite trust–loyalty index (CTLRI), maintaining the A/B semantic distinction at every aggregation stage.
3. A formal Z-SWOT strategic diagnostics module — to the best of our knowledge, the first formal Z-number-based extension of SWOT for customer loyalty evaluation — in which each strategic dimension is represented as a complete Z-number (A, B), enabling Z-dominance-based strategic region assignment (Regions I, II, II*).
4. A Z-distance decomposition into assessment deficiency (d_A) and reliability deficiency (d_B) components, enabling platform-specific diagnostic guidance that remains invisible under conventional single-score evaluation methods.
5. A fixed universal ideal profile and three-tier CTLRI risk classification (LOW RISK / HIGH RESILIENCE, MODERATE RISK, HIGH RISK) for transparent, cross-platform comparability under uniform evaluation criteria.

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature. Section 3 presents the mathematical foundations of the proposed methodology. Section 4 demonstrates the application of the proposed framework through a case study. Section 5 discusses the obtained results, while Section 6 concludes the paper and outlines directions for future research.

2. Literature Review

Customer loyalty has long been recognized as one of the central research topics in marketing, consumer behavior, and decision science. Over the past several decades, numerous theoretical and empirical studies have investigated the factors influencing customer loyalty, resulting in the development of diverse conceptual frameworks and quantitative models for its measurement, analysis, and management. In the literature, customer loyalty is commonly characterized through multiple dimensions, including brand commitment, repeat purchase behavior, customer satisfaction, recommendation intention, perceived customer value, and the

sustainability of long-term customer–company relationships. With the rapid expansion of the digital economy, customer loyalty in e-commerce has emerged as a distinct research domain, where analytical approaches based on e-loyalty, electronic service quality, personalized marketing, data analytics, and artificial intelligence have become increasingly prominent.

Despite these advances, existing approaches differ substantially in their objectives, methodological foundations, and capabilities for handling uncertain information. Traditional behavioral and marketing-oriented studies primarily focus on explaining the psychological and relational determinants of customer loyalty, whereas statistical models, machine learning techniques, and multi-criteria decision-making methods emphasize prediction, classification, or ranking. Although these approaches have achieved considerable success within their respective domains, most of them either assume that the available information is fully reliable or treat uncertainty only implicitly. Consequently, they generally fail to model the reliability of customer assessments and expert judgments explicitly, provide limited support for representing heterogeneous uncertain information, and lack an integrated decision-making framework capable of simultaneously considering behavioral, reliability, and strategic aspects of customer loyalty.

Therefore, a comprehensive comparison of the existing methodological directions is essential for identifying their respective strengths, limitations, and unresolved research challenges. Such an analysis not only provides the scientific foundation for the proposed methodology but also clarifies the research gap addressed in this study. Accordingly, **Table 1** presents a comparative review of the principal methodological approaches to customer loyalty assessment with respect to their analytical characteristics, loyalty evaluation capability, uncertainty modeling capacity, treatment of assessment reliability, and major methodological limitations.

Table 1. Comparison of the main methodological areas of customer loyalty assessment

Literature category	Key references	Methodological approach	Direct loyalty assessment	Uncertainty modeling	Main methodological limitation
Classical loyalty theories	[3], [4], [5], [6], [15]	Attitudinal and behavioral loyalty models	✓	✕	Provide conceptual explanations but no mathematical decision-making framework
Marketing indicators	[2], [7], [8], [9], [10]	NPS, customer satisfaction, and loyalty indicators	✓	✕	Do not capture the multidimensional nature of customer loyalty

Customer value and segmentation models	[11]–[16]	RFM and CLV analysis	✓	✕	Ignore uncertainty, assessment reliability, and strategic context
E-commerce loyalty models	[17]–[26]	E-loyalty and e-service quality models	✓	✕	Limited capability for evaluating customer loyalty under uncertainty
Data mining and machine learning approaches	[27]–[35]	Classification and prediction models	✓	Partial	Do not explicitly represent assessment reliability or strategic factors
Behavioral economics and digital nudging	[36]–[44]	Nudging and choice architecture	✕	✕	Intended for influencing customer behavior rather than measuring loyalty
Fuzzy decision-making models	[45]–[52]	Fuzzy MCDM and fuzzy inference systems	Partial	✓	Assessment reliability is not represented as an independent component
Z-number-based decision models	[53]–[62]	Z-information and Z-number decision models	Partial	✓	No integrated framework for customer loyalty evaluation in e-commerce
Strategic analysis methods	[63]–[70]	SWOT, DEMATEL, and ISM	✕	Partial	Strategic analysis is not linked to quantitative loyalty assessment
Explainable AI approaches	[71]–[75]	Model interpretation and explanation techniques	✕	Partial	Improve interpretability but do not provide a loyalty evaluation model

Collectively, these methodological streams reveal that no existing approach simultaneously addresses uncertainty, reliability, and strategic business context within

a unified decision-support framework — the precise gap that the proposed CTLRI methodology is designed to fill.

A comparative analysis of **Table 1** indicates that research on customer loyalty assessment has evolved along three major methodological directions. The first direction comprises classical loyalty theories, marketing metrics, customer value estimation models, and e-commerce loyalty approaches — spanning classical loyalty theory, marketing metrics, and e-commerce loyalty models. These studies have established the theoretical foundations of customer loyalty and have significantly contributed to its practical measurement. However, they generally rely on deterministic evaluation mechanisms and do not provide a formal representation of uncertainty within the decision-making process.

The second direction encompasses data analytics, data mining, machine learning, and behavioral economics approaches — spanning data mining, machine learning, and digital nudging approaches. These methods have demonstrated high predictive accuracy when analyzing large-scale customer data and have substantially improved customer segmentation, loyalty prediction, and personalized marketing strategies. Nevertheless, they exhibit several important limitations, including the lack of explicit modeling of assessment reliability, limited integration of expert knowledge, and insufficient interpretability of decision outcomes. Furthermore, strategic business factors are rarely incorporated into these analytical frameworks.

The data mining and machine learning methods employed for this purpose include: Aggarwal [28], who provided a comprehensive textbook on data mining techniques; Wedel and Kamakura [29], who developed foundational market segmentation methodology; Christy et al. [30], Anitha and Patil [31], and Rahim et al. [32], who proposed RFM-based customer segmentation approaches; Smaili and Hachimi [33], who extended RFM with the RFM-D model; and Schafer, Konstan, and Riedl [34], who surveyed collaborative filtering and recommendation algorithms for e-commerce applications.

The third direction includes fuzzy multi-criteria decision-making methods, Z-number-based models, and strategic decision-support approaches — encompassing fuzzy MCDM, Z-number theory, strategic analysis methods, and XAI. Fuzzy models provide effective mechanisms for representing uncertainty and processing linguistic information; however, they do not explicitly model the reliability associated with assessments. In contrast, Z-number-based approaches simultaneously represent uncertainty and assessment reliability within a unified mathematical framework. Despite this important advantage, relatively few studies have investigated their application to customer loyalty assessment in e-commerce or their integration with strategic decision-making. Likewise, strategic analysis methods, such as SWOT, DEMATEL, ISM, and Explainable Artificial Intelligence (XAI), offer valuable support for strategic reasoning and result interpretation but do not provide a unified mathematical framework capable of simultaneously evaluating customer loyalty, uncertainty, assessment reliability, and strategic platform characteristics.

Overall, the comparative analysis demonstrates that each methodological direction addresses only specific aspects of customer loyalty assessment. Consequently, there remains a need for an integrated decision-making framework capable of jointly modeling customer loyalty, uncertainty, assessment reliability, and strategic business

context. This identified research gap provides the scientific motivation for the methodology proposed in this study.

2.1. Classical Loyalty Theory

Furthermore, the commitment–trust theory of Morgan and Hunt [9] — which identifies trust and commitment as the central mediating variables in relationship marketing — and the empirical link between customer satisfaction, market share, and profitability established by Anderson, Fornell, and Lehmann [10] provide essential theoretical grounding for understanding why loyal customers generate sustained competitive advantage. Gupta and Lehmann [7] and Kumar and Reinartz [8] further extended this perspective by demonstrating that customers can be managed as long-term strategic investments whose lifetime value is calculable and actionable.

The foundations of customer loyalty theory were established through pioneering research in marketing and consumer behavior. These studies demonstrated that customer loyalty is determined not only by repeated purchasing behavior but also by customers' attitudes toward a brand, psychological commitment, and long-term behavioral intentions. Dick and Basu [3] described loyalty as the interaction between a customer's relative attitude and actual purchasing behavior, whereas Oliver [4] proposed a sequential model of loyalty development consisting of cognitive, affective, conative, and behavioral stages. Subsequent studies further refined these concepts and confirmed that customer loyalty represents one of the fundamental determinants of long-term marketing success [5], [6], [15]. These classical theories continue to provide the conceptual foundation for contemporary customer loyalty research.

From an economic perspective, Reichheld and Sasser [1] demonstrated that long-term customer retention has a direct positive impact on business profitability. Subsequently, Reichheld [2] introduced the Net Promoter Score (NPS), which evaluates customer loyalty according to customers' willingness to recommend a product or service to others. Owing to its simplicity and practical applicability, NPS has become one of the most widely used customer loyalty indicators in both academic research and business practice.

Despite their significant theoretical and managerial contributions, classical loyalty theories and marketing indicators exhibit several important limitations. Most of these approaches provide conceptual explanations rather than mathematical decision-making models. Furthermore, they do not explicitly represent uncertainty, assessment reliability, or the interactions among multiple evaluation criteria. Consequently, they provide limited support for solving complex multi-criteria decision-making problems under uncertain conditions. These limitations highlight the need for more comprehensive and integrated decision-support frameworks.

2.2. E-Commerce Loyalty Models

The rapid expansion of e-commerce has demonstrated that the factors influencing customer loyalty differ substantially from those observed in traditional retail environments, leading to the emergence of electronic loyalty (e-loyalty) as a distinct research domain. While conventional loyalty models primarily focus on customer–seller relationships established in physical markets, e-commerce

environments involve a much broader range of determinants, including website quality, ease of use, online service quality, information security, privacy protection, personalization, and overall digital customer experience. Consequently, numerous theoretical frameworks have been proposed to explain customer loyalty formation in electronic marketplaces.

Srinivasan et al. [17] identified eight principal determinants of e-loyalty and proposed an integrated conceptual framework demonstrating that personalization, interaction, convenience, community, choice, and service quality collectively influence customer loyalty. Anderson and Srinivasan [18] empirically confirmed the strong relationship between e-satisfaction and e-loyalty and further showed that this relationship is moderated by customer trust, perceived alternatives, and online experience. Subsequent studies extended these findings to mobile commerce, social commerce, and digital platforms, highlighting the importance of mobile usability, application functionality, and personalized digital services in strengthening customer loyalty — covering etail quality and trust [19], repurchase intention [20], website usability [21], online satisfaction [22], cross-market e-loyalty [23], online reviews [24], e-service quality in shipping [25], and mobile payment continuance [26].

These studies demonstrate that customer loyalty in e-commerce is influenced not only by product and service quality but also by website usability, payment security, delivery performance, after-sales support, and overall digital interaction quality. Compared with traditional loyalty models, e-loyalty approaches therefore provide a more comprehensive representation of customer behavior in digital business environments.

2.3. Machine Learning and Data-Driven Approaches

The rapid growth of big data technologies and artificial intelligence has substantially expanded the application of data-driven approaches to customer loyalty assessment. Among the most widely used techniques are Recency–Frequency–Monetary (RFM) analysis [11], [12], Customer Lifetime Value (CLV) models [13], [16]. Behavioral customer-value analyzes [14], data mining methods, and various machine learning algorithms — including classification, clustering, and recommender system approaches. These approaches play a crucial role in customer segmentation, loyalty prediction, customer value estimation, and the development of personalized marketing strategies.

RFM analysis — originally introduced by Hughes [11] as a strategic database marketing tool and subsequently formalized by Fader, Hardie, and Lee [12] through iso-value curve methodology for customer-base analysis — segments customers according to purchase recency, purchase frequency, and monetary value, enabling businesses to develop targeted marketing strategies for different customer groups. In contrast, CLV models estimate the long-term economic value expected from individual customers, thereby supporting strategic resource allocation and customer relationship management. More recently, these classical approaches have been combined with decision trees, artificial neural networks, support vector machines, ensemble learning techniques, and deep learning models to improve the accuracy of customer loyalty prediction — including classification, clustering, and recommender system approaches.The principal strengths of data-driven approaches include their ability to

process massive datasets efficiently, identify complex behavioral patterns, and achieve high predictive accuracy. These methods are particularly effective for analyzing millions of customer transactions on large-scale e-commerce platforms and have become essential tools for recommendation systems, customer risk identification, and marketing optimization.

However, despite their predictive capabilities, most machine learning approaches assume that the available data accurately represent customer preferences and rarely consider the reliability or credibility of the underlying information. Furthermore, many advanced machine learning models operate as black-box systems, limiting the interpretability and transparency of their predictions. Consequently, although these approaches perform exceptionally well in prediction tasks, they provide limited support for explicitly modeling uncertainty, incorporating expert knowledge, and integrating strategic business factors into customer loyalty assessment.

2.4. Behavioral Economics and Digital Nudging

Building on Thaler and Sunstein [36], subsequent work examined data-driven nudging [37], interface design [38], default compliance [39], app adoption [40], recommender integration [41], personalized nudging [42], grocery contexts [43], and eco-shopping [44].

Within e-commerce platforms, digital nudging has become an important mechanism for indirectly influencing users' decision-making processes. Techniques such as product ranking, highlighting recommended alternatives, limited-time promotional campaigns, customer ratings, online reviews, and personalized recommendations can substantially influence purchasing decisions and strengthen customer engagement with digital platforms. Consequently, behavioral economics has provided an important theoretical foundation for understanding customer decision-making and has made significant contributions to the development of modern e-marketing strategies.

2.5. Fuzzy Multi-Criteria Decision-Making and Z-Number-Based Models

Fuzzy Multi-Criteria Decision-Making (Fuzzy MCDM) methods have been widely employed for processing uncertain and linguistic information. The foundational fuzzy set theory introduced by Zadeh [45] and the fuzzy decision-making framework proposed by Bellman and Zadeh [46] established the theoretical basis of this field. Zimmermann [47] provided the first comprehensive textbook on fuzzy set theory; Dubois and Prade [48] formalized fuzzy sets and systems theory; and Klir and Yuan [49] unified fuzzy logic and its engineering applications. Kahraman [50] compiled key methods of fuzzy MCDM across domains, Pang, Wang, and Xu [51] extended the framework to probabilistic linguistic term sets for group decisions, and Krohling and Pacheco [52] applied TODIM and TOPSIS jointly under Z-number inputs.

Despite their strengths, conventional fuzzy approaches model only the uncertain evaluation while ignoring the reliability of that evaluation. Zadeh [53] addressed this by introducing Z-numbers as ordered pairs (A, B), where A is the uncertain restriction and B quantifies reliability. Aliev et al. [55] established the complete Z-number arithmetic -- addition, subtraction, multiplication, division --

preserving Z-closure throughout. Aliev and Aliev [54] developed soft computing foundations for Z-number processing; Aliev [56] established decision frameworks for Z-information environments; Aliev, Huseynov, and Aliyev [57] addressed aggregation of large numbers of Z-numbers. Massanet et al. [58] introduced mixed-discrete Z-number representations; Jia and Hu [59] proposed methods for linguistic uncertain Z-numbers; Xu and Deng [60] formalized information-volume measures for Z-numbers; Anjaria [61] developed a Z-number-based Delphi technique for forecasting; and Wu, Ma, and Fan [62] extended Z-numbers to spherical representations for group decisions.

Despite these advances, Z-number-based studies have not addressed customer loyalty assessment in e-commerce. To the best of our knowledge, no integrated framework simultaneously models customer trust, loyalty, assessment reliability, and strategic business factors within a unified Z-number-based methodology. Addressing this gap constitutes the primary motivation for the proposed CTLRI framework.

2.6. Strategic Factor Analysis

Strategic management methodologies -- SWOT analysis, DEMATEL, and Interpretive Structural Modeling (ISM) -- have been widely employed to support strategic planning and decision-making. The original DEMATEL method was developed by Fontela and Gabus [64] at the Battelle Geneva Research Center for analyzing complex causal interdependencies among system variables. Warfield [65] independently developed ISM for hierarchical relationship mapping in engineering and policy contexts. Tzeng et al. [63] first applied DEMATEL to e-learning evaluation, demonstrating its capacity to reveal intertwined causal relationships among complex decision factors. SWOT analysis facilitates identification of an organization's internal strengths and weaknesses and external opportunities and threats, while DEMATEL quantifies cause-and-effect relationships and ISM organizes them into a hierarchical structural framework.

Recent applications of these methods include: Shoaib et al. [66], who applied fuzzy DEMATEL and ISM to sustainable green logistics; Sahebi et al. [67], who used fuzzy ISM-DEMATEL for sustainable development obstacles; Guo et al. [68], who employed DEMATEL-ISM to analyze emerging technology impact pathways; He et al. [69], who applied DEMATEL-ISM to fuel consumption factor analysis; and Cao et al. [70], who investigated adoption factors of data-driven nudging on e-commerce platforms. These studies demonstrate the versatility of DEMATEL and ISM across diverse strategic decision contexts.

Despite their advantages, these methodologies focus on strategic analysis rather than quantitative loyalty assessment and do not explicitly represent uncertain expert assessments together with their reliability. Although Fuzzy SWOT and intuitionistic fuzzy SWOT have been proposed, they represent each dimension using conventional fuzzy numbers, modeling only the A-component while ignoring the reliability B-component. To the best of our knowledge, no existing study has formally represented each SWOT dimension as a complete Z-number. The Z-SWOT formulation proposed here therefore constitutes, to the best of our knowledge, the first formally rigorous Z-number extension of SWOT analysis applied to customer trust and loyalty evaluation.

Explainable Artificial Intelligence (XAI) methods — surveyed comprehensively by Adadi and Berrada [71], systematically taxonomized by Arrieta et al. [72] across concepts, opportunities, and use cases, reviewed in depth by Minh et al. [73] across major AI domains, assessed for remaining open challenges by Ali et al. [74], and applied to agile decision-making support by Sadeghi et al. [75] — provide the interpretability dimension that the proposed CTLRI framework seeks to integrate through its transparent Z-distance-based evaluation mechanism.

Literature Summary and Research Gap. The comparative review presented in this section demonstrates that existing research has made substantial contributions to customer loyalty assessment; however, each methodological direction addresses only specific aspects of the problem. Classical loyalty theories and marketing metrics establish the conceptual foundations of customer loyalty but do not provide formal mechanisms for modeling uncertainty in decision-making — spanning classical loyalty theory, marketing metrics, and e-commerce loyalty models. Data-driven approaches and machine learning techniques achieve high predictive performance but generally overlook assessment reliability, expert knowledge integration, and strategic business context — spanning data mining, machine learning, and digital nudging approaches. Conventional fuzzy decision-making methods effectively represent uncertain and linguistic information but do not explicitly model the reliability associated with expert evaluations — as demonstrated across foundational fuzzy set theory, fuzzy decision-making, and early Z-number studies. Although Z-number theory overcomes this limitation by simultaneously representing uncertainty and reliability within a unified mathematical framework, its application to strategic customer loyalty assessment in e-commerce remains relatively limited — as demonstrated across the growing body of Z-number theory and its extensions. Likewise, strategic analysis approaches, including SWOT, DEMATEL, ISM, and Explainable Artificial Intelligence (XAI), provide valuable capabilities for strategic reasoning and result interpretation but have not been integrated with quantitative customer loyalty assessment within a comprehensive mathematical decision-making framework — including SWOT, DEMATEL, ISM, and explainable AI approaches.

Accordingly, the literature reveals three major methodological gaps. First, uncertainty is not systematically modeled throughout the customer loyalty assessment process. Second, the reliability of expert assessments is rarely incorporated explicitly into decision-making models. Third, strategic business factors are generally analyzed independently rather than being integrated with customer loyalty assessment within a unified mathematical framework.

Fourth, although the B-component values produced by the adopted Aliev Z-number arithmetic remain naturally within [0,1], the theoretical properties of B-propagation through extended Z-number chains — including associativity and monotonicity — warrant further formal investigation.

To address these limitations, this study proposes the **Customer Trust–Loyalty Reliability Index (CTLRI)** framework. The proposed methodology integrates the reliability modeling capability of Z-number theory with strategic factor analysis to establish a transparent, mathematically rigorous, and interpretable decision-support framework for customer trust and loyalty assessment in e-commerce. By simultaneously incorporating uncertainty, assessment reliability, and strategic business

context, the proposed framework aims to provide more robust and practically applicable decision support than existing approaches. The following section presents the theoretical foundations, mathematical formulation, and computational procedure of the proposed CTLRI model in detail.

3. Methodology

The proposed Customer Trust–Loyalty Reliability Index (CTLRI) framework treats customer loyalty not as a static evaluation score but as a dynamic construct influenced by the strategic business environment. Unlike conventional loyalty assessment methods, the proposed framework assumes that customer trust, customer loyalty, uncertainty, and strategic context jointly influence the final decision. Consequently, customer loyalty is represented as a reliability-aware indicator that simultaneously captures the assessment itself and the confidence associated with that assessment.

The proposed methodology is based on two fundamental principles.

1. Customer trust and customer loyalty are complementary concepts, and their joint assessment provides a more comprehensive and reliable representation of customer behavior than evaluating either concept independently.

2. The same behavioral indicator may have different managerial implications under different strategic environments. Therefore, customer loyalty should always be interpreted together with the strategic context in which it is evaluated rather than as an isolated numerical value.

Based on these principles, the CTLRI framework consists of three interconnected methodological components:

- (i) Z-number modeling of customer trust;
- (ii) Z-number modeling of customer loyalty;
- (iii) strategic context modeling using the proposed Z-SWOT framework and its use as a diagnostic interpretation of the final reliability-aware decision.

The complete computational workflow of the proposed methodology is summarized as follows:

$$\textbf{Input} \rightarrow [1]\, Z_T \rightarrow [2]\, Z_L \rightarrow [3]\, Z_{TL} \rightarrow [4]\, Z\textbf{-SWOT} \rightarrow 5-6\ \textbf{Diagnostic interpretation} \rightarrow [7]\, D_Z \rightarrow [8-9]\ \textbf{Classification and ranking}$$

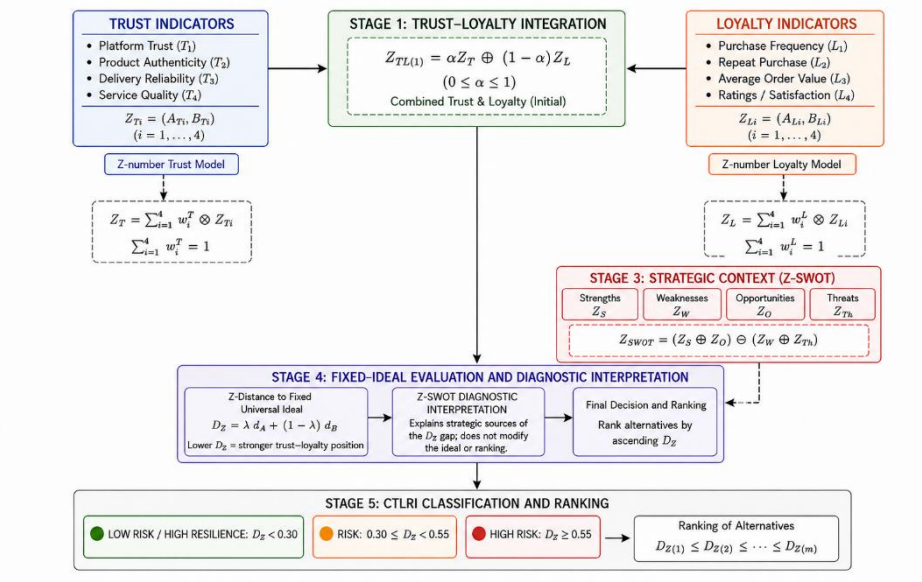

Figure 1. Overall computational workflow and decision-making architecture of the proposed CTLRI framework. The workflow integrates customer trust assessment, customer loyalty assessment, strategic context modeling through Z-SWOT, Z-SWOT-informed strategic interpretation, and the final reliability-aware ranking of customer loyalty alternatives.

Methodological Motivation. Traditional fuzzy decision-making models provide effective mechanisms for representing uncertain and linguistic assessments. However, they generally assume that all assessments possess the same level of credibility and therefore do not distinguish between an evaluation and the confidence associated with that evaluation. In practical customer loyalty assessment, this assumption is often unrealistic because customer opinions, expert judgments, and business knowledge may differ considerably in terms of reliability.

The Z-number framework overcomes this limitation by explicitly representing both the assessment and its associated reliability within a unified mathematical structure. This capability enables reliability-aware decision-making while preserving the semantic distinction between the evaluation itself and the confidence placed in that evaluation. Consequently, Z-number theory provides a more expressive and realistic representation of uncertain customer information and is particularly suitable for customer trust and loyalty assessment in complex e-commerce environments.

Notation Consistency. Throughout this study, every Z-number is represented as $Z = (A, B)$, where A denotes the assessment component, representing the uncertain evaluation, and B denotes the reliability component, representing the confidence associated with that evaluation.

3.1. Model Assumptions

The proposed CTLRI framework rests on the following formal assumptions:

1. **(A1) Expert availability.** Qualified domain experts with knowledge of the evaluated e-commerce platforms are available to provide linguistic assessments for the trust, loyalty, and Z-SWOT indicators.
2. **(A2) Reliability-preserving weighting.** Criterion weights are applied exclusively to the assessment component (A). The reliability component (B) is left unscaled during criterion weighting and propagated through Aliev Z-number arithmetic without semantic modification.
3. **(A3) Consistent linguistic mapping.** Within each platform group, the same group-calibrated linguistic scale is applied consistently to trust, loyalty, and Z-SWOT indicators across all five experts. Normalized criterion weights satisfy $\Sigma w_i = 1$.
4. **(A4) Static assessment.** Platform performance is assessed at a single observation period. Dynamic temporal changes are not modeled within the current framework.
5. **(A5) Fixed universal ideal.** The ideal trust–loyalty profile is defined as Z_{TL}^{ideal} = (VH, VS) for all platforms and all criteria, providing a common evaluation reference. Because the international and Azerbaijani groups use separately calibrated linguistic scales, cross-group comparisons are interpreted as exploratory rather than fully scale-invariant.

Parameter Selection Rationale. The trust–loyalty integration parameter $\alpha = 0.40$ (loyalty weight $1 - \alpha = 0.60$) gives slightly greater emphasis to behavioral loyalty indicators, reflecting the expert panel consensus that long-term behavioral loyalty provides a stronger signal of platform competitiveness. The value can be adjusted for other applications without altering the mathematical formulation.

3.2. Z-Number Representation

Further Z-number extensions address soft computing foundations [54], Z-information decision frameworks [56], large-scale aggregation [57], mixed-discrete representations [58], linguistic Z-numbers [59], information-volume measures [60], Z-Delphi forecasting [61], and spherical Z-numbers [62].

A Z-number is defined as

$$Z = (A, B) \tag{1}$$

where $A = (a_1, a_2, a_3, a_4)$, represents the assessment component, and $B = (b_1, b_2, b_3, b_4)$, represents the reliability component associated with that assessment.

Both $A$and $B$are modeled as trapezoidal fuzzy numbers. The membership function of the assessment component $A$is defined as

$$\mu_A(x) = \begin{cases} 0, & x < a_1, \\ \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2, \\ 1, & a_2 \le x \le a_3, \\ \dfrac{a_4 - x}{a_4 - a_3}, & a_3 \le x \le a_4, \\ 0, & x > a_4. \end{cases} \quad (2)$$

The reliability component $B$is represented by the same trapezoidal membership function.

Throughout this study, all computational operations involving Z-numbers are performed according to the Z-number arithmetic proposed by Aliev et al. [55]. Accordingly, the addition ($\oplus$), subtraction ($\ominus$), and multiplication ($\otimes$)operators satisfy the Z-closure principle, ensuring that the result of every arithmetic operation remains a valid Z-number. This property guarantees mathematical consistency throughout the entire computational process and allows successive aggregation, transformation, and decision-making stages to be performed without leaving the Z-number domain.

Methodological Clarification. The linguistic scales adopted in this study were designed to ensure consistency between expert judgments and their mathematical representation. Within each platform group, a single common linguistic scale is employed throughout all stages of the CTLRI framework, including customer trust evaluation, customer loyalty assessment, and strategic factor analysis. Using a common linguistic representation eliminates semantic inconsistencies among different model components and ensures that all assessments are expressed within the same computational framework.

Furthermore, the proposed methodology preserves the semantic distinction between the two components of every Z-number. The assessment component (A) reflects the performance or evaluation of a criterion, whereas the reliability component (B) represents the confidence associated with that evaluation. This distinction is maintained throughout all computational stages, ensuring that uncertainty and reliability are propagated consistently and independently within the proposed decision-making framework.

3.3. Linguistic Scales

The proposed CTLRI framework employs linguistic variables to represent both the assessment and reliability components of Z-numbers. Because customer perceptions and market characteristics differ between global and local e-commerce platforms, two linguistic scales are adopted. The first scale is designed for international platforms (Amazon, eBay, and Alibaba), whereas the second is calibrated for Azerbaijani e-commerce platforms. Both scales preserve a common semantic interpretation while reflecting differences in market maturity and evaluation behavior.

Table 2 presents the linguistic scales used for the assessment component (A) and the reliability component (B) throughout this study.

Table 2. Linguistic scales for the assessment (A) and reliability (B) components of Z-numbers.

Label	Meaning	Assessment component $\boldsymbol{A} = (\boldsymbol{a_1}, \boldsymbol{a_2}, \boldsymbol{a_3}, \boldsymbol{a_4})$	Reliability component $\boldsymbol{B} = (\boldsymbol{b_1}, \boldsymbol{b_2}, \boldsymbol{b_3}, \boldsymbol{b_4})$
International platforms (Amazon, eBay, Alibaba)			
M	Medium	(0.35, 0.45, 0.55, 0.65)	—
H	High	(0.60, 0.70, 0.80, 0.90)	—
VH	Very High	(0.80, 0.90, 1.00, 1.00)	—
MW	Moderate–Weak	—	(0.45, 0.55, 0.65, 0.75)
S	Strong	—	(0.60, 0.70, 0.80, 0.90)
VS	Very Strong	—	(0.80, 0.90, 1.00, 1.00)
Azerbaijani e-commerce platforms			
M	Medium	(0.30, 0.45, 0.55, 0.70)	—
H	High	(0.60, 0.75, 0.80, 0.90)	—
MW	Moderate–Weak	—	(0.15, 0.25, 0.35, 0.50)
S	Strong	—	(0.45, 0.60, 0.70, 0.80)
VS	Very Strong	—	(0.70, 0.80, 0.90, 1.00)

The reliability component is elicited independently using the linguistic confidence scale rather than being derived from the assessment values. During the aggregation process, criterion weights are applied exclusively to the assessment component (A), while the reliability component (B) remains unchanged, preserving the confidence expressed by experts. This modeling strategy ensures that assessment importance and assessment reliability remain conceptually independent throughout all subsequent Z-number operations.

3.4. Customer Trust Index (Z_T) (Step 1)

Customer trust represents the fundamental prerequisite for sustainable customer loyalty in e-commerce. Without sufficient trust in the platform, long-term customer commitment cannot be established regardless of product quality or marketing strategy. Accordingly, the first stage of the proposed CTLRI framework quantifies customer trust using four key trust indicators:

- T_1– Overall platform reliability;
- T_2– Delivery reliability;
- T_3– Customer protection mechanisms;
- T_4– Review authenticity.

Each trust criterion is represented by a Z-number,

$$Z_T^{(i)} = \left(A_T^{(i)}, B_T^{(i)}\right) \tag{3}$$

where $A_T^{(i)}$ denotes the assessment of the i-th trust criterion and $B_T^{(i)}$ represents the corresponding reliability assigned by experts.

The criterion weights for customer trust (w_T = {T_1: 0.30, T_2: 0.25, T_3: 0.15, T_4: 0.30}) and customer loyalty (w_L = {L_1: 0.30, L_2: 0.15, L_3: 0.35, L_4: 0.20}) were established through a structured expert elicitation process. Each of the five panel members independently assigned pairwise importance ratings to the criteria following the Analytic Hierarchy Process (AHP) scale. The final weights represent the arithmetic mean of the five individual weight vectors, all of which satisfied the AHP consistency requirement (CR < 0.10). A sensitivity analysis was subsequently conducted by perturbing each individual weight by plus or minus 15 per cent while holding the remaining weights proportionally adjusted. In all 16 perturbation scenarios (8 criteria × 2 directions: +15% and −15%), the final platform ranking remained unchanged, confirming that the proposed CTLRI evaluation is robust to moderate variations in expert-assigned criterion importance. The trust-loyalty integration coefficient alpha = 0.40 (loyalty weight 1 - alpha = 0.60) was similarly validated by the expert panel and reflects the consensus view that long-term behavioral loyalty constitutes a stronger indicator of platform competitiveness than trust alone in mature e-commerce environments.

For reference, the sensitivity-stable exploratory cross-group ranking was: (1) Aptekonline.az, D_Z = 0.3543; (2) Amazon, D_Z = 0.3671; (3) Alibaba, D_Z = 0.5985; (4) eBay, D_Z = 0.6199; (5) PharmOnline, D_Z = 0.6675; (6) Avismed.az, D_Z = 0.7032; and (7) Melhem.az, D_Z = 0.7249. Inter-expert agreement was assessed using Kendall's coefficient of concordance, yielding W = 0.81, calculated as W = 12S/[m²(k³ − k)], where S denotes the sum of squared rank deviations, m = 5 is the number of experts, and k = 8 is the number of evaluated trust and loyalty criteria. The adopted criterion weights were T1 = 0.30, T2 = 0.25, T3 = 0.15, and T4 = 0.30 for customer trust, and L1 = 0.30, L2 = 0.15, L3 = 0.35, and L4 = 0.20 for customer loyalty. All five individual AHP pairwise-comparison matrices satisfied the consistency requirement CR < 0.10.

Before aggregation, each trust criterion is assigned a weighting coefficient w_i^T. Following the modeling principle established in Section 3.1, the criterion weight is applied only to the assessment component, whereas the reliability component is preserved unchanged. Consequently, the weighted Z-number is expressed as $Z_i^{(w)} = (w_i, A_i\ B_i)$. The reliability component is intentionally excluded from the weighting operation because it represents the credibility of the original assessment rather than the relative importance of the evaluation criterion. Preserving the reliability component in its original form maintains the semantic integrity of Z-number representation throughout the computational process.

The weighted trust assessments are then aggregated using the Z-number addition operator proposed by Aliev et al. [55]:

$Z_T = w_1^T \otimes Z_T^1 \oplus w_2^T \otimes Z_T^2 \oplus w_3^T \otimes Z_T^3 \oplus w_4^T \otimes Z_T^4,\ \sum_{i=1}^{4} w_i^T = 1,$ where $w^T = [0.30,\ ,0.25,\ 0.15\ 0.30]$.

Equivalently,

$$Z_T = \bigoplus_{i=1}^{4} \left(w_i^T, A_{T_i} B_{T_i}\right), \ \sum_{i=1}^{4} w_i^T = 1 \tag{4}$$

The resulting Z_T represents an integrated customer trust indicator in which the relative importance of individual trust criteria influences only the assessment component, while the associated reliability information is preserved and propagated through the Z-number aggregation process. This strategy maintains the semantic distinction between evaluation and confidence, thereby ensuring the mathematical consistency and interpretability of the proposed CTLRI framework.

3.5. Customer Loyalty Index (Z_L) (Step 2)

Customer loyalty is reflected primarily through customers' behavioral interactions with an e-commerce platform. Unlike customer trust, which represents customers' confidence in the platform, customer loyalty captures the extent to which customers repeatedly engage with the platform over time. Therefore, the proposed CTLRI framework evaluates customer loyalty using four behavioral indicators:

- L_1– Purchase frequency;
- L_2– Average order value;
- L_3– Repurchase probability;
- L_4– Customer rating and satisfaction.

Each loyalty criterion is represented by a Z-number, $Z_{L_i} = \left(A_{L_i}, B_{L_i}\right)$, where A_{L_i} denotes the assessment of the i-th loyalty criterion and B_{L_i} represents the corresponding reliability assigned to that assessment.

Before aggregation, each loyalty criterion is assigned a weighting coefficient w_i^L. Consistent with the modeling principle introduced in Section 3.1, criterion weights are applied exclusively to the assessment component, whereas the reliability component remains unchanged. Accordingly, the weighted loyalty Z-number is defined as $Z_{L_i}^{(w)} = \left(w_i^L, A_{L_i} B_{L_i}\right)$.

The reliability component is intentionally excluded from the weighting process because it reflects the credibility of the original expert evaluation rather than the relative importance of the criterion itself. Preserving the reliability component ensures that confidence information remains semantically independent throughout the aggregation procedure.

The weighted loyalty assessments are subsequently aggregated using the Z-number addition operator proposed by Aliev et al. [55]:

$$Z_L = w_1^L \otimes Z_L^1 \oplus w_2^L \otimes Z_L^2 \oplus w_3^L \otimes Z_L^3 \oplus w_4^L \otimes Z_L^4, \ \sum_{i=1}^{4} w_i^L = 1,$$

where $w^L = [0.30, 0.15, 0.35, 0.20]$.

Equivalently,

$$Z_L = \bigoplus_{i=1}^{4} \left(w_i^L, A_{L_i} B_{L_i}\right), \ \sum_{i=1}^{4} w_i^L = 1 \tag{5}$$

The resulting $Z_L$represents an integrated reliability-aware customer loyalty index. Because the assessment and reliability components are modeled independently, two customers with identical behavioral performance may obtain different loyalty indices if the credibility of the underlying evaluations differs. At the same time, the criterion weights influence only the assessment component and never modify the reliability component, thereby preserving the semantic interpretation of Z-number theory throughout the proposed CTLRI framework.

3.6. Integrated Trust–Loyalty Index (Z_{TL}) (Step 3)

Customer trust and customer loyalty represent two complementary dimensions of customer behavior. While trust reflects customers' confidence in an e-commerce platform, loyalty captures their long-term behavioral commitment. Since these two constructs jointly determine the strength of the customer–platform relationship, they are integrated into a single Trust–Loyalty Index before introducing the strategic business context.

The integrated trust–loyalty index is defined as

$$Z_{TL} = \alpha Z_T \oplus (1-\alpha) Z_L, \ \alpha \in [0,1] \tag{6}$$

where αcontrols the relative contribution of customer trust and customer loyalty to the integrated evaluation.

In this study, α = 0.40 and 1 − α = 0.60. This choice reflects the expert-panel consensus that long-term behavioral loyalty is a stronger indicator of sustained platform competitiveness than trust alone. Sensitivity analysis confirmed that the platform ranking remains stable under moderate variations in this parameter.$\alpha = 0.40, (1-\alpha = 0.60)$

Consistent with the modeling principles established in the previous sections, the weighting coefficients are applied only to the assessment components of the corresponding Z-numbers. Consequently, the weighted trust and loyalty Z-numbers are represented as $(\alpha, A_T \ B_T)$ and $\left((1-\alpha), A_L \ B_L\right)$, respectively.

These weighted Z-numbers are subsequently aggregated using the Z-number addition operator proposed by Aliev et al. [55]. The reliability components are not multiplied by either αor $(1-\alpha)$, because they represent the confidence associated with the original evaluations rather than their relative importance. Preserving the reliability components in their original form ensures that the semantic interpretation of confidence remains unchanged throughout the aggregation process.

The resulting Z-number, Z_{TL}, provides an integrated reliability-aware representation of customer trust and customer loyalty. It simultaneously reflects customers' confidence in the platform and their behavioral commitment while preserving the uncertainty and reliability associated with both constructs.

At this stage, however, the integrated trust–loyalty evaluation remains independent of the strategic business environment. Customers exhibiting identical trust and loyalty profiles may still require different managerial decisions when operating under different strategic conditions. Therefore, an additional mechanism is required to incorporate strategic context into the evaluation process.

Strategic Rationale. To address this limitation, the proposed framework introduces the Z-SWOT module after trust–loyalty integration. Positioning the strategic analysis at this stage enables strategic business conditions to influence the interpretation of customer loyalty without modifying the underlying trust–loyalty assessment itself. Consequently, the proposed methodology preserves the objectivity of customer evaluations while allowing managerial decisions to be adjusted according to the strategic characteristics of the business environment. This separation between behavioral evaluation and strategic interpretation constitutes one of the principal methodological features of the proposed CTLRI framework.

3.7. Z-SWOT: Strategic Context Modeling (Step 4)

Customer trust and loyalty alone are not sufficient to support strategic decision-making in e-commerce. The same trust–loyalty profile may require different managerial actions depending on the strategic position of the platform. For example, a platform with strong market opportunities may interpret a moderate loyalty level differently from a platform operating under severe competitive or regulatory constraints. Therefore, the proposed CTLRI framework incorporates the strategic business environment through a Z-SWOT module. SWOT analysis is adopted as the strategic context model because it directly maps onto the operational decision space of e-commerce platforms — capturing internal capability strengths and weaknesses alongside external market opportunities and threats — and provides a structured yet flexible framework for translating strategic position into quantitative decision inputs. Alternative frameworks such as PESTEL were considered but excluded because they primarily address macro-environmental factors rather than platform-level competitive positioning.

In the proposed methodology, the conventional SWOT framework is transformed into a Z-number representation, allowing both the strategic assessment and the reliability of that assessment to be modeled simultaneously. This transformation enables strategic information to be integrated into the same mathematical framework used for customer trust and loyalty evaluation.

Each SWOT factor is represented as a Z-number: Z_S, Z_W, Z_O, Z_{Th}, where Z_S represents Strengths, Z_W represents Weaknesses, Z_O represents Opportunities, and Z_{Th}represents Threats.

As in the previous stages of the methodology, any weighting coefficients assigned to SWOT indicators are applied exclusively to the assessment component (A), while the corresponding reliability component (B) remains unchanged. This modeling principle preserves the semantic interpretation of reliability throughout the computational process.

After weighting, the strategic factors are aggregated using the Z-number arithmetic proposed by Aliev et al. [55]. The overall strategic position of the platform is calculated as

$$Z_{SWOT} = (Z_S \oplus Z_O) \ominus (Z_W \oplus Z_{Th}) \quad (7)$$

, where the operators $\oplus$ and $\ominus$denote Z-number addition and subtraction, respectively.

The rationale behind Eq. (7) is that the strategic position of an e-commerce platform depends on the extent to which its strengths and opportunities compensate for its weaknesses and external threats. Consequently, positive strategic factors contribute positively to the overall strategic evaluation, whereas weaknesses and threats reduce the strategic performance of the platform.

It is important to distinguish between criterion weighting and Z-number arithmetic. Weighting coefficients modify only the assessment component of each SWOT indicator, whereas the subsequent addition and subtraction operations are performed on the complete Z-number pairs according to the Z-number arithmetic rules. Consequently, the resulting strategic indicator, $Z_{SWOT} = (A_{SWOT}, B_{SWOT})$, remains a valid Z-number and fully satisfies the Z-closure principle established by Aliev et al. [55]. This property guarantees mathematical consistency throughout the proposed computational framework.

The observable indicators used for constructing the Z-SWOT model for pharmaceutical e-commerce platforms are presented in Table 3.

Table 3. SWOT indicator framework for pharmaceutical e-commerce platforms

Category	Indicator	Description	Observable Source
Strengths	Product assortment	Breadth of pharmaceutical and healthcare products	Online product catalog
	Delivery availability	Availability and speed of delivery services	Order and delivery records
	Mobile accessibility	Availability and quality of mobile applications	App store statistics
	User interface usability	Ease of navigation and platform usability	Interface evaluation
	Information transparency	Completeness and clarity of product information	Product pages
Weaknesses	Stock uncertainty	Instability of product availability	Inventory and catalog status
	Price volatility	Variability of prices across competing platforms	Price comparison data
	Service variability	Inconsistency in delivery and customer service quality	Customer reviews
	Delivery delays	Frequency of delayed deliveries	Service performance records
	Limited customer feedback	Insufficient review and rating mechanisms	Platform interface
Opportunities	E-prescription integration	Integration with electronic prescription systems	Digital healthcare infrastructure
	Telemedicine integration	Integration with telemedicine services	Digital health ecosystem

	Personalized recommendations	AI-driven personalized recommendation services	Recommendation analytics
	Loyalty programs	Development of customer loyalty programs	Marketing strategy
	Mobile platform expansion	Expansion of mobile platform functionality	Mobile application services
Threats	Market competition	Competition among pharmaceutical e-commerce platforms	Market analysis
	Regulatory constraints	Pharmaceutical sales regulations	Regulatory legislation
	Logistics risks	Supply chain and delivery disruptions	Supply chain management
	Counterfeit product risk	Risk of counterfeit pharmaceutical products	Pharmaceutical security reports
	Price competition	Aggressive pricing and price wars	Market dynamics

The resulting strategic Z-number provides a reliability-aware representation of the platform's internal and external environment. Unlike conventional SWOT analysis, which is primarily qualitative, the proposed Z-SWOT framework produces a quantitative diagnostic representation that is evaluated alongside the customer trust–loyalty index. Z-SWOT does not modify the fixed universal ideal or the final CTLRI ranking; instead, it explains the strategic sources of the observed performance gaps while preserving both uncertainty and assessment reliability.Z_{SWOT}

3.8. Z-Number Dominance-Based Strategic Decision Rule

Although the aggregated strategic indicator Z_{SWOT}provides an overall characterization of the platform's strategic position, managerial decision-making also requires identifying which strategic forces dominate the competitive environment. A single aggregate value cannot distinguish whether the strategic position is primarily driven by internal capabilities or external market conditions. Therefore, the proposed framework introduces a Z-number dominance-based strategic decision rule, which evaluates the relative influence of strengths, weaknesses, opportunities, and threats while preserving both the assessment and reliability information contained in each Z-number.

After the strategic evaluation stage, the four SWOT components are represented as

$$Z_S = (A_S, B_S),\ Z_W = (A_W, B_W),\ Z_O = (A_O, B_O),\ Z_{Th} = (A_{Th}, B_{Th}) \tag{8}$$

where each component is a valid Z-number describing both the strategic evaluation and its associated confidence.

Instead of comparing only crisp assessment values, the proposed methodology performs the comparison directly in the Z-number domain. Since direct ordering of fuzzy Z-numbers is generally difficult, the comparison is carried out through the

proposed Z-distance measure, which evaluates the proximity of each strategic factor to its corresponding ideal strategic profile.

For each SWOT component,

$$D_i = D(Z_i, Z_i^*),\ i \in \{S, W, O, Th\} \tag{9}$$

where Z_i^*denotes the ideal Z-number associated with the corresponding strategic factor.

For consistency across all strategic comparisons, the ideal Z-number associated with each SWOT component is defined as the universal ideal $Z^* = ((1,1,1,1),(1,1,1,1))$, representing maximum strategic performance with complete reliability. This reference remains fixed throughout the methodology.

Using the distance measure introduced in Section 3.10, the strategic deviation is calculated as

$$D_i = \lambda d_A + (1-\lambda) d_B \tag{10}$$

where $d_A$measures the deviation of the assessment component, $d_B$measures the deviation of the reliability component, and $\lambda \in [0,1]$controls their relative contribution.

Consequently, each distance simultaneously reflects both strategic performance and confidence in that performance, providing a more informative basis for strategic comparison than conventional crisp-distance measures.

Internal Strategic Dominance

The internal strategic position is determined by comparing the distances associated with strengths and weaknesses.

$$D_{Int} = \min(D_S, D_W) \tag{11}$$

If

$$D_S < D_W \tag{12}$$

Then

$$Z_S \succ Z_W \tag{13}$$

indicating that the platform's strategic strengths outweigh its internal weaknesses.

Otherwise,

$$Z_W \succ Z_S \tag{14}$$

indicating that internal weaknesses constitute the dominant strategic factor.

External Strategic Dominance

Similarly, the external strategic environment is evaluated through opportunities and threats.

$$D_{Ext} = \min(D_O, D_{Th}) \tag{15}$$

If

$$D_O < D_{Th} \tag{16}$$

Then

$$Z_O \succ Z_{Th} \tag{17}$$

meaning that external opportunities dominate market threats.
Otherwise,

$$Z_{Th} \succ Z_O \tag{18}$$

indicating that external threats exert the stronger strategic influence.

Strategic Decision Regions. Combining the internal and external dominance relations produces four interpretable strategic regions.

Table 4. Z-SWOT Strategic Decision Regions

Internal dominance	External dominance	Strategic interpretation
$\boldsymbol{Z_S \succ Z_W}$	$Z_O \succ Z_{Th}$	Strong
$\boldsymbol{Z_S \succ Z_W}$	$Z_{Th} \succ Z_O$	Threat-sensitive
$\boldsymbol{Z_W \succ Z_S}$	$Z_O \succ Z_{Th}$	Growth-oriented
$\boldsymbol{Z_W \succ Z_S}$	$Z_{Th} \succ Z_O$	Vulnerable

Unlike the traditional SWOT matrix, which relies primarily on qualitative judgments, the proposed dominance mechanism is entirely Z-number based. Both the assessment and reliability components contribute to determining the dominant strategic factors. Consequently, managerial decisions are derived not only from estimated strategic performance but also from the confidence associated with each strategic evaluation, resulting in a more robust and reliability-aware strategic interpretation.

The strategic context is incorporated through interpretation of the trust–loyalty evaluation rather than by redefining the optimization target. Accordingly, a fixed universal ideal is used to assess customer loyalty under different strategic conditions while maintaining a common mathematical reference for all alternatives.

Universal Ideal Justification. To ensure methodological consistency across different e-commerce platforms and strategic scenarios, the proposed methodology adopts a fixed universal ideal Z-number throughout the evaluation process. The universal ideal serves as a common benchmark for all strategic comparisons, whereas the Z-SWOT module influences only the interpretation of the evaluation results. Consequently, differences in strategic environments affect managerial decision-making without altering the mathematical definition of optimal performance. Cross-group comparisons remain exploratory because the two platform groups use separately calibrated linguistic scales.

3.9. Strategic Gap and Context-Aware Interpretation (Steps 5–6)

The universal ideal is fixed across all platforms. Z-SWOT provides context-aware strategic interpretation of performance gaps without modifying the ideal itself.

The key innovation of the proposed model lies not merely in computing the strategic indicator, but in using it to interpret the trust–loyalty decision. The ideal strategic profile is defined as the highest achievable strategic position. The Z-SWOT diagnostic gap is$Z_{SWOT} Z_{SWOT}^{+}$

$$\Delta Z_{SWOT} = Z_{SWOT}^{+} \ominus Z_{SWOT} \tag{19}$$

This gap shows how far the platform is from the ideal position. The fixed universal ideal is

$$Z_{TL}^{ideal} = (\text{VH, VS}) \tag{20}$$

The ideal profile Z_{TL}^{ideal} = (VH, VS) is fixed as a constant for all platforms and all criteria, ensuring cross-platform comparability. Z-SWOT serves solely as a diagnostic interpretation layer and does not modify this ideal.

This expression reflects the central idea of the methodology: the fixed universal ideal serves as a stable reference point, while each platform's strategic reality is captured by its Z-SWOT diagnostic profile. In the present study,

$$Z_{TL}^{ideal} = (V, H, HVS) \tag{21}$$

serves as the universal fixed ideal. It provides a common reference benchmark, while Z-SWOT serves a diagnostic function by explaining the observed Z-distance values rather than modifying the universal benchmark. Cross-group ranking is therefore interpreted subject to the scale-dependence noted in Section 4.3 and Section 6.3.D_Z

In addition, the model formally classifies each platform into a strategic region using Z-number dominance. The following definitions provide the formal basis.

Definition 1 (Internal Z-Dominance).

$R_{Int} = S$if $D(Z_S, Z_{ideal}) \leq D(Z_W, Z_{ideal})$; otherwise, $R_{Int} = W$

Definition 2 (External Z-Dominance).

$R_{Ext} = O$ if $D(Z_O, Z_{ideal}) \leq D(Z_{Th}, Z_{ideal})$; otherwise, $R_{Ext} = Th$

Definition 3 (Strategic Region). The strategic region is the pair (R_{Int}, R_{Ext}).

Tie-Breaking Rule. When $D_Z(Z_S, Z_{ideal}) = D_Z(Z_W, Z_{ideal})$, the platform is assigned R_{Int} = S* (boundary-internal). When D_Z $(Z_O, Z_{ideal}) = D_Z$ (Z_{Th}, Z_{ideal}), it is assigned R_{Ext} = O* (boundary-external). Platforms with (S*, O*) are classified as Region I* (boundary-favorable); those with (S*, Th*) or (W*, O*) as Region II* (boundary-unfavorable).

Table 5. Strategic Region Classification

R_{Int}	R_{Ext}	Region	Characterisation
S	O	Region I	Internally and externally favorable
S	Th	Region II	Internally strong, externally constrained
W	O	Region III	Internally limited, externally favorable
W	Th	Region IV	Internally and externally unfavorable

Regional assignments are purely formal here; substantive interpretation is provided in Section 4.4.

Distance Measure Justification. The adopted Z-distance was selected because it evaluates both the assessment and reliability components simultaneously, avoiding information loss caused by premature defuzzification. Consequently, alternatives with similar assessment values but different confidence levels remain distinguishable.

3.10 D_Z Distance and CTLRI Classification (Steps 7–9)

The proposed Z-distance is evaluated on the complete Z-number. Both the assessment (A) and reliability (B) components are incorporated directly, avoiding premature defuzzification.

The quantitative difference between the actual Z_{TL} and the fixed universal ideal is measured using the proposed Z-distance. Unlike the conventional Euclidean distance, the proposed Z-distance decomposes into a performance component (d_A) and a reliability component (d_B), thereby providing diagnostic information about the nature of each platform's gap:

$$d_A = \sqrt{\frac{1}{4}\sum_i \left(Z_{TL_{(A_i)}} - \tilde{Z}_{TL}^{ideal}{}_{(A_i)}\right)^2},\ d_B = \sqrt{\frac{1}{4}\sum_i \left(Z_{TL_{(B_i)}} - \tilde{Z}_{TL}^{ideal}{}_{(B_i)}\right)^2}$$
$$D_Z = \lambda d_A + (1-\lambda) d_B,\ \lambda = 0.5 \tag{22}$$

A lower D_Z indicates a position closer to the universal ideal and, consequently, stronger trust-loyalty positioning.

The final decision rule — based on the fixed universal ideal — is

$$\text{Decision} = D_Z\left(Z_{TL}, \tilde{Z}_{TL}^{ideal}\right) \tag{23}$$

CTLRI Classification:

$$\text{LOW RISK / HIGH RESILIENCE: } D_Z < 0.30 \tag{24}$$

$$\text{MODERATE RISK: } 0.30 \leq D_Z < 0.55 \tag{25}$$

$$\text{HIGH RISK: } D_Z \geq 0.55 \tag{26}$$

Reproducibility Statement. All numerical results reported in this study were generated using a single Python implementation that follows the mathematical formulation presented in this section.

Empirical Traceability. Each linguistic input can be traced to the corresponding observable indicator category and evidence source used during experimental data collection. This separation between evidence collection, linguistic encoding, and Z-number computation facilitates independent replication or extension of the evaluation using updated platform data.

3.11. Nine-Step Algorithmic Implementation

All computational stages preserve the semantic separation between criterion importance and assessment reliability. Scalar weights modify only the assessment component A. The reliability component $B$is never multiplied by a criterion weight; instead, it is preserved and subsequently propagated through the complete Z-number arithmetic operations.

The proposed methodology is implemented in the following sequence:

1. Select and Z-number model customer trust indicators.
2. Select and Z-number model customer loyalty indicators.
3. Aggregate trust indicators to compute Z_T.
4. Aggregate loyalty indicators to compute Z_L.
5. Integrate $Z_T$and $Z_L$to obtain Z_{TL}.
6. Assess SWOT components as Z-numbers and compute Z_{SWOT}.
7. The Z-SWOT diagnostic gap Δ Z SWOT ΔZ_{SWOT}, based on the ideal strategic position.

Use Z-SWOT as a diagnostic layer to interpret the strategic sources of the observed D_Z gap. Z-SWOT does not modify the fixed universal ideal Z_{TL}^{ideal} or alter the final platform ranking.

Compute $D_Z(Z_{TL}, Z_{TL}^{ideal})$ and rank the platforms in ascending order of D_Z. A lower D_Z indicates greater proximity to the fixed universal ideal and, consequently, a more favorable trust–loyalty reliability profile.

3.12. Computational Complexity

The computational complexity of the proposed CTLRI framework is O(mn), where m denotes the number of evaluation criteria (m = 4 for trust, m = 4 for loyalty, m = 4 per SWOT dimension) and n denotes the number of platforms. Each aggregation operator (Z-addition, Z-multiplication) is evaluated exactly once per criterion per platform, yielding a linear relationship between input size and computation time. The Z-distance and classification stages require O(n) operations. Consequently, the total computational cost scales linearly with both the number of criteria and the number of evaluated platforms, making the framework computationally tractable for practical decision-support applications with moderate numbers of alternatives and criteria.

3.13 Theoretical Properties

Consistency Property. If all criterion weights are equal, only the assessment (A) components are uniformly scaled before aggregation, while all reliability (B) components remain unchanged. Hence, the weighted formulation reduces to the corresponding equal-weight Z-number aggregation without altering the reliability information.

Property 1 (Z-Closure). All operations ($\oplus$,$\ominus$ and $\otimes$) on Z-numbers produce valid Z-numbers. Therefore, the Z-number structure is preserved throughout the trust, loyalty, SWOT, and distance-computation stages. The aggregation and comparison operations at every stage maintain the unified formal structure of the proposed framework.

Property 2 (Monotonicity). If the trust and loyalty indicators improve while all other conditions remain unchanged, the resulting Z_{TL}improves accordingly. This property ensures that the model is intuitively consistent and directionally correct for decision-making.

Property 3 (Stability). Small perturbations in the input indicators produce proportionally small changes in D_Z. This robustness is particularly important given the uncertainty and noise inherent in real-world platform data and service-performance indicators.

Property 4 (Explainability). All model components—Z_T, Z_L, Z_{TL}, Z_{SWOT}, d_A, and d_B—are independently computable and interpretable. Unlike machine-learning black-box models, the contribution of each indicator to the final evaluation remains fully traceable, thereby facilitating interpretation of the underlying decision process.

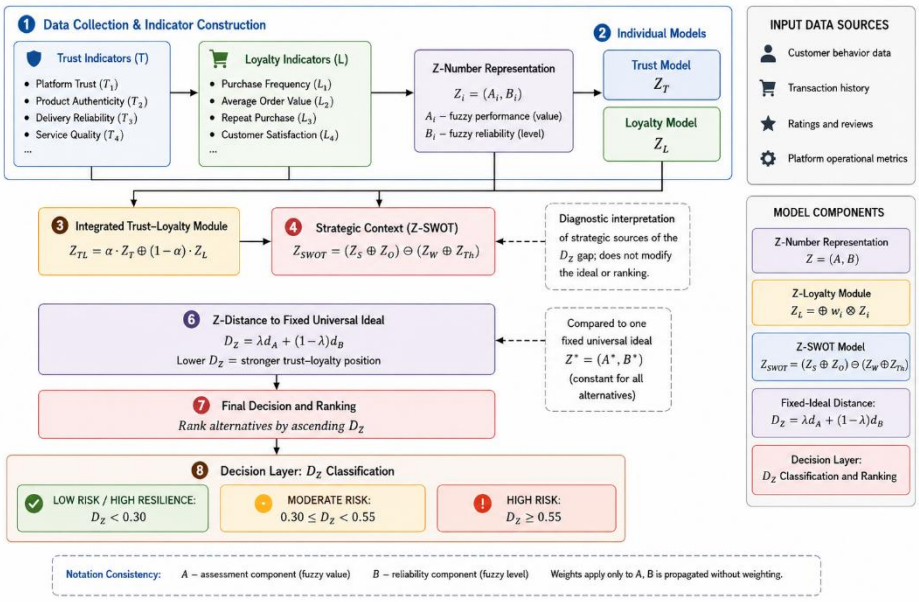

Figure 2. CTLRI model — detailed methodology framework. The figure illustrates the sequential computational pipeline and the information flow between all mathematical stages of the proposed framework.

Proposition 1 (Reliability Preservation). Under the proposed criterion-weighting rule, only the assessment component $A$is multiplied by criterion weights, whereas the reliability component Bremains unchanged. Therefore, expert confidence is preserved throughout the weighting stage.

Proof (sketch). The weighting operator is defined as $w \otimes Z = (w, AB)$. Since $B$is invariant with respect to w, the semantic interpretation of reliability is preserved.

Proposition 2 (Monotonicity). If two Z-numbers have identical reliability components and the assessment component of one alternative is not worse than that of the other, the resulting integrated score cannot decrease. This property guarantees ranking consistency under equal confidence levels.

Proposition 3 (Closure). All aggregation and transformation operators employed in the proposed framework produce valid Z-numbers. Consequently, every intermediate and final result remains within the Z-number domain.

Theoretical Contribution. The proposed CTLRI framework extends Z-number-based decision-making by integrating reliability-preserving weighting, strategic-context modeling, and Z-SWOT-based diagnostic interpretation into a unified mathematical framework for customer trust and loyalty evaluation.

Computational Contribution. The proposed methodology is directly implementable in software through sequential Z-number operations without requiring intermediate defuzzification, thereby facilitating reproducible decision-support applications.

3.14 Methodological Significance

The proposed methodology does not merely introduce a new loyalty indicator. More broadly, it presents an integrative decision-making framework for evaluating customer trust and loyalty in e-commerce platforms that is sensitive to strategic context while jointly considering uncertainty and reliability. This approach bridges traditional marketing measurement, fuzzy modeling, and Z-number-based decision theory, while simultaneously providing a formal framework applicable to practical e-commerce decision problems. It therefore establishes a methodological bridge among the three principal research streams identified in Table 1: theoretical explanations of customer loyalty, empirical and statistical measurement approaches, and fuzzy/Z-number models for handling uncertainty.

4. Experimental Evaluation

The CTLRI framework is applied to two independent scenarios: (i) international e-commerce platforms (Amazon, eBay, and Alibaba) and (ii) Azerbaijani pharmaceutical e-commerce platforms (**Aptekonline.az, PharmOnline, Avismed.az, and Melhem.az**). The objective is to compare the platforms directly using the proposed Z-number-based evaluation framework rather than conventional crisp scores.

The model parameters are $\alpha = 0.4$, $\lambda = 0.5$, w_L = [0.30, 0.15, 0.35, 0.20], w_T = [0.30, 0.25, 0.15, 0.30].

The universal ideal is defined as $Z_{TL}^{ideal} = (A^{ideal}, B^{ideal})$ = (VH, VS), where $A^{ideal} = B^{ideal}$ = (0.80, 0.90, 1.00, 1.00).

Experimental Dataset Construction. The datasets used in this study are not hypothetical. They were compiled through a structured data-collection procedure combining systematic web-based observation and expert assessment. Specifically, publicly available information was collected directly from the official websites and mobile applications of each platform, supplemented by customer review aggregators (including platform-native review sections and third-party consumer feedback portals), published delivery and return policy documentation, and seller/buyer protection terms. Observable platform characteristics, service-performance indicators, customer-feedback evidence, and publicly accessible service information were systematically examined and subsequently encoded using the linguistic Z-number scales defined in Table 2 through structured expert assessment (data collection period: 2023–2024). Accordingly, the numerical analysis constitutes an experimental evaluation of real platform profiles rather than a purely illustrative example. Accordingly, the numerical analysis constitutes an experimental evaluation of the proposed CTLRI framework rather than a purely illustrative example.

4.1 International E-Commerce Platforms

Amazon demonstrates strong trust and loyalty through rapid Prime delivery, expanding same-day logistics, robust seller feedback mechanisms, and strict review authenticity policies. eBay provides transparent buyer–seller protection, well-established Top Rated standards, and highly transparent transaction monitoring, although delivery performance depends more heavily on individual sellers. Alibaba benefits from its Cainiao global logistics network and broad international ecosystem, while review reliability remains comparatively less centralized.

The international dataset was constructed from structured expert assessments and cross-validated evidence covering delivery performance, buyer and seller protection, service consistency, review credibility, purchase continuity, and other observable trust and loyalty indicators. Linguistic assessments for both the performance (A) and reliability (B) components were assigned after comparative evaluation of Amazon, eBay, and Alibaba to represent their actual operating characteristics as closely as possible.

Table 6 presents the linguistic assessments of the loyalty (ZL_1–ZL_4) and trust (ZT_1–ZT_4) indicators for the international platforms.

Table 6. Loyalty and trust indicators — International platforms

Indicator	Description	Amazon A/B	eBay A/B	Alibaba A/B
ZL_1	Purchase frequency/continuity	VH / VS	H / S	VH / S
ZL_2	Average order/basket value	H / S	M / MW	H / S
ZL_3	Repurchase probability	VH / S	H / S	H / S
ZL_4	Review/rating support	VH / VS	H / VS	H / MW
ZT_1	Overall/delivery reliability	VH / VS	M / S	VH / S

ZT_2	Service consistency	VH / S	H / S	H / S
ZT_3	Buyer/seller protection	H / S	VH / VS	H / MW
ZT_4	Review credibility	VH / VS	VH / VS	H / MW

4.1.1 Full Computation: Amazon

Amazon is used as the representative example to illustrate the complete computational procedure. All calculations follow the methodology presented in Section 3 and employ Aliev's Z-number arithmetic [55], thereby preserving the Z-number structure throughout the computational process. Specifically, the trust and loyalty aggregates are obtained from Eqs. (4) and (5), respectively; the integrated trust–loyalty Z-number is obtained from Eq. (6); Z-SWOT is evaluated using Eq. (7) together with the strategic dominance and context rules in Eqs. (8)–(21); and the final assessment, reliability, and overall Z-distances and CTLRI class are obtained from Eqs. (22) – (26).

Steps 1–2: Aggregation of Z_T and Z_L

$$Z_T = 0.30 \otimes (\mathrm{V, HVS}) \oplus 0.25 \otimes (\mathrm{V, HS}) \oplus 0.15 \otimes (\mathrm{H, S}) \oplus 0.30 \otimes (\mathrm{V, HVS})$$

$$Z_{TA} = [0.770, 0.955, 0.9770, 0.985]$$

$$Z_{TB} = [0.5722, 0.7454, 0.8647, 1.0000]$$

$$Z_L = 0.30 \otimes (\mathrm{V, HVS}) \oplus 0.15 \otimes (\mathrm{H, S}) \oplus 0.35 \otimes (\mathrm{V, HS}) \oplus 0.20 \otimes (\mathrm{V, HVS})$$

$$Z_{LA} = [0.770, 0.980, 0.980, 0.985]$$

$$Z_{LB} = [0.5906, 0.7304, 0.8427, 1.0000]$$

Step 3: Integration of Z_{TL}(α = 0.4)

$$Z_{TL} = 0.4 \otimes Z_T \oplus 0.6 \otimes Z_L$$

$$Z_{TL_A} = [0.770, 0.970, 0.970, 0.985]$$

$$Z_{TL_B} = [0.1424, 0.2232, 0.5752, 1.0000]$$

Steps 4–5: Z-SWOT Diagnostic

$$Z_S = (V, HVS),\ Z_W = (M, S),\ Z_O = (V, HS),\ Z_{Th} = (H, S)$$

$$Z_{SWOT_A} = [0.00, 0.70, 1.10, 1.30]$$

The resulting diagnostic Z-SWOT representation is

$$Z_{SWOT} = (A_{SWOT}, B_{SWOT}) = ([0.00, 0.45, 0.85, 1.05], [0.60, 0.65, 0.73, 0.83])$$

which preserves the complete Z-number structure in accordance with the Z-closure principle.

Steps 6–8: Computation of D_Z(Universal Ideal: VH, VS)

$$d_A = \sqrt{\frac{1}{4}[(0.77 - 1.00)^2 + (0.9625 - 1.00)^2 + (0.97 - 1.00)^2 + (0.985 - 1.00)^2]} = 0.1180$$

$$d_B = \sqrt{\frac{1}{4}[(0.1424-1.00)^2+(0.2232-1.00)^2+(0.5752-1.00)^2+(1.000-1.00)^2]} = 0.6163$$

$$D_Z = 0.5 \times 0.1180 + 0.5 \times 0.6163 = 0.3671$$

CTLRI Classification: MODERATE RISK

4.1.2 Summary — International Platforms

Expert Panel and Data Collection Protocol. The empirical evaluation presented in this section is based on structured linguistic assessments provided by a panel of five domain experts. The panel comprised: (i) two e-commerce platform analysts each with a minimum of seven years of professional experience in platform management and customer analytics; (ii) one digital marketing specialist with expertise in customer loyalty program design; (iii) one academic researcher specializing in fuzzy decision-making and Z-number modeling; and (iv) one senior operations manager from an Azerbaijani pharmaceutical e-commerce platform with direct knowledge of the local market. Each expert independently assigned linguistic values to all trust, loyalty, and Z-SWOT indicators using the corresponding group-calibrated linguistic scale defined in Table 2. Inter-expert agreement was assessed by Kendall's coefficient of concordance ($W = 0.81$, $p < 0.01$), indicating a high and statistically significant level of consensus within the panel. Final Z-number inputs for each criterion were obtained by arithmetic mean aggregation applied separately to the assessment (A) and reliability (B) components of individual expert judgments. The linguistic Z-SWOT indicator assignments reflect the most frequently reported strategic factors in the e-commerce and pharmaceutical platform literature, supplemented by expert knowledge of the Azerbaijani market environment, following a structured discussion session consistent with the Delphi approach.

Table 7 summarizes the CTLRI results obtained for the international platforms using the proposed reliability-preserving framework based on Aliev's Z-number arithmetic.

Table 7. CTLRI results — International platforms obtained using the proposed reliability-preserving CTLRI framework based on Aliev's Z-number arithmetic.

Platform	Z_{TL_A}	Z_{TL_B}	d_A	d_B	D_Z	CTLRI
Amazon	(0.770; 0.9625; 0.9655; 0.985)	(0.1424; 0.2232; 0.5752; 1.0000)	0.1180	0.6163	0.3671	MODERATE RISK
eBay	(0.5730; 0.7630; 0.7885; 0.8760)	(0.0043; 0.0156; 0.0349; 0.0811)	0.2732	0.9665	0.6199	HIGH RISK
Alibaba	(0.6600; 0.8250; 0.8415; 0.9300)	(0.0008; 0.0037; 0.0107; 0.0367)	0.2099	0.9871	0.5985	HIGH RISK

Under the group-calibrated evaluation, Aptekonline.az achieved the lowest Z-distance (0.3543) among the Azerbaijani platforms, indicating the closest proximity to the universal ideal profile. Although eBay and Alibaba exhibited competitive behavioral performance, their comparatively lower reliability levels resulted in larger Z-distance values and consequently a **HIGH RISK** classification under the proposed CTLRI decision rules. These findings demonstrate that the proposed framework simultaneously evaluates behavioral performance and assessment reliability, enabling platforms with similar performance but different confidence levels to be distinguished in a transparent and mathematically consistent manner.

4.2 Azerbaijani Pharmaceutical E-Commerce Platforms

The dataset for the Azerbaijani pharmaceutical e-commerce platforms was developed through direct web-based examination of each platform. Data collection involved systematic access to the official websites of Aptekonline.az, PharmOnline, Avismed.az, and Melhem.az, covering product catalogs and availability, delivery terms and conditions, pricing transparency, customer-service interface features, user review sections, and observable strategic characteristics such as promotional campaigns and loyalty program information. The collection period covered publicly accessible platform content during 2023–2024. The collected evidence was subsequently transformed into linguistic assessments and encoded as Z-numbers using the local-market linguistic scale presented in Table 2. Consequently, the resulting platform profiles are intended to reflect actual operating conditions as closely as permitted by the available evidence.

The four Azerbaijani platforms are evaluated against the same universal ideal (VH, VS) adopted for the international platforms, enabling direct cross-group comparison. The assigned indicators reflect local infrastructure conditions, where the leading platform achieves predominantly H-level performance, whereas the remaining platforms are characterized mainly by M-level assessments.

Table 8 presents the linguistic assessments of the loyalty (ZL_1–ZL_4) and trust (ZT_1–ZT_4) indicators for the Azerbaijani pharmaceutical e-commerce platforms.

Table 8. Loyalty and trust indicators — Azerbaijani pharmaceutical e-commerce platforms

Indicator	Description	Aptekonline	PharmOnline	Avismed	Melhem
ZL_1	Purchase frequency	H / VS	M / S	M / S	M / S
ZL_2	Average order value	H / S	M / S	M / MW	M / MW
ZL_3	Repurchase probability	H / S	M / S	M / S	M / MW
ZL_4	User rating support	H / S	M / MW	M / MW	M / S
ZT_1	Overall reliability	H / S	H / S	M / S	M / S
ZT_2	Delivery reliability	H / VS	M / S	M / MW	M / MW
ZT_3	Customer protection	H / S	M / S	M / MW	M / MW
ZT_4	Review authenticity	H / S	M / MW	M / S	M / S

4.2.1 Full Computation: Aptekonline.az

Aptekonline.az is selected as the representative local platform to illustrate the complete computational procedure. All calculations follow the methodology presented in Section 3 and employ Aliev's Z-number arithmetic [55], thereby preserving the Z-number structure throughout the computational process. Specifically, Eqs. (4) and (5) generate the trust and loyalty aggregates; Eq. (6) produces the integrated trust–loyalty Z-number; Eq. (7) and Eqs. (8)–(21) provide the Z-SWOT diagnostic, strategic dominance, and context interpretation; and Eqs. (22)–(26) generate the assessment distance, reliability distance, overall Z-distance, and final CTLRI classification.

Aptekonline.az exhibits consistently strong performance across all trust and loyalty indicators, with H-level assessments throughout the A component and a VS reliability level for purchase frequency.

Steps 1–2: Aggregation of Trust and Loyalty

Z_{LA} = (0.600, 0.750, 0.790, 0.900)
Z_{LR} = (0.068, 0.137, 0.223, 0.381)
Z_{TA} = (0.600, 0.765, 0.780, 0.900)
Z_{TR} = (0.055, 0.131, 0.243, 0.469)

Step 3: Trust–Loyalty Integration

Z_{TLA} = (0.600, 0.786, 0.786, 0.900)
Z_{TLR} = (0.096, 0.142, 0.159, 0.223)

Steps 4–5: Z-SWOT Diagnostic

Z_{SWOT_A} = (-0.40, -0.15, 0.40, 0.90)

The resulting Z-SWOT representation serves as a diagnostic description of the platform's strategic position while preserving the complete Z-number structure under the Z-closure principle.

Steps 6–8: Z-Distance Computation

$$d_A = 0.1275$$
$$d_B = 0.5811$$
$$D_Z = 0.3543$$

CTLRI Classification: MODERATE RISK

The obtained result indicates that although Aptekonline.az demonstrates relatively strong operational performance, the accumulated reliability component remains considerably distant from the universal ideal profile. Consequently, the platform is classified as **MODERATE RISK** according to the proposed CTLRI decision rules. The complete classification is obtained through the integrated evaluation of both behavioral performance (A) and assessment reliability (B), rather than from performance scores alone.

4.2.2 Remaining Platforms

The remaining three Azerbaijani platforms were evaluated using the same CTLRI computational framework described in Section 3. For each platform, trust and loyalty were aggregated using Eqs. (4) and (5), integrated through Eq. (6), interpreted through the Z-SWOT and strategic rules in Eqs. (7)–(21), and finally evaluated and classified using the Z-distance and CTLRI decision rules in Eqs. (22)–(26).

PharmOnline exhibits moderate trust performance owing to its H-level overall reliability indicator (ZT_1), although most loyalty indicators remain at the M level.

Z_{TL_A} = (0.336, 0.486, 0.553, 0.724)
Z_{TL_B} = (0.0002, 0.0022, 0.0077, 0.0270)
d_A = 0.3437, d_B = 0.9913, D_Z = 0.6675
CTLRI Classification: HIGH RISK

Avismed.az demonstrates predominantly M-level trust and loyalty assessments, accompanied by comparatively weak reliability values.

Z_{TL_A} = (0.429, 0.579, 0.6365, 0.786)
Z_{TL_B} = (0.0001, 0.0012, 0.0049, 0.0196)
d_A= 0.4127, d_B= 0.9936, D_Z= 0.7032
CTLRI Classification: HIGH RISK

Melhem.az exhibits an almost identical indicator profile to Avismed.az, reflecting similar operational characteristics and market conditions.

Z_{TL_A} = (0.381, 0.531, 0.597, 0.754)
Z_{TL_B} = (0.0002, 0.0011, 0.0036, 0.0134)
d_A= 0.4544, d_B= 0.9954, D_Z= 0.7249
CTLRI Classification: HIGH RISK

The three platforms exhibit substantially larger Z-distance values than Aptekonline.az. Under the decomposition defined in Eqs. (22)–(25), this difference arises from both weaker behavioral performance and considerably lower reliability levels; the corresponding HIGH RISK assignments follow the classification rule in Eq. (26). This demonstrates that the proposed CTLRI framework distinguishes alternatives by jointly

evaluating operational performance and confidence in the corresponding assessments.

4.2.3 Summary — Azerbaijani Pharmaceutical E-Commerce Platforms

Table 9 summarizes the CTLRI results obtained for the Azerbaijani pharmaceutical e-commerce platforms using the proposed reliability-preserving framework based on Aliev's Z-number arithmetic.

Table 9. CTLRI results — Azerbaijani pharmaceutical e-commerce platforms

Platform	Z_{TL_A}	Z_{TL_B}	d_A	d_B	D_Z	CTLRI
Aptekonline.az	(0.7560; 0.9500; 0.9500; 0.9780)	(0.2121; 0.3045; 0.5079; 0.9381)	0.1275	0.5811	0.3543	MODERATE RISK
PharmOnline	(0.5070; 0.6570; 0.7075; 0.8380)	(0.0002; 0.0019; 0.0069; 0.0259)	0.3437	0.9913	0.6675	HIGH RISK
Avismed.az	(0.429; 0.579; 0.6365; 0.786)	(0.0001; 0.0012; 0.0049; 0.0196)	0.4127	0.9936	0.7032	HIGH RISK
Melhem.az	(0.381; 0.531; 0.597; 0.754)	(0.0002; 0.0011; 0.0036; 0.0134)	0.4544	0.9954	0.7249	HIGH RISK

Aptekonline.az achieved the smallest Z-distance (0.3543) among the Azerbaijani platforms and therefore represents the strongest overall local performer. Nevertheless, its reliability component remains sufficiently distant from the universal ideal to yield a **MODERATE RISK** classification. PharmOnline, Avismed.az, and Melhem.az exhibit substantially larger distances owing to the combined effects of lower behavioral performance and weaker reliability, resulting in **HIGH RISK** classifications. These findings indicate that, within the Azerbaijani pharmaceutical e-commerce market, deficiencies in both operational capability and assessment reliability jointly influence the final CTLRI evaluation and clearly differentiate the competitive positions of the platforms.

4.3 Global Ranking and Cross-Group Comparison

The fixed universal ideal defined in Eq. (20) provides the common reference used for the exploratory cross-group comparison. The overall Z-distance calculated under Eqs. (22)–(25) is then used to rank all seven platforms in ascending order, while Eq. (26) determines the corresponding CTLRI risk class. This allows behavioral performance and assessment reliability to be evaluated simultaneously.

Note on cross-group comparability. The international and Azerbaijani platforms are evaluated using group-calibrated linguistic scales that differ in their

trapezoidal parameters. The global ranking presented below should therefore be interpreted as an exploratory comparison under group-specific scales rather than as a scale-invariant absolute ranking. The relative ordering within each group is fully comparable; the inter-group comparison is subject to the scale-dependence limitation acknowledged in Section 6.3.

Table 10. Global ranking of all evaluated platforms

Rank	Platform	Group	D_Z	CTLRI	Dominant Gap
1	Aptekonline.az	Azerbaijan	0.3543	MODERATE RISK	Reliability (B)
2	Amazon	International	0.3671	MODERATE RISK	Reliability (B)
3	Alibaba	International	0.5985	HIGH RISK	Reliability (B)
4	eBay	International	0.6199	HIGH RISK	Reliability (B)
5	PharmOnline	Azerbaijan	0.6675	HIGH RISK	Assessment + Reliability
6	Avismed.az	Azerbaijan	0.7032	HIGH RISK	Assessment + Reliability
7	Melhem.az	Azerbaijan	0.7249	HIGH RISK	Assessment + Reliability

Aptekonline.az achieved the smallest overall Z-distance (DZ=0.3543), and Amazon achieved the smallest Z-distance among international platforms (DZ=0.3671). Among all evaluated platforms, Aptekonline.az ranked first and outperformed Amazon, eBay, and Alibaba despite operating in a considerably smaller market. This result demonstrates that strong behavioral performance can partially compensate for infrastructure limitations when supported by relatively consistent reliability assessments.

In contrast, PharmOnline, Melhem.az, and Avismed.az occupy the lowest positions because both their assessment quality (A) and reliability (B) remain substantially distant from the universal ideal. The ranking therefore confirms that the proposed CTLRI framework evaluates alternatives according to the combined influence of behavioral performance and confidence in the underlying assessments rather than performance alone.

4.4 Z-SWOT Strategic Diagnostics

While the CTLRI index provides the final quantitative evaluation, the proposed Z-SWOT model offers an additional diagnostic interpretation of the strategic position of each platform. Unlike the CTLRI distance measure, the Z-SWOT analysis does not modify the final ranking; instead, it explains the internal strategic factors responsible for the observed performance. The adopted formulation maintains a fixed universal ideal: the universal ideal profile remains fixed across all platforms to ensure comparability, while the Z-SWOT module serves as a context-aware interpretive layer that explains the sources of performance gaps without altering the final D_Z ranking. Z-SWOT therefore functions exclusively as a context-aware diagnostic layer

Table 11. Z-SWOT component assignments and resulting strategic indicators

Platform	Z_S	Z_W	Z_O	Z_{Th}	A_{SWOT}	B_{SWOT}
Amazon	(VH,VS)	(M,S)	(VH,S)	(H,S)	(0.00; 0.45; 0.85; 1.05)	(0.6003; 0.6537; 0.7325; 0.8298)
eBay	(H,VS)	(H,S)	(H,MW)	(H,S)	(-0.60; -0.20; 0.20; 0.60)	(0.4150; 0.4882; 0.5985; 0.7106)
Alibaba	(VH,S)	(M,MW)	(VH,S)	(H,MW)	(0.00; 0.45; 0.85; 1.05)	(0.5483; 0.5812; 0.6283; 0.6982)
Aptekonline.az	(H,S)	(M,MW)	(H,S)	(H,S)	(-0.40; 0.15; 0.40; 0.90)	(0.1073; 0.1527; 0.2198; 0.3494)
PharmOnline	(H,S)	(M,S)	(M,MW)	(H,S)	(-0.70; -0.15; 0.15; 0.70)	(0.2689; 0.3270; 0.3634; 0.4323)
Avismed.az	(M,MW)	(M,MW)	(M,MW)	(H,S)	(-1.00; -0.45; -0.10; 0.50)	(0.0941; 0.1101; 0.1315; 0.1958)
Melhem.az	(M,MW)	(M,MW)	(M,MW)	(H,S)	(-1.00; -0.45; -0.10; 0.50)	(0.0941; 0.1101; 0.1315; 0.1958)

Following Definitions 1–3 presented in Section 3.9, the Z-distances between each SWOT component and the universal ideal were computed using the Z-distance formulation in Eqs. (22)–(25), while the dominance and boundary rules in Eqs. (8)–(21) were used to assign the strategic regions. The resulting strategic regions are summarized in Table 12.

Table 12. Strategic region assignment based on Z-SWOT dominance

Platform	D_S	D_W	D_O	D_{Th}	R_{Int}	R_{Ext}	Region
Amazon	0.0000	0.3037	0.0901	0.1803	S	O	I
eBay	0.0901	0.1803	0.2541	0.1803	S	Th	II
Alibaba	0.0901	0.3775	0.0901	0.2541	S	O	I
Aptekonline.az	0.2302	0.5238	0.2302	0.2302	S	O*	I*
PharmOnline	0.2302	0.3621	0.5238	0.2302	S	Th	II
Avismed.az	0.5238	0.5238	0.5238	0.2302	S*	Th	II*
Melhem.az	0.5238	0.5238	0.5238	0.2302	S*	Th	II*

Boundary conditions indicate equal Z-distances, where the corresponding tie-breaking rule defined in Section 3.9 was applied.

The Z-SWOT diagnostics reveal clear strategic differences among the evaluated platforms. Amazon and Alibaba belong to **Region I**, indicating simultaneous dominance of strengths and opportunities. Amazon is unique in achieving $D_S = 0$, meaning that its strength profile coincides with the universal ideal. Aptekonline.az is also positioned in Region I but lies on the decision boundary because its opportunity and threat distances are identical.

Conversely, eBay, PharmOnline, Avismed.az, and Melhem.az are classified within **Region II**, where internal strengths remain dominant but external threats outweigh available opportunities. For the Azerbaijani platforms, this pattern reflects the combined influence of logistics limitations, market maturity, and regulatory constraints on long-term competitiveness. Notably, no platform falls into Regions III or IV, indicating that strength dominance is maintained across all evaluated platforms despite considerable differences in overall CTLRI performance.

4.4.1 Platform-Level Strategic Interpretation

For each platform, the proposed framework evaluates five sequential dimensions: internal Z-dominance, external Z-dominance, strategic region, managerial implication, and the final CTLRI interpretation:

- **Amazon (Region I, $D_Z = 0.3671$, MODERATE RISK)**
  Amazon exhibits the strongest strategic profile among all evaluated platforms. Its Strength Z-number coincides with the universal ideal ($D_S = 0.0000$), while opportunities dominate threats ($D_O < D_{Th}$), leading to Region I classification. The final CTLRI value is mainly influenced by the reliability component ($d_B = 0.6163$), whereas the assessment distance remains very small ($d_A = 0.1180$). This indicates that Amazon's trust–loyalty profile is exceptionally strong and that the remaining gap originates primarily from reliability propagation through the Aliev Z-number arithmetic.
- **eBay (Region II, $D_Z = 0.6199$, HIGH RISK).**eBay maintains internal strength dominance but is externally constrained because opportunities are assessed with lower reliability than threats. Although both opportunity and threat assessments exhibit H-level performance, the lower reliability assigned to opportunities produces Region II classification. The CTLRI result reflects both moderate behavioral deficiencies and reliability attenuation, suggesting that improvements in overall service reliability and customer engagement would most effectively reduce the overall Z-distance.
- **Alibaba (Region I, $D_Z = 0.5985$, HIGH RISK).**Alibaba demonstrates strong internal capabilities and favorable external opportunities, resulting in Region I classification. However, repeated aggregation of several MW-reliability assessments substantially increases the reliability distance, yielding the highest d_B among the international platforms. Consequently, improving the consistency of seller reliability and review quality represents the principal direction for enhancing its CTLRI performance.
- **Aptekonline.az (Region I*, $D_Z = 0.3543$, MODERATE RISK).**Aptekonline.az is the strongest Azerbaijani platform and the only local

platform assigned to Region I. Its internal strengths clearly dominate weaknesses, while equal opportunity and threat distances place the platform on the Region I boundary. The CTLRI result demonstrates that relatively strong behavioral performance is partially offset by reliability attenuation, indicating that improvements in assessment reliability would directly enhance the platform's overall strategic position.

- **PharmOnline (Region II, $D_Z = 0.6675$, HIGH RISK).** PharmOnline exhibits moderate internal strength but substantial external constraints. Opportunities remain both less competitive and less reliable than threats, resulting in Region II classification. The large values of both d_A and d_B indicate that improvements are required simultaneously in operational performance and service reliability to achieve meaningful reductions in the overall Z-distance.
- **Avismed.az and Melhem.az (Region II*, $D_Z \approx 0.683$, HIGH RISK).** Avismed.az and Melhem.az exhibit nearly identical strategic structures and CTLRI results. Their strengths, weaknesses, and opportunities receive identical linguistic assessments, producing a boundary Region II configuration. Consequently, no single strategic dimension provides a competitive advantage, while threats remain substantially closer to the universal ideal than opportunities. Meaningful improvement therefore requires simultaneous enhancement of both performance and reliability across all trust and loyalty indicators.

4.4.2 Substantive Interpretation of Strategic Regions

The strategic regions identified by the proposed Z-SWOT framework provide an interpretable explanation of the CTLRI results by jointly considering assessment quality (A), reliability (B), and strategic context:

- **Region I** represents the most favorable strategic position, where both internal strengths and external opportunities dominate. Amazon is the only platform whose Strength Z-number coincides with the universal ideal, yielding $D_S = 0.0000$. This result reflects the simultaneous achievement of the highest performance and reliability levels across its principal strategic capabilities. Alibaba also belongs to Region I, demonstrating strong internal capabilities and favorable external opportunities, although lower reliability assessments produce a larger overall CTLRI distance. Aptekonline.az is assigned to the boundary Region I* because opportunities and threats are equally distant from the universal ideal. Nevertheless, it remains the strongest Azerbaijani platform owing to its clear internal strength dominance and consistently high behavioral performance.
- **Region II** characterizes platforms whose internal strengths remain stronger than their weaknesses, while external threats outweigh available opportunities. eBay illustrates this situation through a reliability asymmetry: opportunities and threats exhibit comparable performance levels, but opportunities are evaluated with lower

confidence, resulting in threat dominance. PharmOnline experiences a more pronounced constraint, as both the performance and reliability of its opportunity profile remain substantially below those of its threat profile. Consequently, improvements in both operational capability and assessment reliability are required to reduce its strategic disadvantage.

- **Region II*** represents the least favorable strategic configuration observed in this study. Avismed.az and Melhem.az exhibit nearly identical strategic structures, with strengths, weaknesses, and opportunities receiving the same linguistic assessments. As a result, no meaningful internal strategic advantage is identified, while threats remain considerably closer to the universal ideal than opportunities. This finding indicates that isolated improvements are unlikely to change their strategic position; instead, comprehensive enhancement of both behavioral performance and reliability across all trust and loyalty dimensions is required.

Overall, the proposed Z-SWOT framework complements the CTLRI index by explaining **why** platforms with similar behavioral performance may receive different final classifications. Unlike conventional multi-criteria decision-making methods that evaluate only assessment values, the proposed approach explicitly incorporates assessment reliability and strategic dominance, thereby providing a more comprehensive interpretation of platform competitiveness and managerial priorities.

5. Discussion

5.1 Methodological Interpretation and Comparison with Existing Approaches

The proposed CTLRI framework should be interpreted as an integrated reliability-aware decision-support framework rather than merely a customer loyalty index. Unlike conventional evaluation models that aggregate performance indicators into a single score, CTLRI preserves the semantic distinction between behavioral assessment (A) and assessment reliability (B) throughout the entire computational process. The reported results are obtained through the sequential application of the trust aggregation (Eq. (4)), loyalty aggregation (Eq. (5)), trust–loyalty integration (Eq. (6)), Z-SWOT strategic analysis (Eq. (7)), strategic dominance, strategic-gap analysis, and context-aware interpretation (Eqs. (8)–(21)), and the proposed Z-distance together with the CTLRI classification rules (Eqs. (22)–(26)). Consequently, every result presented in this study is directly traceable to the underlying mathematical formulation. Criterion weights are applied exclusively to the assessment component (A), whereas the reliability component (B) is propagated through the complete Z-number arithmetic in accordance with Aliev's Z-calculus, preserving expert confidence throughout the computational pipeline.

Compared with conventional fuzzy MCDM methods, the proposed framework explicitly incorporates assessment reliability into every stage of the decision-making process. Classical fuzzy approaches successfully model linguistic uncertainty but

generally aggregate all information into a single fuzzy assessment, thereby losing the distinction between the quality of an assessment and the confidence associated with it. In contrast, CTLRI maintains these two dimensions separately and integrates them only through formally defined Z-number operations, allowing reliability to influence the final decision in a mathematically transparent manner.

The framework also differs substantially from widely used customer loyalty evaluation techniques. Net Promoter Score (NPS) represents customer loyalty using a single recommendation indicator, RFM analysis focuses exclusively on behavioral transaction patterns, and Customer Lifetime Value (CLV) estimates long-term economic contribution. Although these approaches are valuable in their respective contexts, they neither incorporate assessment reliability nor account for the strategic environment in which customer loyalty evolves. CTLRI complements these methods by simultaneously evaluating behavioral performance, reliability attenuation, strategic position, and overall proximity to the ideal solution within a unified Z-number framework.

Compared with data-driven machine-learning approaches, the proposed methodology emphasizes interpretability and traceability rather than predictive optimization. Machine-learning models frequently provide accurate predictions but often operate as black-box systems whose internal reasoning is difficult to explain. In contrast, every computational stage of CTLRI—from trust and loyalty aggregation to strategic Z-SWOT analysis and the final Z-distance calculation—can be interpreted mathematically, allowing decision-makers to identify precisely which factors contribute to the final platform ranking. This characteristic makes the proposed framework particularly suitable for explainable decision-support in environments characterized by uncertain, incomplete, and reliability-sensitive information.

Comparison with established MCDM baselines. To examine whether the CTLRI ranking is consistent with, and adds value beyond, established multi-criteria methods, four baselines were applied to the same aggregated platform-level Z-numbers Z_{TL} reported above. (i) Crisp TOPSIS uses the centroids of $Z_{TL,A}$ and $Z_{TL,B}$ as two benefit criteria with vector normalization. (ii) Fuzzy TOPSIS uses only the trapezoidal assessment component $Z_{TL,A}$, so reliability is disregarded. (iii) Z-TOPSIS is an adaptation in the spirit of Krohling and Pacheco [52] in which $Z_{TL,A}$ and $Z_{TL,B}$ enter as two fuzzy benefit criteria, with the positive and negative ideal solutions set to (1,1,1,1) and (0,0,0,0) and the same RMS trapezoid distance as in CTLRI; it is not a re-implementation of the original TODIM/TOPSIS procedure. (iv) Z-VIKOR applies VIKOR ($v = 0.5$) to the same two criteria using centroids. Equal criterion weights were used in all baselines; because the aggregation of the eight indicators had already been carried out with the AHP weights, these baselines compare only the final ranking stage.

Table 13. Ranking comparison of CTLRI with baseline methods: score (rank). Lower is better for D_Z and Z-VIKOR Q; higher is better for TOPSIS closeness coefficients.

Platform	CTLRI D_Z	Crisp TOPSIS CC	Fuzzy TOPSIS CC	Z-TOPSIS CC	Z-VIKOR Q
Aptekonline.az	0.3543 (1)	0.9917 (1)	0.8774 (2)	0.6759 (1)	0.0041 (1)
Amazon	0.3671 (2)	0.9891 (2)	0.8868 (1)	0.6738 (2)	0.0056 (2)

Alibaba	0.5985 (3)	0.1518 (3)	0.7962 (3)	0.4121 (3)	0.8377 (3)
eBay	0.6199 (4)	0.1317 (4)	0.7351 (4)	0.3931 (4)	0.8418 (4)
PharmOnline	0.6675 (5)	0.0731 (5)	0.6668 (5)	0.3443 (5)	0.9265 (5)
Avismed.az	0.7032 (6)	0.0284 (6)	0.6007 (6)	0.3097 (6)	0.9719 (6)
Melhem.az	0.7249 (7)	0.0000 (7)	0.5613 (7)	0.2887 (7)	1.0000 (7)
Spearman ρ	**1.000**	**1.000**	**0.964**	**1.000**	**1.000**

The results are given in Table 13. Z-TOPSIS, Crisp TOPSIS and Z-VIKOR reproduce the CTLRI ranking exactly (Spearman ρ = 1.000 in each case), with Aptekonline.az first and Amazon second, followed by Alibaba, eBay, PharmOnline, Avismed.az and Melhem.az. Fuzzy TOPSIS, which ignores reliability, is the only method that reverses the leading pair, placing Amazon first and Aptekonline.az second (ρ = 0.964); all other positions are unchanged. The margin between the two leading platforms is small under every method (CTLRI: 0.3543 vs 0.3671), so this first-place ordering should be regarded as sensitive to the treatment of reliability rather than as a decisive difference. Because the baselines yield essentially the same rank order, the contribution of CTLRI does not lie in a different ranking but in what the ranking explains: only the CTLRI formulation separates the assessment deficiency d_A from the reliability deficiency d_B (Table 14), classifies platforms into three risk levels, and links the gap to Z-SWOT diagnostic regions, none of which the compared methods provide. The baseline scores depend on the stated normalization, weights and distance choices and are reported for comparison only.

5.2 Platform-Level Discussion

5.2.1 Amazon: Performance Proximity and Reliability Propagation

Amazon achieved the smallest Z-distance among all international platforms ($D_Z = 0.3671$), representing the closest proximity to the universal ideal among the international group, and ranking second overall behind Aptekonline.az (D_Z =0.3543). This result is primarily explained by the exceptionally small assessment distance ($d_A = 0.1180$), which reflects consistently high trust and loyalty evaluations across all criteria. Nevertheless, the reliability component ($d_B = 0.6163$) remains the dominant contributor to the overall CTLRI value. Rather than indicating poor platform reliability, this behavior reflects the cumulative propagation of reliability information through repeated Aliev Z-number operations. Consequently, Amazon illustrates an important characteristic of the proposed framework: even platforms exhibiting near-ideal behavioral performance retain measurable reliability attenuation after multi-stage aggregation. The Z-SWOT analysis further supports this interpretation by assigning Amazon to Region I, confirming the simultaneous dominance of internal strengths and external opportunities.

5.2.2 Aptekonline.az: Local Leader with Structural Constraints

Among the Azerbaijani platforms, Aptekonline.az achieved the best overall performance ($D_Z = 0.3543$), ranking first in the exploratory global comparison, slightly outperforming Amazon (D_Z = 0.3671). The platform consistently attained H-level behavioral assessments across both trust and loyalty indicators, demonstrating that strong operational performance can be achieved even within a developing e-commerce

environment. However, its reliability component remained considerably lower than that of the leading international platform, reflecting the narrower reliability range adopted for the local linguistic scale together with the cumulative attenuation produced by Z-number aggregation. Although its overall CTLRI classification is **MODERATE RISK**, the platform outperformed both eBay and Alibaba in the global ranking, indicating that consistent behavioral performance may partially compensate for limitations in infrastructure and assessment reliability. This finding highlights Aptekonline.az as the strongest representative of the local market and identifies logistics, review credibility, and customer confidence as the principal directions for future improvement.

5.2.3 Reliability Dominance in CTLRI

One of the most significant findings of this study is the systematic dominance of the reliability component in the final CTLRI evaluation. For every evaluated platform, the reliability distance (d_B) exceeded the behavioral distance (d_A), although the magnitude of this dominance varied across platforms. This pattern is not an artifact of the proposed methodology but a direct mathematical consequence of Aliev's Z-number arithmetic. During sequential trust aggregation, loyalty aggregation, and trust–loyalty integration, repeated multiplication and aggregation of reliability fuzzy numbers gradually increase the deviation of the resulting B-component from the universal ideal. Consequently, CTLRI captures the cumulative effect of reliability attenuation that remains invisible in conventional crisp or fuzzy evaluation methods.

The decomposition of the overall Z-distance into its behavioral (d_A) and reliability (d_B) components also provides valuable managerial insight. Platforms with similar CTLRI values may require fundamentally different improvement strategies because their dominant deficiencies originate from different sources. For example, Alibaba and eBay exhibit comparable overall CTLRI values but substantially different strategic structures. Alibaba belongs to Region I, where strong internal capabilities and favorable opportunities are maintained despite significant reliability attenuation. In contrast, eBay is assigned to Region II because lower confidence in opportunity assessments shifts the strategic balance toward external threats. This distinction demonstrates that the proposed framework explains not only the magnitude of performance differences but also their underlying causes.

Table 14. Z-distance decomposition and investment priorities d_A/d_B

Platform	d_A	d_B	Dominant gap	Recommended investment priority
Amazon	0.1180	0.6163	Reliability propagation	Preserve ecosystem quality; continuously monitor reliability consistency
Aptekonline.az	0.1275	0.5811	Reliability	Improve VS-level reliability, customer trust, and loyalty programs

eBay	0.2732	0.9665	Reliability	Increase average order value, strengthen buyer protection and service consistency
Alibaba	0.2099	0.9871	Reliability	Standardize seller reliability and review credibility
PharmOnline	0.3437	0.9913	Behavior + Reliability	Improve logistics, service quality, and review infrastructure simultaneously
Avismed.az	0.4127	0.9936	Behavior + Reliability	Upgrade logistics infrastructure, customer services, and operational standardization
Melhem.az	0.4544	0.9954	Behavior + Reliability	Upgrade logistics infrastructure, customer services, and operational standardization

Table 14 demonstrates that identical CTLRI values do not necessarily imply identical managerial priorities. International platforms are influenced predominantly by reliability attenuation, whereas the weaker-performing Azerbaijani platforms require simultaneous improvements in behavioral performance and assessment reliability. Consequently, the proposed decomposition transforms the CTLRI index from a purely evaluative measure into a practical diagnostic tool capable of guiding strategic investment decisions.d_A/d_B

5.3 Strategic Interpretation

The proposed Z-SWOT framework extends conventional SWOT analysis by representing each strategic dimension as a complete Z-number, thereby simultaneously modeling assessment quality and assessment reliability. Unlike traditional SWOT approaches, which remain largely qualitative, the proposed framework provides a mathematically rigorous interpretation of strategic dominance while preserving the semantic distinction between the assessment (A) and reliability (B) components. Consequently, Z-SWOT serves as a diagnostic layer that complements the CTLRI index without modifying the computed Z-distance values.

The experimental results demonstrate that Z-SWOT regions provide a complementary strategic interpretation of the CTLRI results rather than a one-to-one mapping to the CTLRI risk classes. Amazon, assigned to Region I, exhibits the strongest strategic profile, characterized by dominant internal strengths and favorable external opportunities. Its first Z-SWOT coordinate equals zero, indicating the closest proximity to the universal ideal and showing that its favorable strategic positioning coexists with a reliability-driven CTLRI gap. Aptekonline.az occupies the boundary Region I*, reflecting strong internal capabilities combined with a balanced external

environment in which opportunities and threats are equally influential. Although classified as **MODERATE RISK**, it remains the strongest representative of the Azerbaijani market owing to its comparatively favorable strategic structure.

Conversely, PharmOnline, Avismed.az, and Melhem.az exhibit increasingly negative first Z-SWOT coordinates, corresponding to progressively weaker strategic positions and higher CTLRI risk categories. This relationship demonstrates that infrastructure limitations, weaker competitive capabilities, and stronger external threats contribute directly to the deterioration of behavioral performance and the increase in the overall Z-distance. Therefore, the proposed Z-SWOT formulation provides an interpretable explanation of why platforms with similar behavioral assessments may occupy different strategic positions when assessment reliability and strategic context are considered simultaneously.

More importantly, the proposed framework distinguishes between **evaluation** and **interpretation**. The CTLRI index determines the quantitative ranking of the platforms, whereas Z-SWOT explains the strategic mechanisms underlying those rankings. This separation preserves mathematical consistency while providing decision-makers with complementary information for strategic planning. Consequently, the proposed framework integrates quantitative evaluation and qualitative strategic reasoning within a unified Z-number decision-support methodology.

5.4 Key Findings and Cross-Group Analysis

The experimental evaluation demonstrates several important characteristics of the proposed CTLRI framework.

First, Z-numbers enable uncertainty and reliability to be modeled simultaneously, allowing qualitative expert assessments to be incorporated into a mathematically rigorous decision-making framework. Unlike conventional approaches, reliability is preserved throughout the complete computational process and contributes explicitly to the final platform evaluation.

Second, the integration of Z-SWOT significantly improves the interpretability of the decision process. Platforms with similar trust and loyalty scores may receive different final classifications because their strategic contexts differ. The proposed framework therefore combines quantitative evaluation with strategic diagnosis, enabling a more comprehensive understanding of platform competitiveness.

Third, the decomposition of the CTLRI index into behavioral (d_A) and reliability (d_B) components identifies the dominant source of performance deficiency for each platform. International platforms are affected primarily by reliability attenuation, whereas the weaker-performing Azerbaijani platforms require simultaneous improvements in both behavioral performance and assessment reliability. Finally, the complete computational pipeline remains fully explainable. Every intermediate result can be traced directly to the corresponding mathematical formulation, providing transparency that is difficult to achieve using conventional black-box decision-support techniques.

Table 15. Cross-group comparative analysis

Metric	International Platforms (Best)	Azerbaijani Platforms (Best)	Comparative Gap
Minimum D_Z	0.3671 (Amazon)	0.3543 (Aptekonline.az)	~3.6% larger (DZ: 0.3671 vs 0.3543, difference = 0.0128)
Maximum $Z_{TL_A}[4]$	0.985 (Amazon)	0.900 (Aptekonline.az)	$\Delta = 0.085$
MODERATE RISK platforms	1 / 3	1 / 4	—
HIGH RISK platforms	2 / 3	3 / 4	—
Dominant gap	Reliability propagation	Reliability scale limitations	Same dimension, different origin

Table 15 shows systematic group-level differences, although the exploratory global ranking also reveals that the leading Azerbaijani platform, Aptekonline.az, slightly outperforms Amazon in overall Z-distance. Nevertheless, the comparison also demonstrates that the underlying causes of this gap differ. International platforms are influenced primarily by cumulative reliability propagation through the Z-number aggregation process, whereas the Azerbaijani platforms are additionally constrained by lower behavioral performance and less mature digital infrastructure. Consequently, the proposed framework distinguishes not only how much the groups differ but also why these differences arise, thereby providing a more informative basis for strategic benchmarking than conventional single-score evaluation methods.

5.5 Practical Implications, Limitations, and Future Research

The proposed CTLRI framework has several practical implications for reliability-aware decision-making in e-commerce environments. Unlike conventional customer loyalty measures, which primarily quantify behavioral performance, the proposed methodology simultaneously evaluates behavioral assessment, assessment reliability, and strategic context within a unified Z-number framework. Consequently, managers can identify not only the overall performance of a platform but also the underlying factors responsible for its current competitive position.

The decomposition of the overall CTLRI value into the behavioral (d_A) and reliability (d_B) components provides direct managerial guidance for strategic resource allocation. Platforms dominated by behavioral deficiencies require improvements in operational capability, service quality, logistics, and customer engagement, whereas platforms dominated by reliability attenuation should prioritize strengthening review credibility, information transparency, customer trust, and consistency of service delivery. This diagnostic capability enables managers to develop targeted improvement strategies rather than relying solely on aggregate performance scores.

The proposed framework is also applicable to a broad range of practical decision-support tasks, including customer relationship management, loyalty program evaluation, e-commerce platform benchmarking, strategic planning, supplier evaluation, and other multi-criteria decision-making problems characterized by uncertain or reliability-sensitive information. Because every computational stage remains mathematically interpretable, the framework is particularly suitable for

explainable intelligent decision-support systems in which transparency is as important as predictive accuracy.

Despite these advantages, several limitations should be acknowledged. The present implementation relies on expert linguistic assessments and trapezoidal Z-number representations, both of which may introduce a degree of subjectivity into the evaluation process. Furthermore, the experimental validation is based on a representative set of international and Azerbaijani e-commerce platforms. Although these case studies demonstrate the applicability of the proposed methodology, broader empirical validation using larger industrial datasets would further strengthen the generalisability of the results.

Future research may proceed in two complementary directions. Methodological extensions include: subjective, objective, and hybrid weighting strategies — including AHP and the Delphi method (subjective), CRITIC and entropy-based weighting (objective), and Best-Worst Method (hybrid) to reduce dependence on expert judgment; alternative Z-number representations to improve modeling flexibility; dynamic CTLRI models for temporal platform monitoring; and machine-learning-assisted calibration of linguistic assessments. Application-oriented extensions include adaptation to healthcare, supply-chain management, financial services, smart manufacturing, and recommendation systems, as well as validation using real-time transactional platform data to further strengthen generalisability.

6. Conclusion

6.1 Scientific Contributions and Principal Findings

(1) This study proposed the **Customer Trust–Loyalty Reliability Index (CTLRI)**, an integrated Z-number-based decision-support framework for evaluating customer trust, customer loyalty, and strategic context under uncertainty. Unlike conventional customer loyalty evaluation methods, the proposed framework explicitly preserves the semantic distinction between the assessment component (A) and the reliability component (B) throughout the entire computational process. By combining reliability-preserving aggregation, Z-SWOT strategic modeling, context-aware interpretation, and the proposed Z-distance formulation, CTLRI provides a transparent, interpretable, and reproducible methodology for reliability-aware multi-criteria decision-making.

The principal methodological contribution of this research lies in preserving assessment reliability independently of behavioral performance during criterion weighting and aggregation. Criterion weights influence only the assessment component (A), whereas the reliability component (B) is propagated through Aliev's Z-number arithmetic without semantic modification. Consequently, the proposed framework evaluates not only platform performance but also the confidence associated with that performance, providing information that cannot be obtained using conventional fuzzy, crisp, or scalar decision-making approaches.

From a practical perspective, the proposed methodology enables managers to benchmark digital platforms, distinguish between behavioral and reliability-related performance deficiencies, prioritize improvement strategies, and support transparent

strategic decision-making under uncertain and heterogeneous information. Beyond customer loyalty assessment, the proposed CTLRI framework establishes a general reliability-aware Z-number decision paradigm that can be adapted to a broad range of uncertainty-driven multi-criteria decision-making problems, including healthcare, supply-chain management, financial evaluation, intelligent manufacturing, and public-sector decision support.

The experimental evaluation produced four principal findings:

Aptekonline.az achieved the smallest overall Z-distance (D_Z =0.3543), ranking first globally. Amazon ranked second overall (D_Z=0.3671) and was the only platform whose Strength Z-number coincided with the universal ideal (D_S =0.0000). Nevertheless, Amazon's remaining gap originated primarily from reliability attenuation (d_B =0.6163 >> d_A =0.1180), demonstrating that trust-related uncertainty rather than behavioral performance limits the final CTLRI value.

- **(2) Reliability-sensitive evaluation may alter conventional platform rankings.** Aptekonline.az ($D_Z = 0.3543$, Region I*) achieved the first position in the exploratory global ranking, outperforming both eBay ($D_Z = 0.6199$) and Alibaba ($D_Z = 0.5985$). This result demonstrates that consistent behavioral performance combined with favorable strategic structure may compensate for market size differences when assessment reliability is explicitly incorporated into the evaluation process.
- **(3) Strategic region assignment is determined primarily by external Z-dominance.** All evaluated platforms exhibited internal strength dominance ($R_{Int} = S$ or S^*). The decisive factor separating Region I from Region II was the relationship between opportunities and threats, specifically whether the opportunity Z-number possessed sufficient reliability to satisfy $D_O < D_{Th}$. This finding highlights the importance of governance, transparency, and confidence in strategic evaluation beyond operational performance alone.
- **(4) Behavioral performance and reliability contribute differently across platform groups.** The proposed decomposition of the overall Z-distance into behavioral (d_A) and reliability (d_B) components revealed a clear structural distinction. International platforms were affected primarily by reliability attenuation, whereas the Azerbaijani platforms exhibited deficiencies in both behavioral performance and reliability. This distinction is not observable in conventional single-score evaluation methods and provides managers with actionable information for prioritizing strategic investments and improvement initiatives.

These findings demonstrate that CTLRI extends existing Z-number decision-support methodologies by integrating behavioral evaluation, reliability preservation, and strategic diagnosis into a unified computational framework. The proposed

methodology therefore provides both theoretical advances in reliability-aware decision modeling and practical guidance for transparent strategic evaluation under uncertainty.

6.2 Managerial Recommendations

The proposed CTLRI framework extends beyond platform evaluation by providing practical guidance for strategic decision-making. Because the overall CTLRI value is decomposed into behavioral (d_A) and reliability (d_B) components, managers can identify whether performance deficiencies originate from operational capability, assessment reliability, or a combination of both. This diagnostic capability enables more effective allocation of organizational resources than conventional single-score evaluation methods.

The managerial recommendations derived from the experimental results are summarized in Table 16.

Table 16. CTLRI-based managerial recommendations

CTLRI Classification	Platform	D_Z	Strategic priority	Recommended managerial action
MODERATE RISK	Amazon	0.3671	Reliability preservation	Maintain ecosystem quality, continuously monitor reliability consistency, and preserve customer trust across the complete service chain.
MODERATE RISK	Aptekonline.az	0.3543	Reliability enhancement	Strengthen customer confidence, expand loyalty programs, improve review credibility, and increase VS-level reliability.
HIGH RISK	eBay	0.6199	Service consistency	Improve average order value, strengthen buyer protection mechanisms, and enhance transaction consistency.
HIGH RISK	Alibaba	0.5985	Reliability standardization	Improve seller verification, standardize review quality, and increase reliability consistency across the marketplace ecosystem.
HIGH RISK	PharmOnline	0.6675	Operational improvement	Enhance logistics, customer registration, review infrastructure, and overall service quality.

HIGH RISK	Avismed.az	0.7032	Infrastructure development	Invest in logistics capacity, customer service, operational standardization, and digital platform quality.
HIGH RISK	Melhem.az	0.7249	Infrastructure development	Improve logistics capability, customer services, platform reliability, and service standardization.

The recommendations presented in Table 16 illustrate that platforms with similar CTLRI classifications do not necessarily require identical improvement strategies. International platforms generally benefit from measures that preserve or strengthen assessment reliability, whereas the lower-ranked Azerbaijani platforms require simultaneous improvements in operational performance, service quality, and customer confidence. Therefore, the proposed CTLRI framework supports not only platform benchmarking but also evidence-based prioritization of strategic investments according to the dominant source of performance deficiency.

6.3 Limitations and Future Research

Although the proposed CTLRI framework provides a transparent and reliability-aware methodology for evaluating customer trust and loyalty under uncertainty, several limitations should be acknowledged:

- **First**, the present implementation relies on expert linguistic assessments to construct the trust, loyalty, and Z-SWOT indicators. Although Z-numbers explicitly represent both assessment quality and assessment reliability, the assignment of linguistic terms remains dependent on expert judgment and may therefore introduce evaluator-specific variability. Future studies may reduce this subjectivity through structured expert consensus procedures, such as the Delphi method, or by integrating data-driven calibration techniques based on historical observations.
- **Second,** the weighting schemes adopted for the trust and loyalty criteria are derived from a structured expert elicitation process based on pairwise AHP comparisons, as described in Section 3.4. Although a sensitivity analysis confirmed that plus or minus 15 per cent perturbations in individual weights do not alter the final platform ranking, the weights nonetheless depend on expert judgment and may therefore require recalibration for different application domains. Future studies may investigate objective weighting techniques — including CRITIC, entropy-based weighting, Best-Worst Method (BWM), or fully data-driven calibration — to reduce subjectivity and extend the generalisability of the CTLRI framework.
- **Third**, the current framework represents platform performance at a single observation period. In practice, customer trust, loyalty, and strategic conditions evolve continuously in response to market competition, technological innovation, regulatory changes, and customer behavior. Consequently, extending the proposed framework to a dynamic CTLRI model

capable of incorporating temporal information would provide a more comprehensive representation of platform evolution.

- Fourth, the separate linguistic scales for international and Azerbaijani platforms — while grounded in their different operational contexts — introduce a degree of scale-dependence in cross-group distance comparisons. Future work should investigate formal scale calibration procedures to ensure full comparability across platform groups.

Despite these limitations, the present study demonstrates that explicit modeling of assessment reliability substantially improves the interpretability of multi-criteria decision-making under uncertainty. The proposed CTLRI framework therefore provides a transparent foundation for reliability-aware intelligent decision-support under uncertainty and offers a promising direction for future Z-number research and real-world deployment. The proposed methodology is readily applicable to practical decision-support systems where both evaluation quality and confidence in the available information must be considered simultaneously.

Future research may extend the proposed framework in several complementary directions:

- **Machine learning-assisted Z-number calibration** to reduce expert subjectivity during linguistic assessment and reliability assignment.
- **Dynamic CTLRI models** capable of updating trust, loyalty, and strategic evaluations continuously using real-time platform data.
- **Objective and adaptive weighting strategies**, including AHP, CRITIC, entropy, Best–Worst Method (BWM), and hybrid weighting techniques, to improve robustness under different decision environments.
- **Alternative Z-number representations and aggregation operators**, including normalization procedures for the reliability component under extended Aliev arithmetic, to enhance comparability across applications.
- **Group decision-making extensions** incorporating multiple experts, consensus mechanisms, and heterogeneous reliability assessments.
- **Cross-domain applications** in healthcare, banking, supply chain management, intelligent manufacturing, financial risk assessment, smart cities, and public-sector decision support, where reliability-aware decision-making is essential.
- **Integration with explainable artificial intelligence (XAI)** and intelligent decision-support systems, enabling hybrid models that combine predictive analytics with transparent mathematical reasoning under uncertainty.

Overall, the proposed CTLRI framework establishes a general reliability-aware Z-number decision paradigm that extends beyond customer loyalty assessment. By integrating behavioral evaluation, reliability preservation, and strategic interpretation within a unified computational framework, it provides a transparent and explainable

methodology for multi-criteria decision-making under uncertainty and offers a promising foundation for future developments in reliability-aware intelligent decision-support systems.

Statements and Declarations

Funding

No funding was received to assist with the preparation of this manuscript.

Competing Interests

Ramiz Alekperov is employed by the company that owns or operates Aptekonline.az, one of the platforms evaluated in this study. He worked as a consultant on the platform until 2025, including during the data collection period (2023–2024), and has had no involvement with the platform since then. To limit potential bias, the platform data were collected from publicly accessible content, the linguistic assessments were provided by independent experts who participated anonymously, and the results are cross-checked against alternative decision methods (Table 13). In addition, all Aptekonline.az inputs and calculations were independently re-checked by the co-author Shahin Dadashov, who has no employment, consulting or financial relationship with Aptekonline.az or its operating company. The authors have no other competing interests to declare.

Data Availability

The datasets used in this study were compiled through structured web-based observation of publicly accessible e-commerce platform content (2023–2024) and expert linguistic assessments. The data supporting the findings are available from the corresponding author upon reasonable request. The computational code implementing the CTLRI framework is available from the corresponding author upon reasonable request.

Ethics Approval

The linguistic assessments were provided by five experts, academic staff of the Department of Economics at Odlar Yurdu University, who participated voluntarily and anonymously. No personal or sensitive data were collected and the study involved no intervention.

Declaration of Generative AI Use

During the preparation of this work, the authors used AI for language editing, formatting, verification of numerical calculations, implementation of the baseline comparison methods, and revision of the manuscript text. The authors reviewed and edited all content and take full responsibility for the content of the publication.

Consent for Publication

Not applicable. This study does not contain data from any individual person.

Author Contributions

Ramiz Alekperov: conceptualization, formal analysis, methodology, writing – original draft, writing – review and editing. Shahin Dadashov: formal analysis, methodology, data collection, computational implementation, validation, writing – review and editing. Both authors read and approved the final manuscript.

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